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Research Article

Pointwise Hemislant Submanifolds in a Complex Space Form

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In this paper, pointwise hemislant submanifolds were introduced in a Kahler manifold. The integrability conditions for the distributions which are involved in the definition of a pointwise hemislant submanifold were investigated. In addition, the necessary and sufficient conditions were given for a pointwise hemislant submanifold to be a pointwise hemislant product.

1. Introduction

The concept of pointwise slant submanifolds appeared under the name of quasi-slant submanifolds by Etayo [1] as a generalization of slant submanifolds introduced by Chen [2, 3]. Then, in [4], Chen and Garay studied pointwise slant submanifolds of almost Hermitian manifolds and proved many interesting results. Later on, pointwise slant submanifolds were investigated on Riemannian manifolds equipped with various structures [5–9].

On the other hand, the notation of hemislant submanifolds was first defined by Carrizo et al. [10, 11], and they named them pseudoslant submanifolds. After that, in [12], Sahin studied hemislant submanifolds and their warped products of Kahler manifolds. Al-Solamy et al. [13] defined the totally umbilical hemislant submanifolds in Kahler manifolds and derived several results. Recently, hemislant submanifolds were studied by different authors in many ambient spaces (see [14–17]). Furthermore, the notation of a quasi-hemislant submanifold was studied by Prasad et al. in [18].

In the present paper, the purpose is to study the geometry of pointwise hemislant submanifolds of a Kahler manifold. Some basic formulas and definitions are recalled in Section 2, which are useful to the next section. Section 3 defines the pointwise hemislant submanifold of a Kahler manifold and gives some basic results on such submanifolds. The integrability condition of the distributions on the pointwise hemislant submanifold of a Kahler manifold is constructed. Following the procedure, the necessary and sufficient conditions are given for a pointwise hemislant submanifold to be a pointwise hemislant product.

2. Preliminaries

Let $\widetilde{M}$ be an almost Hermitian manifold with structure (J, g) where J is a (1, 1) tensor field and g is a Riemannian metric on $\widetilde{M}$ satisfying the following properties:

$$J^2 = -I, \tag{1}$$

$$g(JX, JY) = g(X, Y), \tag{2}$$

for all vector fields X, Y on $\widetilde{M}$. If, in addition to the above relations,

$$(\widetilde{\nabla}_X J)Y = 0, \tag{3}$$

holds, then $\widetilde{M}$ is said to be Kahler manifold, where $\widetilde{\nabla}$ is the Levi-Civita connection of g. The covariant derivative of the complex structure J is given by

$$(\widetilde{\nabla}_X J)Y = \widetilde{\nabla}_X JY - J\widetilde{\nabla}_X Y. \tag{4}$$

Let M be an isometrical immersed submanifold of $\widetilde{M}$ with the induced metric g. Let $\Gamma(\mathrm{TM})$ and $\Gamma(T^{\perp}M)$ be the differential vector fields set tangent and normal to M, respectively. Then, Gauss and Weingarten formulas are, respectively, given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y), \tag{5}$$

and

$$\widetilde{\nabla}_X U = \nabla_X^{\perp} U - A_U X, \tag{6}$$

for all $X, Y \in \Gamma(\mathrm{TM})$ and $U \in \Gamma(T^{\perp}M)$, where ∇ and $\nabla^{\perp}$ are the induced connections on $\Gamma(\mathrm{TM})$ and $\Gamma(T^{\perp}M)$, respectively, σ is the second fundamental form of M, and A_U is the shape operator of the second fundamental form, which is related by

$$g(A_U X, Y) = g(\sigma(X, Y), U). \tag{7}$$

For any orthonormal frame $\{e_1, \ldots, e_n\}$ of M, the mean curvature vector $\overrightarrow{H}(x)$ is given by

$$\overrightarrow{H}(x) = \frac{1}{n}\sum_{i=1}^{n} \sigma(e_i, e_i), \tag{8}$$

where $n = \dim(M)$. The submanifold M is totally geodesic in $\widetilde{M}$ if $\sigma = 0$ and minimal if $\overrightarrow{H} = 0$. If $\sigma(X, Y) = g(X, Y)\overrightarrow{H}$ for all $X, Y \in \Gamma(TM)$, then M is totally umbilical. For any $X \in \Gamma(\mathrm{TM})$ and $U \in \Gamma(T^{\perp}M)$,

$$\mathrm{JX} = \mathrm{TX} + \mathrm{FX}, \tag{9}$$

and

$$\mathrm{JU} = \mathrm{tU} + \mathrm{fU}, \tag{10}$$

where TX and tU are the tangential components and FX and fU are the normal components of JX and JU, respectively. A submanifold M of an almost Hermitian manifold $\widetilde{M}$ is said to be holomorphic (resp. totally real) if $J(T_pM) = T_pM$ (resp. $J(T_pM) \subseteq T_p^{\perp}M \,\forall\, p \in M$ [19]. The covariant derivatives of the tangential and normal components of JX and JU are defined by

$$(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y, \tag{11}$$

$$(\nabla_X F)Y = \nabla_X^{\perp} FY - F\nabla_X Y, \tag{12}$$

$$(\nabla_X t)U = \nabla_X tU - t\nabla_X^{\perp} U, \tag{13}$$

$$(\nabla_X f)U = \nabla_X^{\perp} fU - f\nabla_X^{\perp} U. \tag{14}$$

For any $X, Y \in \Gamma(\mathrm{TM})$, we have $g(X, \mathrm{TY}) = -g(\mathrm{TX}, Y)$. Also, by using (2), (9), and (10), we have $g(U, \mathrm{fV}) = -g(\mathrm{fU}, V)$, for any $U, V \in \Gamma(T^{\perp}M)$. That is, T and f are skew-symmetric tensor fields. Furthermore, the relation between the tensor fields F and t is given by

$$g(\mathrm{FX}, U) = -g(X, \mathrm{tU}), \tag{15}$$

for any $X \in \Gamma(\mathrm{TM})$ and $U \in \Gamma(T^{\perp}M)$.

