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Research Article

Stability Analysis of State Delay Multiagent Systems with Observer-Based Control Protocols

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The consistency problem of multiagent systems with the output feedback and state delay was considered in this paper. First, the reduced-order observer based on the consensus protocol of state delay is designed, and the consensus protocol is proposed by the output information of neighboring agents. Then, by constructing the Lyapunov–Krasovskii functional and combining matrix inequality and the Schur complement lemma, the asymptotic stability of the system can be obtained, and the consistency of state-delay multiagent systems is derived. Finally, the theoretical results are verified by a simulation example.

1. Introduction

Consistency problem of multiagent systems (MASs) has widely application, such as the applications into flocking [1], formation control [2] and multirobot cooperation [3]. Due to the limitation of communication and exchange of information in the MASs, time delays are inevitable, and many works have been studied in this problem [4–12]. However, in most practical systems, if state information cannot be obtained directly, and then the control protocols based on state information cannot be designed. To obtain the state information of the agent, an observer is proposed [13–17].

The delay phenomenon occurs frequently in MASs, which affects the performance of the system and even causes the system to be unsteady. The consensus problem with input time delay and state delays has been discussed in much literature; for example, in [4], a truncated prediction approach is proposed to deal with the consensus problem of Lipschitz nonlinear MASs with input delay. And the time-variant disturbance and communication delays were also studied in [5]. The form of the state delay of MASs can be divided into discrete-time delay and continuous-time delay. In [6], the input delay compensation problem for discrete-time systems is discussed, and a nested predictor feedback controller is designed. In addition, the state feedback and the output feedback protocols were presented in [7]. In [8], the author discussed the finite time domain optimal consistent control problem with unknown MASs and state delay. The consistency of MASs with and without time delay was realized based on the consensus protocol of the impulse observer and Lyapunov function in [9]. Moreover, the distributed state estimation of autonomous dynamic systems with arbitrary large communication and measurement delays are investigated in [10], and a distributed observer framework following low gain method was concerned. In [11, 12], according to the switched Lyapunov function method and the free weighting matrix technique, two delay dependent stability criteria for consistency control and fault estimation are derived, respectively.

In order to obtain the state of the agent, the observer was provided. Because reduced-order observers are more cost-effective than full-order observers, there are a lot of works to be done on how to design a reduced-order observer. In [13], full-order and reduced-order observers were designed based on the output information of adjacent agents. And a distributed reduced-order observer also was proposed in [14]. In [15], the consistency of continuous and discrete-time MASs was analyzed based on the consistency protocol and the algorithms. Moreover, when some key matrices are Hurwitz, the consensus problem of MASs was considered based on a new reduced order observer type dynamic output feedback protocol in [16]. In [17], a new reduced-order observer is used to modify the existing consistency protocol, which does not need the absolute output information of neighbor agents. Based on this foundation, the consistency of MASs with time delay based on reduced-order observers is also studied in [18–20].

From the above, we find that there are many methods to solve the consistency of MASs with input and output delay, and most of them use the reduced-order observers' method to discuss the consistency problem of MASs. However, the consistency of MASs with input and output delay can be converted into a problem without delay by some transformation in [21], this is different from case of state delay, but state delay is seldom considered, and state delay phenomenon exists widely in nature. Therefore, it is meaningful to research consensus problem with state delays, in view of this, on the basis of reference [18–20]. This paper focuses on the reduced-order observer-based consensus problem of the MASs with state delay. The primary contributions of this paper are as follows: (I) A reduced-order observer with state delay is designed to estimate the partial output of the MASs; control protocols are also established. (II) A theoretical stability analysis is proposed for the state-delay MASs with the help of the Lyapunov–Krasovskii function, linear matrix inequalities, and Schur complement lemma.

The rest of the paper is organized as follows: In Section 2, the mathematical model with the reduced-order observer and consensus control protocols is introduced. The consensus results and the stability criterion are derived in Section 3. In Section 4, a simulation example of the MASs with state delay is given. In Section 5, the conclusions are presented.

