Research Article

Thermal Analysis of a Casson Boundary Layer Flow over a Penetrable Stretching Porous Wedge

Dur-e-Shehwar Sagheer, Mohammad Alqudah, Nawal A. Alshehri, M. Sabeel Khan, M. Asif Memon, R. Shehzad, and Amsalu Fenta

1Department of Mathematics, Capital University of Science and Technology, Islamabad 44000, Pakistan
2Department of Basic Sciences, School of Electrical Engineering & Information Technology, German Jordanian University, Amman 11180, Jordan
3Department of Mathematics and Statistics, College of Science, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia
4Department of Mathematics and Social Sciences, Sukkur IBA University, Sukkur 65200, Sindh, Pakistan
5Department of Physics, Mizan Tepi University, P.O. Box 121, Tepi, Ethiopia

Correspondence should be addressed to Amsalu Fenta; amsalu.fenta09@gmail.com

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This work aims to analyze the Casson thermal boundary layer flow over an expanding wedge in a porous medium with convective boundary conditions and ohmic heating. Moreover, the effects of porosity and viscous dissipation are studied in detail and included in the analysis. The importance of this study is due to its applications in biomedical engineering where the analysis of behavior of non-Newtonian blood flow in arteries and veins is desired. Within the context of blood flow, it is also applicable to many other fields, for instance, radiative therapy, MHD generators, soil machines, melt-spinning, and insulation processes. The modeled problem is a set of PDEs, which is nondimensionalized to derive a nonlinear boundary value problem (BVP). The obtained BVP is solved using the shooting technique, endowed with the order four Runge-Kutta and Newton methods. The impact of different parameters on the momentum and temperature fields is investigated along with two important parameters of physical significance, i.e., the Nusselt number and the surface drag force. Results are validated, and an excellent agreement is seen for the parameters of interest using MATLAB built-in function bvp4c. A significant finding is that by increasing the Casson liquid parameter, the velocity decreases as the wedge expands quicker than the free stream velocity at \( R = 2 \). However, the velocity increases for the case when \( R = 0.1 \). A decrease in the Darcy number increases the temperature profile. Furthermore, the convective parameter accelerated the heat transmission rate, and a rise in the Prandtl number thickens the thermal boundary layer. The findings of this investigation contribute to problems in fluid dynamics and heat transfer that involve studying the behavior of a non-Newtonian fluid with Casson rheological properties near a solid porous wedge surface.

1. Introduction

Mixed convection across an impermeable or permeable wedge is used in many manufacturing processes, including extruding molten polymers, fabricating plastic sheets for solar energy, and storing thermal energy. According to Shah et al. [1], mixed convection is a fluid flow phenomenon characterized by the simultaneous effect of forced convection, which is caused by external forces such as a pump or a fan, and natural convection, which is caused by buoyant forces caused by temperature gradients within the fluid. Systems such as technical equipment and natural phenomena such as the movement of temperature stratified masses of air and water on Earth exhibit mixed convection flow [2, 3]. It has uses in heat exchangers [4], thermal design, cooling electronic components, and other fields. It is also pertinent to heat transfer and fluid flow analysis. Karwe and Jaluria [5, 6] explained that the investigation is necessary to check the quality of resulting products and cooling rates. Ishak et al. [2] investigated the mixed convection
magnetohydrodynamics boundary layer across a vertically expanding sheet while the wall temperature remained unchanged. The reader is referred to [1, 7, 8] for a good overview on the mixed convection MHD viscoelastic fluid flow analysis.

Some recent studies have also considered the mixed convective flow in analyzing the porous flow structures. In this connection, thermal analysis in a higher grade Forchheimer porous nanostructure bounded between non-isothermal plates is investigated by Saleem et al. [9]. They utilized a finite difference approach in their analysis. The transient problem of mixed convection over a vertically moving wedge was examined by Ravindran et al. [10]. Numerical results show that the buoyant force enhances the skin friction and Nusselt number. The thermal properties of magnetohydrodynamics of Falkner–Skan Casson flow sideways a moving wedge are analyzed by Ullah et al. [11]. They used finite difference and quasilinearization techniques to derive the problem's solution. It is detected that an augmentation in the values of the Eckert number has a negligible impact on force convection flow. They discovered that the existence of the suction or injection changes the rates of skin friction, concentration, and heat transfer. Unsteady flow with mixed convection dusty flow across a vertical wedge was investigated by Hossain et al. [3].

