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Research Article

A Novel First-Order Fuzzy Rules-Based Forecasting System Using Distance Measures Approach for Financial Market Forecasting

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The precise estimates about finance, atmospheric science, power sector, industries, agriculture, and other science help governments and institutions economically in making the relevant policies regarding import-export, demand, consumption, storage, and local industries. Due to the uncertainty and nondeterministic behavior of data series with respect to time, the foremost challenge is to develop and identify the practical method to handle the above stated complex issues. As an illustration, this study presented an analysis of a new fuzzy time-series (FTS) approach and comparison with traditional forecasting models for prediction of gram pulse production. Taking into consideration the theory of fuzzy sets, FTS, fuzzy rules, triangular membership functions, distance measures, and modified weighted average method, a robust and effective fuzzy rules-based methodology was developed for the prediction of time-series data regarding crop production and share prices. Conventional statistical forecasting methods such as Holt's linear trend, Holt's exponential trend, and Holt's damped exponential trend models were also applied on time-series data for comparison. To identify the primacy of modeling and forecasting, the techniques of root mean squared error (RMSE) and mean absolute error (MAE) were used as a criterion. The numerical values of RMSE and MAE such as 106.51 and 74.8897 clearly demonstrated that the proposed fuzzy rules-based method is robust for forecasting of production and market share prices in the environment of uncertainty.

1. Introduction

Time-series forecasting is the extrapolation in the statistical modeling of the chronologically ordered data. The principal objectives of time-series analysis are the modeling of the vague mechanisms which summarize the observed series and forecasting the future values of that time-series data. Its application spans the area of finance, medicine, business, engineering, and other domains. The precise estimates about different types of production assist governments in making decisions about the import-export volume, the trade at both national and international levels, and the management.

Over the past few decades, the common person's interest in the financial market has grown exponentially [1]. Since investors make decisions on the market in an effort to gain a profit over their investment horizon, it is not surprising that assets valued billions of dollars are traded on financial markets every day [2]. A market player, such as a private or institutional investor, could continuously outperform the market in terms of risk-adjusted returns if they really able to predict the market's behavior properly. However, financial markets are notorious for being highly complex, nonlinear pattern, nonstationarity in recordings, nonparametric, dynamic, and chaotic systems [3], which are susceptible to impact from a number of factors, including as the economic conditions, governmental regulations, and investor psychology [4]. Therefore, forecasting the financial market is highly challenging, and this field of study has provoked considerable global interest in both financial and research communities [5]. Secondly, agriculture is the main sector with a significant contribution to the gross domestic product (GDP) of the world [6]. It contributes to making the prosperous economy of any country. Due to environmental changes, instability in food markets, increase in population and food consumption, pulses have an influential role in the agriculture sector. Many vegetarian and meat diets around the globe rely on pulses as part of a healthy diet. In the last two decades, the world demand for pulses has increased at every level because it meets the requirements of a major part of the population [7]. This study focused on forecasting the gram pulse production whose importance was recognized by the Food and Agriculture Organization of the United Nations (FAO) [8]. The year 2016 has been declared as the International Year of Pulses (IYP) [9]. India is the largest gram producer in the world while Australia and Mexico are leading in gram pulse production after India [10]. In the current scenario, it is obligatory to have significant information regarding the production and cultivated area of the gram and other agricultural commodities for countries to compete in the world food market.

The fuzzy theory was introduced by Zadeh [11] to cope with areas of uncertainty. After this development, many researchers contributed to the extension of fuzzy set theory and operators [12]. The theory of L-fuzzy sets was presented by Goguen [13], which is the further extension and continuation of Zadeh's [11] work. The FTS theory along with the process of fuzzy logic relations was proposed by Song and Chissom [14]. In a further attempt, they developed time-variant forecasting models. Recently, many researchers have contributed to the development of fuzzy modeling after the innovation in a fuzzy theory introduced in [11, 14]. Chen [15] developed a more comprehensive method using arithmetic operations. This model in the field of FTS is projected as the milestone for the development of new models. Furthermore, Chen and Hwang [16] developed a two-factor time-variant model based on FTS. Also, a novel method of forecasting based on clustering methodology and fuzzy logical relationships to decrease forecasting error was presented by Chen and Tanuwijaya [17] Kumar and Gangwar [18] researched the induction of fuzzy sets by using the intuitionistic fuzzy sets theory to develop a robust forecasting model. Additionally, Chen M. Y and Chen B. T [19] introduced the FTS methodology by using the concepts of granular computing and binning based partition. Singh [20, 21] made significant improvisation to develop the forecasting models based on the fuzzy theory. Garg et al. [22] used the fuzzy theory to forecast rice production. Kumar and Kumar [23] applied the fuzzy-based time series to forecast rice production. Furthermore, Iqbal et al. [24–26] developed new FTS models using the theory of hesitant and intuitionistic fuzzy sets to forecast stock index and gram yield production. Singh [27] used different parameters techniques to forecast wheat crop production. Prehanto et al. [28] predicted the soil moisture condition with the sensor technology and fuzzy systems, which signified that fuzzy modeling is a better option to predict soil moisture. Kozlovskyi et al. [29] worked on the estimation of mathematical models in order to estimate the stimulation of agricultural production by utilizing the concept of fuzzy logic. Xiao et al. [30] proposed a fuzzy-based combined approach in relationship with autoregressive integrated moving average (ARIMA) and Holt-Winters statistical models to predict the yield production. Intelligent systems are fitted by adaptive neuro-fuzzy inference and artificial neural networks to forecast the yield production of the potato crop [30]. Garg et al. [31] developed a competent forecasting model by using fuzzy theory and statistical approach. Singh [21] formulated a forecasting model based on three-order time-variant parameter to predict crops production.

The present study was focused on the analysis of the proposed FTS method and comparison with the FTS and conventional statistical models in order to recommend a forecasting approach for data of State Bank of India (SBI) share prices.

To check the robustness of the model, it was also tested on production of a gram food crop. The proposed fuzzy rules-based forecasting method accounts for output by reducing the fluctuations in time-series data which is designed on the basis of stages presented in Chen's [15] work. In references [32, 33], it was observed that the matching rules for the estimation of the forecast values have a significant role in the proposed model, which suggests further work in this domain. So, this study suggested extending the fuzzy rules-based forecasting model to overcome the shortcoming by using the distance measures approach between two fuzzy sets in order to choose the appropriate fuzzy logical relationship to determine the forecast value. Furthermore, in this study, fuzzy rules for the process of defuzzification based on a modified weighted average method are introduced to reduce the forecasting error.

The proposed strategy was termed the fuzzy rules-based time-series (FRBTS) method. For this purpose, firstly, the proposed fuzzy method defines the universe of discourse into seven equal space intervals, and a triangular membership function computes the membership and non-membership values for fuzzification of the time-series data. An approach of distance measures is implemented between two fuzzy sets to select the appropriate fuzzy set. Fuzzy logical relationships are established, and on the basis of fuzzy rules integrated with the modified weighted average method, crisp forecasts are generated. The overall objective of the FRBTS is to develop such a robust forecasting model which can predict the future trend of financial markets accurately and to provide an effective approach to investors of interested parties for deterministic timing on buying and selling shares.

