Hindawi
Journal of Mathematics
Volume 2023, Article ID 9996773, 7 pages
https://doi.org/10.1155/2023/9996773

Research Article

Novel Explosive and Super Fractional Nonlinear Schrödinger Structures

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Received 27 August 2023; Accepted 13 October 2023; Published 27 October 2023

Academic Editor: Mubashir Qayyum

The fractional nonlinear Schrödinger equation solutions have been investigated via fraction space-time derivative sense. We applied the unified technique for this model to extract new structures of waves. The fractional property structures were obtained from the model in form of hyperbolic, soliton, shock, explosive, superperiodic, and trigonometric structures. It was found that increasing fractal factors produces a change in the phase and wave frequency of propagating nonlinear waves. The physical models that explain tidal energy generation are crucial to the development of contemporary green power systems. The parametric description for wave characteristics in this process is created by the solution of nonlinear equations. The obtained solutions are applicable in new communications, energy applications, fractional quantum modes, and in science and complex phenomena in astrophysics. Finally, the proposed technique can be implemented for further fractional physical models.

1. Introduction

In recent years, mathematicians and physicists have become more interested in the exploration of complex wave propagation characterized by a certain type of nonlinear fractional partial differential equations (NFPDEs) [1–4]. These equations play an important role in describing the dynamical processes and physical phenomena in plasma physics, superconductivity, chemical engineering, biochemistry, ocean engineering, networked systems, fiber communications, industrial studies, and many other scientific fields [5–7]. As a result, it is crucial to investigate the exact wave solutions to NFPDEs when looking at nonlinear physical occurrences. Over the years, various sturdy analytical methods have been designed to examine the nature of solutions and comprehend the mechanics of nonlinear difficult phenomena [8–11].

The well-known Riemann–Liouville derivatives have recently been the starting point for numerous writers who have attempted to propose new operators acting on continuous functions. The reason for such broad generalizations is usually that the Riemann–Liouville fractional derivatives fail to account for some intriguing characteristics. For example, the Riemann–Liouville derivatives of a constant function are not zero, or they do not obey Leibniz's relation [12]. Actually, Tarasov [13] clarified that a violation of the Leibniz rule is one of the main characteristic properties of fractional derivatives. Indeed, the prospect of fractional calculus deserves some comments, because they are in contradiction with the classic results [14]. On the other hand, various operators in fractional calculus have been developed to explore NFPDEs, such as local fractional derivative, He's fractional derivative, Caputo's fractional derivatives, and Abel–Riemann fractional derivative [15–19]. The fractional calculus's physical explanation was introduced in [20, 21]. A vital conformable fractional derivative was developed by Khalil et al. [22]. This definition's effectiveness and simplicity have garnered considerable attention. This definition is consistent with the Riemann–Liouville and Caputo polynomial definitions (up to a constant multiple) [22]. As a result, many influential works have been produced using this new definition [3, 23–25]. The definition of the conformable fractional derivative is given in [22].

Definition 1 (see [22]). Let $\mathfrak{U}$: $(0,\infty) \longrightarrow \mathbb{R}$ be a function, and then, the α order of the conformable derivative of $\mathfrak{G}$ is

$$D_t^\alpha(\mathfrak{U}(t)) = \lim_{\kappa \longrightarrow 0} \frac{\mathfrak{U}\left(t+\kappa t^{1-\alpha}\right) - \mathfrak{U}(t)}{\kappa}, \quad t>0,\ 1 \geq \alpha > 0. \tag{1}$$

This definition satisfies the following:

(i) $D_t^\alpha(K_1\mathfrak{G} + K_2\mathfrak{Q}) = K_1 D_t^\delta(\mathfrak{U}) + K_2 D_t^\alpha(\mathfrak{Q}),\ K_1;\ K_2 \in \mathbb{R}$

(ii) $D_t^\alpha(t^\iota) = \iota t^{\iota-\alpha}, \iota \in \mathbb{R}$

(iii) $D_t^\alpha(\mathfrak{U}\mathfrak{Q}) = \mathfrak{Q} D_t^\alpha(\mathfrak{U}) + \mathfrak{U} D_t^\delta(\mathfrak{Q})$

(vi) $D_t^\alpha(\mathfrak{U}/\mathfrak{Q}) = \mathfrak{Q} D_t^\alpha(\mathfrak{U}) - \mathfrak{U} D_t^\alpha(\mathfrak{Q})/\mathfrak{Q}^2$

To know more about the characteristics of a conformable fractional definition, see [22].

