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Research Article

Fuzzy Computational Analysis of Flower Graph via Fuzzy Topological Indices

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Fuzzy graphs have many applications not only in mathematics but also in any field of science where the concept of fuzziness is involved. The notion of fuzziness is suitable in any environment, which favor to predicts the problem and solve this problem in a decent way. As compared to crisp theory, fuzzy graphs are a more beneficial and powerful tool to get better accuracy and precision due to their fuzziness property. A topological index is a numerical value which characterizes the properties of the graph. Topological indices were basically developed for chemical structures, but these are also used for general graphs as well. In chemical graph theory, topological indices are used to extract the chemical properties of the graphs. These indices are also well studied in fuzzy environment. Applications of fuzzy graphs are found in medicines, telecommunications, traffic light control, and many more. Our aim is to find these fuzzy topological indices for flower graphs to strengthen the concepts of fuzziness in general graphs. In this paper, some novel results for $f_{m\times r}$ flower graphs are achieved.

1. Introduction

Humans are facing various problems in real life which can be resolved with a variety of algorithms through graph theory. Therefore, the importance of graph theory in different fields such as chemistry and biochemistry to elaborate chemical structures, in computer science to develop algorithms, in network flow problems to resolve traffic issues, finding the shortest path, scheduling the airlines, railway trains, and busses to connect numerous cities effectively. Traveling of large number of passengers from one place to another in the shortest possible time is a massive problem. While resolving such problems, it has been observed that sometimes solutions of these problems may not be unique nor exact, this means that some solutions may just be a very good approximation. In such cases, approximations are the best outcome of such complex problems. Some real-world problems along with their solution using graph theory are listed below.

Fuzzy graphs are the generalization of the crisp graph. The concept of fuzziness was introduced by Zedah [1]. Information gained through computational perception and cognition, i.e., imprecise, uncertain, partially accurate, vague, or without sharp limitations can be dealt with fuzzy logic. Advance computing techniques based on fuzzy logic can also be effectively used for the development of intelligent systems for identification, decision making, optimization, control, and pattern recognition. Moreover, it provides an effective way to resolve conflict and better assess opportunities and multiple criteria. The environment of fuzziness is favorable in all situations where uncertainty plays a vital role. This is the reason for introducing the concept of fuzzy graphs. Rosenfield [2] demonstrated the notion of fuzzy graphs. Yeh and Bang [3] have established the same notion and portray the importance of clustering analysis. There are certain kinds of centrality measures such as closeness, betweenness, and degree measures which have been discussed by Borgatti [4]. Connectivity is another invariant in graph theory and is also useful in decision-making analysis [5, 6].

The topological molecular conformation index is a nonempirical value that affects the size of the molecular structure and its sequence of branches. In this sense, it can be viewed as a fractional topological index, which describes individual molecular structures with a real number and is examined as a descriptor of the molecule.

There are several topological indicators, several of which are applied in chemistry nowadays. The structural characteristics of the diagrams used to calculate this may be graded; for example, the Hosoya index, the Randic communication index, the Zagreb community index, etc. The Wiener index, which depends on the relative distance of the topological tops in the graph, is the most known and most used topological index mathematicians and chemists study this field. Interest in this subject is therefore being sought worldwide for topological graphic indices.

In 1973, Kaufmann [7] introduced the first definition for a fuzzy graph using Zadeh's [8] fuzzy relations. In 1975, a more detailed description is credited to Rosenfeld [2], who introduced the fuzzy graph theory by considering fuzzy relations on fuzzy sets. He established some relations regarding the properties of path graph, trees, and various graphs. The notion of fuzzy cut, nodes, and fuzzy bridges was introduced by Bhattacharya in [9]. The generalization of a crisp graph is a fuzzy graph. Hence, there exist many similar properties between them and also deviate at several places. In crisp a graph $G = (V, E)$, where V and E are the sets of vertices and edges, respectively, a bijective function $f\colon V \cup E \longrightarrow N$ that allocates a natural number to every vertex and/or edge is known a labeling.

Ameenal [10] develops a new idea of fuzzy magic labeling of cycle and star graph. Vimala and Nagarani [11] gave the generalized concept of energy of fuzzy labeling graph. Nagoor et al. [12] describe various properties of fuzzy labeling of path graph and star graph. Numerous definitions and basic concepts are discussed in [13–15]. Fuzzy topological analysis of pizza graph has been discussed in [16].

