Hindawi
Journal of Mathematics
Volume 2023, Article ID 8140831, 27 pages
https://doi.org/10.1155/2023/8140831

Research Article

Impact of Correlated Measurement Errors on Some Efficient Classes of Estimators

Anoop Kumar,[1] **Shashi Bhushan**,[2] **Shivam Shukla**,[3] **Walid Emam**,[4] **Yusra Tashkandy**,[4] **and Rajesh Gupta**[5]

[1]*Department of Statistics, Amity School of Applied Sciences, Amity University Uttar Pradesh, Lucknow 226028, India*
[2]*Department of Statistics, University of Lucknow, Lucknow 226007, India*
[3]*Department of Mathematics & Statistics, Dr. Shakuntala Misra National Rehabilitation University, Lucknow 226017, India*
[4]*Department of Statistics and Operations Research, Faculty of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia*
[5]*Global Data Insight & Analytics (GDIA), Ford Motor Company, 1 American Rd, Dearborn, MI 48126, USA*

Correspondence should be addressed to Shashi Bhushan; bhushan_s@lkouniv.ac.in

Received 7 July 2023; Revised 12 August 2023; Accepted 23 September 2023; Published 18 October 2023

Academic Editor: Ammar Alsinai

It is well-known that the appearance of measurement errors spoils the traditional results in survey sampling. The concept of correlated measurement errors (CMEs) is true in various practical situations, but this has been seldom considered by researchers in survey sampling. In this article, the influence of the CME under simple random sampling (SRS) has been considered over some prominent classes of estimators for the population mean. The first-order approximated formulae of the mean square error of the introduced estimators are reported, and a comparative analysis has also been conducted with traditional estimators. The theoretical findings are extended by a broad spectrum computational study using real and artificial data.

1. Introduction

In survey research, the primary objective of any surveyor is to enhance the efficiency of the estimation procedures with the help of information on the auxiliary/supplementary variables that are usually associated with the research variable. In this context, the literature contains the ratio, regression, product, exponential methods, and their modified forms for efficiently estimating the parameters of interest. These estimation methods are further extended using two- or multiauxiliary information under different sampling schemes. These estimation methods either consist of a supplementary variable or only on research variable, presupposing that all data are independent from ME, but this presupposition practically never happen. The data are tainted with or have hidden ME due to different types of reasons (readers can refer to Murthy [1] and Cochran [2]). The discrepancy between the true and observed values is known as ME. Many attempts have been made to examine the impact of ME on various parameters of the population such as the mean, variance, total, and distribution function. The effect of ME has been observed over the efficiency of the estimation methods by many authors. Shalabh [3] examined the ME's impact on the classical ratio estimators. Influenced by Shalabh [3], Manisha and Singh [4] studied the ME's impact using a new class of estimators for the population mean. Subsequently, Singh and Karpe [5–7] examined the effect of ME over the parameters of the population using different sampling strategies. The variance computation in the existence of ME was provided by Diana and Giordan [8]. Hussain et al. [9] suggested the estimation of a finite population distribution function with the dual use of supplementary information under nonresponse. Tariq et al. [10] proposed a supplementary information-based variance estimator to tackle the problem of ME. Tariq et al. [11] suggested a generalized variance estimator utilizing supplementary information in the presence and absence of ME. Zahid et al. [12] developed a generalized class of estimators for the sensitive variables in the case of ME and nonresponse. Ahmad et al. [13] discussed the estimation of the finite population mean using a dual supplementary variable for nonresponse under SRS. Tiwari et al. [14] suggested a novel class of efficient estimators to assess the impact of nonresponse and ME. Tiwari and Sharma [15] developed an efficient estimation procedure of the population mean under the joint influence of nonresponse and ME. Bhushan et al. [16] suggested some novel logarithmic estimators for the population mean in the presence of ME. These studies have all taken into account the possibility of uncorrelated measurement errors (UMEs), which can be found in both the supplementary variable and research variable. The subject of our attention is different from the research described above. We assume that both the study and supplementary variables have access to ME. It may be incorrect to assume that both variables are independent of ME because often the same surveyor collects data on the supplementary variable and research variable. This correlation in ME can exist caused by the ulterior inherent propensity of the data. The significance of CME was initially studied by Shalabh and Tsai [17] using the ratio and product estimators of the population mean utilizing the SRS framework. Recently, Bhushan et al. [18, 19] assessed the effectiveness of some new classes of estimators based on the CME.

In the present article, we study the CME's impact on efficient classes of population mean estimators under SRS. The subsequent material is divided into a few sections. Section 2 devotes to the follow-up of the literature related to the CME. Section 3 presents the recommended classes of estimators with their characteristics and conducts a comparative analysis with the class of estimators that is already in use. To support the theoretically obtained results, a numerical and simulation studies along with the important findings are given in Section 4 and Section 5, respectively. Section 6 of the research provides conclusions.

2. Follow-Up of the Literature

It is assumed that $F = (F_1, F_2, \ldots, F_N)$ is a finite population of size N units from which a sample of size n is chosen using simple random sampling with replacement (SRSWR). Let x_i and y_i be the true amount of research variable and supplementary variable for i^{th} units of the population F, respectively. These amounts are unavailable, while they may easily be quantified as (y_i, x_i) having the ME's (u_i, v_i) in the i^{th} unit of the variables, respectively. Suppose we can write x_i and y_i as $y_i = Y_i + u_i$ and $x_i = X_i + v_i, i = 1, 2, \ldots, n$. MEs u_i and v_i are also unobservable such that $E(u) = E(v) = 0$, $V(u) = \sigma_u^2$, and $V(v) = \sigma_v^2$, respectively, and the correlation coefficient between u and v is $\operatorname{Cor}(u, v) = \rho_{uv}$. Let $(\overline{y}, \overline{x})$, (μ_y, μ_x), (σ_y^2, σ_x^2), and (C_y, C_x) be the sample means, population means, population variances, and population coefficient of variations of the research variable and supplementary variable, respectively. Let ρ_{xy} be the correlation coefficient between the supplementary variable and the research variable.

For calculating the mean square error (MSE) of several estimators in the situation of ME, we employ the notations discussed as $w_x = \sum (x_i - \mu_x)/\sqrt{n}$, $w_y = \sum (y_i - \mu_y)/\sqrt{n}$, $w_v = \sum v_i/\sqrt{n}$, and $w_u = \sum u_i/\sqrt{n}$. It is very appealing to observe that $\overline{x} = \mu_x + (w_x + w_v)/\sqrt{n}$ and $\overline{y} = \mu_y + (w_y + w_u)/\sqrt{n}$.

The mean per unit estimator under CME is defined as follows:

$$t_m = \overline{y}. \tag{1}$$

It is found that

$$E(t_m) = \mu_y, \tag{2}$$

$$V(t_m) = \frac{\left(\sigma_y^2 + \sigma_u^2\right)}{n}. \tag{3}$$

Shalabh and Tsai [17] advocated the usual ratio and product estimators in the presence of CME as given by

$$\begin{aligned} t_r &= \overline{y}\frac{\mu_x}{\overline{x}}, \\ t_p &= \overline{y}\frac{\overline{x}}{\mu_x}. \end{aligned} \tag{4}$$

The estimators t_r and t_p have the following MSE:

$$\operatorname{MSE}(t_r) = \frac{(V_r + V_{rm})}{n}, \tag{5}$$

$$\operatorname{MSE}\left(t_p\right) = \frac{\left(V_p + V_{\mathrm{pm}}\right)}{n}, \tag{6}$$

where

$$\begin{aligned} V_r &= \sigma_y^2\left\{1 - 2\left(\frac{\sigma_x\mu_y}{\sigma_y\mu_x}\right)\rho_{xy} + \left(\frac{\sigma_x\mu_y}{\sigma_y\mu_x}\right)^2\right\}, \\ V_{\mathrm{rm}} &= \sigma_u^2\left\{1 - 2\left(\frac{\sigma_v\mu_y}{\sigma_u\mu_x}\right)\rho_{uv} + \left(\frac{\sigma_v\mu_y}{\sigma_u\mu_x}\right)^2\right\}, \\ V_p &= \sigma_y^2\left\{1 + 2\left(\frac{\sigma_x\mu_y}{\sigma_y\mu_x}\right)\rho_{xy} + \left(\frac{\sigma_x\mu_y}{\sigma_y\mu_x}\right)^2\right\}, \\ V_{\mathrm{pm}} &= \sigma_u^2\left\{1 + 2\left(\frac{\sigma_v\mu_y}{\sigma_u\mu_x}\right)\rho_{uv} + \left(\frac{\sigma_v\mu_y}{\sigma_u\mu_x}\right)^2\right\}. \end{aligned} \tag{7}$$

3. Proffered Estimators under CME and Their Properties

The objective of this article is to provide some efficient estimates of the population mean and to study the effect of CME on some prominent classes of estimators. To the best of our knowledge, the impact of the CME has not been attempted so far over any class of estimators. We propose to study the following efficient classes of estimators of the population mean under CME as follows:

$$T_1 = \overline{y} + \beta_1 (\overline{x} - \mu_x),$$
$$T_2 = \overline{y}\left(\frac{\mu_x}{\overline{x}}\right)^{\beta_2}, \tag{8}$$
$$T_3 = \overline{y}\left\{\frac{\mu_x}{\mu_x + \beta_3 (\overline{x} - \mu_x)}\right\},$$

where β_1, β_2, and β_3 are duly selected scalars to be determined.

Considering the notations described in Section 2, we write the estimator T_1 as follows:

$$T_1 = \left(\mu_y + \frac{w_y + w_u}{n^{1/2}}\right) + \beta_1\left(\frac{w_x + w_v}{n^{1/2}}\right). \tag{9}$$

To derive the MSE (T_1), the square and expectation are taken on both sides of (9):

$$\text{MSE}(T_1) = \mu_y^2\left(\frac{\sigma_y^2 + \sigma_u^2}{n\mu_y^2}\right) + \beta_1^2\mu_x^2\left(\frac{\sigma_x^2 + \sigma_v^2}{n\mu_x^2}\right) + 2\beta_1\mu_x\mu_y\left(\frac{\rho_{xy}\sigma_x\sigma_y + \rho_{uv}\sigma_u\sigma_v}{n\mu_x\mu_y}\right). \tag{10}$$

By minimizing (10) in relation to β_1, we obtain the optimum value of β_1 as follows:

$$\beta_{1\,(\text{opt})} = -\frac{\left(\rho_{xy}\sigma_x\sigma_y + \rho_{uv}\sigma_u\sigma_v\right)}{\left(\sigma_x^2 + \sigma_v^2\right)}. \tag{11}$$

Replacing the value of β_1 with $\beta_{1\,(\text{opt})}$ in (10), we obtain the minimum MSE of the estimator T_1 as follows:

$$\text{minMSE}(T_1) = \frac{1}{n}\left[\left(\sigma_y^2 + \sigma_u^2\right) - \frac{\left(\rho_{xy}\sigma_x\sigma_y + \rho_{uv}\sigma_u\sigma_v\right)^2}{\left(\sigma_x^2 + \sigma_v^2\right)}\right]. \tag{12}$$

Again, considering the notations described in Section 2, we write the estimator T_2 as follows:

$$T_2 = \left(\mu_y + \frac{w_y + w_u}{n^{1/2}}\right)\left(\frac{\mu_x}{\mu_x + w_x + w_v/n^{1/2}}\right)^{\beta_2}. \tag{13}$$

To derive the MSE (T_2), the square and expectation are considered on both sides of (13):

$$\text{MSE}(T_2) = \frac{\left(\sigma_y^2 + \sigma_u^2\right)}{n} + \beta_2^2\frac{\mu_y^2}{\mu_x^2}\frac{\left(\sigma_x^2 + \sigma_v^2\right)}{n} - 2\beta_2\frac{\mu_y}{\mu_x}\frac{\left(\rho_{xy}\sigma_x\sigma_y + \rho_{uv}\sigma_u\sigma_v\right)}{n}. \tag{14}$$