Many of the above-described studies addressed the flow convection over a vertical porous wedge. It is worth mentioning that there is limited research about convection flow in the presence of suction/injection, viscous and ohmic dissipation upon a permeable/impermeable wedge. The numerical solution of thermal radiation influence on magnetohydrodynamics with forced convective flow adjacent to a wedge with a heat source/sink [12] was computed by Chamkha et al. [13] employing the implicit finite difference technique. Anwar Hossain et al. [14] have explored the problem of MHD flow via a wedge with variable surface temperature. Additionally, perturbation solutions for different values of τ dimensions of time are discovered. Su et al. [15] investigated the MHD mixed convective flow of heat transfer across a permeable stretched wedge using numerical and analytic (DTM) methods. Also, the authors demonstrated that as the domain becomes unbounded, the DTM-BF solutions diverge. The influence of chemical reaction on the mixed convection magnetohydrodynamics and heat transfer across a porous wedge was investigated by Kandasamy et al. [16]. This research concludes that in a mixed convective regime, there is an increase in the hydrodynamic layer. In contrast, the concentration and temperature boundary layers decrease by lowering the values of the suction parameter. The issue of MHD flow through a porous nonisothermal wedge was examined by Prasad et al. [17]. They observed that among all variables, the mixed convection parameter substantially impacts the surface drag force, thermal rate, normal velocity of the wall, and temperature profile of the MHD flow through a porous nonisothermal wedge. Transient Falkner–Skan liquid flow of Carreau microliquid via a still/moving wedge was demonstrated by Khan et al. [18].

Due to the numerous applications in radiative therapy, metal casting technology, and ceramic engineering, several studies [4, 19] have explored the influence of radiative effect at the boundary layer with the transfer of heat characteristics. In biomedical engineering, within the context of blood flow [20, 21], the Casson parameter has been considered in analyzing the behavior of non-Newtonian flow in arteries [22–24]. For a detailed review of the use and applications of Casson nanofluid flow, the reader is also referred to the work of Nayak et al. [25] who have investigated the Casson nanofluid flow subjected to biomedicine applications and chemical reactions [26–28]. For some other important applications related to heat transfer enhancement, the reader is referred to the work of [29] and references therein. Ullah et al. [30] have investigated the Casson liquid flow across a nonstationary wedge using the effect of thermophoresis and Brownian diffusion. The transportation of heat for homogeneous/heterogeneous Casson liquid in a porous medium was studied by Bilal et al. [31]. The velocity, temperature, and concentration of solutions for homogeneous and heterogeneous reactions are fundamental limitations of the basic subsidized flow parameters. Several researchers investigated the impact of heat radiation on different flow problems [32–34] using finite element strategies.

Viscous dissipation can significantly affect the temperature distribution and overall energy balance of the system. Similarly, porous media have a large contact surface with fluids, which can significantly increase the heat transfer effect. The porous medium changes the flow field conditions, and the conduction heat transfer coefficient is usually higher than that of the fluid studied. The main aim of this study is to extend the work of Hussain et al. [35] by considering the following additional physical effects:

(i) The Casson thermal boundary layer flow rheology is assumed
(ii) The effects of porosity and viscous dissipation are taken into account

In this respect, two important parameters of physical interest arise are the viscous dissipation parameter $\varepsilon_r$ and the porosity parameter $D_n$. The specific goals and objectives of the study are to investigate the following research questions:

(i) How does a porous wedge flow's thermal reaction to Casson flow rheology behave?
(ii) How does the porosity affect the velocity and thermal profiles in the presence of Casson rheology?
(iii) How does the viscous dissipation affect the hydrodynamic profiles of the present rheology?
(iv) How do the viscous dissipation and porosity of the Casson fluid affect the skin-friction coefficient and Nusselt number?