3. Pointwise Hemislant Submanifolds of a Kahler Manifold

In this section, a brief introduction of pointwise hemislant submanifolds of a Kahler manifold is given. We shall obtain some results.

Chen defined slant and pointwise slant submanifolds as follows.

For a nonzero vector $X \in T_pM$, $p \in M$, the angle $\theta(X)$ between JX and T_pM is called the Wirtinger angle of X. A submanifold M is said to be slant if the Wirtinger angle $\theta(X)$ is constant on M; i.e., it is independent of the choice of $X \in T_pM$ and $p \in M$ [2, 3]. In this case, θ is called the slant angle of M.

A submanifold M is said to be pointwise slant if the Wirtinger angle $\theta(X)$ can be regarded as a function on M, which is known as the slant function in [4]. A pointwise slant submanifold with a slant function θ is simply called a pointwise θ slant submanifold. Clearly, a pointwise slant submanifold M is a slant submanifold if its slant function θ is a constant function on M.

Holomorphic and totally real submanifolds are slant submanifolds with slant angles 0 and $\pi/2$, respectively. A slant submanifold is called proper slant if it is neither holomorphic nor totally real.

We recall the following basic result from [4] for pointwise slant submanifolds of an almost Hermitian manifold.

Theorem 1. *Let M be a submanifold of an almost Hermitian manifold. Then, M is pointwise slant if and only if*

$$T^2 X = -\left(\cos^2\theta\right)X, \tag{16}$$

for some real-valued function θ defined on M.

Following relations are straightforward consequence of equation (16):

$$g(\mathrm{TX}, \mathrm{TY}) = \cos^2\theta\, g(X, Y), \tag{17}$$

$$g(\mathrm{FX}, \mathrm{FY}) = \sin^2\theta\, g(X, Y), \tag{18}$$

for all $X, Y \in \Gamma(\mathrm{TM})$. Clearly, we also have

$$\mathrm{tFX} = -\sin^2\theta X, \; f\mathrm{FX} = -\mathrm{FTX}. \tag{19}$$

In [20], Uddin and Stankovic defined a pointwise hemislant submanifold as follows.

Definition 2. A submanifold M of a Kahler manifold $\widetilde{M}$ is said to be a pointwise hemislant submanifold if there exist two orthogonal complementary distributions $\mathscr{D}^{\theta}$ and $\mathscr{D}^{\perp}$ such that:

(i) The tangent space TM admits the orthogonal direct decomposition $\mathrm{TM} = \mathscr{D}^{\theta} \oplus \mathscr{D}^{\perp}$
(ii) The distribution $\mathscr{D}^{\theta}$ is pointwise slant with a slant function θ
(iii) The distribution $\mathscr{D}^{\perp}$ is a totally real, i.e., $J\mathscr{D}^{\perp} \subseteq T^{\perp}M$

If the dimensions of the distributions $\mathscr{D}^{\theta}$ and $\mathscr{D}^{\perp}$ are denoted by m_1 and m_2, respectively, then the following cases are obtained:

(i) If $m_1 = 0$, then M is totally real submanifold
(ii) If $m_2 = 0$, then M is a pointwise slant submanifold
(iii) If $m_2 = 0$ and $\theta = 0$, then M is a holomorphic submanifold

(iv) If θ is constant on M, then M is a hemislant submanifold with a slant angle θ

(v) If $m_1 \neq 0$ and θ is not constant, then M is a proper pointwise hemislant submanifold

We mention the following example of pointwise hemislant submanifolds in the Euclidean space.

Example 1. Let $\mathbb{R}^6$ be the Euclidean 6-space with the cartesian coordinates $(x_1, y_1, x_2, y_2, x_3, y_3)$, and the almost comlex structure J is defined by

$$J\left(\frac{\partial}{\partial x_i}\right) = -\frac{\partial}{\partial y_i}, J\left(\frac{\partial}{\partial y_j}\right) = \frac{\partial}{\partial x_j}, \quad 1 \leq i, j \leq 3, \tag{20}$$

and the standard Euclidean metric g is on $\mathbb{R}^6$. Consider a submanifold M of $\mathbb{R}^6$ given by the immersion ψ as follows:

$$\psi(u, v, \theta) = \left(-u\cos\theta, u\sin\theta, \frac{u^2}{2}, 0, v, v\right), \tag{21}$$

for nonvanishing real valued functions u, v on M. Then, the tangent bundle of M is spanned by the following tangent vectors:

$$\begin{aligned} Z_1 &= -\cos\theta\frac{\partial}{\partial x_1} + \sin\theta\frac{\partial}{\partial y_1} + u\frac{\partial}{\partial x_2},\\ Z_2 &= \frac{\partial}{\partial x_3} + \frac{\partial}{\partial y_3},\\ Z_3 &= u\sin\theta\frac{\partial}{\partial x_1} + u\cos\theta\frac{\partial}{\partial y_1}. \end{aligned} \tag{22}$$

Then,

$$\begin{aligned} \mathrm{JZ}_1 &= \cos\theta\frac{\partial}{\partial y_1} + \sin\theta\frac{\partial}{\partial x_1} - u\frac{\partial}{\partial y_2},\\ \mathrm{JZ}_2 &= -\frac{\partial}{\partial y_3} + \frac{\partial}{\partial x_3},\\ \mathrm{JZ}_3 &= -u\sin\theta\frac{\partial}{\partial y_1} + u\cos\theta\frac{\partial}{\partial x_1}. \end{aligned} \tag{23}$$

Clearly, JZ_2 is an orthogonal to TM; hence, $\mathscr{D}^\perp = \mathrm{Span}\{Z_2\}$ is a totally real distribution, and $\mathscr{D}^\theta = \mathrm{Span}\{Z_1, Z_3\}$ is a proper pointwise slant distribution with a slant function $\alpha = \cos^{-1} 1/\sqrt{1+u^2}$. Thus, M is proper pointwise hemislant submanifold of $\mathbb{R}^6$.