Topological indices are a powerful tool in the field of chemical graph theory. The topological index describes the physicochemical properties of molecular structures. Many topological indices are available in the literature with respect to degree, distance, and spectrum. The very first topological index is the Wiener index which was established in 1947 when Wiener was working on the boiling point of paraffin [17]. This index got a very rich impact due to its use in the application of different fields such as chemical graph theory and spectral graph theory. In the current research, people are working in fuzzy topological indices. For instance, Binu has introduced the Wiener index [18] and connectivity index [19] for fuzzy graphs. Islam et al. [20] studied the Wiener index for saturated fuzzy graphs. Islam et al. [21] also shed a light on the notion of hyper-Wiener index. In 1972, Gutman and Trinajstic [22] introduced the degree-based topological indices and used these indices to find total π-electron energy for saturated hydrocarbons.

2. Motivation

The topological indices are getting great importance these days. Researchers are motivated to work in this direction, where the application part of topological indices plays a critical role. Many fuzzy topological indices established recently and modification in these indices are also introduced [20, 21, 23–25]. We have filled the gap in research of fuzzy topological indices. We have found the fuzzy topological indices of general structures which have not been discussed before. These results are beneficial in chemical graph theory and any other field where the graphs are isomorphic to some structures. Mufti et al. [26] have found general results for chemical graphs using fuzzy topological indices such as the first and second fuzzy Zagreb indices, the fuzzy harmonic index, and the Randic index. In this paper, using the abovementioned fuzzy topological indices, we have found the general results for the flower graph $f_{m\times r}$.

Topological indices have many advantages in a crisp environment as these indices give physiochemical properties of chemical structures, but in a fuzzy environment, it gets more benefits and advantages. In a recent research [27], a new model of fuzzy topological indices has been proposed to investigate the cybercrimes. Our proposed model is beneficial for chemical industries where they can use this model for the chemical documentation of chemicals.

3. Flower Graph $f_{m\times r}$

A graph H is named as $(m \times r)$-flower graph, if it carries m vertices forming m-cycle and m sets with $r-2$ vertices forming r-cycles around the m cycle, so that each r-cycle uniquely intersects with the m-cycle on a single edge and is symbolized by $f_{m\times r}$. Clearly, $f_{m\times r}$ contains $m \times (r-1)$ vertices together with mr edges while the r-cycles represents the petals and m-cycles the center of $f_{m\times r}$ [28]. For example, Figure 1 shows the diagram of flower graphs.

4. Preliminaries

In this section, we will define some basic definitions.

Definition 1 (see [29]). An undirected finite graph H is fuzzy graph (having no loops or parallel edges), having set of finite vertices $V(H)$ and set of finite edges $E(H)$. Two mappings σ and μ, where $\sigma\colon V(H) \longrightarrow [0,1]$ and $\mu\colon V(H) \times V(H) \longrightarrow [0,1]$ such that $\mu(v_1 v_2) \le \sigma(v_1) \wedge \sigma(v_1)$ for every pair of vertices $v_1, v_2 \in V(H)$.

Definition 2 (see [29]). The degree of vertex of a fuzzy graph H can be defined as the sum of all the weights of edges corresponding to vertex v;mathematically, it is represented as

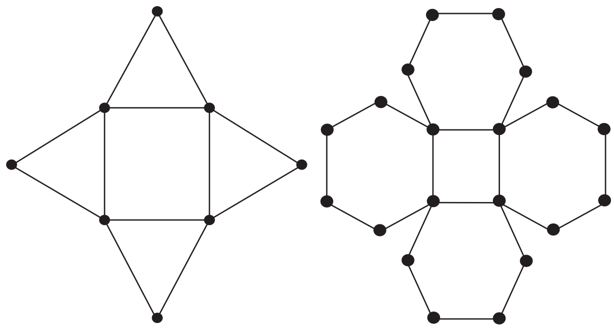

Figure 1: $f_{4\times 3}$ and $f_{4\times 6}$, respectively.

$$d(v) = \sum_{n=1}^{j} \mu(vu), \quad where\ v \neq u. \tag{1}$$

Definition 3 (see [29]). The order $O(H)$ of a fuzzy graph H can be defined as the sum of all the weights of vertices; mathematically, it is represented as

$$O(H) = \sum \sigma(v). \tag{2}$$

Definition 4 (see [29]). The size $S(H)$ of a fuzzy graph H can be defined as the sum of all the weights of edges; mathematically, it is represented as

$$S(H) = \sum \mu(vu), \tag{3}$$

for all vertices $u, v \in V(H)$ and $v \neq u$.