The next section presents the modeling aspects of the modeled problem which is first transformed to a boundary value problem in ordinary differential equations. The problem is then solved numerically by the shooting method, and obtained results are discussed.
2. Mathematical Modeling

This problem considers an electrically conducting thermal radiative 2D Casson fluid flow with mixed convection on a moving wedge as shown in Figure 1. The conservation equations for mass, momentum, and energy for viscous incompressible fluid in the presence of the viscous dissipation, Rosseland radiation, and porosity constitute the

\[ \rho C_p (V \cdot V)T = \alpha V^2 T + \alpha \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] + \frac{\partial}{\partial y} \left( \frac{\rho_f}{C_p} \frac{du}{dy} \right). \]  

where in equation (2), the symbol \( S \) represents the stress tensor of the considered fluid, which is represented by

\[ S = -\nabla p + \tau - \sigma B^2 V. \]

The rheological equation for the incompressible flow of the Casson fluid is given by [37]

\[ \tau_{ij} = \begin{cases} 
2\left( \mu + \frac{p_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c, \\
2\left( \mu + \frac{p_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi < \pi_c,
\end{cases} \]

where \( e_{ij} \) are the components of the strain tensor defined as

\[ e = \frac{1}{2}(\nabla V + \nabla V^T). \]

In equation (5), \( \pi \) is the product of the component of the deformation rate with itself, \( \pi = e_{ij}e_{ij} \), \( \pi_c \) is a critical value of this product based on the non-Newtonian model, and \( p_y \) is the yield stress of the fluid. The Cartesian coordinates’ system is denoted by \((x, y)\) from which \( x \)-direction is along the wedge and \( y \)-direction is normal. The velocity of the wedge is represented by \( U_{oxm} \) and the velocity that is distant from the wedge is \( U \), which depends on \( x \), i.e., \( U = U_{oxm} \). The stretching wedge has suction and injection velocity, denoted by \( v_w \). During the heating of the wedge, a hot fluid with temperature \( T_f \) produces a high heat transfer coefficient \( h_f \), where \( T_{cco} \) is the temperature of the fluid away from the wedge. The uniform magnetic field is denoted by \( B \), acting along \( y \)-direction and equivalent to \( B = B_0 x^{m-1/2} \). The wedge angle is denoted by \( \Omega = \beta \pi \).

2.1. Dimensional Equations. Connected flow equations with boundary conditions are (see for reference [15])

\[ u_x + v_y = 0, \]

\[ uu_x + uu_y = U \frac{dU}{dx} + \gamma \left( 1 + \frac{1}{\delta} \right) u_{yy} + g\beta_p \left( T - T_{cco} \right) \sin \frac{\Omega}{2} - \frac{\sigma B^2}{\rho} (u - U) \]

\[ \frac{v}{\kappa} = \frac{\partial U}{\partial y}, \]

\[ \rho C_p (u T_x + v T_y) = -u \left( \rho U \frac{dU}{dx} + \alpha B^2 U \right) + \alpha T_{yy} \left( q_r \right)_y + \sigma B^2 u^2 + \left( 1 + \frac{1}{\delta} \right) \frac{\mu}{\rho C_p} (u_y)^2. \]

The boundary conditions are described as follows:

\[ u_{l(x,0)} = RU, \]  
\[ v_{l(x,0)} = v_w = -\left( \nu U_{oxm} \right)^{1/2} C \left( \frac{m + 1}{2} \right) x^{m+1/2}, \]

\[ T_{y,l(x,0)} = \frac{h}{\kappa} (T - T_f). \]