Lemma 3. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then, $J(\mathscr{D}^\perp) \perp F(\mathscr{D}^\theta)$.*

Proof. For any $X \in \Gamma(\mathscr{D}^\theta)$ and $Z \in \Gamma(\mathscr{D}^\perp)$, by (9), we have

$$g(\mathrm{JX}, \mathrm{JZ}) = g(\mathrm{TX} + \mathrm{FX}, \mathrm{JZ}) = g(\mathrm{FX}, \mathrm{JZ}). \tag{24}$$

But, from (2), we have

$$g(\mathrm{JX}, \mathrm{JZ}) = g(X, Z) = 0. \tag{25}$$

Thus,

$$g(\mathrm{FX}, \mathrm{JZ}) = 0, \tag{26}$$

which means that $J(\mathscr{D}^\perp) \perp F(\mathscr{D}^\theta)$.

From the above lemma, the normal bundle $T^\perp M$ can be decomposed as

$$T^\perp M = J(\mathscr{D}^\perp) \oplus F(\mathscr{D}^\theta) \oplus \mu, \tag{27}$$

where μ is the invariant distribution of $T^\perp M$ under J. □

Lemma 4. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then, we have*

$$T(\mathscr{D}^\perp) = \{0\} \text{ and } T(\mathscr{D}^\theta) = \mathscr{D}^\theta. \tag{28}$$

Proof. The proof is direct, and it can be obtained by using (1), (2), (9), and (16). □

Lemma 5. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then, we have*

$$(\nabla_X T)Y = A_{\mathrm{FY}}X + t\sigma(X, Y), \tag{29}$$

and

$$(\nabla_X F)Y = f\sigma(X, Y) - \sigma(X, \mathrm{TY}), \tag{30}$$

for all $X, Y \in \Gamma(\mathrm{TM})$.

Proof. In a Kahler manifold, we have that

$$(\widetilde{\nabla}_X J)Y = 0, \tag{31}$$

which gives that

$$\widetilde{\nabla}_X JY - J\widetilde{\nabla}_X Y = 0. \tag{32}$$

From (5) and (9), we obtain

$$\widetilde{\nabla}_X TY + \widetilde{\nabla}_X FY - J\nabla_X Y - J\sigma(X, Y) = 0. \tag{33}$$

Again, by (5), (6), (9), and (10), we can write

$$\begin{aligned} &\nabla_X \mathrm{TY} + \sigma(X, \mathrm{TY}) - A_{\mathrm{FY}}X + \nabla_X^\perp \mathrm{FY} - T\nabla_X Y - F\nabla_X Y\\ &\quad - t\sigma(X, Y) - f\sigma(X, Y) = 0. \end{aligned} \tag{34}$$

Comparing the tangential and normal parts with using (11) and (12), we get the required results. Hence, the lemma is proved completely. □

By a similar argument, we have the following Lemma.

Lemma 6. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then, we have*

$$(\nabla_X t)U = A_{\mathrm{fU}}X - TA_U X, \tag{35}$$

and

$$(\nabla_X f)U = -(\sigma(\mathrm{tU}, X) + FA_U X), \tag{36}$$

for all $X \in \Gamma(\mathrm{TM})$ *and* $U \in \Gamma(T^{\perp}M)$.

Lemma 7. *Let* M *be a pointwise hemislant submanifold of a Kahler manifold* $\widetilde{M}$. *Then,*

$$A_{\mathrm{JZ}}W = A_{\mathrm{JW}}Z, \tag{37}$$

for all $Z, W \in \Gamma(\mathscr{D}^{\perp})$.

Proof. For any $X \in \Gamma(\mathrm{TM})$ and $Z, W \in \Gamma(\mathscr{D}^{\perp})$, using (3), (5), (6), and (7), we have

$$\begin{aligned} g\left(A_{\mathrm{JZ}}W - A_{\mathrm{JW}}Z, X\right) &= g(\sigma(W,X), JZ) - g(\sigma(Z,X), \mathrm{JW}) \\ &= g(\widetilde{\nabla}_X W, \mathrm{JZ}) - g(\widetilde{\nabla}_X Z, \mathrm{JW}) \\ &= -g(J\widetilde{\nabla}_X W, Z) + g(J\widetilde{\nabla}_X Z, W) \\ &= -g(\widetilde{\nabla}_X \mathrm{JW}, Z) + g(\widetilde{\nabla}_X \mathrm{JZ}, W) \\ &= g\left(A_{\mathrm{JW}}X, Z\right) - g\left(A_{\mathrm{JZ}}X, W\right) \\ &= g\left(A_{\mathrm{JW}}Z - A_{\mathrm{JZ}}W, X\right). \end{aligned} \tag{38}$$

Therefore,

$$A_{\mathrm{JZ}}W = A_{\mathrm{JW}}Z. \tag{39}$$

□

It follows from (28) that

$$A_{\mathrm{FZ}}W = A_{\mathrm{FW}}Z, \tag{40}$$

for any $Z, W \in \Gamma(\mathscr{D}^{\perp})$.

Theorem 8. *Let* M *be a pointwise hemislant submanifold of a Kahler manifold* $\widetilde{M}$. *Then, the covariant derivation of the endomorphism* T *is skew-symmetric, i.e.,*

$$g((\nabla_X T)Y, Z) = -g((\nabla_X T)Z, Y), \tag{41}$$

for any $X, Y, Z \in \Gamma(\mathrm{TM})$.