The contributions of this study are given as follows: (1) it was proposed to use the distance measures between two fuzzy sets to choose the appropriate fuzzy set that makes the model of the FTS applicable for real-life applications where it is not possible the exact match of the fuzzy set; (2) it proposed to introduce the fuzzy rules for the process of defuzzification; (3) it proposed to use the new weighted average method and comparison of the FRBTS with conventional statistical models; (4) this structure will easily produce accurate forecasts for a time-series data of agricultural commodities, which is a significant benefit and of major contribution to agricultural applications; (5) also, the proposed research spans several fields, primarily finance and computer science applications and artificial intelligence. The methodological contributions are related to computer science and artificial intelligence, while the empirical contributions are related to finance, specifically to the field of financial market forecasting.

2. Basic Theories

2.1. Fuzzy Sets. The ideology of fuzzy sets is presented with the basic definition of fuzzy logic, which is relevant to fuzzy sets and their features [11]. Fuzzy sets A_i are described by the membership information associated with the components of the variable by assigning a value between 0 and 1.

Definition 1. Fuzzy set on the universe of discourse $U = \{v_1, v_2, v_3, \ldots, v_n\}$ can be defined as $A = \{\langle v_1, v_A(v_1)\rangle, \langle v_2, v_A(v_2)\rangle, \langle v_3, v_A(v_3)\rangle, \ldots, \langle v_k, v_A(v_k)\rangle\}$ where v_i $(i = 1, 2, .., k)$ are linguistic values of U; $v_A(v_i)$ is the membership grade of v_i in fuzzy set A with $v_A(v_i) \in [0,1]$ and $1 \leq k \leq n$.

Definition 2. The triangular membership function for a fuzzy set A can be described as v_A: $Y \longrightarrow [0, 1]$, where components of Y are assigned values from 0 to 1. The assigned value is termed the membership degree. An equation of triangular membership for fuzzy sets can be written as follows:

$$v_A(Y) = \left\{\begin{array}{cc} 0 & y \leq a \\ \left(\dfrac{y-a}{m-a}\right) & a < y \leq m \\ \left(\dfrac{b-y}{b-m}\right) & m < y < b \\ 0 & y \geq b \end{array}\right\}, \tag{1}$$

where a is the lower limit; b is the upper limit; y is the value of variable; m is the center point $(a < m < b)$.

Definition 3. If $F(t-1)$ and $F(t)$ are related to each other in terms of fuzzy, then this relationship can be presented as $F(t) = F(t-1)$, $R(t, t-1)$, and $F(t-1) \longrightarrow F(t)$, where $R(t, t-1)$ describes the fuzzy relationship and "∘" is the symbol for composition operator.

2.2. Fuzzy Time Series. Fuzzy set theory [11] provided a prudent avenue to cope with the constraints of haziness in the desired information. This theory is significant in the areas of ambiguous information, complex historical data, and circumstances of uncertainty. Song and Chissom [14] introduced the concept of fuzzy FTS as a significant process under the reference theory of fuzzy sets and fuzzy logic relations.

Definition 4. Let $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots.)$, a subset of actual numbers, be the universe of discourse that defines fuzzy sets $f_i(t)$ $(i = 1, 2, \ldots, n)$. If $F(t)$ is a fixed collection $f_i(t)$ $(i = 1, 2, \ldots, n)$, then $F(t)$ on $Y(t)$ $(t = 0, 1, 2, \ldots, n)$ is called a fuzzy time series. Observations are referred to as real numbers in prevalent time-series analysis, but they are referred to as fuzzy sets in FTS.

3. Proposed Fuzzy Rules-Based Forecasting System

This section presents a stepwise description of the proposed fuzzy rules-based time-series (FRBTS) approach and Holt's smoothing models to forecast the time-series data. For this objective, time-series data regarding gram production of Pakistan were extracted from the website of FAO. The proposed fuzzy and statistical forecasting models are explained as follows.

3.1. Proposed Fuzzy Rules-Based Time-Series Method. The proposed FRBTS model is developed based on the following steps:

Step 1. Equal space data partitioning techniques have been adopted in many studies [15, 34–37] with different approaches establishing fuzzy logical relationships and defuzzification. Also, in the literature of fuzzy time-series modeling, data are partitioned without removing the trend component [14, 17, 38, 39]. In this study for the given time-series data, the universe of discourse is defined as if $E_{\min}$ and $E_{\max}$ are the minimum and the maximum figures for the given data set, respectively, then the universe of discourse is $U = [E_{\min} - E_1, E_{\max} + E_2]$, where E_1 and E_2 are two positive integers. This method made the universe of discourse acceptable and optimal for the observed data series to be further evaluated.

Step 2. This step involves the partitioning of the universe of discourse into fixed equal spaced intervals for production. If U is the universe of discourse, then $U1, U2, \ldots, UN$ are equal length intervals for U where universe of discourse U is partitioned with equal space intervals with length l defined as

$$l = \frac{[(E_{\min} - E_1) - (E_{\max} + E_2)]}{n}. \tag{2}$$

Now define the fuzzy sets in corresponding intervals as fuzzy intervals. Each interval is associated with the characteristics of the variable as

Ã1: poor yield production

Ã2: below average yield production

Ã3: average yield production

Ã4: good yield production

Ã5: very good yield production

Ã6: excellent yield production

Ã7: bumper yield production

Step 3. This step considers the process of assigning membership values to the variables. The procedure of assigning membership values with correspondence to the probability density function to the variable of interest can be done in many ways [40]. Membership functions are the key part of the fuzzy set theory because the fuzziness in a fuzzy set on a suitable domain can be computed by a membership function. Different membership functions can be used for fuzzification process according to nature of data; for example, trapezoidal membership function is employed in many studies [41–43] for fuzzification process. Also, Bose and Mali [33] combined trapezoidal and triangular functions to improve the forecast accuracy. In our study, triangular membership function is used to assign membership values, which is defined in the previous section as Definition 2.

Step 4. The distance measures $d_1, d_2, \ldots, d_n$ between y of the reported fuzzified readings of the time-series data and the related centers $c_1, c_2, \ldots, c_7$ of equal space intervals $u_1, u_2, \ldots, u_7$ are determined, where $d_{ik}\ (y, c) = |y_i - c_k|$, it contains some well know properties: $d_{ik}\,(y, c) \geq 0$, for all y and c; $d_{ik}\ (y, c) = 0$ only if $y = c$; $d_{ik}\ (y, c) = d_{ik}\ (y, c)$ for all values of y and $c.d_{ik}\ (y, c)$ represents the distance measure between the time-series values y_i and the middle of the related interval.

For example, if two fuzzy sets A_3 and A_4 are developed for i-th observation of time-series data y, then the distance measurement for the fuzzy sets A_3 and A_4 can be calculated as $d_{i3}\ (y, c) = |y_i - c_3|$, $d_{i4}\ (y, c) = |y_i - c_4|$, where c_3 and c_4 are the center points of the respective intervals of equal lengths.

Step 5. This step contains the construction of fuzzy logical relationships (FLRs) and fuzzy logical relationship groups (FLRGs) after assigning the membership information which is defined in Definition 3.