Many methods have been used to solve nonlinear and fractional equations in order to find mathematical structures suitable for interpreting the phenomena described by these equations and also useful in cases of energy conversion that occurs in coupled systems, symmetric potentials, higher order nonlinearity in locked fiber laser modes, and femtosecond optical soliton. Examples of these methods are the Hirota bilinear method [26], similarity transformations [27], the square operator method [28], and the network method [29].

NFPDEs are suggested for comprehending and evaluating real-world models since their behavior is influenced by their past states. In recent years, the investigation of nonlinear fractional Schrödinger equations (NLFSEs) is very wonderful. This equation is extremely important in fractional quantum mechanics [30–32]. Darvishi et al. [33] developed three NLFSE equations as space-time fractional types, and they presented optical soliton solutions for these models using the sine-cosine approach. After that, Darvishi and Najafi [34] applied the semi-inverse variational principle to produce some new soliton solutions for these equations. This paper is concerned with the following NLFSE [33]:

$$iD_t^\alpha q + D_x^{2\beta} q - 2\mu |q|^2 q = 0, \quad 1 \geq \alpha, \beta > 0, \tag{2}$$

where $\mu \in \mathbb{R} - \{0\}$ and $q(x,t)$ is a complex valued function.

In fractal complex media such as space plasma, superconductors, chemical engineering, ocean waves, fiber communication, industrial applications, and many other scientific domains, NLFSEs are crucial for characterizing dynamical processes and physical phenomena. The energy and wave propagation in these media are impacted by fractal features in these complex modes, which may result in wave forcing, wave turbulence, and trapping [5–7, 33, 34]. Using Fourier spectral techniques, it is possible to study the stability of solutions for NLFSEs numerically [35, 36]. NLSE mass conservation and dispersion relations are maintained by this approach. Furthermore, research on the numerical instability of NLSEs and NLFSEs has been conducted [37–40].

In [41], we created a reliable solver approach to solve NFPDEs based on the Jacobian elliptic function approach [42, 43]. This solver explicitly offers the unified structure of solitary waves of different types of NFPDEs. It is also simple, dependable, and effective. This solver was used in this study to generate some novel solitary waves for equation (2). Namely, some fractional structures for this model are presented in the forms of hyperbolic, shock, soliton, explosive, superperiodic, and trigonometric structures. With regard to optical fiber communications [44, 45] and fractional quantum mechanics [30–32], the presented waves are essential for describing complex but crucial processes. To the best of our knowledge, this method has not yet been used in any scientific research.

The rest parts of this article are scheduled as follows: Section 2 introduces some new solitary waves for the three models of space-time fractional NLSE. Section 3 describes the physical interpretation for the obtained results. Finally, Section 4 gives a conclusion remark about the acquired results.

2. Optical Fiber Solitary Waves

Using the wave transformation [33], we obtain

$$q(x,t) = e^{i\varphi} Q(\xi), \xi = k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}, \varphi = r\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha}. \tag{3}$$

Substituting equation (3) into equation (2) yields $w = 2\mathrm{rk}$ and

$$\Gamma_1 Q'' + \Gamma_2 Q^3 + \Gamma_3 Q = 0, \tag{4}$$

where $\Gamma_1 = k^2, \Gamma_2 = -2\mu$ and $\Gamma_3 = c - r^2$.

The equation expressed in equation (4) is solved using many mathematical techniques [46–48].

In the light of the solver technique [41], the solutions of equation (2) are as follows.