2. Problem Formulation and Preliminaries

In this section, we consider the continuous linear multiagent systems (MASs) with state delays, we suppose the system contain N agents, the dynamic system of the ith agent can be described as follows:

$$\begin{aligned}\dot{x}_i(t) &= Ax_i(t-\tau) + Bu_i(t),\\ y_i(t) &= Cx_i(t),\end{aligned} \tag{1}$$

where $x_i(t) \in R^n, u_i(t) \in R^p$, and $y_i(t) \in R^r$ denote the state vector, control input vector, and the output vector, respectively, $A \in R^{n\times n}, B \in R^{n\times p}$ and $C \in R^{r\times n}$ are the constant matrices, $\tau > 0$ is the state delay.

Since the process of studying the consistency of MASs, the state of each agent should be known completely. As we all know, the input and output of the system can be measured, but in many cases the output is incomplete known due to some factors (e.g. cost). Therefore, since their states cannot be obtained directly, a method of state reconstruction will be adopted, that is, by constructing the reduced-order observer to estimate the unknown output of MASs. In order to investigate the consensus of MASs (1) with the state delay, first, the reduced-order observer $s_i(t)$ need to be established, and second, the distributed consensus protocol $u_i(t)$ also need to be designed.

In the following, we need to make some assumptions.

Assumption 1. Let $D \in R^{(n-r)\times(n-r)}$, $E \in R^{(n-r)\times r}$ and $F \in R^{(n-r)\times n}$ satisfy:

(a) Characteristic equation: $\det(\lambda I - e^{-\lambda\tau}D) = 0$, has negative real roots, and (A, D) is controllable and observable.

(b) The equation $FA - DF = EC$ is hold.

(c) The matrix $\begin{bmatrix} C \\ F \end{bmatrix}$ is nonsingular.

Then, the reduced-order observer can be designed by

$$\dot{s}_i(t) = Ds_i(t-\tau) + Ey_i(t-\tau) + FBu_i(t), \tag{2}$$

where $s_i(t) \in R^{(n-r)}$ is the state vector of the observer, $D \in R^{(n-r)\times(n-r)}$, $E \in R^{(n-r)\times r}$ and $F \in R^{(n-r)\times n}$ are constant matrices.

Based on the above assumption, let $\overline{y}_i(t) = \begin{bmatrix} y_i(t) \\ \widehat{y}_i(t) \end{bmatrix}$ be the total output of MASs (1), where $\widehat{y}_i(t) = Fx_i(t)$ are defined as the unknown output of (1). And let $z_i(t) = s_i(t) - \widehat{y}_i(t)$ denote the errors of between the observer $s_i(t)$ and unknown output $\widehat{y}_i(t)$. Then, we have the result in the following.

Theorem 1. *Suppose that Assumption 1 be satisfied, then the errors $z_i(t)$ are asymptotic stability.*

Proof. From (1), the relation of between $\overline{y}_i(t)$ and $\widehat{y}_i(t)$ can be expressed as follows:

$$\overline{y}_i(t) = \begin{bmatrix} y_i(t) \\ \widehat{y}_i(t) \end{bmatrix} = \begin{bmatrix} Cx_i(t) \\ Fx_i(t) \end{bmatrix} = \begin{bmatrix} C \\ F \end{bmatrix} x_i(t), \tag{3}$$

where $F \in R^{(n-r)\times n}$, and C, F satisfy Assumption 1 (c), it is observed from (3) that

$$\widehat{y}_i(t) = Fx_i(t), \tag{4}$$

differentiating both sides of (4) respect to t, one has

$$\dot{\widehat{y}}_i(t) = F\dot{x}_i(t) = FAx_i(t-\tau) + FBu_i(t). \tag{5}$$

Then, by taking the difference between (2) and (5), and according to (1) and (4), we have

$$\dot{s}_i(t) - \dot{\widehat{y}}_i(t) = D\big(s_i(t-\tau) - \widehat{y}_i(t-\tau)\big) + (DF + EC - FA)x_i(t-\tau), \tag{6}$$

because of $z_i(t) = s_i(t) - \widehat{y}_i(t)$, we can rewrite (6) as follows:

$$\dot{z}_i(t) = Dz_i(t-\tau) + (DF + EC - FA)x_i(t-\tau). \tag{7}$$

According to Assumption 1, it is known that $z_i(t)$ is asymptotically stable, hence, Theorem 1 is proved. □

From MASs (1), the distributed control protocol can be designed by

$$u_i(t) = cKG_1 \sum_{j\in N_i} a_{ij}\big(y_i(t) - y_j(t)\big) + cKG_2 \sum_{j\in N_i} a_{ij}\big(s_i(t) - s_j(t)\big), \tag{8}$$

where $G_1 \in R^{n\times r}, G_2 \in R^{n\times(n-r)}$ and $\begin{bmatrix} G_1 & G_2 \end{bmatrix} = \begin{bmatrix} C \\ F \end{bmatrix}^{-1}$, $K \in R^{p\times n}$ is the feedback gain matrix, c is the coupling strength.