Step 6. This step includes a process of defuzzification to extract expected values for data from FTS. The concepts of defuzzification in literature have been greatly improved with time in order to achieve accuracy in prediction. Song and Chissom [14, 38] defuzzification approach comprises three principles on the basis of membership information, and Chen's model [15] relied on three rules for defuzzification: (1) if Fj has no relationship, then the defuzzification value is the center point of that relating interval u_i; (2) if $F_j \longrightarrow F_k$ has the relationship, then defuzzification value is the center point of u_k interval; (3) if $F_j \longrightarrow F_{k1}, F_{k2}, \ldots, F_{kq}$ establishes the fuzzy logical relationships group, then defuzzification value is the arithmetic mean of center values of corresponding intervals $u_{k1}, u_{k2}, \ldots, u_{kq}$. Also, Li et al. [44] employed deterministic forecasting model. Huang et al. [45] combined global and local information in their forecasting algorithms. These mentioned studies and many prior researches have presented defuzzification methods to cope with uncertain and ambiguous data. One disadvantage of these designs is that they do not properly account for the weights of fuzzy relations. As a result, in order to address the issue of the weight of fuzzy relations, the proper weight must be assigned. Therefore, in this study, a specific defuzzification design is proposed to enhance accuracy in forecasting.

On the basis of FLR's and FLRG's, compute the significant weighted constant value relating to each actual value in time-series data, and determine the crisp forecast using the following equation:

$$\widehat{y}_{ti} = h_i c_k + \sum_{i=1}^{r} \frac{(1 - h_i)}{r} \left(c_{kq}\right), \tag{3}$$

where h_i is the weighted constant value for the current year of actual time-series data; $\widehat{y}_t$ is the forecasted values for the current year; r is the number of FLR's; c_k is the middle value of a respective fuzzy set. To enhance the accuracy in forecasting, the following rules for the defuzzification phase are illustrated to find out the estimated output.

Rule 1. If the actual yield production value for the year t_i ($i = 1, 2, \ldots, n$) has the membership information h_i, and F_j ($j = 1, 2, \ldots, q$) is the fuzzy set relating to i-th value of time-series data and there is only one fuzzy logical relationship such as $F_j \longrightarrow F_j$, then the forecast of actual value for i-th time period is calculated as

$$\widehat{y}_{ti} = h_i y_{ti} + (1 - h_i) c_k, \tag{4}$$

where h_i is the weighted constant for the corresponding year of the actual data series y_{ti} which is related to fuzzy set F_j, whereas $\widehat{y}_t$ is the forecasted value for the time t_i; c_k is the middle value of the corresponding interval related to F_k.

Rule 2. If the actual yield production value for the year t_i has the membership information h_i and F_j is the fuzzy set related to i-th value of time-series data and there exists an FLRG such as $F_j \longrightarrow F_{k1}, F_{k2}, \ldots, F_{kq}$, then the forecasted value for i-th time period is computed using equation (3) as

$$\widehat{y}_{ti} = h_i c_k + \sum_{i=1}^{r} \frac{(1 - h_i)}{r} \left(c_{kq}\right), \tag{5}$$

where hi is the weighted constant for the corresponding year of actual time-series data y_{ti}; $\widehat{y}_t$ is the forecasted value for the year t_i; c_{kq} is the middle value of the interval corresponding to F_{kq}; r is the number of fuzzy logical relationships, for example, if $F_j \longrightarrow F_{k1}, F_{k2}$, which shows a relationship of F_j with F_{k1}, F_{k2}, then the value of r will be equals to 2.

Rule 3. If the actual yield production value for the year t_i ($i = 1, 2, \ldots$, n) has the membership information h_i, and F_j ($j = 1, 2, \ldots$, q) is the fuzzy set relating to i-th value of time-series data and there is no FLR, then the forecasted value of the for the i-th time period will be the center value of the corresponding interval.

3.2. Exponential Smoothing Approach. Exponential smoothing is a popular technique for forecasting time-series data. This optimal approach was developed by Brown and Meyer [46] and Holt [47] in the era of the 1950s and applied in various empirical studies. Gardner [48] clarified simple models relating to the terminology of Holt-Winters, general exponential smoothing models with a brief description regarding properties, parameters, the initial phase of modeling, and accurate forecasting.

The formation of forecasting equations for exponential weighted moving averages covering the nonseasonal component was also described by Holt [49] with no trend. Some of the exponential trend methods were formulated as exponential function classification [50].

3.2.1. Holt's Linear Trend Method. Holt [49] expanded the simple exponential smoothing by adding a concept that accounts for the chances of a trend as when the series expressed some aspect of the trend. This model includes three equations are used, one for forecasting and the other two for smoothing (level and trend). The model for Holt's linear trend method can be written as

$$l_t = \alpha y_t + (1-\alpha)\left(l_{t-1} + b_{t-1}\right), \tag{6}$$

$$b_t = \beta\left(l_t - l_{t-1}\right) + (1-\beta)b_t - 1, \tag{7}$$

$$\hat{y}_{t+k}(t) = l_t + kb_t\,(k = 1, 2, 3, \ldots, n), \tag{8}$$

where α and β are known as smoothing parameters, and their ranges are from 0 to 1; l_t is used for a level of the series and b_t for the best estimate of the trend; $\hat{y}_{t+k}(t)$ is the point forecast for the period t, and k is for the number of next year's forecasts.

3.2.2. Holt's Exponential Trend Method. In contrast to Holt's linear trend method, in Holt's exponential trend method, level and slope are multiplied instead of being added. Its equation can be written as

$$l_t = \alpha y_t + (1-\alpha)\left\{\left(l_{t-1}\right)\left(b_{t-1}\right)\right\}, \tag{9}$$

$$b_t = \beta \frac{l_t}{l_{t-1}} + (1-\beta)\, b_{t-1}, \tag{10}$$

$$\hat{y}_{t+k}(t) = l_t b_t^k, \tag{11}$$

where b_t shows the growth rate which is to be estimated.

3.2.3. Holt's Damped Exponential Trend Method. The forecasts from Holt linear method indicate a constant trend, while an exponential trend is seen by the exponential trend method. The previous research indicated that in some cases, these previously mentioned approaches appear to over-forecast. In the exponential trend process, a damping parameter was introduced in [51] that dampened the trend to a smooth line. Its equation is defined as

$$l_t = \alpha y_t + (1-\alpha)\left\{\left(l_{t-1}\right)\left(b_{t-1}^{\Phi}\right)\right\}, \tag{12}$$

$$b_t = \beta \frac{l_t}{l_{t-1}} + (1-\beta) b_{t-1}^{\Phi}, \tag{13}$$

$$\hat{y}_{t+k}(t) = l_t + b_t^{\left(\Phi^1 \Phi^2 \Phi^3\right)}. \tag{14}$$

3.3. Performance Evaluation Techniques. After the identification process, the next step is to specify the model with parameters estimation. It was achieved by using some information criterion which is explained in this section.

3.3.1. Mean Absolute Error. Mean absolute error (MAE) is the output assessment approach used to evaluate the forecast error. It is an absolute difference between the actual values and the predicted values, which is estimated in the same unit as the actual time-series data contains. The equation for the MAE is presented as

$$\text{MAE} = \frac{1}{n} \sum_{i=1}^{n} \left| y_{if} - y_{io} \right|, \tag{15}$$

where y_{if} is the forecasted value; y_{io} is the observed value of historical data.

3.3.2. Root Mean Squared Error. Root mean squared error (RMSE) tests how residuals are distributed and how data from concentrates around the best line fit. The RMSE equation is given as

$$\text{RMSE} = \sqrt{\sum_{i=1}^{n} \frac{\left(y_{if} - y_{io}\right)^2}{n}}. \tag{16}$$

3.4. Comparison of Models. In the time-series analysis, the most important step is a selection of the robust method after fitting all the proposed methods to the observed data series. In this study, the fuzzy-based time-series and Holt's smoothing methods were employed to the historical gram production data and SBI share prices at SBE, India. In the end, the best-fitted model was selected on the basis of minimum values of accuracy measures, such as RMSE and MAE.