Proof. For any $X, Y, Z \in \Gamma(\mathrm{TM})$, using (7), (15), and (21), we get

$$\begin{aligned} g((\nabla_X T)Y, Z) &= g(A_{\mathrm{FY}}X, Z) + g(t\sigma(X,Y), Z) \\ &= g(\sigma(X,Z), \mathrm{FY}) - g(\sigma(X,Y), \mathrm{FZ}) \\ &= -g(t\sigma(X,Z), Y) - g(A_{\mathrm{FZ}}X, Y) \\ &= -g(A_{\mathrm{FZ}}X + t\sigma(X,Z), Y) \\ &= -g((\nabla_X T)Z, Y). \end{aligned} \tag{42}$$

This proves our assertion. □

Theorem 9. *Let* M *be a pointwise hemislant submanifold of a Kahler manifold* $\widetilde{M}$. *Then, we have*

$$g((\nabla_X t)U, Y) = -g((\nabla_X F)Y, U), \tag{43}$$

for any $X, Y \in \Gamma(\mathrm{TM})$ *and* $U \in \Gamma(T^{\perp}M)$.

Proof. For any $X, Y \in \Gamma(\mathrm{TM})$ and $U \in \Gamma(T^{\perp}M)$, using (7), (22), and (23), we obtain

$$\begin{aligned} g((\nabla_X t)U, Y) &= g(A_{\mathrm{fU}}X, Y) - g(TA_U X, Y) \\ &= g(\sigma(X,Y), \mathrm{fU}) + g(A_U X, \mathrm{TY}) \\ &= -g(f\sigma(X,Y), U) + g(\sigma(X, \mathrm{TY}), U) \\ &= -g((\nabla_X F)Y, U), \end{aligned} \tag{44}$$

which verifies our assertion. □

Theorem 10. *Let* M *be a pointwise hemislant submanifold of a Kahler manifold* $\widetilde{M}$. *Then, the tensor* T *is parallel if and only if*

$$A_{\mathrm{FY}}X = A_{\mathrm{FX}}Y, \tag{45}$$

for any $X, Y \in \Gamma(\mathrm{TM})$.

Proof. For any $X, Y \in \Gamma(\mathrm{TM})$, using (7), (15), and (21), we have

$$\begin{aligned} g((\nabla_X T)Y, Z) &= g(A_{\mathrm{FY}}X, Z) + g(t\sigma(X,Y), Z) \\ &= g(\sigma(X,Z), \mathrm{FY}) - g(\sigma(X,Y), \mathrm{FZ}) \\ &= g(A_{\mathrm{FY}}Z, X) - g(A_{\mathrm{FZ}}Y, X). \end{aligned} \tag{46}$$

Hence, the result is obtained. □

Theorem 11. *Let* M *be a pointwise hemislant submanifold in a Kahler manifold* $\widetilde{M}$. *Then, the tensor* F *is parallel if and only if*

$$A_U \mathrm{TY} = -A_{\mathrm{fU}}Y, \tag{47}$$

for any $Y \in \Gamma(\mathrm{TM})$ *and* $U \in \Gamma(T^{\perp}M)$.

Proof. For any $X, Y \in \Gamma(\mathrm{TM})$ and $U \in \Gamma(T^{\perp}M)$, using (7) and (22), we have

$$\begin{aligned} g((\nabla_X F)Y, U) &= g(f\sigma(X,Y), U) - g(\sigma(X, \mathrm{TY}), U) \\ &= -g(\sigma(X,Y), \mathrm{fU}) - g(A_U \mathrm{TY}, X) \\ &= -g\left(A_{fU}Y, X\right) - g(A_U \mathrm{TY}, X). \end{aligned} \tag{48}$$

The proof is completed. □

Theorem 12. *Let* M *be a pointwise hemislant submanifold of a Kahler manifold* $\widetilde{M}$. *Then, the covariant derivation of the endomorphism* f *is skew-symmetric, that is,*

$$g((\nabla_X f)U, V) = -g((\nabla_X f)V, U), \tag{49}$$

for any $X \in \Gamma(\mathrm{TM})$ *and* $U, V \in \Gamma(T^{\perp}M)$.

Proof. For any $X \in \Gamma(\mathrm{TM})$ and $U, V \in \Gamma(T^{\perp}M)$, using (7), (15), and (24), we get

$$\begin{aligned} g((\nabla_X f)U,V) &= -g(\sigma(tU,X),V) - g(FA_U X,V) \\ &= -g(A_V X, tU) + g(A_U X, \mathrm{tV}) \\ &= g(FA_V X, U) + g(\sigma(X,\mathrm{tV}),U) \\ &= -g(-FA_V X - \sigma(X,\mathrm{tV}),U) \\ &= -g((\nabla_X f)V,U), \end{aligned} \tag{50}$$

which is the required result. □

Theorem 13. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then, the tensor f is parallel if and only if*

$$A_U \mathrm{tV} = A_V \mathrm{tU}, \tag{51}$$

for all $U, V \in \Gamma(T^\perp M)$.

Proof. Let $X \in \Gamma(\mathrm{TM})$ and $U, V \in \Gamma(T^\perp M)$, using (7), (15), and (24), we get

$$\begin{aligned} g((\nabla_X f)U,V) &= -g(\sigma(\mathrm{tU},X),V) - g(FA_U X,V) \\ &= -g(A_V \mathrm{tU}, X) + g(A_U X, \mathrm{tV}) \\ &= g(A_U \mathrm{tV} - A_V \mathrm{tU}, X). \end{aligned} \tag{52}$$

This proves our assertion. □

Theorem 14. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then,*

$$\nabla^\perp_Z \mathrm{FW} - \nabla^\perp_W \mathrm{FZ} \in F(D^\perp), \tag{53}$$

for any $Z, W \in \Gamma(D^\perp)$.