Kalathian et al. [30] introduce the concept of fuzzy Zagreb indices defined as follows.

Definition 5. The first Zagreb index is denoted by $M(H)$ and defined as

$$M(H) = \sum_{k=1}^{q} \sigma(u_k)[du_k]^2. \tag{4}$$

Definition 6. The second Zagreb index is denoted by $M^*(H)$ and defined as

$$M^*(H) = \frac{1}{2} \sum_{kl\in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)]. \tag{5}$$

Definition 7. The Randic index of fuzzy graph H is defined as

$$R(H) = \frac{1}{2} \sum_{kl\in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)]^{-1/2}. \tag{6}$$

Definition 8. The harmonic index of fuzzy graph H is defined as

$$P(H) = \frac{1}{2} \sum_{kl\in E(G)} \left[\frac{1}{\sigma(u_k)(du_k) + \sigma(v_l)(dv_l)}\right]. \tag{7}$$

5. Application of Fuzzy Graphs

One cannot deny the importance of fuzzy graphs in real life. Fuzzy graphs having various applications in numerous fields of life such as the telecom sector, social networking, control of traffic signals, air network, and several others. Fuzzy graphs are also used in optimization techniques to solve real-world problems in the field of statistic, economics, decision theory, and many more. Some applications have already been discussed in the earlier section, and a few are discussed in [31, 32].

6. Zagreb Indices of Flower Graph $f_{m\times 3}$

Suppose that $H = f_{m\times 3}$ be a flower graph shown in Figure 1, $f_{m\times 3}$ consist upon $2m$ vertices and $3m$ edges for all $m \in N$.

Theorem 9. *Let $H = f_{m\times 3}$ be a fuzzy flower graph, then the first fuzzy Zagreb index of $H = f_{m\times 3}$ flower graph is $M(H) = 0.748\,m$.*

Proof. Let $H = f_{m\times 3}$ fuzzy flower graph shown in Figure 1. The partition of the vertex set is as follows.

Weight of all vertices u_i's $\in m$-cycle is 0.3 with cardinality m and has vertex weight 0.6, where the remaining v_i's $\in r$-cycle has a weight of 0.4 with cardinality m and has a vertex weight of 0.4; therefore, the total count of vertices is $2m$.

$$\begin{aligned} M(H_1) &= \sum_{k=1}^{q} \sigma(u_k)[du_k]^2 \\ &= (0.3)\left[m(0.6)^2\right] + (0.4)\left[m(0.4)^2\right] \\ &= (0.3)[0.36m] + (0.4)[0.16m] \\ &= 0.108m + 0.64m \\ &= 0.748m. \end{aligned} \tag{8}$$

□

Theorem 10. *Let $H = f_{m\times 3}$ be a fuzzy flower graph, then the second fuzzy Zagreb index of $f_{m\times 3}$ flower graph is $M^*(H) = 0.045\,m$.*

Proof. Let $H = f_{m\times 3}$ flower graph shown in Figure 1. The partition of the edge sets is as follows.

The edge set $E_i \in m$-cycle with end vertices weight (0.3, 0.3) has one (01) partition having degree type (0.6, 0.6) with cardinality m. The edge set $E_j \in r$-cycle with end vertices weight (0.3, 0.4) has one (01) partition having degree type (0.6, 0.4) with cardinality $2m$.

$$
\begin{aligned}
M^*(H) &= \frac{1}{2}\sum_{kl\in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)] \\
&= \frac{1}{2}[(m)(0.3)(0.6)(0.3)(0.6)] + \frac{1}{2}[(2m)(0.3)(0.6)(0.4)(0.4)] \\
&\quad + \frac{1}{2}[(0.0324)(m) + (0.0288)(2m)] \\
&= \frac{1}{2}[0.0324m + 0.0576m] \\
&= \frac{1}{2}[0.9m] \\
&= 0.045m.
\end{aligned} \tag{9}
$$