By minimizing (14) in relation to β_2, we obtain the optimum value of β_2 as follows:

$$\beta_{2\,(\text{opt})} = \frac{\mu_x}{\mu_y}\frac{\left(\rho_{xy}\sigma_x\sigma_y + \rho_{uv}\sigma_u\sigma_v\right)}{\left(\sigma_x^2 + \sigma_v^2\right)}. \tag{15}$$

Replacing the value of β_2 with $\beta_{2\,(\text{opt})}$ in (14), we obtain the minimum MSE of the estimator T_2 as follows:

$$\text{minMSE}(T_2) = \frac{1}{n}\left[\left(\sigma_y^2 + \sigma_u^2\right) - \frac{\left(\rho_{xy}\sigma_x\sigma_y + \rho_{uv}\sigma_u\sigma_v\right)^2}{\left(\sigma_x^2 + \sigma_v^2\right)}\right]. \tag{16}$$

Now, we write the estimator T_3 by using the notations described in Section 2 as follows:

$$T_3 = \left(\mu_y + \frac{w_y + w_u}{n^{1/2}}\right)\left\{\frac{\mu_x}{\mu_x + (w_x + w_v)/n^{1/2} + \beta_3\left(\mu_x + (w_x + w_v)/n^{1/2} - \mu_x\right)}\right\}. \tag{17}$$

To derive the MSE (T_3), we square and take the expectation on both sides of (17):

$$MSE(T_3) = \frac{\left(\sigma_y^2 + \sigma_u^2\right)}{n} + (\beta_3 + 1)^2\frac{\mu_y^2}{\mu_x^2}\frac{\left(\sigma_x^2 + \sigma_v^2\right)}{n} - 2(\beta_3 + 1)\frac{\mu_y}{\mu_x}\frac{\left(\rho_{xy}\sigma_x\sigma_y + \rho_{uv}\sigma_u\sigma_v\right)}{n}. \tag{18}$$

By minimizing (18) in relation to β_3, we obtain the optimum value of β_3 as follows:

$$\beta_{3(\mathrm{opt})} = \frac{\mu_x}{\mu_y}\frac{\left(\rho_{xy}\sigma_x\sigma_y + \rho_{uv}\sigma_u\sigma_v\right)}{\left(\sigma_x^2 + \sigma_v^2\right)} - 1. \tag{19}$$

Replacing the value of β_3 with $\beta_{3(\mathrm{opt})}$ in (18), we obtain the minimum MSE of the estimator T_3 as follows:

$$\mathrm{minMSE}(T_3) = \frac{1}{n}\left[\left(\sigma_y^2 + \sigma_u^2\right) - \frac{\left(\rho_{xy}\sigma_x\sigma_y + \rho_{uv}\sigma_u\sigma_v\right)^2}{\left(\sigma_x^2 + \sigma_v^2\right)}\right]. \tag{20}$$

It is to be noted that the minimum MSE expressions of the proposed estimator $T_i, i = 1, 2, 3$ are same.

Furthermore, we present the theoretical comparisons of the proffered estimators $T_i, i = 1, 2, 3$ with other existing estimators in the presence of CME and obtain the following conditions:

(i) Comparing (3) with (12), (16), and (20), we get

$$V(t_m) - \mathrm{MSE}(T_i) > 0 \text{ if } \rho_{xy}\sigma_x\sigma_y > -\rho_{uv}\sigma_u\sigma_v. \tag{21}$$

(ii) Comparing (5) with (12), (16), and (20), we get

$$\mathrm{MSE}(t_r) - \mathrm{MSE}(T_i) > 0 \text{ if } \rho_{xy}\sigma_x\sigma_y > \sqrt{\left\{\left(V_r + V_{rm}\right) - \left(\sigma_y^2 + \sigma_u^2\right)\right\}\left(\sigma_x^2 + \sigma_v^2\right)} - \rho_{uv}\sigma_u\sigma_v. \tag{22}$$

(iii) Comparing (6) with (12), (16), and (20), we get

$$\mathrm{MSE}\left(t_p\right) - \mathrm{MSE}(T_i) > 0 \text{ if } \rho_{xy}\sigma_x\sigma_y > \sqrt{\left\{\left(V_p + V_{pm}\right) - \left(\sigma_y^2 + \sigma_u^2\right)\right\}\left(\sigma_x^2 + \sigma_v^2\right)} - \rho_{uv}\sigma_u\sigma_v. \tag{23}$$

The suggested estimators will surpass the existing estimators under the above conditions.

4. Numerical Study

This section presents a numerical study using two real populations which are discussed as follows:

Population 1: origin: The book of U.S. Census Bureau (1986)

X_i = actual size of farm, x_i = quantified size of farm, Y_i = real sale price of the item, and y_i = quantified value of the goods sold

Population 2: origin: Gujarati and Sangeetha [20]

X_i = actual income, x_i = quantified income, Y_i = actual consumption costs, and y_i = quantified consumption cost

The parameters of these populations are given in Table 1.

The percent relative efficiency (PRE) is calculated by utilizing the following expression:

$$\mathrm{PRE} = \frac{V(t_m)}{\mathrm{MSE}(T^*)} \times 100, \tag{24}$$

where $T^* = t_m, t_p, t_r, T_1, T_2$, and T_3.

The numerical results (PREs) are given in Table 2 that demonstrate the outperformance of the proposed estimators against the existing estimators in each population. The results in Table 2 also demonstrate that the proposed estimators $T_i, i = 1, 2, 3$ perform equally in each population. Moreover, these results are further generalized by simulation.

5. Simulation

To assess the cogency of the theoretical results in practice, we consider an extensive simulation study over an artificially drawn population of size $N = 1000$ from the normal distribution with population parameters $\mu_y = 30$, $\mu_x = 20$, $\sigma_y^2 = 1$ and 3, $\sigma_x^2 = 1$ and 3, $\rho_{xy} = -0.9, -0.5, -0.1, 0, 0.5$, and 0.9, $\sigma_u^2 = 1$ and 3, $\sigma_v^2 = 1$ and 3, and $\rho_{uv} = -0.9, -0.5, 0, 0.5$, and 0.9. The steps used in the simulation study are as follows:

Step 1: draw the data hypothetically from R software with y, x, v, and u utilizing a 4-variate multivariate normal distribution as $Z = (X, Y, u, v)'$ with the mean vector $\mu_z = (\mu_x, \mu_y, 0, 0)'$ and the covariance matrix:

$$\begin{pmatrix} \sigma_x^2 & \sigma_x\sigma_y\rho_{xy} & 0 & 0 \\ \sigma_x\sigma_y\rho_{xy} & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_u^2 & \sigma_u\sigma_v\rho_{uv} \\ 0 & 0 & \sigma_u\sigma_v\rho_{uv} & \sigma_v^2 \end{pmatrix}. \tag{25}$$

Step 2: quantify a sample of size $n = 100$ from the above population utilizing SRSWR.

Step 3: calculate the required statistics.

Step 4: using 10000 replications, calculate the PRE of several estimators regarding the usual mean estimator t_m utilizing the expression given as follows:

Table 1: Population parameters.

Parameters	N	n	μ_x	μ_y	σ_y^2	σ_x^2	σ_u^2	σ_v^2	ρ_{xy}	ρ_{uv}
Population 1	56	15	75.79	61.59	577.44	155.5	16	16	−0.508	−0.418
Population 2	10	4	170	127	1278	3300	36	36	0.964	0.800

Table 2: Numerical results of several estimators.

Estimators	t_m	t_r	t_p	$T_i, i = 1, 2, 3$
Population 1	100.000	61.684	131.486	132.024
Population 2	100.000	751.622	21.273	1254.989

$$\mathrm{PRE} = \frac{V(t_m)}{\mathrm{MSE}(T^*)} \times 100 = \frac{1/10000 \sum_{i=1}^{10,000} (t_m - E(t_m))^2}{1/10000 \sum_{i=1}^{10,000} (T^* - \mu_y)^2} \times 100. \tag{26}$$

Subsequently, we have computed the PREs of the estimators for several values of parameters σ_y^2, σ_x^2, σ_u^2, σ_v^2, ρ_{xy}, and ρ_{uv}. The simulation results are presented in Tables 3–10. Moreover, we have also calculated the PRE of estimators for several values of measurement errors such as 10%, 15%, 20%, and 25% and presented the results in Tables 11–18.

5.1. Main Findings. The computed results of the PRE for the proffered estimators are presented in Tables 3–18. The comparative studies of several estimators are presented in terms of the PRE in Tables 3–10 for several parameters σ_x^2, σ_y^2, σ_u^2, σ_v^2, ρ_{xy}, and ρ_{uv}. The comparisons of different estimators are also presented in terms of the PRE in Tables 11–18 for different amounts of ME. The important results of the proffered estimators are as follows:

(1) From Table 3, concerning to the values of $\sigma_y^2 = 1$, $\sigma_x^2 = 1$ along with the positive correlation $\rho_{xy} = 0, 0.5, 0.9$, $\sigma_u^2 = 1$, and $\sigma_v^2 = 1$, we can observe that

  (i) As the value of ρ_{xy} rises, the percent relative efficiency of the traditional ratio estimator t_r also rises. In addition, the values of ρ_{uv} affect the rate and size of this growth. This can also be observed from Figures 1–3.

  (ii) As ρ_{xy} fluctuates between 0 and 0.9, the percent relative efficiency of the traditional product estimator t_p drops. The percent relative efficiency likewise reduces when ρ_{uv} fluctuates from −0.9 to +0.9. This can easily be observed from Figures 1–3.

  (iii) When the values of ρ_{xy} rises between 0 and 0.9, the percent relative efficiency of the proposed estimators T_1, T_2, and T_3 rises. The size and rate of this rise both rely on the sign and value of ρ_{uv}, and they both drop as ρ_{uv} values decline from 0.9 to −0.9.

  (iv) The primary influence of the correlated measurement errors over the percent relative efficiency of the suggested estimators may be seen by comparing the percent relative efficiency of the estimators at $\rho_{uv} = 0$ and $\rho_{uv} = \pm 0.9$. This effect can easily be observed from Figures 1–3.

  (v) The percent relative efficiency of the suggested estimators $T_i, i = 1, 2, 3$ is higher for positively correlated measurement errors, and the percent relative efficiency decreases as the valuation of ρ_{uv} varies from −0.9 to 0 and increases and the values of ρ_{uv} vary from 0 to 0.9. This effect can easily be observed from Figures 1–3.

  (vi) For various combinations of σ_u^2 and σ_v^2, the same trend can be seen in the percent relative efficiency values of the proffered estimators.

(2) The same pattern in the percent relative efficiency like Table 3 can also be observed from Tables 5, 7, and 9 consisting of different combinations of σ_y^2 and σ_x^2 and the positive correlation coefficient ρ_{xy}. The graphs for the same can be provided, if required.

(3) From Table 4, concerning to the values of $\sigma_y^2 = 1$, $\sigma_x^2 = 1$ with negative correlation $\rho_{xy} = -0.9, -0.5, -0.1$, $\sigma_u^2 = 1$ and $\sigma_v^2 = 1$, we can observe that

  (i) As ρ_{xy} fluctuates from −0.1 to −0.9 over sequential decrements of 0.4, the percent relative efficiency of the traditional ratio estimator t_r declines. The percent relative efficiency also drops as the value of ρ_{uv} is successively decreased from +0.9 to −0.9. This can easily be observed from Figures 4–6.

  (ii) As ρ_{xy} fluctuates from −0.1 to −0.9 across sequential decrements of 0.4, the percent relative efficiency of the traditional product estimator t_p rises. The percent relative efficiency also rises when the value of ρ_{uv} is successively decreased from +0.9 to −0.9. This can easily be observed from Figures 4–6.