Proof. For any $Z, W \in \Gamma(D^\perp)$ and $V \in \mu$, using (3), (5), (6), (9), and (20), we have

$$\begin{aligned} g(\nabla^\perp_Z \mathrm{FW} - \nabla^\perp_W \mathrm{FZ}, V) &= g\big(\widetilde{\nabla}_Z \mathrm{JW} + A_{\mathrm{JW}} Z - \widetilde{\nabla}_W \mathrm{JZ} - A_{\mathrm{JZ}} W, V\big) \\ &= g(J\widetilde{\nabla}_Z W, V) - g(J\widetilde{\nabla}_W Z, V) \\ &= g(\widetilde{\nabla}_W Z, \mathrm{JV}) - g(\widetilde{\nabla}_Z W, \mathrm{JV}) \\ &= g(\nabla_W Z, \mathrm{JV}) + g(\sigma(W,Z), \mathrm{JV}) \\ &\quad - g(\nabla_Z W, \mathrm{JV}) - g(\sigma(Z,W), \mathrm{JV}) = 0. \end{aligned} \tag{54}$$

Thus, the result is concluded. □

Theorem 15. *Let M be a proper pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. If the tensor f is parallel, then, M is a totally geodesic submanifold of $\widetilde{M}$.*

Proof. Suppose that f is parallel, then making use of (9) and (24), we have

$$\sigma(\mathrm{tU}, X) + JA_U X = 0, \tag{55}$$

for all $X \in \Gamma(\mathrm{TM})$ and $U \in \Gamma(T^\perp M)$. Applying J to the above relation with using (1) and (10), we find

$$t\sigma(\mathrm{tU}, X) + f\sigma(\mathrm{tU}, X) - A_U X = 0. \tag{56}$$

Taking the inner product with $Y \in \Gamma(\mathrm{TM})$ and then using (7), (15), (19), and (92), we obtain

$$\begin{aligned} g(A_U X, Y) &= g(t\sigma(\mathrm{tU}, X), Y) = -g(\sigma(\mathrm{tU}, X), \mathrm{FY}) \\ &= -g(A_{\mathrm{FY}} tU, X) = -g(A_U \mathrm{tFY}, X) \\ &= \sin^2\theta\, g(A_U Y, X) = \sin^2\theta\, g(A_U X, Y), \end{aligned} \tag{57}$$

which implies that

$$\cos^2\theta\, g(\sigma(X,Y),U) = 0. \tag{58}$$

As M is a proper pointwise hemislant submanifold, we obtain $\sigma(X,Y) = 0$, which means that M is a totally geodesic submanifold of $\widetilde{M}$. □

Definition 16. A pointwise hemislant submanifold M of a Kahler manifold $\widetilde{M}$ is said to be $\mathscr{D}^\theta$-geodesic (resp. $\mathscr{D}^\perp$-geodesic) if $\sigma(X,Y) = 0$, for any $X, Y \in \Gamma(\mathscr{D}^\theta)$ (resp. $\sigma(Z,W) = 0$, for any $Z, W \in \Gamma(\mathscr{D}^\perp)$), and M is called a mixed geodesic submanifold if $\sigma(X,Z) = 0$, for any $X \in \Gamma(\mathscr{D}^\theta)$ and $Z \in \Gamma(\mathscr{D}^\perp)$.

Theorem 17. *Let M be a proper pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. If the tensor t is parallel, then, M is a mixed geodesic submanifold of $\widetilde{M}$.*

Proof. If t is parallel, then from Theorem 9 and (28) with (22), we obtain

$$f\sigma(X,Z) = 0, \tag{59}$$

for any $X \in \Gamma(D^\theta)$ and $Z \in \Gamma(D^\perp)$. Also, we can write

$$f\sigma(Z,X) - \sigma(Z,TX) = 0. \tag{60}$$

Putting $X = \mathrm{TX}$ with using (16), we find

$$f\sigma(Z,\mathrm{TX}) + \cos^2\theta\,\sigma(Z,X) = \cos^2\theta\,\sigma(X,Z) = 0. \tag{61}$$

Since M is a proper pointwise hemislant submanifold, we conclude $\sigma(X,Z) = 0$. That is, M is a mixed geodesic submanifold of $\widetilde{M}$. □

Theorem 18. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. If the tensor t is parallel, then M is either $D^\perp$-geodesic or a totally real submanifold of $\widetilde{M}$.*

Proof. Suppose that t is parallel, then if we put $U = \mathrm{FZ}$ in (23), we obtain

$$TA_{\mathrm{FZ}}W = 0, \tag{62}$$

for any $Z, W \in \Gamma(D^{\perp})$. This tells us that M is either totally real or $A_{\mathrm{FZ}}W = 0$. Again by (35), we derive

$$\begin{aligned} g(A_{\mathrm{fU}}W, Z) - g(TA_U W, Z) = g(\sigma(W, Z), \\ fU) + g(A_U W, \mathrm{TZ}) = 0, \end{aligned} \tag{63}$$

which implies that

$$g(\sigma(W, Z), \mathrm{fU}) = 0, \tag{64}$$

for any $U \in \Gamma(T^{\perp}M)$. That is, M is a $D^{\perp}$-geodesic or a totally real submanifold of $\widetilde{M}$. □

Theorem 19. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then, the totally real distribution $D^{\perp}$ is always integrable, and its maximal integral submanifold is totally real submanifold of $\widetilde{M}$.*