Family 2

$$Q_1(x,t) = \pm \frac{k}{\sqrt{\mu}} \operatorname{msn}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right). \tag{5}$$

At $m \longrightarrow 1$, equation (5) converts to

$$Q_1(x,t) = \pm \frac{k}{\sqrt{\mu}} \tanh\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right). \tag{6}$$

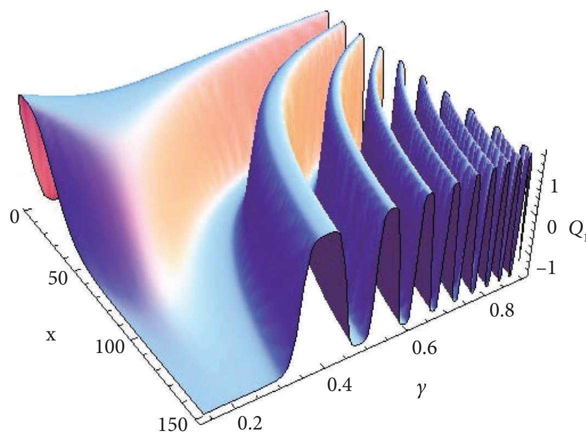

FIGURE 1: Graph of $Q_1(x,t)$ with x and γ.

Thus, the solutions of equation (2) are

$$q_1(x,t) = \pm \frac{k}{\sqrt{\mu}} e^{i\left(r\left(x^\beta/\beta\right) - c\left(t^\alpha/\alpha\right)\right)} \tanh\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right), \quad (7)$$

$c = r^2 + 2k^2$.

Family 3

$$\begin{aligned} Q_2(x,t) = {} & \pm \frac{k}{2\sqrt{\mu}} \mathrm{msn}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right) \\ & + i\frac{k}{2\sqrt{\mu}} \mathrm{mcn}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right). \end{aligned} \quad (8)$$

At $m \longrightarrow 1$, equation (8) converts to

$$\begin{aligned} Q_2(x,t) = {} & \pm \frac{k}{2\sqrt{\mu}} \tanh\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right) \\ & + i\frac{k}{2\sqrt{\mu}} \mathrm{sech}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right). \end{aligned} \quad (9)$$

Thus, the solutions of equation (2) are

$$q_2(x,t) = \left(\pm \frac{k}{2\sqrt{\mu}} \tanh\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right) + i\frac{k}{2\sqrt{\mu}} \mathrm{sech}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right)\right) e^{i\left(r\left(x^\beta/\beta\right) - c\left(t^\alpha/\alpha\right)\right)}, \quad (10)$$

$c = r^2 + 1/2k^2$.

Family 4

$$\begin{aligned} Q_3(x,t) = {} & \pm \frac{k}{2\sqrt{\mu}} \mathrm{msn}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right) \\ & - i\frac{k}{2\sqrt{\mu}} \mathrm{mcn}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right). \end{aligned} \quad (11)$$

At $m \longrightarrow 1$, equation (11) converts to

$$\begin{aligned} Q_3(x,t) = {} & \pm \frac{k}{2\sqrt{\mu}} \tanh\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right) \\ & - i\frac{k}{2\sqrt{\mu}} \mathrm{sech}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right). \end{aligned} \quad (12)$$

Thus, the solutions of equation (2) are

$$q_3(x,t) = \left(\pm \frac{k}{2\sqrt{\mu}} \tanh\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right) - i\frac{k}{2\sqrt{\mu}} \mathrm{sech}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right)\right) e^{i\left(r\left(x^\beta/\beta\right) - c\left(t^\alpha/\alpha\right)\right)}, \quad (13)$$

$c = r^2 + 1/2k^2$.

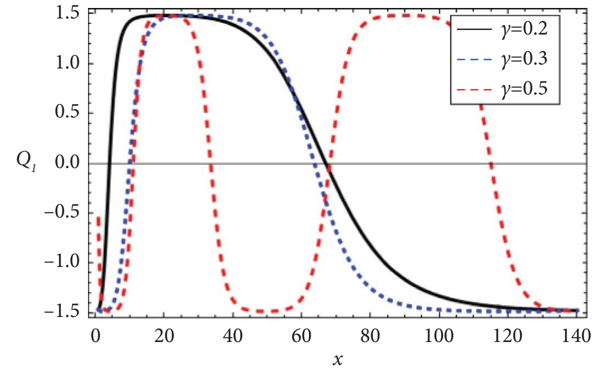

FIGURE 2: Graph of $Q_1(x,t)$ with x and different values of γ.