□

Theorem 11. *Let $H = f_{m\times 3}$ be a fuzzy flower graph, then the Randic fuzzy Zagreb index of $H = f_{m\times 3}$ flower graph is $R(H) = 8.665\,m$.*

Proof. Let $H = f_{m\times 3}$ be a flower graph shown in Figure 1. The partition of all types of edges in the form of sets is as follows: The edge set $E_i \in m$-cycle having weights of the vertices (0.3, 0.3) has one (01) type of partition with degree type (0.6, 0.6) has cardinality m. The edge set $E_j \in r$-cycle having weights of the vertices (0.3, 0.4) has one (01) type of partition with degree type (0.6, 0.4) has cardinality $2m$.

$$
\begin{aligned}
R(H) &= \frac{1}{2}\sum_{kl\in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)]^{-1/2} \\
&= \frac{1}{2}\left[(m)[(0.3)(0.6)(0.3)(0.6)]^{-1/2}\right] + \frac{1}{2}\left[(2m)[(0.3)(0.6)(0.4)(0.4)]^{-1/2}\right] \\
&= \frac{1}{2}\left[(m)[0.0324]^{-1/2} + (2m)[0.0288]^{-1/2}\right] \\
&= \frac{1}{2}[5.55m + 5.89(2m)] \\
&= 8.665m.
\end{aligned} \tag{10}
$$

□

Theorem 12. *Let $H = f_{m\times 3}$ be a fuzzy flower graph, then the harmonic fuzzy Zagreb index of $H = f_{m\times 3}$ flower graph is $P(H) = 4.33\,m$.*

Proof. Let $H = f_{m\times 3}$ be a flower graph shown in Figure 1. The partition of all types of edges in the form of sets is as follows.

The edge set $E_i \in m$-cycle having weights of the vertices (0.3, 0.3) has one (01) type of partition with degree type (0.6, 0.6) has cardinality m. The edge set $E_j \in r$-cycle having weights of the vertices (0.3, 0.4) has one (01) type of partition with degree type (0.6, 0.4) has cardinality $2m$.

$$P(H) = \frac{1}{2}\sum_{kl\in E(G)}\left[\frac{1}{\sigma(u_k)(du_k)+\sigma(v_l)(dv_l)}\right]$$
$$= \frac{1}{2}\left[\frac{m}{(0.3)(0.6)+(0.3)(0.6)}+\frac{2m}{(0.3)(0.6)+(0.4)(0.4)}\right]$$
$$= \frac{1}{2}\left[\frac{m}{0.36}+\frac{2m}{0.34}\right]$$
$$= \frac{1}{2}[2.78m+5.88m]$$
$$= 4.33m. \quad (11)$$

□

7. Zagreb Indices of Fuzzy Flower Graph $f_{m\times r}$

Let $H_1 = f_{m\times r}$ be a fuzzy flower graph shown in Figure 1, consist upon $m(r-1)$ vertices and mr edges. (4×3)-flower graph $f_{4\times 3}$ and (4×6)-flower graph $f_{4\times 6}$, having 8 vertices and 12 edges and 20 vertices and 24 edges, respectively.

Theorem 13. *Let $H_1 = f_{m\times r}$ be a fuzzy flower graph, then the first fuzzy Zagreb index of $H_1 = f_{m\times r}$ fuzzy flower graph is $M(H_1) = (0.64mr - 1.172m)$.*

Proof. Let $H_1 = f_{m\times r}$ be a fuzzy flower graph shown in Figure 1. The partition of the vertex set is as follows.