By substituting the control protocol (8) into (1) and (2), we get

$$\begin{aligned}\dot{x}_i(t) = {} & Ax_i(t-\tau) + cBKG_1 \sum_{j\in N_i} a_{ij}\big(y_i(t) - y_j(t)\big) \\ & + cBKG_2 \sum_{j\in N_i} a_{ij}\big(s_i(t) - s_j(t)\big),\end{aligned} \tag{9}$$

$$\begin{aligned}\dot{s}_i(t) = {} & Ds_i(t-\tau) + ECx_i(t-\tau) + cFBKG_1 \sum_{j\in N_i} a_{ij}\big(y_i(t) - y_j(t)\big) \\ & + cFBKG_2 \sum_{j\in N_i} a_{ij}\big(s_i(t) - s_j(t)\big).\end{aligned} \tag{10}$$

For simplicity of use, we define the augmented vector as follows:

$$\gamma_i(t) = \left[x_i^T(t) s_i^T(t)\right]^T, \gamma(t) = \left[\gamma_1^T(t)\gamma_2^T(t)\ldots\gamma_N^T(t)\right]^T. \tag{11}$$

Taking advantage of the Kronecker product of matrix, the dynamics system (1) consisting of (9) and (10) can be written as follows:

$$\dot{\gamma}(t) = (cL\otimes M_1)\gamma(t) + (I_N \otimes M_2)\gamma(t-\tau), \tag{12}$$

where $L \in R^N$ is the Laplacian matrix of the MASs, and

$$M_1 = \begin{bmatrix} BKG_1C & BKG_2 \\ FBKG_1C & FBKG_2 \end{bmatrix}, M_2 = \begin{bmatrix} A & 0 \\ EC & D \end{bmatrix}. \tag{13}$$

In order to consider stability of (12), some definition and lemmas are presented in the following.

Definition 1 (see [22]). The system (1) is said to be consistent with respect to U, if exists $u_i(t) \in U$ such that,

$$\lim_{t\longrightarrow\infty} \left\| x_j(t) - x_i(t) \right\| = 0, \quad i, j = 1, 2, \ldots, N, \tag{14}$$

hold, for any initial value $x_i(0)$.

Lemma 1 (see [23]). *Let* $A \in R^{m\times n}, B \in R^{p\times q}, C \in R^{r\times s}, D \in R^{k\times h}$, *then the Kronecker product has the following properties:*

(1) $(A+B)\otimes C = A\otimes C + B\otimes C$;

(2) $(A\otimes B)(C\otimes D) = (AC)\otimes(B\ D)$;

(3) $(A\otimes B)^T = A^T \otimes B^T$.

Lemma 2 (see [24]) (Schur complement property). *Let* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$ *be symmetric matrix, and* S_{11}, S_{12}, S_{22} *be the block matrix,* S_{11} *be a square matrix, the following conditions are equivalent:*

(1) $S < 0$;

(2) $S_{22} < 0, S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$;

(3) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$.

From the above lemma, we have the following theorem.

Theorem 2. *Suppose that* $a(t), b(t)$ *are vectors, and* Ω, X, Y, Z *are matrices, if* $\begin{bmatrix} I_N\otimes X & I_N\otimes Y \\ I_N\otimes Y^T & I_N\otimes Z \end{bmatrix} \geq 0$, *then the following inequality holds*

$$-2a^T(t)(I_N\otimes\Omega)b(t) \leq \begin{bmatrix} a^T(t) \\ b^T(t) \end{bmatrix}^T \begin{bmatrix} I_N\otimes X & I_N\otimes(Y-\Omega) \\ I_N\otimes(Y-\Omega)^T & I_N\otimes Z \end{bmatrix} \begin{bmatrix} a(t) \\ b(t) \end{bmatrix}. \tag{15}$$