  (iii) As ρ_{xy} fluctuates from −0.9 to −0.1 over the sequential increase of 0.4, the percent relative efficiency of the suggested estimators $T_i, i = 1, 2, 3$ declines. The direction and value of ρ_{uv} also affect the size and rate of this decline. It is greater when the correlated measurement

Table 3: PRE of different estimators for $\rho_{xy} = (0, 0.5, 0.9)$ and $(\sigma_x^2, \sigma_y^2) = (1, 1)$.

σ_u^2	σ_v^2	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
			−0.9	49.185	121.952	130.264
			−0.5	56.865	94.217	110.077
		0	0	69.863	68.85	101.857
			0.5	92.104	56.234	109.474
			0.9	120.321	49.455	128.848
			−0.9	58.821	83.048	106.242
			−0.5	70.055	68.913	102.166
1	1	0.5	0	89.585	54.092	109.597
			0.5	136.962	46.184	142.84
			0.9	208.844	41.231	213.634
			−0.9	69.846	68.505	103.072
			−0.5	83.267	59.013	106.159
		0.9	0	118.119	47.512	129.236
			0.5	208.911	41.152	213.634
			0.9	431.07	37.601	585.79
			−0.9	34.585	123.149	148.367
			−0.5	40.329	80.376	114.204
		0	0	53.427	51.713	101.928
			0.5	75.647	40.33	111.943
			0.9	118.473	34.389	145.469
			−0.9	38.823	83.983	117.943
			−0.5	46.6	60.308	103.621
1	3	0.5	0	63.893	43.698	105.684
			0.5	105.657	34.636	136.985
			0.9	204.305	30.053	226.852
			−0.9	43.323	68.609	107.93
			−0.5	53.255	52.179	102.393
		0.9	0	77.088	39.281	114.53
			0.5	145.268	31.898	173.659
			0.9	421.448	28.195	438.291
			−0.9	57.417	145.767	151.952
			−0.5	65.747	110.956	115.064
		0	0	82.937	80.879	101.92
			0.5	108.247	67.582	112.689
			0.9	143.216	58.234	148.875
			−0.9	63.556	114.153	118.944
			−0.5	74.719	91.055	103.818
3	1	0.5	0	96.23	72.163	105.56
			0.5	134.262	59.356	136.979
			0.9	190.874	52.271	228.155
			−0.9	69.591	99.089	108.609
			−0.5	82.647	81.359	102.348
		0.9	0	109.53	66.029	113.993
			0.5	161.198	55.196	172.414
			0.9	251.721	48.916	440.066
			−0.9	43.104	189.33	193.096
			−0.5	50.052	105.911	119.853
		0	0	71.088	67.963	102.438
			0.5	103.966	50.499	118.507
			0.9	187.516	42.58	190.812
			−0.9	46.342	139.937	145.718
			−0.5	56.115	91.454	109.586
3	3	0.5	0	80.239	61.047	104.704
			0.5	136.733	46.639	142.805
			0.9	282.116	39.181	304.15
			−0.9	49.178	117.897	127.958
			−0.5	60.585	81.647	105.162
		0.9	0	89.189	56.898	108.916
			0.5	165.101	44.059	169.293
			0.9	430.55	37.567	585.295

Table 4: PRE of different estimators for ρ_{xy} = (−0.9, −0.5, −0.1) and $(\sigma_x^2, \sigma_y^2) = (1, 1)$.

σ_u^2	σ_v^2	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
			−0.9	37.666	429.518	585.79
			−0.5	41.306	208.675	214.101
		−0.9	0	47.158	119.665	130.187
			0.5	58.617	82.828	106.242
			0.9	68.599	69.931	103.072
			−0.9	41.344	208.726	214.101
			−0.5	46.338	137.121	142.851
1	1	−0.5	0	54.906	91.939	109.988
			0.5	68.292	69.44	102.174
			0.9	83.197	58.968	106.159
			−0.9	46.063	136.355	142.665
			−0.5	52.976	101.819	115.329
		−0.1	0	62.788	72.477	102.375
			0.5	83.342	58.262	105.766
			0.9	106.757	50.823	119.928
			−0.9	28.37	428.032	444.16
			−0.5	32.079	149.362	176.481
		−0.9	0	38.968	78.794	115.862
			0.5	51.441	53.423	102.551
			0.9	68.18	43.29	107.851
			−0.9	30.454	209.776	229.724
			−0.5	34.586	107.711	138.534
1	3	−0.5	0	43.016	64.722	106.406
			0.5	59.029	46.393	103.528
			0.9	82.071	38.58	117.038
			−0.9	32.794	137.795	163.208
			−0.5	38.036	85.609	119.2
		−0.1	0	48.832	55.268	102.258
			0.5	69.258	41.062	108.664
			0.9	105.273	34.766	135.977
			−0.9	48.79	253.371	445.717
			−0.5	54.816	163.139	175.066
		−0.9	0	65.56	111.06	115.122
			0.5	81.122	83.502	102.485
			0.9	99.007	69.944	108.5
			−0.9	51.728	194.273	233.211
			−0.5	58.731	135.555	138.284
3	1	−0.5	0	71.387	97.296	106.17
			0.5	90.493	75.288	103.681
			0.9	113.599	64.301	118.149
			−0.9	55.393	155.837	164.633
			−0.5	63.208	116.35	119.519
		−0.1	0	78.916	86.687	102.244
			0.5	102.384	69.173	109.018
			0.9	133.61	59.311	137.418
			−0.9	37.841	424.865	584.988
			−0.5	43.659	164.9	169.524
		−0.9	0	59.748	83.198	106.043
			0.5	81.108	60.097	105.194
			0.9	117.697	48.999	128.015
			−0.9	39.773	279.073	304.295
			−0.5	48.594	128.386	135.149
3	3	−0.5	0	64.641	75.688	103.196
			0.5	92.454	56.975	109.6
			0.9	139.312	45.682	145.451
			−0.9	41.847	207.412	212.573
			−0.5	51.315	110.848	121.682
		−0.1	0	69.836	68.921	102.349
			0.5	103.698	53.007	116.632
			0.9	171.852	43.022	175.262

Table 5: PRE of different estimators for $\rho_{xy} = (0, 0.5, 0.9)$ and $(\sigma_x^2, \sigma_y^2) = (1, 3)$.

σ_u^2	σ_v^2	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
1	1	0	−0.9	65.343	110.972	115.122
			−0.5	71.329	97.243	106.17
			0	80.567	83.286	101.658
			0.5	96.117	72.036	105.56
			0.9	109.442	65.885	113.993
		0.5	−0.9	80.945	83.333	102.485
			−0.5	90.472	75.248	103.681
			0	107.081	66.402	112.382
			0.5	134.276	59.25	136.979
			0.9	161.346	55.077	172.414
		0.9	−0.9	98.892	69.785	108.5
			−0.5	113.62	64.307	118.149
			0	142.499	57.13	147.495
			0.5	191.189	52.168	228.155
			0.9	252.416	48.828	440.066
1	3	0	−0.9	51.463	110.614	120.199
			−0.5	57.723	89.43	107.131
			0	69.513	69.959	101.259
			0.5	86.334	58.224	105.893
			0.9	108.142	51.585	118.685
		0.5	−0.9	60.623	82.891	104.501
			−0.5	69.981	69.475	101.54
			0	85.659	57.623	106.334
			0.5	118.361	49.193	126.909
			0.9	159.378	44.55	161.614
		0.9	−0.9	70.089	69.459	102.327
			−0.5	80.818	60.132	105.023
			0	106.882	50.503	120.355
			0.5	159.058	43.878	163.713
			0.9	247.89	40.335	261.88
3	1	0	−0.9	67.047	126.426	130.523
			−0.5	74.338	107.142	110.308
			0	86.996	89.236	101.753
			0.5	105.189	75.941	108.667
			0.9	124.828	67.835	128.543
		0.5	−0.9	77.322	100.546	106.583
			−0.5	87.925	87.392	102.085
			0	105.16	75.845	108.667
			0.5	133.853	65.342	140.657
			0.9	165.695	59.626	209.205
		0.9	−0.9	87.734	86.647	103.029
			−0.5	99.714	77.974	106.398
			0	124.871	67.734	128.543
			0.5	165.904	59.544	209.205
			0.9	221.137	54.376	583.474
3	3	0	−0.9	53.273	145.928	148.082
			−0.5	61.614	106.712	113.46
			0	78.163	76.03	101.923
			0.5	106.964	61.531	113.68
			0.9	144.232	52.977	146.125
		0.5	−0.9	59.492	112.701	118.115
			−0.5	70.042	87.653	103.719
			0	89.554	66.701	105.133
			0.5	135.526	54.623	137.984
			0.9	202.611	47.772	222.468
		0.9	−0.9	65.151	95.508	107.896
			−0.5	78.165	77.118	102.409
			0	104.325	60.031	114.036
			0.5	169.537	50.347	177.384
			0.9	291.306	44.533	440.319

Table 6: PRE of different estimators for $\rho_{xy} = (-0.9, -0.5, -0.1)$ and $(\sigma_x^2, \sigma_y^2) = (1, 3)$.

σ_u^2	σ_v^2	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
			−0.9	48.567	255.146	445.717
			−0.5	51.645	194.483	233.211
		−0.9	0	56.215	145.122	149.954
			0.5	63.342	114.074	118.944
			0.9	69.417	98.975	108.609
			−0.9	54.592	163.454	175.066
			−0.5	58.662	135.549	138.284
1	1	−0.5	0	64.488	109.746	114.568
			0.5	74.566	90.94	103.818
			0.9	82.489	81.216	102.348
			−0.9	62.834	118.815	121.525
			−0.5	68.338	103.24	109.561
		−0.1	0	76.655	87.638	102.407
			0.5	90.801	75.25	103.418
			0.9	102.648	68.544	109.432
			−0.9	40.446	253.224	269.97
			−0.5	43.328	162.605	166.992
		−0.9	0	49.594	108.413	121.559
			0.5	58.95	81.024	105.261
			0.9	68.569	70.166	102.39
			−0.9	44.509	163.134	165.231
			−0.5	49.317	119.482	127.677
1	3	−0.5	0	55.905	87.33	107.489
			0.5	69.054	70.136	101.691
			0.9	81.43	60.683	104.302
			−0.9	49.884	118.51	126.051
			−0.5	55.733	94.552	109.952
		−0.1	0	64.911	72.837	101.62
			0.5	81.917	60.373	104.049
			0.9	101.396	53.239	114.283
			−0.9	54.316	221.569	585.374
			−0.5	58.771	168.634	212.848
		−0.9	0	66.857	126.493	130.523
			0.5	77.165	100.461	106.583
			0.9	86.704	87.998	103.132
			−0.9	58.999	168.06	212.848
			−0.5	65.089	134.445	141.457
3	1	−0.5	0	74.176	107.091	110.308
			0.5	87.306	88.443	102.285
			0.9	99.782	78.092	106.398
			−0.9	65.241	133.264	139.945
			−0.5	72.122	112.019	114.393
		−0.1	0	84.019	92.419	102.285
			0.5	100.856	78.33	105.962
			0.9	118.772	69.734	121.561
			−0.9	44.766	289.313	442.798
			−0.5	50.43	169.283	177.213
		−0.9	0	59.055	105.844	114.934
			0.5	77.224	78.203	102.435
			0.9	95.417	65.007	107.974
			−0.9	48.089	203.863	226.799
			−0.5	54.728	135.27	137.752
3	3	−0.5	0	65.02	91.237	105.972
			0.5	87.827	70.034	103.836
			0.9	111.567	59.09	117.754
			−0.9	52.134	155.053	158.215
			−0.5	60.094	111.582	116.94
		−0.1	0	72.978	79.917	101.995
			0.5	102.457	63.122	110.763
			0.9	136.135	54.139	138.138

Table 7: PRE of different estimators for $\rho_{xy} = (0, 0.5, 0.9)$ and $(\sigma_x^2, \sigma_y^2) = (3, 1)$.