Proof. For any $X \in \Gamma(D^{\theta})$ and $Z, W \in \Gamma(D^{\perp})$, by (2), (3), (5), (6), and (9), we have

$$\begin{aligned} g([Z, W], X) &= g(\widetilde{\nabla}_Z W - \widetilde{\nabla}_W Z, X) = g(\widetilde{\nabla}_W X, Z) - g(\widetilde{\nabla}_Z X, W) \\ &= g(J\widetilde{\nabla}_W X, \mathrm{JZ}) - g(J\widetilde{\nabla}_Z X, \mathrm{JW}) \\ &= g(\widetilde{\nabla}_W \mathrm{JX}, \mathrm{JZ}) - g(\widetilde{\nabla}_Z \mathrm{JX}, \mathrm{JW}) \\ &= g(\widetilde{\nabla}_W \mathrm{TX} + \widetilde{\nabla}_W \mathrm{FX}, \mathrm{FZ}) \\ &\quad - g(\widetilde{\nabla}_Z \mathrm{TX} + \widetilde{\nabla}_Z \mathrm{FX}, \mathrm{FW}) \\ &= g(\sigma(W, \mathrm{TX}), \mathrm{FZ}) + g(\nabla^{\perp}_W \mathrm{FX}, \mathrm{FZ}) \\ &\quad - g(\sigma(Z, \mathrm{TX}), \mathrm{FW}) - g(\nabla^{\perp}_Z \mathrm{FX}, \mathrm{FW}). \end{aligned} \tag{65}$$

Now, from (7), (12), (19), (28), (30), and (40), we derive

$$\begin{aligned} g([Z, W], X) &= g(A_{\mathrm{FZ}}W - A_{\mathrm{FW}}Z, \mathrm{TX}) + g(\nabla^{\perp}_W \mathrm{FX}, \mathrm{FZ}) \\ &\quad - g(\nabla^{\perp}_Z \mathrm{FX}, \mathrm{FW}) \\ &= g((\nabla_W F)X + F\nabla_W X, \mathrm{FZ}) \\ &\quad - g((\nabla_Z F)X + F\nabla_Z X, \mathrm{FW}) \\ &= g(f\sigma(W, X) - \sigma(W, \mathrm{TX}), \mathrm{FZ}) \\ &\quad - g(f\sigma(Z, X) - \sigma(Z, \mathrm{TX}), \mathrm{FW}) \\ &\quad + g(F\nabla_W X, \mathrm{FZ}) - g(F\nabla_Z X, \mathrm{FW}) \\ &= -g(\sigma(W, \mathrm{TX}), \mathrm{FZ}) + g(\sigma(Z, \mathrm{TX}), \mathrm{FW}) \\ &\quad + g(F\nabla_W X, \mathrm{FZ}) - g(F\nabla_Z X, \mathrm{FW}). \end{aligned} \tag{66}$$

Thus, by (7), (18), and (20), we can write

$$\begin{aligned} g([Z, W], X) &= g(-A_{\mathrm{FZ}}W + A_{\mathrm{FW}}Z, \mathrm{TX}) + \sin^2\theta\, g(\nabla_W X, Z) - \sin^2\theta\, g(\nabla_Z X, W) \\ &= \sin^2\theta\, g(\nabla_Z W, X) - \sin^2\theta\, g(\nabla_W Z, X) \\ &= \sin^2\theta\, g([Z, W], X), \end{aligned} \tag{67}$$

which implies that

$$\cos^2\theta\, g([Z, W], X) = 0. \tag{68}$$

Therefore, $[Z, W] \in \Gamma(D^{\perp})$ for any $Z, W \in \Gamma(D^{\perp})$, which means that the totally real distribution $D^{\perp}$ is always integrable, and its maximal integral submanifold is a totally real submanifold of $\widetilde{M}$. Hence, the proof is completed. □

Corollary 20. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then, we have*

$$(\nabla_Z T)W = (\nabla_W T)Z, \tag{69}$$

for any $Z, W \in \Gamma(D^{\perp})$.

Proof. Since the ambient manifold $\widetilde{M}$ is Kahler, for any $Z, W \in \Gamma(D^{\perp})$, we have

$$(\widetilde{\nabla}_Z J)W = 0, \tag{70}$$

which can be written as

$$\widetilde{\nabla}_Z JW - J\widetilde{\nabla}_Z W = 0. \tag{71}$$

Now, using (5) and (9), we find

$$\widetilde{\nabla}_Z \mathrm{FW} - J\nabla_Z W - J\sigma(Z, W) = 0. \tag{72}$$

Hence, by (6), (9), and (10), we deduce

$$\nabla_Z^{\perp}\mathrm{FW} - A_{\mathrm{FW}}Z - T\nabla_Z W - F\nabla_Z W - t\sigma(Z, W) - f\sigma(Z, W) = 0. \tag{73}$$

If we take the tangential components of the above equation, we obtain

$$A_{\mathrm{FW}}Z + T\nabla_Z W + t\sigma(Z, W) = 0. \tag{74}$$

Similarly, we get

$$A_{\mathrm{FZ}}W + T\nabla_Z W + t\sigma(Z, W) = 0. \tag{75}$$

Thus, by (26), (35), and (36), we derive

$$(\nabla_Z T)W = (\nabla_W T)Z. \tag{76}$$

□

Theorem 21. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then, the slant distribution D^{θ} is integrable if and only if*

$$\sigma(X, \mathrm{TY}) - \sigma(Y, \mathrm{TX}) + \nabla_X^{\perp}\mathrm{FY} - \nabla_Y^{\perp}\mathrm{FX} \in F\left(D^{\theta}\right) \oplus \mu, \tag{77}$$

for any $X, Y \in \Gamma(D^{\theta})$.