Proof. From the right-hand side of (15), by means of the technique of matrix inequality and Lemma 1, it yields that

$$\begin{bmatrix} a^T(t) \\ b^T(t) \end{bmatrix}^T \begin{bmatrix} I_N \otimes X & I_N \otimes (Y-\Omega) \\ I_N \otimes (Y-\Omega)^T & I_N \otimes Z \end{bmatrix} \begin{bmatrix} a(t) \\ b(t) \end{bmatrix}$$
$$= \begin{bmatrix} a^T(t) \\ b^T(t) \end{bmatrix}^T \begin{bmatrix} I_N \otimes X & I_N \otimes Y \\ (I_N \otimes Y)^T & I_N \otimes Z \end{bmatrix} \begin{bmatrix} a(t) \\ b(t) \end{bmatrix}$$
$$- \begin{bmatrix} a^T(t) \\ b^T(t) \end{bmatrix}^T \begin{bmatrix} 0 & I_N \otimes \Omega \\ (I_N \otimes \Omega)^T & 0 \end{bmatrix} \begin{bmatrix} a(t) \\ b(t) \end{bmatrix}$$
$$= \begin{bmatrix} a^T(t) \\ b^T(t) \end{bmatrix}^T \begin{bmatrix} I_N \otimes X & I_N \otimes Y \\ (I_N \otimes Y)^T & I_N \otimes Z \end{bmatrix} \begin{bmatrix} a(t) \\ b(t) \end{bmatrix} - 2a^T(t)(I_N \otimes \Omega)b(t). \tag{16}$$

Since $\begin{bmatrix} I_N \otimes X & I_N \otimes Y \\ I_N \otimes Y^T & I_N \otimes Z \end{bmatrix} \geq 0$, the following inequality holds

$$\begin{bmatrix} a^T(t) \\ b^T(t) \end{bmatrix}^T \begin{bmatrix} I_N \otimes X & I_N \otimes Y \\ [I_N \otimes Y]^T & I_N \otimes Z \end{bmatrix} \begin{bmatrix} a(t) \\ b(t) \end{bmatrix} \geq 0. \tag{17}$$

Therefore, (15) is holds, this completes the proof of Theorem 2. □

3. Stability Analysis

In this section, the consensus of MASs (1) is obtained by stability analysis's result for (12).

Theorem 3. *Let P be positive definite matrix, Y be matrix, Q, X, Z be the symmetric matrices, if there exists $\begin{bmatrix} I_N \otimes X & I_N \otimes Y \\ I_N \otimes Y^T & I_N \otimes Z \end{bmatrix} \geq 0$, such that*

$$\begin{bmatrix} \Xi & I_N \otimes (PM_2 - Y) & \tau cL \otimes M_1^T Z \\ I_N \otimes (PM_2 - Y)^T & -I_N \otimes Q & \tau I_N \otimes M_2^T Z \\ \tau cL \otimes ZM_1 & \tau I_N \otimes ZM_2 & -\tau Z \end{bmatrix} \leq 0, \tag{18}$$

where $\Xi = cL \otimes (M_1^T P + PM_1) + I_N \otimes (Y + Y^T + \tau Z + Q)$, then the system (12) is asymptotically stable.

Proof. By the definition of Newton–Leibnitz formula in [25], we have $\gamma(t-\tau) = \gamma(t) - \int_{t-\tau}^{t} \dot{\gamma}(s)ds$, then (12) can be described

$$\dot{\gamma}(t) = (cL \otimes M_1 + I_N \otimes M_2)\gamma(t) - (I_N \otimes M_2) \int_{t-\tau}^{t} \dot{\gamma}(s)\mathrm{d}s. \tag{19}$$

Since matrices P, Q and Z satisfy (18), and the Lyapunov–Krasovskii function can be constructed as follows:

$$V(\gamma(t)) = V_1(\gamma(t)) + V_2(\gamma(t)) + V_3(\gamma(t)), \tag{20}$$

where

$$\begin{aligned} V_1(\gamma(t)) &= \gamma^T(t)(I_N \otimes P)\gamma(t), \\ V_2(\gamma(t)) &= \int_{-\tau}^{0} \int_{t+\varphi}^{t} \dot{\gamma}^T(s)(I_N \otimes Z)\dot{\gamma}(s)ds\, d\varphi, \\ V_3(\gamma(t)) &= \int_{t-\tau}^{t} \gamma^T(s)(I_N \otimes Q)\gamma(s)\mathrm{d}s. \end{aligned} \tag{21}$$

The next is to show that $\dot{V}(\gamma(t)) \leq 0$.