4. Forecasting and Model Evaluation

4.1. Crop Production Data. The current method was implemented on one of the crop yield production data. The motivation to use data of food crop production was that food estimation is one of the real problems because of the uncertainty in known and some unknown variables. For this reason, the gram (chickpea) production data of Pakistan were taken from the United Nations Food and Agriculture Organization website [52].

4.1.1. The New Proposed Fuzzy Time-Series Method. The overview of the gram yield production data in Pakistan is provided in Table 1. The maximum crop production statistic for Pakistan was 868.300 kt, and the minimum production was 284.304 kt in the years 1987 to 2013. The gram time-series data are represented in Figure 1.

Step 7. After the initial process of computing maximum and minimum quantities of the observed gram production data, the universe of discourse was defined as $U = [284.3 - E_1, 868.3 + E_2]$. Setting $E_1 = 24.3$ and $E_2 = 21.7$, the universe of discourse is $U = [260, 890]$. The values of production are expressed in kt in the whole study.

Step 8. The universe of discourse is partitioned into seven intervals of equal lengths using the equation (1), $u_1 = [260, 350]$, $u_2 = [350, 440]$, $u_3 = [440, 530]$, $u_4 = [530, 620]$, $u_5 = [620, 710]$, $u_6 = [710, 800]$, $u_7 = [800, 890]$. The fuzzy sets A_i are defined into seven intervals of equal length for the gram production, such as

$\tilde{A}_1$: poor yield production

$\tilde{A}_2$: below average yield production

$\tilde{A}_3$: average yield production

$\tilde{A}_4$: good yield production

$\tilde{A}_5$: very good yield production

$\tilde{A}_6$: excellent yield production

$\tilde{A}_7$: bumper yield production

Step 9. In this step, triangular fuzzy sets were established, and the values of membership grades and nonmembership grades were calculated by the triangular membership function. The equation for the triangular membership function is given in equation (2). Seven triangular fuzzy sets on the basis of equal length of intervals defined on universe of discourse are given as $F_{u1} = [260, 350, 440]$, $F_{u2} = [350, 440, 530]$, $F_{u3} = [440, 530, 620]$, $F_{u4} = [530, 620, 710]$, $F_{u5} = [620, 710, 800]$, $F_{u6} = [710, 800, 890]$, $F_{u7} = [800, 890, 890]$.

A sample calculation for the year 1987 production is as follows: for the year 1987, the membership and nonmembership grades are calculated. The production value of 493 falls in fuzzy sets F_2 and F_3. If $F_2 = [350, 440, 530]$, then using a prescribed membership function, the membership and nonmembership grades are calculated for the corresponding market price.

From equation (1), the value of $y_1 = 493$ satisfies the condition of $\{(b - y/b - m)\text{QUOTE}, m < y < b\}$ for triangular fuzzy set $F_3 = [350, 440, 530]$ and also satisfies the condition of $\{(y - a/m - a)\ \text{QUOTE} \quad a < y \le m\}$ for triangular fuzzy set F_2. Then, the value of membership grades a_2 and a_3 is calculated as $a_2 = 0.411$, $a_3 = 0.589$.

Similarly, the further calculations of membership degrees and nonmembership degrees regarding a given dataset of gram production are computed. On the basis of this information, fuzzy sets for both membership and nonmembership degrees are computed.

The actual value of gram yield production for the year 1987 was 493, which lies in triangular fuzzy sets F_2 and F_3. Therefore, two fuzzy sets, A_2 and A_2, are constructed for the production value 493 kt using the triangular membership function with membership information of 0.411 and 0.589, respectively.

Step 10. In this step, the distances $d_1, d_2, \ldots, d_k$ between the fuzzified data of the gram production and the center values $c_1, c_2, \ldots, c_7$ of the corresponding intervals $u_1, u_2, \ldots, u_7$ are calculated. On the basis of the minimum value between the values of d_{k-1} and d_k, choose the fuzzy set A_i. The process of computing distance measures and selected fuzzy sets is presented in Table 2.

Step 11. Following the instructions given in step 4, select the appropriate fuzzy sets in respect of time-series data of crop production and establish the FLRs and FLRGs which are shown in Table 3.

Step 12. Membership and nonmembership information are taken as weights in the proposed weighted average approach on the basis of fuzzy FLRs and FLRGs. The equation of the stated approach is described in equation (3). For example, the computations regarding the year 1987 were analyzed, and for gram production of the year 1987, the fuzzy set is $F_3 \le 493$, $\{0.589, 0.411\}>$ and FLRG corresponding to F_3 is $F_3 \longrightarrow F_1, F_2, F_4, F_6, F_7$. Since the midpoints of the optimized length of intervals $u_1, u_2, \ldots, u_7$ are 305, 395, 485, 575, 755, and 845, respectively, then forecast production using equation (3) was calculated and got $\hat{y}_1 = 534.1$.

Other predictions are also examined and presented in Table 4 in a similar pattern. The graph of comparison (Figure 2) indicates that the predicted results are very close to the observed values.

For visual presentation, forecasted values plotted in Figure 2 indicate significantly that lines of actual production and proposed method pass each other very closely.

4.1.2. Holt's Smoothing Models

(1) Holt's Method with Linear Trend. In this method, the value of trend was smoothed by the weights computed by R software. Here, the most important concern is the optimization of the combination of smoothing parameters which is a complex procedure as compared to one parameter estimation. The value of α $(\alpha = 0.5)$ is fitted on a subjective

Table 1: Descriptive statistics regarding production of gram pulse.

Variable	Mean	Standard error mean	Standard deviation	Median	Kurtosis	Skewness
Production (kt)	488.90	29.60	153.80	456.40	−0.83	0.36

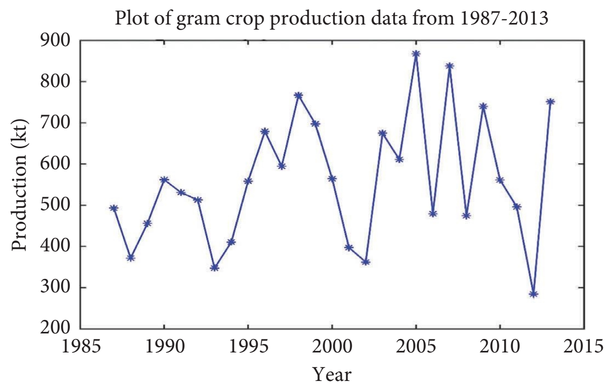

Figure 1: Gram crop production data from 1987 to 2013.

Table 2: Fuzzification of observed values of gram production.