Proof. Let $X, Y \in \Gamma(D^{\theta})$ and $Z \in \Gamma(D^{\perp})$, by using (2) and (3), we have

$$\begin{aligned} g([X, Y], Z) &= g(\widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X, Z) \\ &= g(J\widetilde{\nabla}_X Y - J\widetilde{\nabla}_Y X, \mathrm{JZ}) \\ &= g(\widetilde{\nabla}_X \mathrm{JY} - \widetilde{\nabla}_Y JX, \mathrm{FZ}). \end{aligned} \tag{78}$$

Then, by (5), (6), and (9), we obtain

$$\begin{aligned} g([X, Y], Z) &= g(\widetilde{\nabla}_X \mathrm{TY} + \widetilde{\nabla}_X \mathrm{FY} - \widetilde{\nabla}_Y \mathrm{TX} - \widetilde{\nabla}_Y \mathrm{FX}, \mathrm{FZ}) \\ &= g(\sigma(X, TY) + \nabla_X^{\perp}FY - \sigma(Y, \mathrm{TX}) - \nabla_Y^{\perp}\mathrm{FX}, \mathrm{FZ}). \end{aligned} \tag{79}$$

Since $\mathrm{FZ} \in J(D^{\perp}) \subseteq T^{\perp}M$, we deduce the result. □

Theorem 22. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then, the pointwise slant distribution D^{θ} is integrable if and only if*

$$TA_{\mathrm{FZ}}X + A_{\mathrm{FZ}}\mathrm{TX} = 0, \tag{80}$$

for any $X \in \Gamma(D^{\theta})$ and $Z \in \Gamma(D^{\perp})$.

Proof. For any $X, Y \in \Gamma(D^{\theta})$ and $Z \in \Gamma(D^{\perp})$, by using (2), (3), and (9), we have

$$\begin{aligned} g([X, Y], Z) &= g(\widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X, Z) = g(J\widetilde{\nabla}_X Y, JZ) - g(J\widetilde{\nabla}_Y X, \mathrm{JZ}) \\ &= g(\widetilde{\nabla}_X \mathrm{JY}, \mathrm{FZ}) - g(\widetilde{\nabla}_Y \mathrm{JX}, \mathrm{FZ}) = g(\widetilde{\nabla}_Y \mathrm{FZ}, \mathrm{JX}) - g(\widetilde{\nabla}_X \mathrm{FZ}, \mathrm{JY}) \\ &= g(\widetilde{\nabla}_Y \mathrm{FZ}, \mathrm{TX}) + g(\widetilde{\nabla}_Y \mathrm{FZ}, \mathrm{FX}) - g(\widetilde{\nabla}_X \mathrm{FZ}, \mathrm{TY}) - g(\widetilde{\nabla}_X \mathrm{FZ}, \mathrm{FY}). \end{aligned} \tag{81}$$

Thus, by (6), (12), and (22), we find

$$\begin{aligned} g([X,Y],Z) &= g(A_{\mathrm{FZ}}X,\mathrm{TY}) - g(A_{\mathrm{FZ}}Y,\mathrm{TX}) + g(\nabla^{\perp}_{Y}\mathrm{FZ},\mathrm{FX}) - g(\nabla^{\perp}_{X}\mathrm{FZ},\mathrm{FY}) \\ &= g(A_{\mathrm{FZ}}TY,X) + g(TA_{\mathrm{FZ}}Y,X) + g((\nabla_Y F)Z + F\nabla_Y Z,\mathrm{FX}) \\ &\quad - g((\nabla_X F)Z + F\nabla_X Z,\mathrm{FY}) \\ &= g(A_{FZ}\mathrm{TY},X) + g(TA_{\mathrm{FZ}}Y,X) + g(f\sigma(Y,Z),\mathrm{FX}) + g(F\nabla_Y Z,\mathrm{FX}) \\ &\quad - g(f\sigma(X,Z),FY) - g(F\nabla_X Z,FY) \\ &= g(A_{\mathrm{FZ}}\mathrm{TY} + TA_{\mathrm{FZ}}Y,X) + \sin^2\theta\{g(\nabla_Y Z,X) - g(\nabla_X Z,Y)\} \\ &= g(A_{FZ}\mathrm{TY} + TA_{FZ}Y,X) + \sin^2\theta\{g(\nabla_X Y - \nabla_Y X,Z)\} \\ &= g(A_{FZ}\mathrm{TY} + TA_{\mathrm{FZ}}Y,X) + \sin^2\theta\, g([X,Y],Z). \end{aligned} \tag{82}$$

Hence,

$$\cos^2\theta\, g([X,Y],Z) = g(A_{\mathrm{FZ}}\mathrm{TY} + TA_{\mathrm{FZ}}Y,X), \tag{83}$$

which is the required result. □

Theorem 23. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then, the slant distribution D^θ is minimal if and only if the normal bundle is parallel and*

$$g\left(A_{\mathrm{FZ}}X + \sec^2\theta\, A_{\mathrm{FZ}}X + \sec^2\theta\tan\theta\, X(\theta)A_{\mathrm{FZ}}X,\mathrm{TX}\right) = 0, \tag{84}$$

for any $X \in \Gamma(D^\theta)$ and $Z \in \Gamma(D^\perp)$.

Proof. For any $X \in \Gamma(D^\theta)$ and $Z \in \Gamma(D^\perp)$, we have

$$g(\nabla_X X + \nabla_{\sec\theta\mathrm{TX}}\sec\theta TX, Z) = g(\widetilde{\nabla}_X X, Z) + g(\widetilde{\nabla}_{\sec\theta\mathrm{TX}}\sec\theta\mathrm{TX}, Z). \tag{85}$$