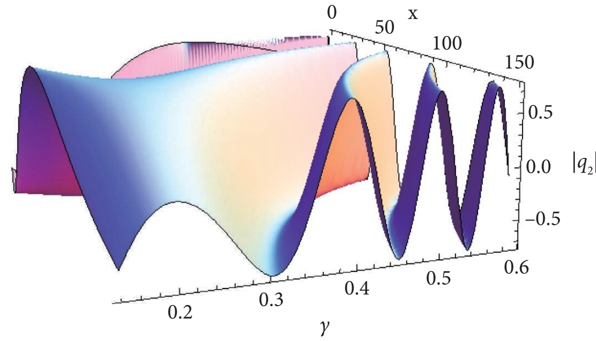

FIGURE 5: Graph of $q_2(x,t)$ with x and γ.

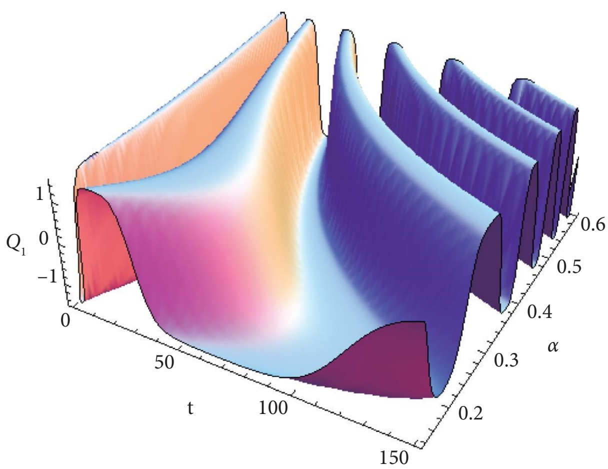

FIGURE 3: Graph of $Q_1(x,t)$ with t and α.

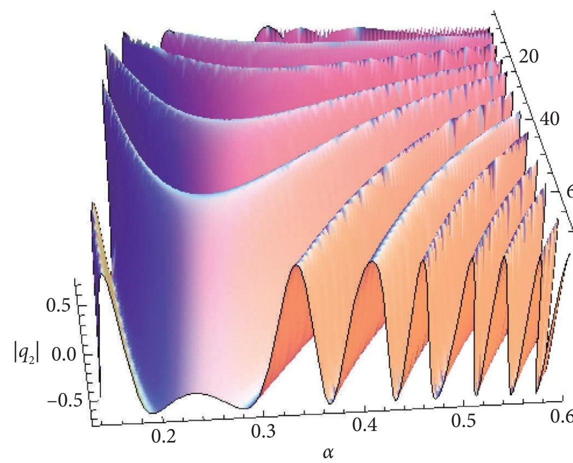

FIGURE 6: Graph of $q_2(x,t)$ with t and α.

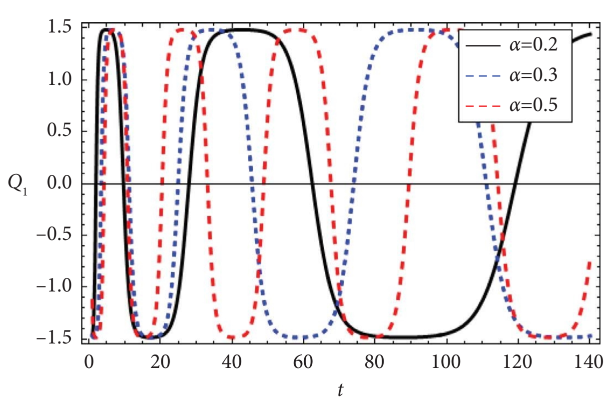

FIGURE 4: Graph of $Q_1(x,t)$ with t and different values of α.

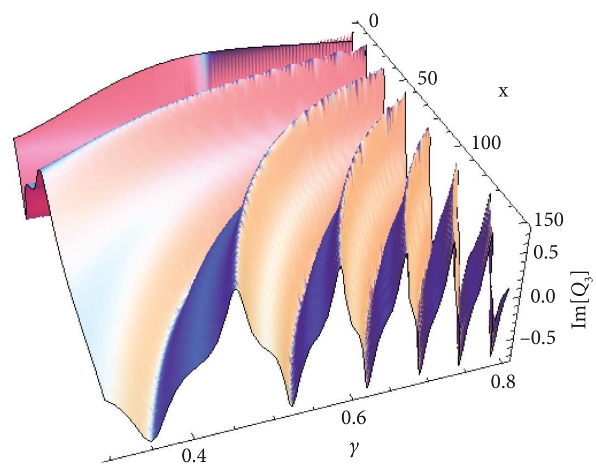

FIGURE 7: Graph of $\mathrm{Im}Q_3(x,t)$ with x and γ.