Weight of all the vertices $u_i's \in m$-cycle is 0.3 with cardinality m and has a vertex weight of 0.6 where remaining $v_i's \in r$-cycle is 0.4 having count of $m(r-1)-m$ and has vertex weight of 0.4; therefore, the total count of vertices is $m(r-1)$.

$$M(H_1) = \sum_{k=1}^{q}\sigma(u_k)[du_k]^2$$
$$= (0.3)\left[m(0.6)^2\right]+(0.4)\left[(m(r-1)-m)(0.4)^2\right]$$
$$= (0.3)[0.36m]+(0.4)[0.16(mr-2m)] \quad (12)$$
$$= 0.108m+0.64(mr-2m)$$
$$= 0.64mr-1.172m.$$

□

Theorem 14. *Let $H_1 = f_{m\times r}$ be a fuzzy flower graph, then the second fuzzy Zagreb index of $H_1 = f_{m\times r}$ fuzzy flower graph is $M(H_1) = (0.0066m + 0.0128mr)$.*

Proof. Let $H_1 = f_{m\times r}$ be a fuzzy flower graph shown in Figure 1. The partition of all types of edges in the form of sets is as follows.

The edge set $E_i \in m$-cycle having weights of the vertices (0.3, 0.3) has one (01) type of representations all having degree type (0.6, 0.6) with cardinality m. The edge set $E_j \in r$-cycle having has two (02) type of representations, $2m$ vertices have weights (0.3, 0.4) having degree type (0.6, 0.4), and remaining $mr-3m$ vertices have weights (0.4, 0.4) having degree type (0.4, 0.4); therefore, the total count of edges is mr.

$$M^*(H_1) = \frac{1}{2}\sum_{kl\in E(G)}[\sigma(u_k)(du_k)\sigma(v_l)(dv_l)]$$
$$= \frac{1}{2}[(m)(0.3)(0.6)(0.3)(0.6)]+\frac{1}{2}[(2m)(0.3)(0.6)(0.4)(0.4)]$$
$$+\frac{1}{2}[(mr-3m)(0.3)(0.6)(0.4)(0.4)] \quad (13)$$
$$= \frac{1}{2}[(m)(0.0324)]+\frac{1}{2}[(2m)(0.0288)]+\frac{1}{2}[(mr-3m)(0.0256)]$$
$$= \frac{1}{2}[0.0324m+0.0576m+0.0256mr-0.0768m)]$$
$$= 0.0066m+0.0128mr.$$

□

Theorem 15. *Let $H_1 = f_{m\times r}$ be a fuzzy flower graph, then the Randic fuzzy Zagreb index of $H_1 = f_{m\times r}$ fuzzy flower graph is $R(H_1) = (3.125mr - 0.71\,m)$.*

Proof. Let $H_1 = f_{m\times r}$ be a flower graph shown in Figure 1. The partition of all types of edges in the form of sets is as follows.

The edge set $E_i \in m$-cycle having weights of the vertices (0.3, 0.3), has one (01) type of representations, all having degree type (0.6, 0.6) with cardinality m. The edge set $E_j \in r$-cycle having two (02) type of representations, $2m$ vertices have weights (0.3, 0.4) having degree type (0.6, 0.4), and remaining $mr - 3m$ vertices have weights (0.4, 0.4) with degree type (0.4, 0.4); therefore, the total count of edges is mr.

$$\begin{aligned}
R(H_1) &= \frac{1}{2}\sum_{kl\in E(G)} \left[\sigma(u_k)(du_k)\sigma(v_l)(dv_l)\right]^{-1/2},\\
&= \frac{1}{2}\left[(m)\left[(0.3)(0.6)(0.3)(0.6)\right]^{-1/2}\right] + \frac{1}{2}\left[(2m)\left[(0.3)(0.6)(0.4)(0.4)\right]^{-1/2}\right]\\
&\quad + \frac{1}{2}\left[\left[(mr-3m)(0.3)(0.6)(0.4)(0.4)\right]^{-1/2}\right]\\
&= \frac{1}{2}\left[(m)[0.0324]^{-1/2} + (2m)[0.0288]^{-1/2} + +(mr-3m)[0.0256]^{-1/2}\right]\\
&= \frac{1}{2}\left[5.55m + 5.89(2m) + 6.25(mr-3m)\right]\\
&= 3.125mr - 0.71m.
\end{aligned} \tag{14}$$

□

Theorem 16. *Let $H_1 = f_{m\times r}$ be a fuzzy flower graph, then the harmonic fuzzy Zagreb index of $H_1 = f_{m\times r}$ fuzzy flower graph is $H(H_1) = (1.5625mr - 0.3575\,m)$.*

Proof. Let $H_1 = f_{m\times r}$ be a fuzzy flower graph shown in Figure 1. The partition of all types of edges in the form of sets is as follows.