For the physical description of these boundary conditions, the reader is referred to the work of Su et al. [15]. Here, \( v, g, \beta, \alpha, \rho, C_p, \) and \( \sigma \) denote the kinematic viscosity, gravity field, coefficient of thermal expansion, conductivity of fluid due to electric current, fluid density, heat capacity, and thermal conductivity, respectively. The term \( \partial q_r / \partial y \) here \( q_r \) is radiative heat flux and is equal to \(-4{\sigma \epsilon}/(3\kappa) \partial T^4 / \partial y \), where
\( \sigma_o \) and \( k^* \) are known as the Stefan–Boltzman parameter and mean absorption coefficient, respectively. \( \beta = \mu_B \sqrt{2\pi_\gamma / \rho_y} \) is the parameter of Casson fluid.

2.2. Similarity Transformations. Here, in this subsection, the similar similarity transformations are introduced. Similar transformations are employed [15] to simplify the set of governing equations of a complex physical problem by converting them dimensionless and making them similar in structure.

\[
\begin{align*}
\psi &= \sqrt{U} \nu x f, \\
u &= \frac{\partial \psi}{\partial y}, \\
v &= \frac{\partial \psi}{\partial x} \\
\eta &= \sqrt{\frac{U_o}{v}} \alpha^{-1/2} y, \\
\theta(\eta) &= \frac{T - T_{\infty}}{T_f - T_{\infty}}
\end{align*}
\]

These are particularly useful for analyzing the flow and heat transfer phenomenon scales with changing parameters like Reynolds and Nusselt numbers. There are a lot of situations when one needs to introduce nonsimilar transformations [38] to convert this physical system into a nondimensional form. Such situations arise when non-standard boundary conditions [39–41] are introduced in the mathematical description of the flow problems.

2.3. Nondimensional Equations. Equations (8)–(13) are nondimensionalized using transformations in Section 2.2. The resulting equations take the form:

\[
f'' = \left( \frac{\delta}{1 + \delta} \right) \left[ -\frac{m + 1}{2} \Omega'' - (m + M) + (m + \alpha_i) f'' + (M + (Da)^{-1}) f'' \right] - \lambda \sin \frac{\Omega}{2} \theta'
\]

\[
\theta'' = \frac{-1}{(1 + Nr)} \left[ Pr \left( \frac{m + 1}{2} \right) f' \theta'' - \text{MeCPr} f'' - \text{MeCPr} f'' + \text{MeCPr} f''^2 \right] + \left( 1 + \frac{1}{\delta} \right) \text{PrEcf} f''',
\]

Figure 1: Geometry of the problem.
along with the dimensionless BCs as
\[
\begin{align*}
\theta' (0) &= -B_1 + B_2 \theta(0), \theta(\omega) = 0, \\
f' (0) &= R, f'(\omega) = 1, f(0) = C \\
\end{align*}
\] (17)

2.4. Dimensionless Parameters. The associated dimensionless parameters are
\[
\begin{align*}
M &= \frac{\sigma B^2}{\rho U_o}, \lambda = \frac{Gr_x}{Re_x^2}, Gr_x = \frac{\beta_0 (T_f - T_\infty) x^3}{\nu^2}, \\
Re_x &= \frac{U x}{\nu}, Nr = \frac{16 T_\infty^3 \beta_\alpha}{3 K^* \alpha}, Ec = \frac{U^2}{c_p (T_f - T_\infty)}, \\
Pr &= \frac{\mu c_p}{\alpha} B_i = \frac{h}{k} \sqrt{\frac{\nu}{U}}, Da^{-1} = \frac{\Phi}{k U_o x^{m-r}}, \alpha_1 = \frac{C_\alpha \Phi x}{\sqrt{k}}.
\end{align*}
\] (18)

3. Quantities of Interest

3.1. Surface Drag Force. The dimensional form of the skin friction is as follows:
\[
C_f = \frac{\mu_B (1 + \frac{1}{2} \sqrt{2}) \partial u/\partial y|_{y=0}}{\rho U^2},
\] (19)

where \(\mu_B\) represents the viscosity of the liquid and \(p_y\) is the liquid yield stress. The nondimensional form of surface drag force is given as
\[
(Re_x)^{1/2} C_f = \left(1 + \frac{1}{\delta}\right) f''(0).
\] (20)