Using (2), (6), and (20), we derive

$$\begin{aligned} g(\nabla_X X + \nabla_{\sec\theta\mathrm{TX}}\sec\theta TX, Z) &= g(\widetilde{\nabla}_X \mathrm{JX},\mathrm{JZ}) + \sec^2\theta\, g(\widetilde{\nabla}_X \mathrm{JX},\mathrm{JZ}) \\ &\quad + \sec^2\theta\tan\theta X(\theta) g(\widetilde{\nabla}_X JX, JZ) \\ &= g(\widetilde{\nabla}_X \mathrm{TX},\mathrm{FZ}) + g(\widetilde{\nabla}_X \mathrm{FX},\mathrm{FZ}) \\ &\quad + \sec^2\theta\, g(\widetilde{\nabla}_X \mathrm{TX},\mathrm{FZ}) + \sec^2\theta\, g(\widetilde{\nabla}_X \mathrm{FX},\mathrm{FZ}) \\ &\quad + \sec^2\theta\tan\theta X(\theta) g(\widetilde{\nabla}_X \mathrm{TX},\mathrm{FZ}) \\ &\quad + \sec^2\theta\tan\theta X(\theta) g(\widetilde{\nabla}_X \mathrm{FX},\mathrm{FZ}) \\ &= -g(\widetilde{\nabla}_X \mathrm{FZ},\mathrm{TX}) + g(\widetilde{\nabla}_X \mathrm{FX},\mathrm{FZ}) \\ &\quad - \sec^2\theta\, g(\widetilde{\nabla}_X \mathrm{FZ},\mathrm{TX}) + \sec^2\theta\, g(\widetilde{\nabla}_X \mathrm{FX},\mathrm{FZ}) \\ &\quad - \sec^2\theta\tan\theta X(\theta) g(\widetilde{\nabla}_X \mathrm{FZ},\mathrm{TX}) \\ &\quad + \sec^2\theta\tan\theta X(\theta) g(\widetilde{\nabla}_X \mathrm{FX},\mathrm{FZ}) \\ &= g(A_{\mathrm{FZ}}X,\mathrm{TX}) + g(\nabla^{\perp}_X \mathrm{FX},\mathrm{FZ}) \\ &\quad + \sec^2\theta\, g(A_{\mathrm{FZ}}X,\mathrm{TX}) + \sec^2\theta\, g(\nabla^{\perp}_X \mathrm{FX},\mathrm{FZ}) \\ &\quad + \sec^2\theta\tan\theta X(\theta) g(A_{\mathrm{FZ}}X,\mathrm{TX}) \\ &\quad + \sec^2\theta\tan\theta X(\theta) g(\nabla^{\perp}_X \mathrm{FX},\mathrm{FZ}). \end{aligned} \tag{86}$$

Hence, we conclude that D^θ is minimal if and only if the normal bundle is parallel and

$$g\left(A_{\mathrm{FZ}}X + \sec^2\theta A_{\mathrm{FZ}}X + \sec^2\theta\tan\theta X(\theta)A_{\mathrm{FZ}}X,\mathrm{TX}\right) = 0. \tag{87}$$

□

Definition 24. A pointwise hemislant submanifold M of a Kahler manifold $\widetilde{M}$ is said to be pointwise hemislant product if the distributions D^θ and $D^\perp$ are totally geodesic in M.

Theorem 25. *Let M be a pointwise hemislant submanifold of a Kahler manifold $\widetilde{M}$. Then, M is a pointwise hemislant* *product if and only if the second fundamental form of M satisfies the following condition:*

$$t\sigma(X, N) = 0, \tag{88}$$

for any $X \in \Gamma(D^{\theta})$ and $N \in \Gamma(TM)$.

Proof. For all $X, Y \in \Gamma(D^{\theta})$ and $Z, W \in \Gamma(D^{\perp})$, we have

$$g(\nabla_X Y, Z) = -g(\nabla_X Z, Y) = -g(\tilde{\nabla}_X Z, Y) = -g(\tilde{\nabla}_X JZ, JY). \tag{89}$$

Now, by (6), (9), (12), and (28), we have

$$\begin{aligned} g(\nabla_X Y, Z) &= -g(\tilde{\nabla}_X FZ, TY) - g(\tilde{\nabla}_X FZ, FY) \\ &= g(A_{FZ}X, TY) - g(\nabla_X^{\perp} FZ, FY) \\ &= g(A_{FZ}X, TY) - g((\nabla_X F)Z, FY) - g(F\nabla_X Z, FY). \end{aligned} \tag{90}$$

Thus, using (7), (18), and (22), we find

$$\begin{aligned} g(\nabla_X Y, Z) &= g(\sigma(X, TY), FZ) - g(f\sigma(X, Z), FY) - \sin^2\theta\, g(\nabla_X Z, Y) \\ &= g(\sigma(X, TY), FZ) + \sin^2\theta\, g(\nabla_X Y, Z), \end{aligned} \tag{91}$$

which gives that

$$\cos^2\theta\, g(\nabla_X Y, Z) = g(\sigma(X, TY), FZ) = -g(t\sigma(X, TY), Z). \tag{92}$$

By similar argument, we obtain

$$\begin{aligned} g(\nabla_W Z, X) &= -g(\tilde{\nabla}_W Z, X) = -g(\tilde{\nabla}_W X, Z) = -g(\tilde{\nabla}_W JX, JZ) \\ &= -g(\tilde{\nabla}_W TX, FZ) - g(\tilde{\nabla}_W FX, FZ) \\ &= -g(\nabla_W TX + \sigma(W, TX), FZ) - g(\nabla_W^{\perp} FX - A_{FX}W, FZ) \\ &= -g(\sigma(W, TX), FZ) - g(\nabla_W^{\perp} FX, FZ) \\ &= -g(\sigma(W, TX), FZ) - g((\nabla_W F)X, FZ) - g(F\nabla_W X, FZ) \\ &= -g(\sigma(W, TX), FZ) + g(\sigma(W, TX), FZ) \\ &\quad - g(f\sigma(W, X), FZ) - \sin^2\theta\, g(\nabla_W X, FZ) \\ &= -g(f\sigma(W, X), FZ) + \sin^2\theta\, g(\nabla_W Z, X). \end{aligned} \tag{93}$$

Hence,

$$\cos^2\theta\, g(\nabla_W Z, X) = -g(f\sigma(W, X), FZ) = g(t\sigma(W, X), Z). \tag{94}$$

So, from (92) and (94), we conclude that D^{θ} and $D^{\perp}$ are totally geodesic if and only if (74) is satisfied. □

Data Availability

No underlying data were collected or produced in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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