Differentiating the Lyapunov function $V_1(\gamma(t)) = \gamma^T(t)(I_N \otimes P)\gamma(t)$ with respect to t, and with (19), we have

$$\begin{aligned} \dot{V}_1(\gamma(t)) &= \dot{\gamma}^T(t)(I_N \otimes P)\gamma(t) + \gamma^T(t)(I_N \otimes P)\dot{\gamma}(t) \\ &= \left[(cL \otimes M_1 + I_N \otimes M_2)\gamma(t) - (I_N \otimes M_2)\int_{t-\tau}^{t} \dot{\gamma}(s)\mathrm{d}s\right]^T (I_N \otimes P)\gamma(t) \\ &\quad + \gamma^T(t)(I_N \otimes P)\left[(cL \otimes M_1 + I_N \otimes M_2)\gamma(t) - (I_N \otimes M_2)\int_{t-\tau}^{t} \dot{\gamma}(s)\mathrm{d}s\right] \\ &= 2\gamma^T(t)(I_N \otimes P)(cL \otimes M_1 + I_N \otimes M_2)\gamma(t) - \int_{t-\tau}^{t} 2\gamma^T(t)(I_N \otimes PM_2)\dot{\gamma}(s)\mathrm{d}s. \end{aligned} \tag{22}$$

Let $a(t) = \gamma(t), b(t) = \dot{\gamma}(s)$, and $\Omega = PM_2$, by Theorem 2, we can obtain

$$\begin{aligned}&-2\gamma^T(t)(I_N \otimes PM_2)\dot{\gamma}(s) \le \begin{bmatrix}\gamma^T(t)\\ \dot{\gamma}^T(s)\end{bmatrix}^T H \begin{bmatrix}\gamma(t)\\ \dot{\gamma}(s)\end{bmatrix}\\ &= \gamma^T(t)(I_N \otimes X)\gamma(t) + 2\gamma^T(t)[I_N \otimes (Y - PM_2)]\dot{\gamma}(s)\\ &\quad + \dot{\gamma}^T(s)(I_N \otimes Z)\dot{\gamma}(s),\end{aligned} \tag{23}$$

where

$$H = \begin{bmatrix}(I_N \otimes X) & (I_N \otimes (Y - PM_2))\\ (I_N \otimes (Y - PM_2))^T & I_N \otimes Z\end{bmatrix}. \tag{24}$$

The above inequality (23) can be integrated in the time interval $[0, \tau]$, and by substituting the result of the integration into (22), we obtain the inequality

$$\begin{aligned}\dot{V}_1(\gamma(t)) \le{}& \gamma^T(t)\left[cL \otimes \left(M_1^T P + PM_1\right) + I_N \otimes \left(Y + Y^T + \tau X\right)\right]\gamma(t)\\ &- 2\gamma^T(t)[I_N \otimes (Y - PM_2)]\gamma(t-\tau) + \int_{t-\tau}^{t} \dot{\gamma}(s)^T(s)(I_N \otimes Z)\dot{\gamma}(s)ds.\end{aligned} \tag{25}$$

From (12), we have the following formula

$$\begin{aligned}&\dot{\gamma}^T(t)(I_N \otimes \tau Z)\dot{\gamma}(t)\\ &= [(cL \otimes M_1)\gamma(t) + (I_N \otimes M_2)\gamma(t-\tau)]^T (I_N \otimes \tau Z)\\ &\quad \times [(cL \otimes (M_1)\gamma(t) + (I_N \otimes M_2)\gamma(t-\tau)]\\ &= \gamma^T(t)\left(\tau c^2 L^2 \otimes M_1^T Z M_1\right)\gamma(t) + \gamma^T(t)\left(\tau c L M_1^T Z M_2\right)\gamma(t-\tau)\\ &\quad + \gamma^T(t-\tau)\left(cL \otimes M_2^T Z M_1\right)\gamma(t) + \gamma^T(t-\tau)\left(\tau I_N \otimes M_2^T Z M_2\right)\gamma(t-\tau).\end{aligned} \tag{26}$$