3.2. Local Nusselt Number. The local Nusselt parameter is described as
\[
Nu_x = \frac{x q_w}{a(T_f - T_\infty)}
\] (21)

where \(q_w\) is the conductive heat flux from the surface of the wedge.
\[
(Re_x)^{-1/2} Nu_x = -\theta'(0).
\] (22)

4. Solution Method through Shooting and bvp4c

The above nonlinear ODEs are now solved through the shooting method. This numerical technique is used for finding the solution to a boundary value problem. Using the following derivatives, the boundary value problem is transformed into a first-order initial value problem (see Table 1).

Using the above notations, equations (15) and (16) are transformed into the following fifteen ODEs:

\[
y_1' = y_2; y_1(0) = C, \\
y_2' = y_3; y_2(0) = R, \\
y_3' = \frac{1}{1 + \delta} \left[ -\frac{m + 1}{2} y_1 y_3 - (m + M) + (m + \alpha_1) y_2^2 + (M + (Da)^{-1}) y_2 - \lambda \sin \frac{\Omega}{2} y_4 \right], \\
y_4(0) = s, \\
y_4' = y_5; y_4(0) = q, \\
y_5' = \frac{-1}{1 + \delta} \left[ Pr \left(\frac{m + 1}{2}\right) y_1 y_5 - M Ec P r y_2 - M Ec P r y_2^2 + M Ec P r^2 \right] + \left(1 + \frac{1}{\delta}\right) Pr Ec y_5^2, \\
y_5(0) = -Bi (1 - q), \\
y_6' = y_7; y_6(0) = 0, \\
y_7' = y_8; y_7(0) = 0,
\]
where $s$ and $q$ are the missing initial conditions. Choose the missing initial conditions that satisfy the above boundary conditions. The RK-4 method is applied to solve the first-order initial value problem. Newton’s numerical technique is used to refine the starting values. This iteration is repeated until the values meet the required accuracy, achieved by getting sufficiently close to the given boundary conditions. The stopping criteria for Newton’s method are given as follows:

$$\max(|\phi_1(\eta_{co}, s_n, q_n)|, |\phi_2(\eta_{co}, s_n, q_n)|) < \epsilon, \quad (24)$$

Here, $\epsilon = 10^{-8}$.

The solution of the above model boundary value problem can also be calculated through the built-in MATLAB function bvp4c. This built-in function is suitable for solving higher-order boundary value problems in fluid mechanics. The criteria behind the bvp4c function are the three-stage Lobatto IIIa formula, implemented by the finite difference algorithm [42]. This formula uses collocation, and the collocation polynomial offers a $C^1$-continuous solution that is consistently fourth-order accurate across the integration interval of the problem.

The residual of the continuous solution serves as the foundation for mesh selection and error control in the model problem. The integration interval is split into smaller intervals using the collocation technique and a mesh of points. The solver can arrive at a numerical solution by resolving a global system of algebraic equations caused by the boundary conditions and the collocation conditions placed on each subinterval. The solver then determines how much each subinterval’s numerical solution error is. The solver adjusts the mesh and restarts the procedure if the solution does not meet the tolerance conditions. In addition to an initial approximation of the solution at the mesh points, one must supply the initial mesh points.

5. Analysis of Results

This research is focused on analyzing the following aspects:

1. The steady nonlinear conducting radiative Casson flow through a penetrable wedge is examined with ohmic heating, thermal behavior, suction/injection, and convective BC.
(2) The effects of porosity and viscous dissipation are considered.

(3) Drag caused by the friction of a fluid against the surface and heat transfer due to pure conduction is examined by calculating the numerical values of the skin-friction coefficient and Nusselt number by varying the values of wedge angle \( \beta \) and the magnetic parameter \( M \).