σ_u^2	σ_v^2	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
			−0.9	65.343	110.972	115.122
			−0.5	43.268	65.09	106.406
		0	0	49.59	51.802	101.7
			0.5	64.117	43.848	105.684
			0.9	77.323	39.4	114.53
			−0.9	51.428	53.421	102.551
			−0.5	59.299	46.603	103.528
1	1	0.5	0	73.217	38.616	112.368
			0.5	105.885	34.715	136.985
			0.9	145.514	31.957	173.659
			−0.9	68.089	43.238	107.851
			−0.5	82.252	38.661	117.038
		0.9	0	116.238	32.903	147.539
			0.5	204.06	30.009	226.852
			0.9	421.643	28.215	438.291
			−0.9	28.587	77.217	130.441
			−0.5	32.54	56.831	110.202
		0	0	40.154	40.473	101.755
			0.5	53.57	32.439	108.704
			0.9	74.235	28.419	128.854
			−0.9	35.34	52.389	106.642
			−0.5	41.857	41.465	102.077
1	3	0.5	0	53.631	32.474	108.704
			0.5	85.671	27.212	140.647
			0.9	141.671	24.373	209.489
			−0.9	43.129	42.331	103.024
			−0.5	51.192	35.391	106.357
		0.9	0	74.321	28.443	128.854
			0.5	141.6	24.365	209.489
			0.9	418.102	22.39	583.383
			−0.9	49.744	108.591	121.559
			−0.5	56.043	87.486	107.489
		0	0	68.577	70.7	101.247
			0.5	85.703	57.669	106.334
			0.9	106.903	50.543	120.355
			−0.9	59.166	81.25	105.261
			−0.5	68.247	68.874	101.57
3	1	0.5	0	86.276	58.17	105.893
			0.5	116.513	49.194	125.704
			0.9	159.08	43.949	163.713
			−0.9	69.427	68.385	102.278
			−0.5	81.303	60.579	104.302
		0.9	0	108.086	51.528	118.685
			0.5	159.41	44.521	161.614
			0.9	251.228	40.024	265.014
			−0.9	38.434	132.243	149.173
			−0.5	44.949	86.791	113.682
		0	0	61.743	59.814	101.916
			0.5	87.066	44.876	113.95
			0.9	129.838	37.988	148.545
			−0.9	43.868	94.008	119.097
			−0.5	52.689	68.653	103.928
3	3	0.5	0	69.618	49.108	105.091
			0.5	119.642	39.703	137.707
			0.9	217.651	34.156	226.917
			−0.9	49.055	76.935	108.553
			−0.5	60.432	59.477	102.345
		0.9	0	84.813	44.01	113.821
			0.5	164.693	36.645	175.56
			0.9	429.474	32.038	444.726

Table 8: PRE of different estimators for ρ_{xy} = (−0.9, −0.5, −0.1) and $(\sigma_x^2, \sigma_y^2) = (3, 1)$.

σ_u^2	σ_v^2	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
1	1	−0.9	−0.9	28.418	428.396	444.16
			−0.5	30.628	210.615	229.724
			0	33.238	121.391	150.303
			0.5	38.907	84.119	117.943
			0.9	43.486	68.835	107.93
		−0.5	−0.9	32.13	149.599	176.481
			−0.5	34.808	108.331	138.534
			0	38.764	78.159	114.64
			0.5	46.792	60.538	103.621
			0.9	53.45	52.358	102.393
		−0.1	−0.9	37.295	87.499	122.434
			−0.5	41.133	70.966	109.859
			0	46.782	55.785	102.445
			0.5	59.475	46.476	103.509
			0.9	70.698	41.498	109.86
1	3	−0.9	−0.9	22.39	418.816	585.357
			−0.5	24.87	149.311	213.131
			0	28.726	77.649	130.441
			0.5	35.471	52.585	106.642
			0.9	41.813	42.766	103.137
		−0.5	−0.9	24.708	147.966	213.131
			−0.5	27.248	86.712	141.435
			0	32.67	57.079	110.202
			0.5	40.708	41.52	102.29
			0.9	51.034	35.296	106.357
		−0.1	−0.9	27.618	85.767	139.872
			−0.5	31.262	61.45	114.27
			0	37.968	43.863	102.244
			0.5	50.027	33.975	105.973
			0.9	67.713	29.566	121.757
3	1	−0.9	−0.9	39.756	253.174	266.509
			−0.5	44.534	163.2	165.231
			0	51.538	110.705	120.199
			0.5	60.743	83.001	104.501
			0.9	68.246	69.501	102.342
		−0.5	−0.9	43.408	162.664	166.992
			−0.5	48.044	120.01	128.846
			0	57.771	89.487	107.131
			0.5	68.192	70.316	101.622
			0.9	80.771	60.115	105.023
		−0.1	−0.9	48.247	116.649	127.512
			−0.5	54.137	92.68	110.308
			0	65.983	73.956	101.612
			0.5	81.349	59.866	104.429
			0.9	100.183	52.231	115.758
3	3	−0.9	−0.9	32.119	424.396	439.174
			−0.5	36.527	164.185	175.382
			0	43.967	87.443	114.861
			0.5	59.313	60.213	102.376
			0.9	76.429	48.622	108.651
		−0.5	−0.9	34.655	219.372	227.247
			−0.5	39.626	119.096	137.412
			0	48.763	72.561	105.989
			0.5	68.592	52.484	104.039
			0.9	92.567	43.386	118.952
		−0.1	−0.9	37.551	144.186	159.29
			−0.5	43.69	91.899	117.102
			0	54.88	61.083	102.049
			0.5	82.449	46.208	111.054
			0.9	119.867	38.959	140.242

Table 9: PRE of different estimators for $\rho_{xy} = (0, 0.5, 0.9)$ and $(\sigma_x^2, \sigma_y^2) = (3, 3)$.

σ_u^2	σ_v^2	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
			−0.9	59.849	83.34	106.043
			−0.5	64.918	75.993	103.196
		0	0	71.353	68.242	102.438
			0.5	80.445	61.255	104.704
			0.9	89.372	57.082	108.916
			−0.9	81.508	60.456	105.162
			−0.5	91.274	55.957	109.572
1	1	0.5	0	103.873	50.433	118.507
			0.5	136.645	46.544	142.805
			0.9	165.052	43.954	169.293
			−0.9	117.588	48.907	128.015
			−0.5	139.356	45.7	145.451
		0.9	0	187.757	41.651	191.269
			0.5	282.56	39.085	304.15
			0.9	430.785	37.552	585.295
			−0.9	46.565	90.846	114.505
			−0.5	51.593	75.954	105.878
		0	0	59.988	61.879	101.74
			0.5	73.169	52.105	104.874
			0.9	88.475	46.776	113.197
			−0.9	59.364	60.252	102.376
			−0.5	68.634	52.507	104.039
1	3	0.5	0	87.064	44.869	113.95
			0.5	119.547	39.658	137.707
			0.9	164.51	36.574	175.56
			−0.9	76.659	48.767	108.651
			−0.5	92.646	43.411	118.952
		0.9	0	129.798	37.959	148.545
			0.5	217.515	34.097	226.917
			0.9	429.313	32.051	444.726
			−0.9	61.385	108.297	115.557
			−0.5	67.43	93.794	106.249
		0	0	78.269	79.02	101.795
			0.5	92.006	69.031	105.172
			0.9	106.685	62.257	114.15
			−0.9	77.004	77.987	102.435
			−0.5	87.631	69.818	103.836
3	1	0.5	0	106.831	61.321	113.68
			0.5	135.514	54.437	137.984
			0.9	169.715	50.184	177.384
			−0.9	95.299	64.88	107.974
			−0.5	111.461	58.941	117.754
		0.9	0	144.258	52.87	146.125
			0.5	202.866	47.692	222.468
			0.9	292.052	44.462	440.319
			−0.9	61.385	108.297	115.557
			−0.5	67.43	93.794	106.249
		0	0	78.269	79.02	101.795
			0.5	92.006	69.031	105.172
			0.9	106.685	62.257	114.15
			−0.9	77.004	77.987	102.435
			−0.5	87.631	69.818	103.836
3	3	0.5	0	106.831	61.321	113.68
			0.5	135.514	54.437	137.984
			0.9	169.715	50.184	177.384
			−0.9	95.299	64.88	107.974
			−0.5	111.461	58.941	117.754
		0.9	0	144.258	52.87	146.125
			0.5	202.866	47.692	222.468
			0.9	292.052	44.462	440.319

Table 10: PRE of different estimators for ρ_{xy} = (−0.9, −0.5, −0.1) and (σ_x^2, σ_y^2) =(3, 3).

σ_u^2	σ_v^2	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
			−0.9	37.93	422.739	584.988
			−0.5	39.999	277.703	304.295
		−0.9	0	42.4	189.437	193.128
			0.5	46.467	139.956	145.718
			0.9	49.364	118.04	127.958
			−0.9	44.216	165.111	169.293
			−0.5	46.771	136.834	142.804
1	1	−0.5	0	50.301	106.192	119.853
			0.5	56.317	91.666	109.586
			0.9	60.776	81.841	105.162
			−0.9	56	94.169	110.986
			−0.5	60.385	84.913	105.845
		−0.1	0	65.818	75.499	102.738
			0.5	73.341	67.169	102.505
			0.9	80.611	62.223	104.573
			−0.9	32.381	422.386	439.174
			−0.5	34.881	219.679	227.247
		−0.9	0	38.628	132.619	149.173
			0.5	44.043	94.268	119.097
			0.9	49.303	77.245	108.553
			−0.9	36.81	164.791	175.382
			−0.5	39.92	119.693	137.412
1	3	−0.5	0	45.246	87.263	113.682
			0.5	52.975	68.99	103.928
			0.9	60.704	59.735	102.345
			−0.9	42.433	101.845	123.455
			−0.5	46.714	82.959	110.578
		−0.1	0	53.887	63.794	102.389
			0.5	64.559	54.621	102.494
			0.9	76.885	48.56	107.705
			−0.9	44.822	288.771	442.798
			−0.5	48.157	203.668	226.799
		−0.9	0	53.387	145.894	148.082
			0.5	59.641	112.762	118.115
			0.9	65.274	95.577	107.896
			−0.9	50.477	169.278	177.213
			−0.5	54.784	135.303	137.752
3	1	−0.5	0	61.686	106.758	113.46
			0.5	70.135	87.718	103.719
			0.9	78.273	77.208	102.409
			−0.9	57.004	119.035	124.022
			−0.5	62.314	101.386	110.742
		−0.1	0	72.361	83.436	102.37
			0.5	83.509	72.501	102.609
			0.9	95.761	64.902	108.174
			−0.9	37.544	431.646	585.831
			−0.5	41.986	207.955	214.025
		−0.9	0	47.924	120.526	130.187
			0.5	59.766	84.075	106.18
			0.9	68.203	69.567	103.081
			−0.9	41.698	208.325	214.025
			−0.5	46.267	137.042	142.851
3	3	−0.5	0	54.972	92.01	109.988
			0.5	68.606	69.762	102.175
			0.9	83.106	58.901	106.159
			−0.9	46.564	136.799	142.644
			−0.5	53.033	101.867	115.329
		−0.1	0	62.699	72.377	102.375
			0.5	84.574	59.35	105.812
			0.9	106.822	50.875	119.928

Table 11: PRE of different estimators for ρ_{xy} = (0, 0.5, 0.9) and % of ME = (10, 15, 20, 25).