Year	Gram production (kt)	Fuzzy sets	Distance measures set (d_{k-1}, d_k), $(k = 1, 2, \ldots, 7)$	Selected fuzzy sets	Fuzzified values
1987	493	$(A2, A3)$	(98, 8)	(0.589, 0.411)	A_3
1988	371.5	$(A1, A2)$	(66.5, 23.5)	(0.233,0.767)	A_2
1989	456	$(A2, A3)$	(61, 29)	(0.178, 0.822)	A_3
1990	561.9	$(A3, A4)$	(76.9, 13.1)	(0.345, 0.655)	A_4
1991	531	$(A3, A4)$	(46, 44)	(0.012, 0.988)	A_4
1992	512.8	$(A2, A3)$	(117.8, 27.8)	(0.809, 0.191)	A_3
1993	347.3	$(A1)$	—	(0.97, 0.03)	A_1
1994	410.7	$(A1, A2)$	(105.7, 15.7)	(0.675, 0.325)	A_2
1995	558.5	$(A3, A4)$	(73.5, 16.5)	(0.317, 0.683)	A_4
1996	679.6	$(A4, A5)$	(104.6, 14.6)	(0.663, 0.337)	A_5
1997	594.4	$(A3, A4)$	(109.4, 19.4)	(0.716, 0.284)	A_4
1998	767.1	$(A5, A6)$	(102.1, 12.1)	(0.635, 0.365)	A_6
1999	697.9	$(A4, A5)$	(122.9, 32.9)	(0.866, 0.134)	A_5
2000	564.5	$(A3, A4)$	(79.5, 10.5)	(0.383, 0.617)	A_4
2001	397	$(A1, A2)$	(92, 2)	(0.523, 0.477)	A_2
2002	362.1	$(A1, A2)$	(57.1, 32.9)	(0.135, 0.865)	A_2
2003	675.2	$(A4, A5)$	(100.2, 10.2)	(0.614, 0.386)	A_5
2004	611.1	$(A3, A4)$	(126.1, 36.1)	(0.902, 0.098)	A_4
2005	868.3	$(A6, A7)$	(113.3, 23.3)	(0.759, 0.241)	A_7
2006	479.5	$(A2, A3)$	(84.5, 5.5)	(0.439, 0.561)	A_3
2007	838	$(A6, A7)$	(83, 7)	(0.423, 0.577)	A_7
2008	474.6	$(A2, A3)$	(79.6, 10.4)	(0.385, 0.615)	A_3
2009	740.5	$(A5, A6)$	(75.5, 14.5)	(0.339, 0.661)	A_6
2010	561.5	$(A3, A4)$	(76.5, 13.5)	(0.35, 0.65)	A_4
2011	496	$(A2, A3)$	(101, 11)	(0.623, 0.377)	A_3
2012	284.304	$(A1)$	—	(0.27, 0.73)	A_1
2013	751.223	$(A5, A6)$	(86.223, 3.77)	(0.458, 0.512)	A_6

Note. d_k is the distance between the actual fuzzified data of the gram production and the center values of the corresponding intervals.

Table 3: Fuzzy logical relationships of the fuzzified production.

Construction	Fuzzified gram production data
FLR	$F3 \longrightarrow F2$, $F2 \longrightarrow F3$, $F3 \longrightarrow F4$, $F4 \longrightarrow F4$, $F4 \longrightarrow F3$, $F3 \longrightarrow F1$, $F1 \longrightarrow F2$, $F2 \longrightarrow F4$, $F4 \longrightarrow F5$, $F5 \longrightarrow F4$, $F4 \longrightarrow F6$, $F6 \longrightarrow F5$, $F5 \longrightarrow F4$, $F4 \longrightarrow F2$, $F2 \longrightarrow F2$, $F2 \longrightarrow F5$, $F5 \longrightarrow F4$, $F4 \longrightarrow F7$, $F7 \longrightarrow F3$, $F3 \longrightarrow F7$, $F7 \longrightarrow F3$, $F3 \longrightarrow F6$, $F6 \longrightarrow F4$, $F4 \longrightarrow F3$, $F3 \longrightarrow F1$, $F1 \longrightarrow F6$
FLRG	$F1 \longrightarrow F2$, $F1 \longrightarrow F6$, $F2 \longrightarrow F3$, $F2 \longrightarrow F4$, $F2 \longrightarrow F5$, $F3 \longrightarrow F1$ $F3 \longrightarrow F2$, $F3 \longrightarrow F4$, $F3 \longrightarrow F6$, $F3 \longrightarrow F7$, $F4 \longrightarrow F2$, $F4 \longrightarrow F3$, $F4 \longrightarrow F5$, $F4 \longrightarrow F6$, $F4 \longrightarrow F7$, $F5 \longrightarrow F4$, $F6 \longrightarrow F4$, $F6 \longrightarrow F5$, $F7 \longrightarrow F3$

Note. FLR: fuzzy logical relationships; FLRG: fuzzy logical relationship groups.

Table 4: Forecasting results regarding gram production (kt).

Year	Actual production	Forecasted production
1987	493.0	534.100
1988	371.5	531.910
1989	456.0	558.980
1990	561.9	610.370
1991	531.0	628.352
1992	512.8	502.190
1993	347.3	313.100
1994	410.7	452.925
1995	558.5	611.882
1996	679.6	634.670
1997	594.4	590.336
1998	767.1	705.725
1999	697.9	652.940
2000	564.5	608.318
2001	397.0	480.860
2002	362.1	550.125
2003	675.2	630.260
2004	611.1	580.292
2005	868.3	758.240
2006	479.5	535.429
2007	838.0	637.280
2008	474.6	540.350
2009	740.5	665.765
2010	561.5	610.100
2011	496.0	518.930
2012	284.304	502.100
2013	751.223	663.230

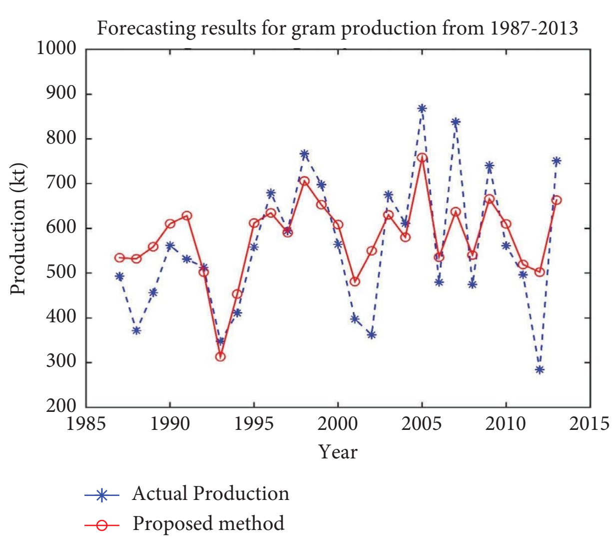

Figure 2: Actual and forecasted values for FTS method.