(4) Thickness of the momentum boundary layer is examined for magnetic parameter \( M \), wedge range \( \beta \), Casson parameter \( \delta \), Prandtl number \( Pr \), and radiative heat transfer parameter \( N_r \).

(5) Thermal boundary layer is analyzed through the numerical results by varying the Eckert number \( Ec \), Biot number \( B_i \), Darcy number \( Da \), Prandtl number \( Pr \), and radiative heat transfer parameter \( N_r \).

6. Results and Discussion

This section describes the outcomes of the model presented above through the discussed shooting technique. Table 2 shows the values of the heat rate parameter \( |\theta'(0)| \) and drag force parameter \( f''(0) \) for various values of the wedge angle \( \beta \) and velocity ratio parameter \( R \). When \( R \) is less than or more than 1, surface drag force increases as the angle \( \beta \) increases. It is also seen that by an increase in \( \beta \), the temperature gradient coefficient of Nusselt number \( |\theta'(0)| \) decreases. The results acquired by the shooting approach are verified by BVP4C as well. Here, the values of different parameters are fixed as \( \delta = 1, m = 0.1, \lambda = 0.8, B_i = 5, M = 0, N_r = 1, Pr = 1, Ec = 0.5, C = 1, (Da)^{-1} = 0.01, \) and \( a_i = 0.01 \).

The numerical effects of drag force and Nusselt number for various values of the velocity ratio parameter \( R \) and the magnetic parameter \( M \) are shown in Table 3. The values of the Nusselt coefficient \( \theta'(0) \), surface drag force coefficient \( f''(0) \), and magnetic coefficient \( M \) rise when \( R \) is less than 1. When \( R \) is more than 1, the skin-friction coefficient and \( |\theta'(0)| \) both drop as the magnetic coefficient \( M \) values increase. The outcomes obtained using the shooting approach are likewise met by bvp4c.

<table>
<thead>
<tr>
<th>( R )</th>
<th>( \beta )</th>
<th>( f''(0) )</th>
<th>( \theta'(0) )</th>
<th>( Da )</th>
<th>( Pr )</th>
<th>( Ec )</th>
<th>( B_i )</th>
<th>( M )</th>
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</table>

6.2. The Temperature Profile. The influence of the Casson coefficient \( \delta \) on the temperature profile \( \theta(\eta) \), at \( R = 0.1 \) and \( R = 1.5 \), is shown in Figures 5(a) and 6. The temperature profile decreases in all scenarios when \( \delta \) increases. Figure 5(b) illustrates how changing the magnetic parameter \( M \) affects the momentum boundary layer’s behavior. The graph indicates that \( \theta(\eta) \) decreases for rising values of \( M \) for \( R = 0.3 \) and exhibits opposite behavior at \( R = 2 \). A direct relationship between the wedge angle \( \beta \) and the temperature profile is seen in Figure 5(c). Three important effects on thermal profiles are depicted in Figure 7. Here, Figure 6(a) demonstrates how the thickness of the thermal boundary layer decreases as the radiation parameter \( N_r \) decreases. The thermal profile \( \theta(\eta) \) decreases as the Prandtl number accumulates, as seen in Figure 6(b). The temperature profile \( \theta(\eta) \) decreases as the velocity ratio parameter \( R \) increases, as
The temperature’s relationship with the Eckert number is seen in Figure 8(a). When $Ec$ rises, the temperature profile decreases. The effect of convective number $Bi$ is seen in Figure 8(b). Increasing $Bi$ levels increases the body’s internal conductive resistance. The behavior of temperature $\theta(\eta)$ is steepened. Figure 8(c)

![Figure 2](image-url)
illustrates how $\theta(\eta)$ increases along with a drop in the Darcy number $Da$. The following fixed values apply to the selection of various parameters: $\delta = 1, m = 0.2, \lambda = 0.8, B_i = 5, M = 0, Nr = 1, Pr = 1, Ec = 0.5, C = 1$, $(Da)^{-1} = 0.01$, and $a_1 = 0.01$.