Differentiating the Lyapunov function $V_2(\gamma(t))$ with respect to t, we have

$$\begin{aligned}\dot{V}_2(\gamma(t)) &= \left(\int_{-\tau}^{0}\int_{t+\varphi}^{t} \dot{\gamma}^T(s)(I_N \otimes Z)\dot{\gamma}(s)ds\, d\varphi\right)'\\ &= \int_{-\tau}^{0} \dot{\gamma}^T(t)(I_N \otimes Z)\dot{\gamma}(t) - \dot{\gamma}^T(t+\varphi)(I_N \otimes Z)\dot{\gamma}(t+\varphi)d\varphi\\ &= \dot{\gamma}^T(t)(I_N \otimes \tau Z)\dot{\gamma}(t) - \int_{t-\tau}^{t} \dot{\gamma}^T(s)(I_N \otimes Z)\dot{\gamma}(s)\mathrm{d}s.\end{aligned} \tag{27}$$

By substituting (26) into (27), (27) can be written as follows:

$$\begin{aligned}\dot{V}_2(\gamma(t)) ={}& \gamma^T(t)\left(\tau c^2 L^2 \otimes M_1^T Z M_1\right)\gamma(t) + 2\gamma^T(t)\left(\tau c L \otimes M_1^T Z M_2\right)\gamma(t-\tau)\\ &+ \gamma^T(t-\tau)\left(\tau I_N \otimes M_2^T Z M_2\right)\gamma(t-\tau) - \int_{t-\tau}^{t} \dot{\gamma}^T(s)(I_N \otimes Z)\dot{\gamma}(s)\mathrm{d}s.\end{aligned} \tag{28}$$

In the same way, the time-derivative of the Lyapunov function

$$V_3(\gamma(t)) = \int_{t-\tau}^{t} \gamma^T(s)(I_N \otimes Q)\gamma(s)\mathrm{d}s, \tag{29}$$

can be evaluated as follows:

$$\begin{aligned}\dot{V}_3 &= \left(\int_{t-\tau}^{t} \gamma^T(s)(I_N \otimes Q)\gamma(s)\mathrm{d}s\right)' \\ &= \gamma^T(t)(I_N \otimes Q)\gamma(t) - \gamma^T(t-\tau)(I_N \otimes Q)\gamma(t-\tau).\end{aligned} \tag{30}$$

Hence, it follows from (25), (28) and (30) that the Lyapunov–Krasovskii functions $\dot{V}(\gamma(t))$ satisfy the following equation:

$$\begin{aligned}\dot{V}(\gamma(t)) &= \dot{V}_1(\gamma(t)) + \dot{V}_2(\gamma(t)) + \dot{V}_3(\gamma(t)) \\ &= \gamma^T(t)\left[cL \otimes \left(M_1^T P + PM_1\right) + I_N \otimes \left(Y + Y^T + \tau Z + Q\right) + \tau c^2 L^2 \otimes M_1^T Z M_1\right]\gamma(t) \\ &\quad + 2\gamma^T(t)\left[\tau cL \otimes M_1^T Z M_1 - I_N \otimes (Y - PM_2)\right]\gamma(t-\tau) \\ &\quad + \gamma^T(t-\tau)\left(\tau I_N \otimes M_2^T Z M_2 - I_N \otimes Q\right)\gamma(t-\tau).\end{aligned} \tag{31}$$

For ease of notations, let $\Psi = [\gamma^T(t)\,\gamma^T(t-\tau)]$, and with $\Xi = cL \otimes (M_1^T P + PM_1) + I_N \otimes (Y + Y^T + \tau Z + Q)$. Then, from (31), we have $\dot{V}(\gamma(t)) = \Psi^T W \Psi$, where matrix W is given by

$$W = \begin{bmatrix} \Xi + \tau c^2 L^2 \otimes M_1^T Z M_1 & \tau cL \otimes M_1^T Z M_2 - I_N \otimes (Y - PM_2) \\ \left(\tau cL \otimes M_1^T Z M_2 - I_N \otimes (Y - PM_2)\right)^T & \tau I_N \otimes M_2^T Z M_2 - I_N \otimes Q \end{bmatrix}. \tag{32}$$