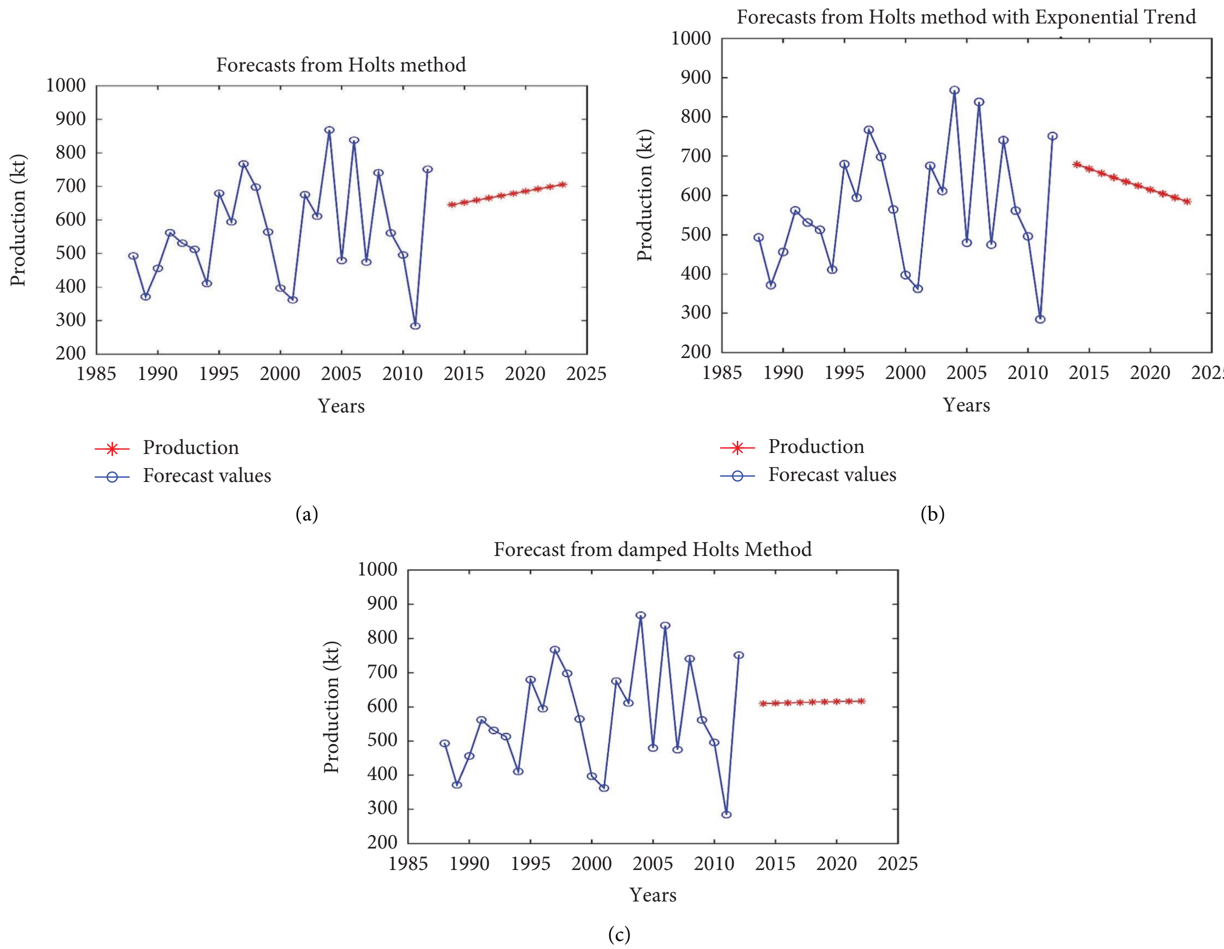

Figure 3: Forecasting results from different Holt's smoothing methods: (a) plot of Holt's method with linear trend, (b) Holt's method with exponential trend, and (c) damped Holt's method with exponential trend.

basis, and the value of β is determined by the application designed for this study. The initial values of the level and growth rates are estimated by the R software and are fitted in the least square trend line which is presented as

$$\hat{y}_t = 440.8906 + 5.6661t, \tag{17}$$

where $l_o = 440.8906$ and $b_o = 5.6661$. Then, using equation (17), the point forecasts are calculated with the process $\hat{y}_{t+k}(t) = l_t + kb_t$ $(t = 0,\ k = 1)$, $\hat{y}_1(0) = 446.5567$.

In order to determine forecasts of the next year, the values l_t and b_t are estimated by putting the values of smoothing parameters, α and β in equations (6) and (7). Then, the estimated l_t, b_t and the forecasts for next years are given as $l_1 = 469.7783$, $b_1 = 6.091$.

In this study, the time-series data were used for the period between 1987 and 2013, and the forecasts were computed by estimating the smoothing parameters for all given years. Now, for future prediction of the gram production, k ahead forecasts are calculated by putting the values in equation (8). If $k = 1$, then for the year 2015, forecasting equation is given as

$$\hat{y}_{27}(26) = l_{26} + b_{26}. \tag{18}$$

To compute three-year ahead of production value, the value of $k = 3$ put in equation (8). The pattern and trend of the next few years forecast by this method can be seen in Figure 3(a).

(2) Holt's Method with Exponential Trend. In this method, the local growth rate b_t was modelled by the smoothing ratios, l_t/l_{t-1}, of the level. This method simulated the trend in a multiplicative way, and forecasts were formed by the multiplication of the level and slope. The software generated the values of level, growth rate, smoothing parameters, and the forecasting accuracy measures, where $l_o = 451.1233$, $b_o = 1.0421$, $\alpha = 0.0358$, and $\beta = 0.0358$. After fitting Holt's exponential method on the dataset, the actual and forecasted trend lines are shown in Figure 3(b). The procedure of calculations was repeated here as it was considered in Holt's linear method, and we get the parameter values as $l_1 = 470.9348, b_1 = 1.0421$

In order to compute the point forecast, the values of l_1 and b_1 computed from equation (9) and (10) were put in

Table 5: Model selection for gram production.

Model	MAE	RMSE	Rank
Proposed fuzzy time-series method	74.8897	93.1683	1
Holt's method (linear trend)	126.2541	156.1272	3
Holt's method (exponential trend)	128.7694	157.9536	4
Holt's method (damped exponential trend)	120.0311	146.8991	2

Table 6: Actual SBI share prices/INR at BSE, India.

Month/year	Actual SBI shares
04/2008	1819.95
05/2008	1840
06/2008	1496.7
07/2008	1567.5
08/2008	1638.9
09/2008	1618
10/2008	1569.9
11/2008	1375
12/2008	1325
01/2009	1376.4
02/2009	1205.9
03/2009	1132.25
04/2009	1355
05/2009	1891
06/2009	1935
07/2009	1840
08/2009	1886
09/2009	2235
10/2009	2500
11/2009	2394
12/2009	2374
01/2010	2315
02/2010	2059.95
03/2010	2120.05

Note. SBI: State Bank of India; BSE: Bombay Stock Exchange.

equation (11) and get the forecasted value for the observation $\hat{y}_2(1) = 490.7611$.

The forecasts for the other time period are calculated by putting the values of the growth rate and level in equation (11).

(3) Holt's Method with Damped Exponential Trend. Pegels [50] suggested that multiplicative trend methods are more applicable than additive trend methods in real-life applications. Gardner [48] introduced the damping parameter Φ which can be used in Holt's methods to manage the trend extrapolation. In this method, the growth rate further undergoes the process of damping for each future value. The value of Φ lies between 0 and 1, which indicates that the multiplicative trend is damped. The increase or decrease in the value of phi indicates the degree of damping. In the present study, the computed value of the damping parameter is 0.9287 which shows that the degree of damping is decreased. The R software estimated the damping, smoothing, and other parameters. Using these estimated values, the same procedure and calculations were repeated by using the equations (10)–(12) as it was done in Holt's linear and exponential trend methods, where $l_o = 463.5722$, $b_o = 1.0256$, $\alpha = 1\text{e}{-04}$, $\beta = 0.0188$, and $\Phi = 0.9287$, and then, the point forecast of y_1 of $t = 0$ is computed as $\hat{y}_{t+k}(t) = l_t + b_t^{(\Phi^1\Phi^2\Phi^3)}$, get $\hat{y}_1(0) = 464.5959$.