6.3. Nusselt Number and Surface Drag Force Coefficient. The relationship between the Nusselt number and Pr and Bi is evident in Figure 7(a). On the other hand, Figure 7(b) shows that skin friction rises with increasing Bi values but

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**Figure 3:** Impacts of the velocity ratio parameter $R$, Prandtl number $Pr$, and radiative heat transfer parameter $Nr$ on the velocity profile. (a) Effects of $R$ on $f'(\eta)$. (b) Effects of $Pr$ on $f'(\eta)$. (c) Influence of $Nr$ on $f'(\eta)$. 
Figure 4: Impacts of Biot number $B_i$, Eckert number $E_c$, and Darcy parameter $(Da)^{-1}$ on the velocity profile. (a) $f'(\eta)$ for various values of $B_i$. (b) $f'(\eta)$ for various values of $E_c$. (c) $f'(\eta)$ for various values of $(Da)^{-1}$. 
Figure 5: Influence of the Casson parameter $\delta$, magnetic parameter $M$, and wedge angle $\beta$ on the temperature profile. (a) $\theta(\eta)$ for various values of $\delta$. (b) $\theta(\eta)$ for various values of $M$. (c) $\theta(\eta)$ for various values of $\beta$. 

$\delta=0, R=0.1$
$\delta=0.5, R=0.1$
$\delta=0, R=1.5$
$\delta=0.5, R=1.5$

$M=0, R=0.3$
$M=1, R=0.3$
$M=1.5, R=0.3$
$M=0, R=2$
$M=4, R=2$
$M=4.5, R=2$

$\delta=0.5, m=0.2, \lambda=0.8, \beta=0.1, Nr=1, Ec=0.5, C=1, R=0.3,$
$Da^{-}=0.01, a_{f}=0.01$

$\beta=0, R=0.3$
$\beta=2/3, R=0.3$
$\beta=1, R=0.3$
$\beta=0, R=2$
$\beta=2/3, R=2$
$\beta=1, R=2$

$1.15, 1.155, 1.16$
$0.31, 0.32, 0.33, 0.34$
Figure 6: Impacts of the radiative heat transfer parameter \( N_r \), Prandtl number \( Pr \), and velocity ratio parameter \( R \) on the temperature profile. (a) Impact of \( N_r \) at \( \theta(\eta) \). (b) Impact of \( Pr \) at \( \theta(\eta) \). (c) Impact of \( R \) at \( \theta(\eta) \).
Figure 7: Effects of Biot number $B_i$ and Casson parameter $\delta$ on drag force and Nusselt number. (a) Effects of $B_i$ on Nusselt number. (b) Effects of $B_i$ on surface drag force. (c) Effects of $\delta$ on Nusselt number. (d) Effects of $\delta$ on surface drag force.
decreases as Pr increases. The Nusselt number is an increasing function of Pr, as Figure 7(c) illustrates, but for a constant value of Pr, it decreases with decreasing values of the Casson parameter $\delta$. Furthermore, Figure 7(d) shows that skin friction rises with the increasing values of $\delta$ for a given value of Pr.

7. Conclusions

This paper considers magnetohydrodynamics thermal flow and boundary layer analysis using Casson fluid over an extending wedge by considering porosity, ohmic heating, convective boundary layer conditions, and viscous dissipation. The mathematical model of the problem is a system of nonlinear PDEs. This system is transformed into a set of ODEs with suitable transformations. The obtained boundary value problem is numerically solved by using the shooting method. Tables and graphs give a detailed picture of the different thermodynamic profiles.

The following observations are of notable significance:

(i) As Casson parameter $\delta$ increases when $R = 0.3$, $f'(\eta)$ increases and increase in $\delta$ when $R = 2$, reduces the depth of momentum boundary layer.

(ii) When $M$ rises with $R < 1$, $f'(\eta)$ increases and when $R = 2$, $f'(\eta)$ decreases.

(iii) At $R = 0.3$ and $R = 2$, the momentum boundary layer decreases by an increase in $\text{Da}^{-1}$.