% of ME	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
		−0.9	38.745	293.546	319.998
		−0.5	49.88	120.178	129.272
	0	0	71.855	67.435	103.28
		0.5	119.322	50.029	128.609
		0.9	282.814	39.836	312.147
		−0.9	40.468	243.835	256.165
		−0.5	51.336	112.421	123.283
10	0.5	0	74.979	64.871	103.815
		0.5	128.05	48.626	135.812
		0.9	360.663	38.179	435.539
		−0.9	41.294	217.876	224.644
		−0.5	52.553	107.269	119.517
	0.9	0	77.596	63.13	104.526
		0.5	135.591	47.674	142.368
		0.9	430.217	37.545	587.841
		−0.9	38.271	323.154	364.692
		−0.5	49.525	122.535	131.343
	0	0	71.991	67.343	103.469
		0.5	119.295	47.289	131.219
		0.9	310.869	39.35	357.263
		−0.9	38.893	284.976	308.045
		−0.5	50.515	116.92	126.841
15	0.5	0	74.132	65.587	103.803
		0.5	125.573	46.376	136.408
		0.9	354.899	38.748	438.775
		−0.9	40.141	256.905	273.292
		−0.5	51.322	113.053	123.845
	0.9	0	75.873	64.362	104.198
		0.5	130.803	45.745	140.86
		0.9	430.157	37.541	588.554
		−0.9	38.029	341.12	394.11
		−0.5	49.342	123.8	132.479
	0	0	72.062	67.295	103.574
		0.5	123.325	49.418	132.099
		0.9	327.827	39.102	387.211
		−0.9	38.504	307.901	341.416
		−0.5	50.098	119.407	128.883
20	0.5	0	73.698	65.967	103.814
		0.5	128.049	48.719	136.063
		0.9	363.939	38.657	458.246
		−0.9	38.889	286.74	310.84
		−0.5	50.703	116.32	126.417
	0.9	0	75.004	65.023	104.078
		0.5	131.879	48.223	139.364
		0.9	396.641	38.346	534.025
		−0.9	37.881	353.184	414.936
		−0.5	46.852	121.963	133.609
	0	0	72.106	67.266	103.64
		0.5	124.201	49.291	132.884
		0.9	339.184	38.951	408.534
		−0.9	38.27	323.901	366.298
		−0.5	47.447	118.274	130.579
25	0.5	0	73.434	66.202	103.827
		0.5	128.049	48.738	136.12
		0.9	369.717	38.602	471.174
		−0.9	38.578	304.776	337.069
		−0.5	47.919	115.654	128.459
	0.9	0	74.481	65.436	104.022
		0.5	131.124	48.34	138.764
		0.9	396.478	38.354	534.234

Table 12: PRE of different estimators for ρ_{xy} = (−0.9, −0.5, −0.1) and % of ME = (10, 15, 20, 25).

% of ME	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
10	−0.9	−0.9	37.864	423.685	587.049
		−0.5	47.737	135.635	142.368
		0	67.34	72.218	103.362
		0.5	107.338	52.619	119.517
		0.9	218.15	41.087	224.902
	−0.5	−0.9	38.355	358.516	435.04
		−0.5	48.696	128.126	135.809
		0	69.332	69.966	103.17
		0.5	112.508	51.409	123.28
		0.9	244.776	40.112	256.428
	−0.1	−0.9	39.169	307.81	348.352
		−0.5	49.717	121.391	130.241
		0	71.483	67.846	103.23
		0.5	118.221	50.27	127.686
		0.9	277.967	39.472	301.164
15	−0.9	−0.9	37.868	423.474	587.655
		−0.5	48.097	133.173	140.364
		0	68.869	70.59	103.377
		0.5	110.322	48.999	124.117
		0.9	257.611	39.93	273.658
	−0.5	−0.9	37.77	366.584	439.469
		−0.5	48.714	128.08	135.971
		0	70.228	69.049	103.346
		0.5	114.226	48.183	127.144
		0.9	279.475	39.947	307.745
	−0.1	−0.9	38.235	328.919	374.689
		−0.5	49.45	123.431	132.047
		0	71.777	67.662	103.434
		0.5	118.523	47.477	130.539
		0.9	305.138	39.489	348.255
20	−0.9	−0.9	37.388	412.649	535.428
		−0.5	48.287	131.926	139.364
		0	69.694	69.77	103.444
		0.5	116.383	50.768	126.417
		0.9	280.839	39.987	310.51
	−0.5	−0.9	37.647	377.033	459.082
		−0.5	48.724	128.057	136.061
		0	70.705	68.579	103.46
		0.5	119.415	50.103	128.882
		0.9	300.725	39.545	340.993
	−0.1	−0.9	38.023	345.737	403.082
		−0.5	49.312	124.52	133.031
		0	71.932	67.567	103.546
		0.5	122.762	49.593	131.562
		0.9	322.745	39.229	378.982
25	−0.9	−0.9	37.387	412.625	535.648
		−0.5	46.051	128.734	139.257
		0	70.21	69.277	103.505
		0.5	118.48	50.396	128.117
		0.9	297.419	39.679	336.643
	−0.5	−0.9	38.596	369.748	471.238
		−0.5	46.369	125.509	136.565
		0	71.001	68.294	103.538
		0.5	120.978	49.837	130.205
		0.9	315.457	39.302	365.773
	−0.1	−0.9	38.927	345.537	422.214
		−0.5	46.848	122.588	134.071
		0	72.027	67.508	103.616
		0.5	123.76	49.453	132.441
		0.9	334.589	39.071	401.038

Table 13: PRE of different estimators for $\rho_{xy} = (0, 0.5, 0.9)$ and % of ME = (10, 15, 20, 25).

% of ME	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
		−0.9	56.243	194.616	323.514
		−0.5	68.012	122.935	127.062
	0	0	89.011	84.95	103.236
		0.5	127.315	65.48	131.753
		0.9	193.364	56.504	322.323
		−0.9	57.379	181.592	258.083
		−0.5	69.567	118.242	121.667
10	0.5	0	91.721	82.663	103.719
		0.5	132.985	64.101	139.347
		0.9	207.923	55.321	430.803
		−0.9	58.275	172.688	225.162
		−0.5	70.832	114.677	117.999
	0.9	0	93.984	80.864	104.41
		0.5	137.904	62.991	146.647
		0.9	220.346	54.456	582.782
		−0.9	55.594	202.43	374.73
		−0.5	67.616	124.293	128.956
	0	0	89.112	84.843	103.421
		0.5	129.149	64.981	134.284
		0.9	200.943	55.865	372.753
		−0.9	56.348	192.578	309.404
		−0.5	68.659	120.966	124.912
15	0.5	0	90.955	83.262	103.719
		0.5	133.102	64.039	139.696
		0.9	211.565	55.059	468.678
		−0.9	56.934	185.62	274.028
		−0.5	69.493	118.386	122.002
	0.9	0	92.465	81.999	104.099
		0.5	136.448	63.271	144.601
		0.9	220.291	54.462	583.267
		−0.9	55.26	206.784	409.528
		−0.5	67.411	125.013	129.995
	0	0	89.165	84.788	103.522
		0.5	124.729	67.494	129.644
		0.9	205.159	55.535	406.934
		−0.9	55.822	198.887	347.094
		−0.5	68.194	122.437	126.769
20	0.5	0	90.56	83.578	103.736
		0.5	127.444	66.734	133.244
		0.9	213.526	54.922	491.586
		−0.9	56.255	193.206	311.505
		−0.5	68.815	120.419	124.378
	0.9	0	91.692	82.604	103.987
		0.5	129.705	66.11	136.404
		0.9	220.262	54.465	583.553
		−0.9	55.656	204.554	408.275
		−0.5	67.286	125.459	130.65
	0	0	89.198	84.753	103.587
		0.5	130.737	64.567	136.568
		0.9	210.55	54.636	415.141
		−0.9	56.084	199.017	362.978
		−0.5	67.912	123.359	127.97
25	0.5	0	90.319	83.774	103.752
		0.5	133.201	63.988	140.003
		0.9	217.004	54.221	478.067
		−0.9	55.845	198.193	341.104
		−0.5	68.405	121.702	125.946
	0.9	0	91.224	82.98	103.937
		0.5	135.244	63.509	142.964
		0.9	220.244	54.466	583.739

Table 14: PRE of different estimators for ρ_{xy} = (−0.9, −0.5, −0.1) and % of ME = (10, 15, 20, 25).

% of ME	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
10	−0.9	−0.9	54.243	221.919	588.684
		−0.5	65.501	131.993	139.395
		0	84.711	89.229	103.381
		0.5	118.613	68.02	121.78
		0.9	171.558	58.603	224.067
	−0.5	−0.9	55.091	209.343	433.667
		−0.5	66.567	127.818	133.37
		0	86.536	87.267	103.16
		0.5	122.285	66.855	125.739
		0.9	180.417	57.68	257.104
	−0.1	−0.9	56.008	197.481	341.312
		−0.5	67.712	123.893	128.241
		0	88.495	85.408	103.189
		0.5	126.259	65.752	130.433
		0.9	190.591	56.745	306.371
15	−0.9	−0.9	54.238	221.943	589.479
		−0.5	65.897	130.497	137.434
		0	86.113	87.725	103.372
		0.5	122.874	66.679	126.571
		0.9	184.185	57.263	272.375
	−0.5	−0.9	54.817	213.145	472.4
		−0.5	66.631	127.672	133.409
		0	87.397	86.415	103.318
		0.5	125.555	65.905	129.73
		0.9	191.125	56.652	307.798
	−0.1	−0.9	55.437	204.55	391.476
		−0.5	67.413	124.962	129.809
		0	88.757	85.156	103.385
		0.5	128.402	65.165	133.307
		0.9	198.871	56.027	357.43
20	−0.9	−0.9	54.235	221.956	589.928
		−0.5	66.105	129.73	136.458
		0	86.863	86.961	103.426
		0.5	120.322	68.848	124.282
		0.9	191.578	56.585	309.379
	−0.5	−0.9	54.674	215.194	495.871
		−0.5	66.665	127.596	133.436
		0	87.853	85.977	103.424
		0.5	122.219	68.234	126.514
		0.9	197.264	56.128	344.96
	−0.1	−0.9	55.142	208.462	424.897
		−0.5	67.258	125.527	130.662
		0	88.895	85.025	103.494
		0.5	124.21	67.642	128.981
		0.9	203.505	55.658	392.678
25	−0.9	−0.9	54.234	221.964	590.216
		−0.5	66.233	129.264	135.873
		0	87.33	86.498	103.479
		0.5	126.71	65.589	131.311
		0.9	196.432	56.175	338.576
	−0.5	−0.9	55.241	210.223	465.137
		−0.5	66.686	127.549	133.455
		0	88.136	85.71	103.497
		0.5	128.449	65.125	133.521
		0.9	204.723	55.032	369.22
	−0.1	−0.9	55.571	205.678	418.506
		−0.5	67.163	125.876	131.199
		0	88.979	84.945	103.563
		0.5	130.265	64.679	135.927
		0.9	209.339	54.715	404.772

Table 15: PRE of different estimators for ρ_{xy} = (0, 0.5, 0.9) and % of ME = (10, 15, 20, 25).