This process continued and was used for computing the forecasts for all the years. Similarly, the forecasts for the year 2015 were calculated by putting the information in equations (12)–(14), as follows:

$$\hat{y}_{27}(26) = l_{26} + b_{26}^{(\Phi^1\Phi^2\Phi^3)}. \tag{19}$$

The plot of Holt's damped method with an exponential trend is shown in Figure 3(c), which illustrates that the forecasted series is damped with the decrease of the forecasting errors as compared to the previous Holt's exponential method. All the results were but on the basis of minimum values of Akaike information criterion (AIC) and Bayesian information criterion (BIC), Holt's method with the exponential trend was found to be the best fit method among all the Holt's smoothing methods within this study. The values of the accuracy measures are given in Table 5.

Table 7: Descriptive statistics regarding SBI share prices (INR).

Variable	Mean	Minimum	Standard deviation	Median	Maximum
SBI share prices	1786	1132	399.4764	1786	2500

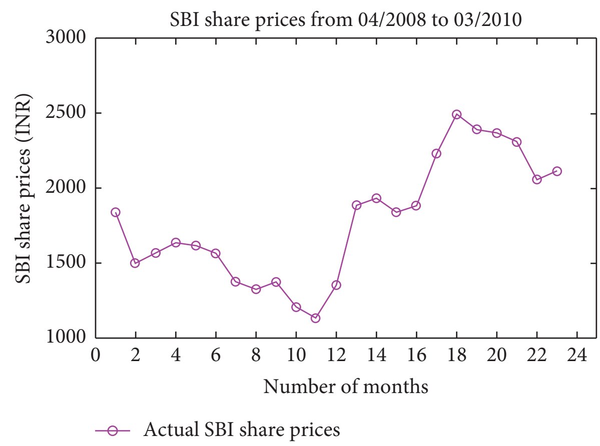

Figure 4: Actual SBI share prices/INR from April 2008 to March 2010.

Table 8: Distance measures and selected fuzzy sets for SBI share prices (INR).

Actual SBI shares	Fuzzy sets	Distance measures (d_{ik-1}, d_{ik}), ($k = 1, 2, \ldots, 7$)	Selected fuzzy sets	Fuzzified values
1819.95	($A3$, $A4$)	(207.45, 2.45)	(0.512, 0.488)	$A4$
1840	($A3$, $A4$)	(227.5, 22.5)	(0.610, 0.390)	$A4$
1496.7	($A1$, $A2$)	(294.2, 89.2)	(0.932, 0.068)	$A2$
1567.5	($A2$, $A3$)	(160, 45)	(0.281, 0.719)	$A3$
1638.9	($A2$, $A3$)	(231.4, 26.4)	(0.629, 0.371)	$A3$
1618	($A2$, $A3$)	(210.5, 5.5)	(0.527, 0.473)	$A3$
1569.9	($A2$, $A3$)	(162.4, 42.6)	(0.293, 0.707)	$A3$
1375	($A1$, $A2$)	(172.5, 32.5)	(0.342, 0.658)	$A2$
1325	($A1$, $A2$)	(122.5, 82.5)	(0.098, 0.902)	$A2$
1376.4	($A1$, $A2$)	(173.9, 31.1)	(0.342, 0.652)	$A2$
1205.9	($A1$)	—	(0.512, 0.488)	$A1$
1132.25	($A1$)	—	(0.157, 0.843)	$A1$
1355	($A1$, $A2$)	(152.5, 52.5)	(0.756, 0.244)	$A2$
1891	($A3$, $A4$)	(278.5, 73.5)	(0.858, 0.142)	$A4$
1935	($A4$, $A5$)	(117.5, 87.5)	(0.073, 0.927)	$A5$
1840	($A3$, $A4$)	(227.5, 22.5)	(0.610, 0.390)	$A4$
1886	($A3$, $A4$)	(273.5, 68.5)	(0.834, 0.166)	$A4$
2235	($A5$, $A6$)	(212.5, 7.5)	(0.537, 0.463)	$A6$
2500	($A6$, $A7$)	(272.5, 67.5)	(0.829, 0.171)	$A7$
2394	($A6$, $A7$)	(166.5, 38.5)	(0.312, 0.688)	$A7$
2374	($A6$, $A7$)	(166.5, 38.5)	(0.215, 0.785)	$A7$
2315	($A5$, $A6$)	(292.5, 87.5)	(0.927, 0.073)	$A6$
2059.95	($A4$, $A5$)	(242.45, 37.45)	(0.683, 0.317)	$A5$
2120.05	($A4$, $A5$)	(302.55, 97.55)	(0.976, 0.024)	$A5$

4.1.3. Comparison of Proposed Fuzzy Time Series with the Other Models. The performances of the FTS method, Holt's linear trend method, Holt's exponential trend, and Holt's damped exponential trend method were evaluated by using the forecasting accuracy measures, RMSE and MAE. The computed values of the above-mentioned accuracy measures are presented in Table 6. The findings demonstrated that the value of MAE for the proposed fuzzy-based time-series method was 57.65837, which was significantly lower than all the values of Holt's smoothing models. The computed value of RMSE for the proposed fuzzy model was 81.18085 which was also meaningfully lower than RMSE values of

Table 9: Forecasts and comparison for SBI share prices (INR).

Month/Year	Actual SBI shares	Chen [15]	Huarng [54]	Joshi and Kumar [55]	Kumar and Gangwar [56]	Gupta and Kumar [57]	Proposed method
04/2008	1819.95	—	—	—	—	—	1852.73
05/2008	1840	1900	1855	1777.8	1725.98	1860.08	1844.15
06/2008	1496.7	1900	1855	1865.7	1725.98	1860.08	1418.34
07/2008	1567.5	1500	1575	1531.5	1512.39	1452.59	1465.11
08/2008	1638.9	1500	1505	1531.5	1512.39	1452.59	1536.45
09/2008	1618	1600	1610	1777.8	1574.35	1544.29	1515.54
10/2008	1569.9	1600	1505	1531.5	1574.35	1544.29	1467.57
11/2008	1375	1500	1482	1531.5	1512.39	1452.59	1495.88
12/2008	1325	1433	1155	1504.23	1305.52	1682.31	1532.32
01/2009	1376.4	1433	1365	1504.23	1665.9	1682.31	1486.62
02/2009	1205.9	1433	1482	1504.23	1305.52	1682.31	1302.54
03/2009	1132.25	1433	1155	1258.23	1294.27	1264.98	1375.32
04/2009	1355	1300	1365	1258.23	1294.27	1264.98	1434.67
05/2009	1891	1433	1482	1504.23	1665.9	1682.31	1825.32
06/2009	1935	1900	1890	1865.71	2006.51	2138.31	1832.46
07/2009	1840	1900	1890	1883.93	2006.51	1853.54	1844.15
08/2009	1886	1900	1855	1865.71	1725.98	1860.08	1826.96
09/2009	2235	1900	1855	1865.71	2006.51	2138.21	2227.5
10/2009	2500	2300	2485	2142.04	2520	2466.99	2397.45
11/2009	2394	2300	2415	2245.65	2420	2328.48	2291.46
12/2009	2374	2300	2345	2191.75	2365.99	2321.66	2271.58
01/2010	2315	2300	2205	2191.75	2365.99	2321.66	2227.50
02/2010	2059.95	2300	2205	2142.04	2020	2070.4	1957.52
03/2010	2120.05	2100	2135	1883.93	2120	2138.21	2017.58

Holt's linear, exponential, and damped exponential methods. Accuracy measures showed that forecasts with the proposed fuzzy time-series method were the closest to the actual observations with minimum error as compared to other statistical models.