Family 5

$$Q_4(x,t) = \pm i\frac{k}{\sqrt{\mu}}\mathrm{mcn}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right). \tag{14}$$

At $m \longrightarrow 1$, equation (14) converts to

$$q_4(x,t) = \pm i\frac{k}{\sqrt{\mu}}\mathrm{sech}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right). \tag{15}$$

Thus, the solutions of equation (2) are

$$q_4(x,t) = \pm i\frac{k}{\sqrt{\mu}}e^{i\left(r\left(x^\beta/\beta\right) - c\left(t^\alpha/\alpha\right)\right)}\,\mathrm{sech}\left(k\frac{x^\beta}{\beta} - w\frac{t^\alpha}{\alpha}\right), \tag{16}$$

$c = r^2 - k^2$.

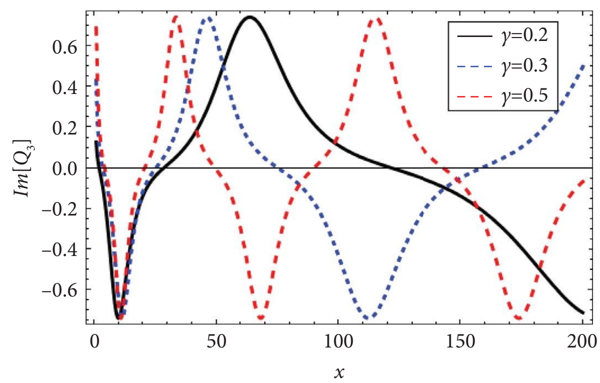

Figure 8: Graph of $\mathrm{Im}Q_3(x,t)$ with x and different values of γ.

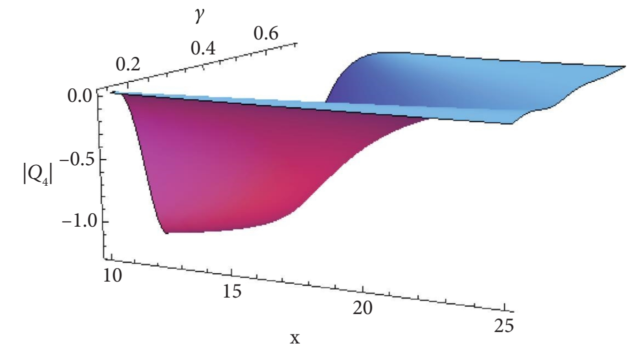

Figure 11: Graph of $|Q_4(x,t)|$ with x and γ.

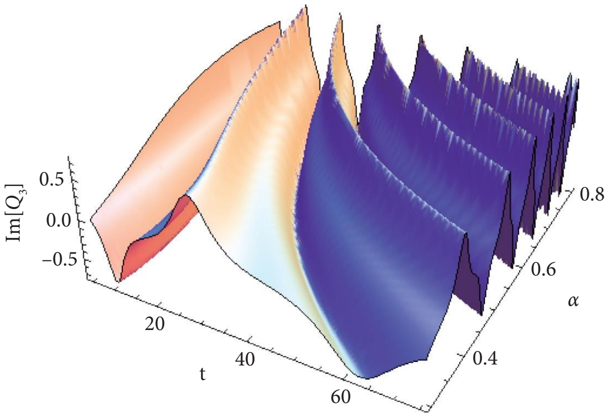

Figure 9: Graph of $\mathrm{Im}Q_3(x,t)$ with t and α.