% of ME	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
		−0.9	23.176	228.538	321.951
		−0.5	29.236	73.385	127.292
	0	0	44.746	41.791	103.28
		0.5	82.695	30.14	131.934
		0.9	229.227	23.169	322.925
		−0.9	23.91	181.587	257.024
		−0.5	30.111	68.271	121.842
10	0.5	0	46.764	40.134	103.798
		0.5	89.745	29.284	139.557
		0.9	307.748	22.685	431.589
		−0.9	24.516	157.018	224.255
		−0.5	30.928	64.751	118.134
	0.9	0	48.628	38.972	104.518
		0.5	96.496	28.69	146.859
		0.9	411.494	22.357	584.229
		−0.9	23.098	271.462	374.598
		−0.5	28.984	74.873	129.217
	0	0	44.845	41.736	103.469
		0.5	74.452	29.048	128.746
		0.9	265.741	22.89	373.526
		−0.9	23.596	224.672	309.32
		−0.5	29.578	71.171	125.129
15	0.5	0	46.231	40.604	103.793
		0.5	78.568	28.464	133.456
		0.9	333.597	22.572	469.695
		−0.9	24.003	198.505	273.918
		−0.5	30.124	68.525	122.185
	0.9	0	47.479	39.791	104.194
		0.5	82.323	28.057	137.71
		0.9	411.037	22.355	584.857
		−0.9	22.956	296.607	409.377
		−0.5	28.854	75.674	130.272
	0	0	44.896	41.708	103.574
		0.5	86.257	29.754	135.894
		0.9	289.851	22.746	407.844
		−0.9	23.336	252.872	346.981
		−0.5	29.308	72.785	127.011
20	0.5	0	45.959	40.852	103.807
		0.5	90.159	29.328	140.112
		0.9	348.933	22.514	492.758
		−0.9	23.644	227.197	311.375
		−0.5	29.72	70.677	124.591
	0.9	0	46.902	40.233	104.076
		0.5	93.633	29.027	143.808
		0.9	410.798	22.354	585.219
		−0.9	22.638	287.574	409.233
		−0.5	28.775	76.176	130.939
	0	0	44.928	41.691	103.64
		0.5	87.045	29.674	136.787
		0.9	283.798	22.661	403.639
		−0.9	22.865	256.675	363.718
		−0.5	29.144	73.813	128.229
25	0.5	0	45.793	41.007	103.822
		0.5	90.246	29.337	140.234
		0.9	323.996	22.449	463.775
		−0.9	23.428	249.387	340.962
		−0.5	29.478	72.067	126.18
	0.9	0	46.554	40.508	104.02
		0.5	93.054	29.098	143.198
		0.9	423.156	22.558	590.205

Table 16: PRE of different estimators for ρ_{xy} = (−0.9, −0.5, −0.1) and % of ME = (10, 15, 20, 25).

% of ME	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
		−0.9	22.279	407.343	584.171
		−0.5	27.828	84.212	139.741
	−0.9	0	41.637	44.958	103.364
		0.5	72.854	31.716	121.958
		0.9	156.219	24.426	224.42
		−0.9	22.633	305.588	430.986
		−0.5	28.462	79.12	133.655
10	−0.5	0	42.992	43.543	103.17
		0.5	76.938	31.031	125.91
		0.9	181.205	23.845	257.562
		−0.9	23.05	241.156	339.578
		−0.5	29.079	74.488	128.482
	−0.1	0	44.382	42.14	103.226
		0.5	81.461	30.32	130.609
		0.9	217.537	23.292	306.953
		−0.9	22.477	419.243	589.367
		−0.5	28.012	82.254	137.778
	−0.9	0	42.644	43.84	103.377
		0.5	68.307	30.105	122.059
		0.9	193.35	23.712	272.899
		−0.9	22.724	340.28	472.24
		−0.5	28.452	78.857	133.71
15	−0.5	0	43.612	42.911	103.342
		0.5	70.92	29.649	124.791
		0.9	219.268	23.333	308.426
		−0.9	23.011	283.357	391.33
		−0.5	28.876	75.652	130.077
	−0.1	0	44.591	41.972	103.429
		0.5	73.709	29.17	127.896
		0.9	254.815	22.97	358.178
		−0.9	22.48	419.258	589.814
		−0.5	28.108	81.266	136.8
	−0.9	0	43.189	43.278	103.443
		0.5	80.315	30.522	129.633
		0.9	220.878	23.355	310.042
		−0.9	22.669	356.373	495.714
		−0.5	28.446	78.721	133.745
20	−0.5	0	43.943	42.588	103.456
		0.5	82.851	30.192	132.235
		0.9	246.338	23.074	345.726
		−0.9	22.889	307.303	424.736
		−0.5	28.772	76.275	130.946
	−0.1	0	44.701	41.885	103.541
		0.5	85.543	29.843	135.117
		0.9	279.859	22.806	393.56
		−0.9	22.482	419.267	590.101
		−0.5	28.168	80.67	136.215
	−0.9	0	43.53	42.94	103.504
		0.5	82.082	30.283	131.528
		0.9	247.851	23.348	341.151
		−0.9	22.435	325.42	466.416
		−0.5	28.443	78.637	133.769
25	−0.5	0	44.149	42.392	103.534
		0.5	84.212	30.021	133.735
		0.9	253.64	22.881	359.624
		−0.9	22.597	294.494	419.518
		−0.5	28.708	76.663	131.492
	−0.1	0	44.768	41.832	103.612
		0.5	86.454	29.744	136.143
		0.9	277.075	22.706	393.701

Table 17: PRE of different estimators for ρ_{xy} = (0, 0.5, 0.9) and % of ME = (10, 15, 20, 25).

% of ME	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
10	0	−0.9	39.047	295.11	324.725
		−0.5	47.553	117.518	129.595
		0	71.794	67.38	103.28
		0.5	119.258	49.982	128.609
		0.9	293.336	39.079	321.982
	0.5	−0.9	40.244	244.438	256.428
		−0.5	49.225	109.946	123.524
		0	75.235	65.109	103.818
		0.5	128.256	48.83	135.809
		0.9	360.038	38.224	435.573
	0.9	−0.9	41.027	218.264	224.902
		−0.5	50.201	104.474	119.722
		0	77.601	63.135	104.526
		0.5	135.592	47.678	142.368
		0.9	430.066	37.551	587.841
15	0	−0.9	38.232	323.36	364.692
		−0.5	49.478	122.473	131.343
		0	71.93	67.288	103.469
		0.5	119.227	47.243	131.219
		0.9	311.054	39.311	357.263
	0.5	−0.9	39.037	284.167	307.978
		−0.5	50.68	117.103	126.84
		0	74.333	65.774	103.805
		0.5	125.751	46.539	136.392
		0.9	353.236	38.888	438.851
	0.9	−0.9	40.147	256.869	273.292
		−0.5	51.327	113.056	123.845
		0	75.878	64.367	104.198
		0.5	130.803	45.75	140.86
		0.9	430.007	37.547	588.554
20	0	−0.9	37.99	341.374	394.11
		−0.5	49.295	123.739	132.479
		0	72.001	67.241	103.574
		0.5	120.623	47.008	132.487
		0.9	328.057	39.062	387.211
	0.5	−0.9	38.627	306.984	341.35
		−0.5	50.239	119.561	128.882
		0	73.871	66.127	103.815
		0.5	125.69	46.515	136.498
		0.9	362.387	38.778	458.317
	0.9	−0.9	38.893	286.7	310.84
		−0.5	50.708	116.322	126.417
		0	75.01	65.028	104.078
		0.5	129.482	45.901	139.858
		0.9	396.549	38.35	534.025
25	0	−0.9	37.842	353.47	414.936
		−0.5	49.184	124.528	133.197
		0	72.045	67.212	103.64
		0.5	121.497	46.865	133.29
		0.9	339.446	38.911	408.534
	0.5	−0.9	38.379	322.923	366.234
		−0.5	49.972	121.122	130.205
		0	73.59	66.346	103.828
		0.5	125.652	46.5	136.565
		0.9	368.246	38.71	471.238
	0.9	−0.9	38.582	304.73	337.069
		−0.5	50.336	118.421	128.117
		0	74.487	65.441	104.022
		0.5	128.683	45.994	139.257
		0.9	396.387	38.358	534.234

Table 18: PRE of different estimators for $\rho_{xy} = (-0.9, -0.5, -0.1)$ and % of ME = (10, 15, 20, 25).

% of ME	ρ_{xy}	ρ_{uv}	t_r	t_p	$T_i, i = 1, 2, 3$
		−0.9	37.384	432.801	587.896
		−0.5	45.38	133.293	142.855
	−0.9	0	67.151	72.049	103.363
		0.5	107.152	52.462	119.516
		0.9	218.331	40.851	224.883
		−0.9	38.037	362.049	435.573
		−0.5	46.398	125.644	136.201
10	−0.5	0	69.255	69.895	103.17
		0.5	112.427	51.347	123.28
		0.9	244.823	40.051	256.428
		−0.9	38.941	309.35	348.599
		−0.5	47.47	118.846	130.579
	−0.1	0	71.522	67.881	103.23
		0.5	118.245	50.303	127.686
		0.9	277.703	39.507	301.164
		−0.9	37.648	427.499	587.59
		−0.5	47.927	132.998	140.364
	−0.9	0	68.652	70.396	103.378
		0.5	110.107	48.828	124.112
		0.9	258.224	39.7	273.644
		−0.9	37.72	366.993	439.469
		−0.5	48.655	128.004	135.971
15	−0.5	0	70.15	68.978	103.346
		0.5	114.14	48.123	127.144
		0.9	279.62	39.895	307.745
		−0.9	38.263	328.532	374.689
		−0.5	49.483	123.452	132.047
	−0.1	0	71.817	67.697	103.434
		0.5	118.545	47.51	130.539
		0.9	304.831	39.518	348.255
		−0.9	37.238	415.007	535.333
		−0.5	48.105	131.738	139.363
	−0.9	0	69.462	69.563	103.444
		0.5	113.376	48.183	126.727
		0.9	281.513	39.829	310.525
		−0.9	37.598	377.481	459.082
		−0.5	48.664	127.98	136.061
20	−0.5	0	70.627	68.509	103.46
		0.5	116.61	47.662	129.224
		0.9	300.929	39.494	340.993
		−0.9	38.052	345.291	403.082
		−0.5	49.345	124.54	133.031
	−0.1	0	71.972	67.602	103.546
		0.5	120.137	47.254	131.949
		0.9	322.383	39.258	378.982
		−0.9	37.231	415.094	535.571
		−0.5	48.215	130.976	138.764
	−0.9	0	69.968	69.061	103.505
		0.5	115.484	47.795	128.455
		0.9	298.303	39.515	336.658
		−0.9	37.523	384.23	472.108
		−0.5	48.67	127.966	136.119
25	−0.5	0	70.923	68.224	103.538
		0.5	118.183	47.383	130.571
		0.9	315.706	39.251	365.773
		−0.9	37.923	356.455	423.007
		−0.5	49.261	125.217	133.651
	−0.1	0	72.067	67.543	103.616
		0.5	121.135	47.099	132.846
		0.9	334.188	39.099	401.038

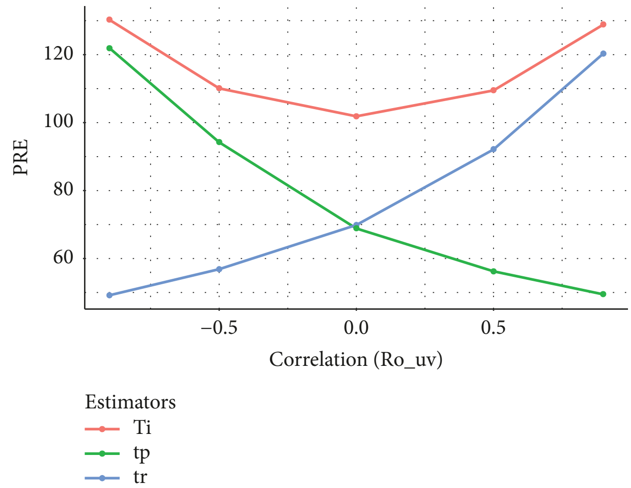

Figure 1: PRE of Table 3 when $(\sigma_u^2, \sigma_v^2) = (1, 1)$, $\rho_{xy} = 0$, and $\rho_{uv} = (-0.9, -0.5, 0, 0.5, 0.9)$.