The edge set $E_i \in m$-cycle having weights of the vertices (0.3, 0.3) has one (01) type of representations all having degree type (0.6, 0.6) with cardinality m. The edge set $E_j \in r$-cycle having has two (02) type of representations, $2m$ vertices have weights (0.3, 0.4) having degree type (0.6, 0.4), and remaining $mr - 3m$ vertices have weights (0.4, 0.4) having degree type (0.4, 0.4); therefore, the total count of edges is mr.

$$\begin{aligned}
H(H_1) &= \frac{1}{2}\sum_{kl\in E(G)}\left[\frac{1}{\sigma(u_k)(du_k)+\sigma(v_l)(dv_l)}\right]\\
&= \frac{1}{2}\left[\frac{m}{(0.3)(0.6)+(0.3)(0.6)} + \frac{mr-m}{(0.3)(0.6)+(0.4)(0.4)}\right.\\
&\quad \left. + \frac{mr-3m}{(0.4)(0.4)+(0.4)(0.4)}\right]\\
&= \frac{1}{2}\left[\frac{m}{0.36} + \frac{2m}{0.34} + + \frac{mr-3m}{0.32}\right]\\
&= \frac{1}{2}\left[2.78m + (2.94)(2m) + (3.125)(mr-3m)\right]\\
&= 1.5625mr - 0.3575m.
\end{aligned} \tag{15}$$

□

8. Conclusion

Topological indices have many applications in different areas, especially in chemical graph theory. Researchers are increasing the impact of topological indices in both directions such as crisp graph theory and fuzzy graph theory. We have established results for general chemical structures rather than working on a single unit of graph. We took a flower graph and generalize its structure and found the topological indices for generalized structures. This way of conducting research will motivate other researchers to work for other generalized structures.

9. Open Problems

Fuzzy Zagreb indices of subdivided $H = f_{m\times r}$ flower graph are still an open problem.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally and significantly in writing this paper. All authors have read and agreed to the published version of the manuscript.

References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] A. Rosenfeld, "Fuzzy graph," in *Fuzzy Sets and Their Applications to Cognitive and Decision Process*, L. A. Zadeh, K. S. Fu, and M. Shimura, Eds., Academic Press, New York, NY, USApp. 77–95, 1975.

[3] R. T. Yeh and S. Y. Bang, "Fuzzy relations, fuzzy graphs, and their applications to clustering analysis," in *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*, pp. 125–149, Academic Press, New York, NY, USA, 1975.

[4] S. P. Borgatti, "Centrality and network flow," *Social Networks*, vol. 27, no. 1, pp. 55–71, 2005.

[5] M. Akram and F. Zafar, "A new approach to compute measures of connectivity in rough fuzzy network models," *Journal of Intelligent and Fuzzy Systems*, vol. 36, no. 1, pp. 449–465, 2019.

[6] M. Binu, S. Mathew, and J. N. Mordeson, "Connectivity status of fuzzy graphs," *Information Sciences*, vol. 573, pp. 382–395, 2021.

[7] A. Kaufmann, *Introduction a la theorie des Sour-ensembles flous*, Masson et Cie, Paris, France, 1973.

[8] L. A. Zadeh, "Similarity relations and fuzzy orderings," *Information Sciences*, vol. 3, no. 2, pp. 177–200, 1971.

[9] P. Bhattacharya, "Some remarks on fuzzy graphs," *Pattern Recognition Letters*, vol. 6, no. 5, pp. 297–302, 1987.

[10] K. A. Bibi and M. Devi, "Fuzzy Bi-magic labeling on cycle graph and star graph," *Global Journal of Pure and Applied Mathematics*, vol. 13, pp. 1975–1985, 2017.

[11] S. Vimala and A. Nagarani, "Energy of fuzzy labeling graph [EFl(G)]- Part I," *International Journal of Scientific Engineering and Research*, vol. 7, 2016.

[12] A. Nagoor Gani, M. Akram, and D. R. a Subahashini, "Novel properties of fuzzy labeling graphs," *Journal of Mathematics*, vol. 2014, Article ID 375135, 6 pages, 2014.