The next goal is to show that $W \leq 0$, by (32), we get

$$\begin{aligned}&\begin{bmatrix} \Xi + \tau c^2 L^2 \otimes M_1^T Z M_1 & \tau cL \otimes M_1^T Z M_2 - I_N \otimes (Y - PM_2) \\ \left(\tau cL \otimes M_1^T Z M_2 - I_N \otimes (Y - PM_2)\right)^T & \tau I_N \otimes M_2^T Z M_2 - I_N \otimes Q \end{bmatrix} \\ &= \begin{bmatrix} \Xi & I_N \otimes (PM_2 - Y) \\ I_N \otimes (PM_2 - Y)^T & -I_N \otimes Q \end{bmatrix} \\ &\quad + \begin{bmatrix} \tau c^2 L^2 \otimes M_1^T Z M_1 & \tau cL \otimes M_1^T Z M_2 \\ \tau cL \otimes M_2^T Z M_1 & \tau I_N \otimes M_2^T Z M_2 \end{bmatrix} \\ &= \begin{bmatrix} \Xi & I_N \otimes (PM_2 - Y) \\ I_N \otimes (PM_2 - Y)^T & -I_N \otimes Q \end{bmatrix} \\ &\quad - \begin{bmatrix} \tau cL \otimes M_1^T Z \\ \tau I_N \otimes M_2^T Z \end{bmatrix}\left(-\tau^{-1} Z^{-1}\right)\left[\tau cL \otimes M_1\ \tau I_N \otimes Z M_2\right],\end{aligned} \tag{33}$$

according to (33) and Lemma 2, thus yields

$$\begin{bmatrix} \Xi & I_N \otimes (PM_2 - Y) \\ I_N \otimes (PM_2 - Y)^T & -I_N \otimes Q \end{bmatrix} - \begin{bmatrix} \tau cL \otimes M_1^T Z \\ \tau I_N \otimes M_2^T Z \end{bmatrix}\left(-\tau^{-1} Z^{-1}\right)\left[\tau cL \otimes M_1 \quad \tau I_N \otimes Z M_2\right] \leq 0. \tag{34}$$

It is apparent from Lemma 2, the following matrix can be derived

$$\begin{bmatrix} \Xi & I_N \otimes (PM_2 - Y) & \tau cL \otimes M^T Z \\ I_N \otimes (PM_2 - Y)^T & -I_N \otimes Q & \tau I_N \otimes M_2^T Z \\ \tau cL \otimes Z M_1 & \tau I_N \otimes Z M_2 & -\tau Z \end{bmatrix} \leq 0. \tag{35}$$

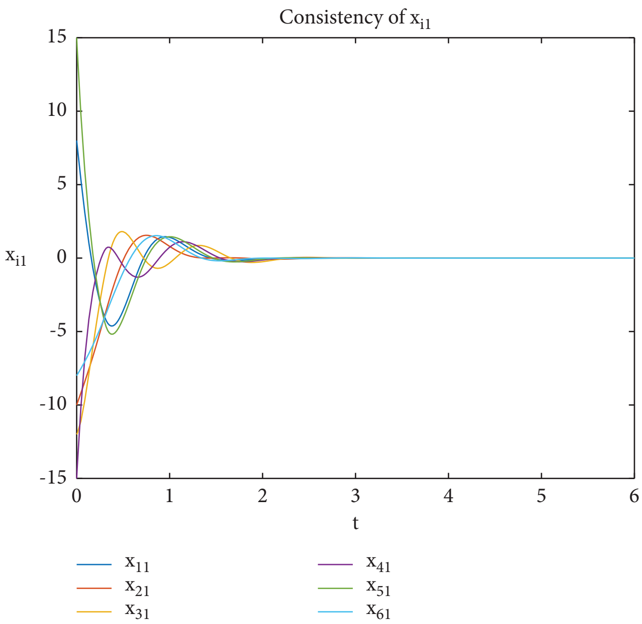

FIGURE 1: The evolution of states $x_{i1}(t)$ with $\tau = 0.3$.