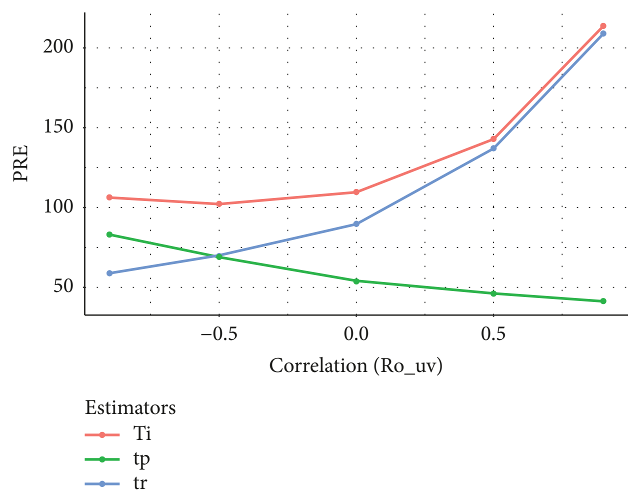

Figure 2: PRE of Table 3 when $(\sigma_u^2, \sigma_v^2) = (1, 1)$, $\rho_{xy} = 0.5$, and $\rho_{uv} = (-0.9, -0.5, 0, 0.5, 0.9)$.

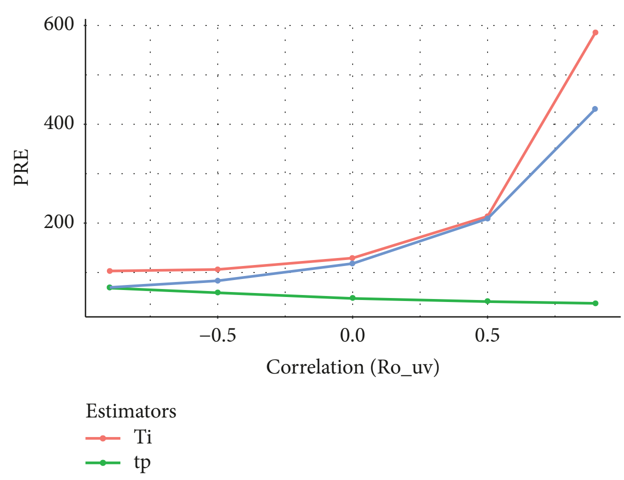

Figure 3: PRE of Table 3 when $(\sigma_u^2, \sigma_v^2) = (1, 1)$, $\rho_{xy} = 0.9$, and $\rho_{uv} = (-0.9, -0.5, 0, 0.5, 0.9)$.

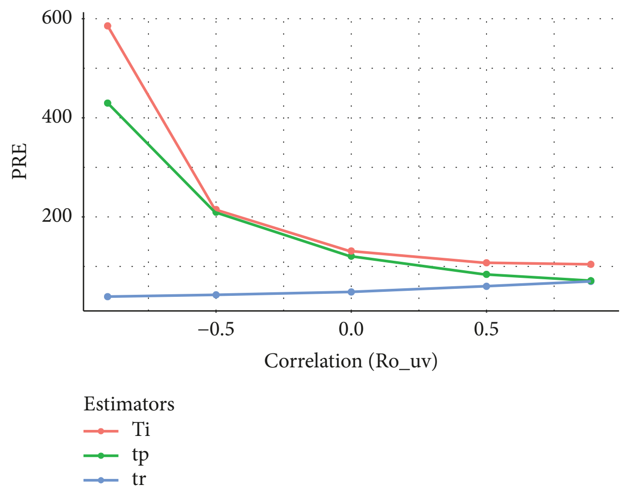

FIGURE 4: PRE of Table 4 when $(\sigma_u^2, \sigma_v^2) = (1,\ 1)$, $\rho_{xy} = -0.9$, and $\rho_{uv} = (-0.9, -0.5, 0, 0.5, 0.9)$.

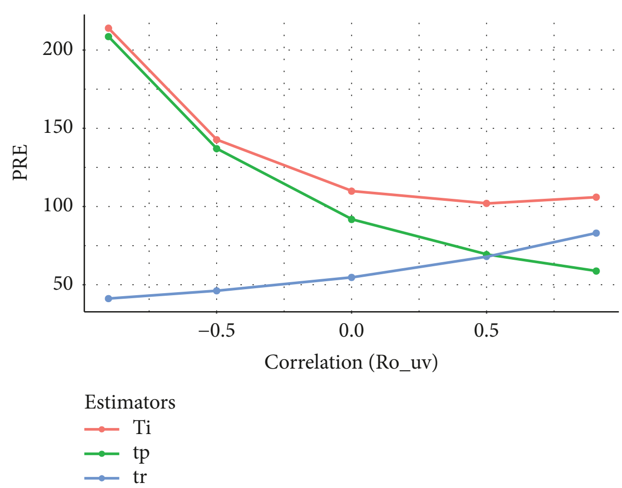

FIGURE 5: PRE of Table 4 when $(\sigma_u^2, \sigma_v^2) = (1,\ 1)$, $\rho_{xy} = -0.5$, and $\rho_{uv} = (-0.9, -0.5, 0, 0.5, 0.9)$.

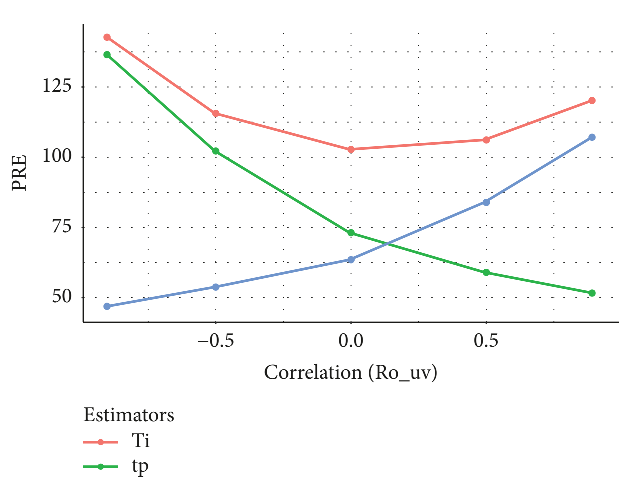

FIGURE 6: PRE of Table 4 when $(\sigma_u^2, \sigma_v^2) = (1,\ 1)$, $\rho_{xy} = -0.1$, and $\rho_{uv} = (-0.9, -0.5, 0, 0.5, 0.9)$.

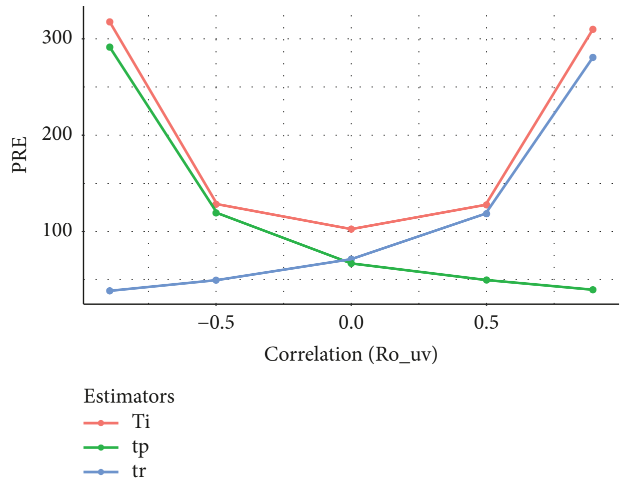

Figure 7: PRE of Table 11 when ME = 10%, $\rho_{xy} = 0$, and $\rho_{uv} = (-0.9, -0.5, 0, 0.5, 0.9)$.

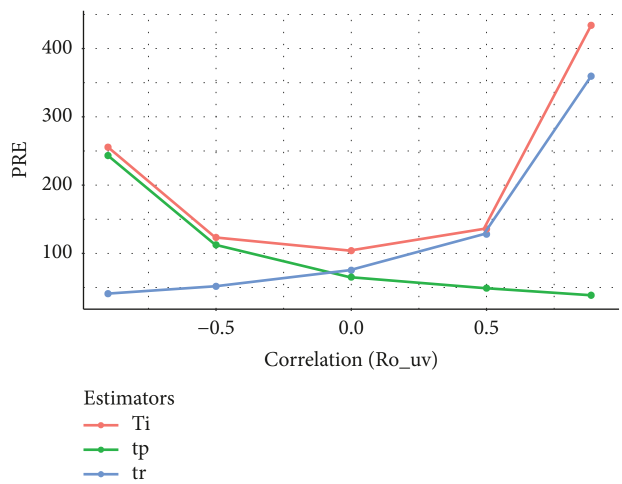

Figure 8: PRE of Table 11 when ME = 10%, $\rho_{xy} = 0.5$, and $\rho_{uv} = (-0.9, -0.5, 0, 0.5, 0.9)$.

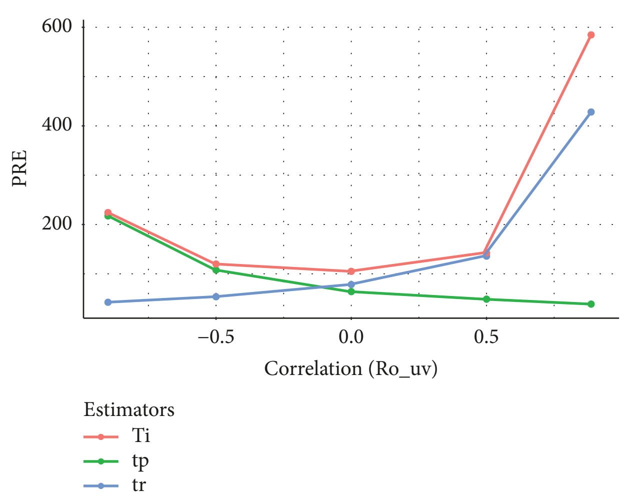

Figure 9: PRE of Table 11 when ME = 10%, $\rho_{xy} = 0.9$, and $\rho_{uv} = (-0.9, -0.5, 0, 0.5, 0.9)$.

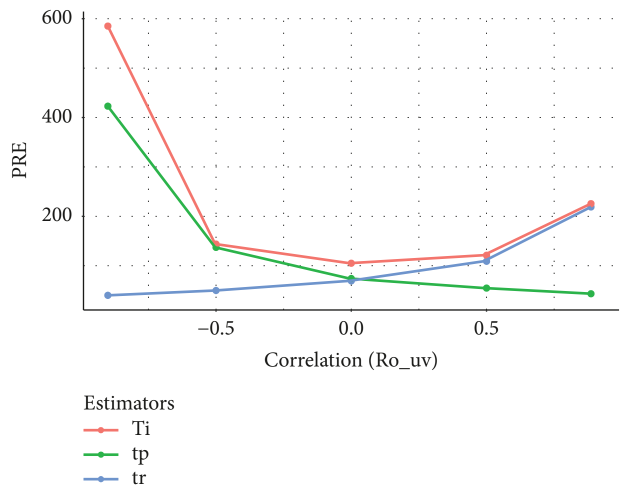

Figure 10: PRE of Table 12 when ME = 10%, ρ_{xy} = −0.9, and ρ_{uv} = (−0.9, −0.5, 0, 0.5, 0.9).

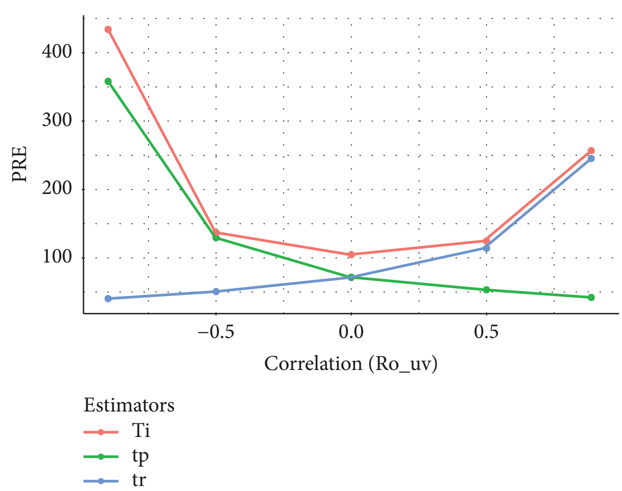

Figure 11: PRE of Table 12 when ME = 10%, ρ_{xy} = −0.5, and ρ_{uv} = (−0.9, −0.5, 0, 0.5, 0.9).