[13] M. Akram, "Bipolar fuzzy graphs," *Information Sciences*, vol. 181, no. 24, pp. 5548–5564, 2011.

[14] M. Akram and W. A. Dudek, "Interval-valued fuzzy graphs," *Computers and Mathematics with Applications*, vol. 61, no. 2, pp. 289–299, 2011.

[15] M. Akram and W. A. Dudek, "Intuitionistic fuzzy hypergraphs with applications," *Information Sciences*, vol. 218, pp. 182–193, 2013.

[16] Z. S. Mufti, A. Tabraiz, Q. Xin, B. Almutairi, and R. Anjum, "Fuzzy topological analysis of pizza graph," *AIMS Mathematics*, vol. 8, no. 6, pp. 12841–12856, 2023.

[17] H. Wiener, "Structural determination of paraffin boiling points," *Journal of the American Chemical Society*, vol. 69, no. 1, pp. 17–20, 1947.

[18] M. Binu, S. Mathew, and J. N. Mordeson, "Wiener index of a fuzzy graph and application to illegal immigration networks," *Fuzzy Sets and Systems*, vol. 384, pp. 132–147, 2020.

[19] M. Binu, S. Mathew, and J. N. Mordeson, "Connectivity index of a fuzzy graph and its application to human trafficking," *Fuzzy Sets and Systems*, vol. 360, pp. 117–136, 2019.

[20] S. R. Islam and M. Pal, "Further development of F-index for fuzzy graph and its application in Indian railway crime," *Journal of applied mathematics and computing*, vol. 69, pp. 321–353, 2022.

[21] S. R. Islam and M. Pal, "Hyper-Wiener index for fuzzy graph and its application in share market," *Journal of Intelligent and Fuzzy Systems*, vol. 41, no. 1, pp. 2073–2083, 2021.

[22] I. Gutman and N. Trinajstić, "Graph theory and molecular orbitals. Total π-electron energy of alternant hydrocarbons," *Chemical Physics Letters*, vol. 17, no. 4, pp. 535–538, 1972.

[23] S. R. Islam and M. Pal, "Second Zagreb index for fuzzy graphs and its application in mathematical chemistry," *Iranian Journal of Fuzzy Systems*, vol. 20, no. 1, pp. 119–136, 2023.

[24] S. R. Islam and M. Pal, "An investigation of edge F-index on fuzzy graphs and application in molecular chemistry," *Complex & Intelligent Systems*, vol. 9, no. 2, pp. 2043–2063, 2022.

[25] S. R. Islam and M. Pal, "First Zagreb index on a fuzzy graph and its application," *Journal of Intelligent and Fuzzy Systems*, vol. 40, no. 6, pp. 10575–10587, 2021.

[26] Z. S. Mufti, E. Fatima, R. Anjum, F. Tchier, Q. Xin, and M. M. Hossain, "Computing first and second fuzzy Zagreb indices of linear and multiacyclic hydrocarbons," *Journal of Function Spaces*, vol. 2022, Article ID 6281592, 8 pages, 2022.

[27] U. Ahmad, N. K. Khan, and A. B. Saeid, "Fuzzy topological indices with application to cybercrime problem," *Granular Computing*, vol. 8, no. 5, pp. 967–980, 2023.

[28] E. Mphako-Banda, "Some polynomials of flower graphs," *International Mathematical Forum*, vol. 2, no. 51, pp. 2511–2518, 2007.

[29] M. Pal, S. Samanta, and G. Ghorai, *Modern Trends in Fuzzy Graph Theory*, Springer, Berlin, Germany, 2020.

[30] S. Kalathian, S. Ramalingam, S. Raman, and N. Srinivasan, "Some topological indices in fuzzy graphs," *Journal of Intelligent and Fuzzy Systems*, vol. 39, no. 5, pp. 6033–6046, 2020.

[31] J. N. Mordeson and S. Mathew, "Non-deterministic flow in fuzzy networks and its application in identification of human trafficking chains," *New Mathematics and Natural Computation*, vol. 13, no. 03, pp. 231–243, 2017.

[32] V. Obraztsov and V. V. Krasnoproshin, "Management optimization problems on fuzzy graphs," *Investigaciones europeas de direccion y economia de la empresa*, vol. 1, no. 3, pp. 63–70, 1995.