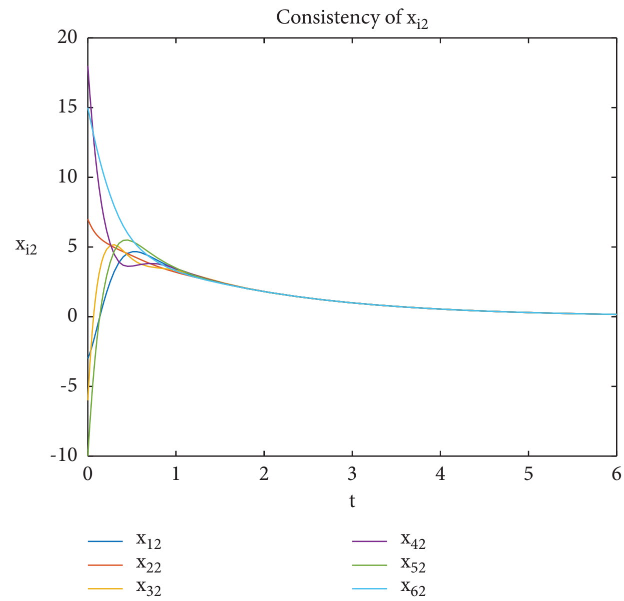

FIGURE 2: The evolution of states $x_{i2}(t)$ with $\tau = 0.3$.

Therefore, the system (12) is asymptotically stable, which completes the proof of Theorem 3. □

4. Numerical Simulation

In this section, an example is used to illustrate the theoretical results. Consider the network topology of the MASs with six agents, and suppose the directed weighted adjacency matrix is represented as follows:

$$A_1 = \begin{bmatrix} 0 & 4 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 3 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{36}$$

The dynamic system of each agent is as follows:

$$\begin{aligned} x_i(t) &= \begin{bmatrix} -2 & 0 \\ 0 & -\dfrac{1}{2} \end{bmatrix} x_i(t-\tau) + \begin{bmatrix} -1 \\ -1 \end{bmatrix} u_i(t), \\ y_i(t) &= \begin{bmatrix} \dfrac{3}{2} & 0 \end{bmatrix} x_i(t). \end{aligned} \tag{37}$$

The reduced-order observe of order 1 in (2) can be established to achieve consensus. Let

$$D = -\frac{1}{2}, E = -1, F = \begin{bmatrix} 1 & \dfrac{3}{2} \end{bmatrix}. \tag{38}$$

Then, D satisfies the characteristic equation: $\det(\lambda I - De^{\lambda\tau}) = 0$, and characteristic equation have a negative real part, it can easily be seen that F satisify the Assumption 1 (b), and $\begin{bmatrix} C \\ F \end{bmatrix}$ is invertible, then, for the convenience of calculation, choose the coupling strength $c = 2$, and the state delay parameter $\tau = 0.3$.

The initial state of 6 agents are given by $x_1(0) = \begin{bmatrix} 8 & -3 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} -10 & 7 \end{bmatrix}^T$, $x_3(0) = \begin{bmatrix} -12 & 6 \end{bmatrix}^T$, $x_4(0) = \begin{bmatrix} -15 & 18 \end{bmatrix}^T$,$x_5(0) = \begin{bmatrix} 15 & 10 \end{bmatrix}^T$, $x_6(0) = \begin{bmatrix} -8 & 15 \end{bmatrix}^T$.

Through the above parameters information, combined with the given theory, the consistency of the multiagent system can be realized, as shown in Figures 1 and 2.

In this example, the state of 6 agents is considered in two dimensions. In Figure 1, it is clear to see that the consensus of $x_{i1}(t)$ is achieved. Similarly, in Figure 2, we can also see that the state of $x_{i2}(t)$ also asymptotically tend to zero and remain stable.

5. Conclusions

The output feedback consistency problem for the MASs with state delay was studied. A reduced-order observer was introduced, and a distributed, consistent control protocol was designed. Then, the Lyapunov–Krasovskii function was constructed to analyze the stability of the system; the matrix inequality and Schur complement lemma were also used, which indirectly solve the consistency problem of MASs with state delay. Finally, the theoretical results were illustrated by a simulation example. In this paper, the time-varying delays, event triggering or switching topology were not considered. However, these issues are very interesting topics and deserve in further study.

Data Availability

All data used are included inside the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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