4.2. Prediction of State Bank of India Shares Prices by the Proposed Method. In this part of the section, proposed model is fitted on the benchmark dataset of State Bank of India (SBI) share prices at Bombay Stock Exchange (BSE), India [53], presented in Table 6.

Description of the data regarding market prices of SBI share, at BSE, India is presented in Table 7. The maximum value for the market price was 2500 INR, and the minimum value was 1132 INR. The time-series data of market prices of SBI share are plotted in Figure 4.

After the initial process of computing maximum and minimum quantities of the observed time-series data regarding SBI share price, the universe of discourse is defined as $U = [1132 - \text{E1},\ 2500 + \text{E2}]$. Setting $\text{E1} = 32$ and $\text{E2} = 35$, the universe of discourse is $U = [1100, 2535]$. The universe of discourse is partitioned into seven intervals of equal lengths using equation (1). In the next step, seven triangular fuzzy sets are defined on the universe of discourse using equation (2).

Furthermore, calculated the distances $d_1, d_2, \ldots, d_k$ between the actual fuzzified data of the gram production y and the center values, $c_1, c_2, \ldots, c_7$, of the corresponding intervals, $u_1, u_2, \ldots, u_7$. On the basis of the minimum value between the values of d_{ik-1} and d_{ik}, choose the fuzzy set A_i. The process of computing distance measures and selected fuzzy sets is presented in Table 8.

Following the instructions given in Step 4 of the proposed method, selected the appropriate fuzzy sets in respect of time-series data of share prices, and established the FLRs and FLRGs. At the final stage, membership and non-membership information was taken as weights in the proposed weighted average approach on the basis of fuzzy FLRs and FLRGs. The equation of the stated approach is described in equation (3). For example, the computations regarding time in November 2008 are analyzed as, for the share price of time November 2008, the fuzzy set is F_2 $\leq$1375, {0.342, 0.658}> and FLRG corresponding to F_2 is $F_2 \longrightarrow F_1, F_3, F_4$. Since the midpoints of the optimized length of intervals $u_1, u_2, u_2\, and u_4$ are 1205.5, 1407.5, 1612.5 and 1817.5, respectively. Then, forecast share price for the month of November 2008 using equation (3) is given as $\hat{y}_8 = 1495.88$.

Other forecasted values of share prices were also determined similarly on the basis of Rule 1 and Rule 2 and are reported in Table 9. The comparison shown in Figure 3 indicates that the predictions are very similar to the actual values.

For visual presentation, forecasted values are plotted in Figure 5, which significantly indicates that lines of actual share prices and the proposed method pass each other very closely.

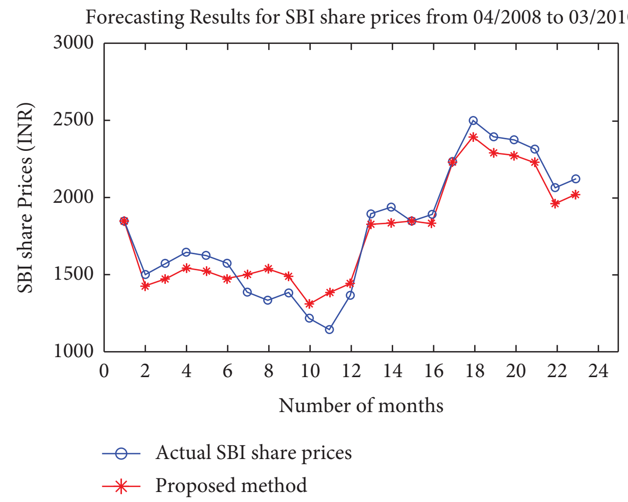

FIGURE 5: Comparison of forecasting results with actual values of SBI share prices.

TABLE 10: Accuracy measures for evaluation of proposed method with existing fuzzy time-series models.

Models	RMSE	Rank
Chen [15]	187.26	5
Huarng [54]	164.04	3
Joshi and Kumar [55]	200.17	6
Kumar and Gangwar [56]	134.24	2
Gupta and Kumar [57]	182.98	4
Proposed method	106.51	1

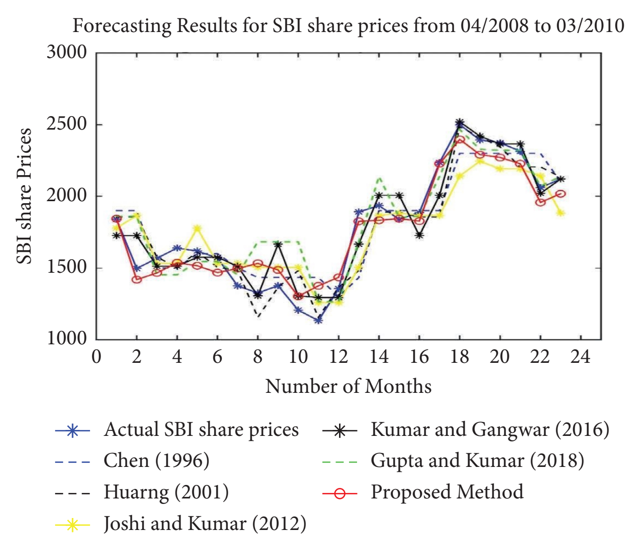

FIGURE 6: Comparison of proposed model with the existing fuzzy time-series models.

4.3. Comparison of the Proposed FRBTS Method with the Existing Methods. The primacy and efficiency of the proposed FTS model is compared with different existing benchmark models [15, 54–57], in this subsection. Table 10 and Figure 6 represents the measured values of the forecast accuracy in the form of RMSE. For the proposed fuzzy system, the numerical values of accuracy measures are 106.51 with respect to RMSE. These results showed that the current method outerperforms the existing FTS methods with minimal error.

5. Conclusions

In this research, a robust fuzzy rules-based method for the prediction of crop production and SBI share prices was proposed in order to manage the prediction situations when uncertainty, hesitancy, and decision-making problems occur in practice. Different studies have been conducted regarding the FTS and conventional statistical methods in order to develop methodologies for prediction, but insufficient work was done for developing the methodologies regarding financial markets data such as share prices, stock index, and volatility. A fuzzy rules-based time-series (FRBTS) forecasting method was proposed using triangular membership function which determined the membership and non-membership grades. A distance measuring technique was applied to select appropriate fuzzy sets, and a modified weighted average approach was used to develop fuzzy rules in order to generate forecasts. Additionally, statistical Holt's smoothing approach for prediction was also applied for comparison purpose. The results were analyzed and compared on the basis of statistical accuracy measures. The calculations revealed that the proposed fuzzy rules-based time-series approach is much better and more accurate than the aforementioned FTS and Holt's smoothing models for the prediction of time-series data. This fruitful procedure can be employed as a precise computational method to forecast the time-series data in other domains. This achievement will help the policymakers in planning, management, import-export, and other areas. For future works, new models will develop based on unequal length of intervals and high order to obtain higher accuracy in forecasting.

Data Availability

The datasets presented in this study can be found online on Food and Agriculture Organization of the United Nations. https://www.fao.org/faostat/en/#data/TP

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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