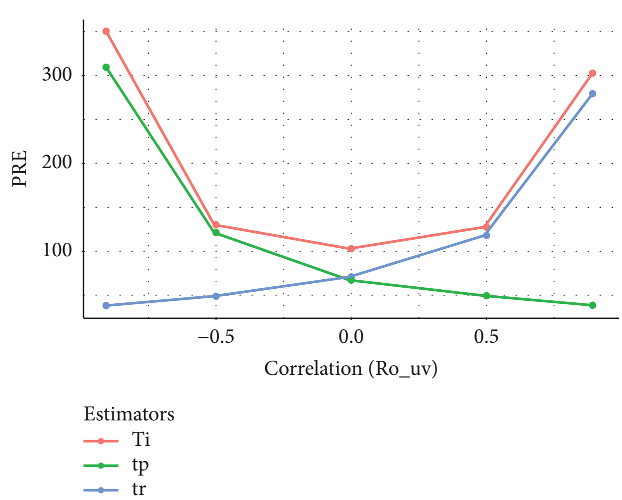

Figure 12: PRE of Table 12 when ME = 10%, ρ_{xy} = −0.1, and ρ_{uv} = (−0.9, −0.5, 0, 0.5, 0.9).

errors are negative, and it gets less as ρ_{uv} values increase incrementally from −0.9 to +0.9. This tendency can easily be observed from Figures 4–6.

(iv) The considerable influence of the correlated measurement errors over the percent relative efficiency of the suggested estimators can be seen by comparing the percent relative efficiency of the estimators at $\rho_{uv} = 0$ and $\rho_{uv} = \pm 0.9$. This tendency can easily be observed from Figures 4–6.

(v) For different combinations of σ_u^2 and σ_v^2, the same trend in the percent relative efficiency values of the proffered estimators may be seen.

(4) The same trend in the fluctuation of percent relative efficiency that is shown in Table 4 can also be seen in Tables 6, 8, and 10, consisting of different values of σ_x^2, σ_y^2, and negative correlation coefficient ρ_{xy}. The graphs for the same can be provided, if required.

(5) From Table 11, concerning to the values of $\sigma_y^2 = 1$, $\sigma_x^2 = 1$, the positive correlation coefficient $\rho_{xy} = 0, 0.5, 0.9$, and for the level of ME = 10%, we can observe that

(i) As ρ_{xy} fluctuates from 0 to 0.9, the percent relative efficiency of the traditional ratio estimator t_r grows. In addition, when the value of ρ_{uv} rises from −0.9 to +0.9, the percent relative efficiency rises as well, which can also be observed from Figures 7–9.

(ii) As the value of ρ_{xy} changes between 0 and 0.9, the percent relative efficiency of the traditional product estimator t_p declines. The percent relative efficiency declines as well when the value of ρ_{uv} increases incrementally from −0.9 to +0.9, which can also be observed from Figures 7–9.

(iii) The percent relative efficiency of the proffered estimators $T_i, i = 1, 2, 3$ rises as ρ_{xy}'s value changes from 0 to 0.9. The direction and value of ρ_{uv} also affect the size and rate of this decline. The percent relative efficiency is greater for correlated measurement errors that are negative, and it reduces as ρ_{uv} varies from −0.9 to 0 and rises as it fluctuates from 0 to 0.9. This effect can easily be observed from Figures 7–9.

(iv) The considerable influence of the correlated measurement errors over the percent relative efficiency of the recommended estimators can be shown by comparing the percent relative efficiency of the suggested estimators at $\rho_{uv} = 0$ and $\rho_{uv} = \pm 0.9$. This pattern can easily be observed from Figures 7–9.

(v) For additional levels of ME, namely, at 15%, 20%, and 25%, an analogous pattern in the percent relative efficiency of the proffered estimators may be seen.

(vi) As the amount of ME rises, the percent relative efficiency of several estimators drops for $\rho_{uv} = 0$.

(vii) At the smallest amount of ME, the percent relative efficiency of the ratio estimator t_r is larger, and it drops as the amount of ME changes from 10% to 25% over sequential increments of 5%. However, for the product estimator t_p, when the amount of ME rises, the percent relative efficiency rises for $\rho_{uv} = -0.9, -0.5$ and falls for $\rho_{uv} = 0, 0.5, 0.9$. In addition, for the proposed estimators, the percent relative efficiency rises at the upper side of ρ_{uv} and falls for the remaining values of ρ_{uv} as the amount of ME rises.

(6) The same trend in the fluctuation in the percent relative efficiency that is shown in Table 11 can also be seen from Tables 13, 15, and 17 for different combinations of σ_x^2, σ_y^2, and positive correlation coefficient ρ_{xy}. The graphs for the same can be provided, if required.

(7) From Table 12, consisting of the values of $\sigma_y^2 = 1$, $\sigma_x^2 = 1$ along with negative correlation $\rho_{xy} = -0.9, -0.5, -0.1$, and the level of ME as 10%, we observe that

(8) The same trend in the fluctuation in percent relative efficiency that is shown in Table 12 can also be seen from Tables 14, 16, and 18 consisting of different amounts of σ_y^2, σ_x^2, and negative correlation coefficient ρ_{xy}. The graphs for the same can be provided, if required.

(9) Furthermore, Tables 3–18 rely on several values of $\sigma_y^2, \sigma_x^2, \sigma_u^2, \sigma_v^2, \rho_{xy}, \rho_{uv}$, and the percentage of ME, and the percent relative efficiency of the suggested estimators surpasses the percent relative efficiency of the usual mean estimator t_m, classical ratio, and product estimators t_r and t_p, respectively.

6. Conclusion

This article has introduced few efficient classes of estimators of the population mean in the existence of correlated measurement errors under simple random sampling. The mean square error of the suggested classes of estimators is obtained with the approximation of order one. The theoretical comparison of the suggested estimators and the existing estimators has been performed. Subsequently, numerical and simulation studies have been conducted to substantiate the theoretical findings. The effects of correlated measurement errors on the performance of the suggested estimators have also been looked at, and the percent relative efficiency has been provided in Tables 2–18. The numerical results given in Table 2 show that the proposed estimators perform better than the traditional estimators with higher percent relative efficiency in each real population. Moreover, the simulation findings reported in Tables 3–10 exhibit that the percent relative efficiency of the proffered estimators $T_i, i = 1, 2, 3$ in case of correlated measurement errors for several values of σ_y^2, σ_x^2, σ_u^2, and σ_v^2 increases as the value of ρ_{xy} moves between 0 and 0.9 and declines as the value of ρ_{xy} moves between −0.9 and −0.1 for the sequential increment of 0.4. The sign and magnitude of ρ_{uv} have an impact on the percent relative efficiency as well. Likewise, a pattern in the percent relative efficiency of the proffered estimators can be seen from Tables 11–18 that are concerned to the various percentage of ME, namely, 10, 15, 20, and 25. In the cases of uncorrelated and correlated measurement errors, the percent relative efficiency of the suggested estimators is significantly different. Furthermore, the suggested estimators surpass the traditional estimators for various values of σ_y^2, σ_x^2, σ_u^2, σ_v^2, ρ_{xy}, ρ_{uv}, and various amounts of measurement errors. Since the proposed classes of estimators perform superior than their counterparts, therefore, they are enthusiastically recommended to the surveyors for computing the population mean under correlated as well as uncorrelated measurement errors.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors extend their appreciation to the Deputyship for Research and Innovation, Ministry of Education in Saudi Arabia, for funding this research work through the project no. IFKSUOR3-216-3.

References

[1] M. N. Murthy, *Sampling: Theory and Methods*, Statistical Publishing Society, Calcutta, India, 1967.

[2] W. G. Cochran, "Errors of measurement in statistics," *Technometrics*, vol. 10, no. 4, pp. 637–666, 1968.

[3] Shalabh, "Ratio method of estimation in the presence of measurement errors," *Journal of the Indian Society of Agricultural Statistics*, vol. 50, no. 2, pp. 150–155, 1997.

[4] Manisha and R. K. Singh, "An estimation of population mean in the presence of measurement errors," *Journal of the Indian Society of Agricultural Statistics*, vol. 54, no. 1, pp. 13–18, 2001.

[5] H. P. Singh and N. Karpe, "Estimation of population variance using auxiliary information in the presence of measurement errors," *Statistics in Transition-New Series*, vol. 9, no. 3, pp. 443–470, 2008.

[6] H. P. Singh and N. Karpe, "On the estimation of ratio and product of two population means using supplementary information in presence of measurement errors," *Statistica, Anno*, vol. 69, no. 1, pp. 27–47, 2009.

[7] H. P. Singh and N. Karpe, "Estimation of mean, ratio and product using auxiliary information in the presence of measurement errors in sample surveys," *Journal of Statistical Theory and Practice*, vol. 4, no. 1, pp. 111–136, 2010.

[8] G. Diana and M. Giordan, "Finite population variance estimation in presence of measurement errors," *Communications in Statistics- Theory and Methods*, vol. 41, no. 23, pp. 4302–4314, 2012.

[9] S. Hussain, S. Ahmad, S. Akhtar, A. Javed, and U. Yasmeen, "Estimation of finite population distribution function with dual use of auxiliary information under non-response," *PLoS One*, vol. 15, no. 12, Article ID e0243584, 2020.

[10] M. U. Tariq, M. N. Qureshi, and M. Hanif, "Variance estimators in the presence of measurement errors using auxiliary information," *Thailand Statistician*, vol. 19, no. 3, pp. 606–616, 2021a.

[11] M. U. Tariq, M. N. Qureshi, and M. Hanif, "Generalized variance estimator using auxiliary information in the presence and absence of measurement error," *Scientia Iranica B*, vol. 29, no. 4, pp. 1868–1879, 2021b.

[12] E. Zahid, J. Shabbir, S. Gupta, R. Onyango, and S. Saeed, "A generalized class of estimators for sensitive variable in the presence of measurement error and non-response," *PLoS One*, vol. 17, no. 1, Article ID e0261561, 2022.

[13] S. Ahmad, S. Hussain, M. Aamir, F. Khan, M. N. Alshahrani, and M. Alqawba, "Estimation of finite population mean using dual auxiliary variable for non-response using simple random sampling," *Aims Mathematics*, vol. 7, no. 3, pp. 4592–4613, 2022.

[14] K. K. Tiwari, S. Bhougal, S. Kumar, and R. Onyango, "Assessing the effect of nonresponse and measurement error using a novel class of efficient estimators," *Journal of Mathematics*, vol. 2022, Article ID 4946265, 9 pages, 2022.

[15] K. K. Tiwari and V. Sharma, "Efficient estimation of population mean in the presence of non-response and measurement error," *Statistics in Transition- new series*, vol. 24, no. 3, pp. 95–116, 2023.

[16] S. Bhushan, A. Kumar, and S. Shukla, "Novel logarithmic type estimators in presence of measurement errors," *Journal of Statistical Theory and Practice*, vol. 17, no. 3, pp. 35–26, 2023a.

[17] Shalabh and J. R. Tsai, "Ratio and product methods of estimation of population mean in the presence of correlated measurement errors," *Communications in Statistics- Simulation and Computation*, vol. 46, no. 7, pp. 5566–5593, 2017.

[18] S. Bhushan, A. Kumar, and S. Shukla, "Performance evaluation of novel logarithmic estimators under correlated measurement errors," *Communications in Statistics- Theory and Methods*, pp. 1–12, 2023b.

[19] S. Bhushan, A. Kumar, and S. Shukla, "On classes of robust estimators in presence of correlated measurement errors," *Measurement*, pp. 1–23, 2023c.

[20] D. N. Gujarati and Sangeetha, *Basic Econometrics*, Tata McGraw Hill Publishing Company Limited, New Delhi, India, 2007.