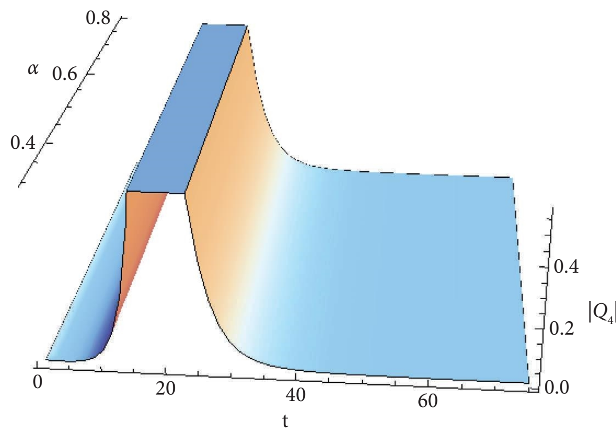

Figure 12: Graph of $|Q_4(x,t)|$ with t and α.

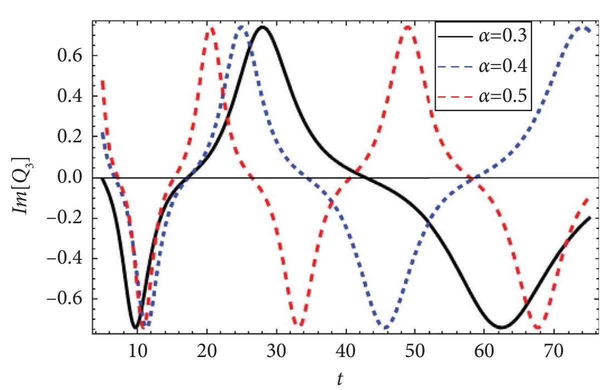

Figure 10: Graph of $\mathrm{Im}Q_3(x,t)$ with t and different values of α.

3. Results and Discussion

The NLS fractional approach discussed here is very crucial in the energy generation of various environmental physical modes. The fractional new NLS structures were derived using a novel analytical solver using mathematical and algebraic techniques. However, it would be more significant for us to examine at how fractal variables influence the features, geometries, and dynamics of nonlinear wave forms.

The output fractional solutions appears in forms of rational solitons, superperiodic shock, shock like-soliton, and rational, dispersive, and cnoidal waves. Equation (5) characterizes shock waves and supershock periodical waves as shown in Figures 1–4. In Figure 2, a shock wave is formed for $\gamma = 0.2$, but increasing γ generates a new super shock profile. The change of equation (5) with t and α is given in Figure 4, and the increasing of α causes an increase in the wave frequency of the produced wave. Variations of equation (10) with t, x, γ, α are characterized by periodic cnoidal and geometrical solitons with phase changes as shown in Figures 5 and 6. Figures 7–10 show the variations of Q_3 and Q_4 with t, x, γ, α. They represent an important wave form called super periodic solitary formations. For $\gamma = 0.2$ and $\alpha = 0.3$, the deformed supersolitons have been obtained, and by raising γ and α, the periodic form has been generated as shown in Figures 8 and 10. The change in equation (16) for q_4 with t, x, γ, α produces a geometrical shock wave with x, γ and rational shock formation with t, α as plotted in Figures 11 and 12.

The authors in [33, 34] obtained the sech-type and bright solitons for pace-time fractional NLSE and other forms of dark, bright, and singular solutions of the same space-time fractional model. In our work, new shock rationals, super shock, and supersolitons have been produced for the fractional space-time NLS model. From a physical point of view, our results are important for wave energy applications. For special cases of m values, some of our results reduce to solutions given in [33, 34].

Finally, the presented solutions of this work admit a very characteristic prospect. Namely, the various transformation formulae of elliptic functions can be used to relate these solutions ([49], Chapter 16). Actually, this technique will generate vital families of elliptic functions.

4. Conclusions

New rational shocks, super shocks, and periodic super-soliton generation have been produced for the fraction space-time NLS model. Wave structures, phase, and frequencies have been impacted by fractional time-space modulations. The fractional model mechanisms in energy generation over many physical modes are applicable in many communication and new engineering aspects. Our findings may be utilized to enhance fluid models that explain tidal lagoons, which are a potential new model for tidal power production research.

Data Availability

No data were used to support this research.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Authors' Contributions

All the authors equally contributed to this work.

Acknowledgments

The authors extend their appreciation to Prince Sattam Bin Abdulaziz University for funding this research work through the project number (PSAU/33803/01/2023).

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