(iv) As the velocity ratio parameter $R$ increases, $f'(\eta)$ also increases.

(v) $\theta(\eta)$ increases as $\delta$ rises at $R = 0.5$, and $\theta(\eta)$ decreases when $R = 2.1$.

Figure 8: Influence of Eckert number $Ec$, Biot number $Bi$, and Darcy parameter $(\text{Da})^{-1}$ on the temperature profile. (a) $\theta(\eta)$ for various values of $Ec$. (b) $\theta(\eta)$ for various values of $Bi$. (c) $\theta(\eta)$ for various values of $(\text{Da})^{-1}$.
(vi) At $R = 0.3$ and $R = 2$, the thermal boundary’s thickness is directly related to the angle parameter $\beta$.

(vii) It is also detected that the temperature profile reduces with increasing values of the Eckert number $Ec$.

With a rise in the Prandtl number $Pr$, the depth of the thermal boundary layer decreases.

(i) When the value of $R$ rises, $\theta(\eta)$ decreases.
(ii) Decrease in the Darcy number $Da$ increases the temperature profile $\theta(\eta)$.
(iii) Nusselt number and drag force are the increasing functions of Biot number $Bi$.
(iv) Increasing the Casson parameter enhances the skin friction and reduces the Nusselt number.

The findings of this investigation contribute to problems in fluid dynamics and heat transfer that involve studying the behavior of a non-Newtonian fluid with Casson rheological properties near a solid porous wedge surface. Such problems are common in various engineering and scientific applications, including chemical engineering, biomedical engineering, and materials processing. In biomedical engineering, within the context of blood flow, the Casson parameter considered in this analysis describes the behavior of non-Newtonian blood flow in arteries and veins. In the pharmaceutical industry, the Casson parameter characterizes the rheological properties of pharmaceutical suspensions and emulsions, thus playing a vital role in the formulation and manufacturing of drugs. In polymer suspensions, the Casson parameter is vital in extrusion and injection molding processes. The findings of this investigation and the data obtained through numerical calculations delineate the variational changes in the physical parameters of importance, i.e., the skin-friction coefficient and Nusselt number. Therefore, it is recommended to reinvestigate some related physical problems in the field of applications as mentioned earlier based on the findings of the present investigation.

**Nomenclature**

\[(u, v) \ [\text{ms}^{-1}]\]: $x$ and $y$ components of velocity  
\[(x, y)\]: Cartesian coordinates  
$T$ [K]: Temperature  
$T_f$ [K]: Temperature of hot fluid  
$U$ [ms$^{-1}$]: Velocity far away from wedge  
$U_w$ [ms$^{-1}$]: Velocity of expanding wedge  
$v_w$ [ms$^{-1}$]: Suction/injection velocity  
$B$ [M$^1$T$^{-2}$T$^{-1}$]: Uniform magnetic field  
$\Omega$ [-]: Total angle of the wedge  
$\beta$ [-]: Wedge’s angle  
$h_f$ [W/m$^2$/K]: Heat transfer coefficient  
$g$ [ms$^{-2}$]: Gravity field  
$\sigma$ [M$^{-1}$L$^{-3}$T$^{-2}$A$^2$]: Conductivity of fluid due to electric current  
$q_r$ [kWm$^{-2}$]: Radiative heat flux  
$\psi$: Stream function temperature  
$c_p$ [M$^0$L$^2$T$^{-2}$K$^{-1}$]: Casson parameter  
$\delta$ [-]: Drag coefficient  
$C_o$: Skin friction  
$Nu$: Nusselt number  
$M$ [-]: Mixed convective parameter  
$\lambda$: Magnetic parameter  
$Gr$: Grashof number  
$Re$: Reynolds number  
$Nu_r$: Radiative heat transfer parameter  
$Ec$: Eckert number  
$Pr$: Prandtl number  
$Bi$: Biot number  
$R$: Velocity ratio parameter  
$Da$: Darcy inverse  
$\alpha_1$: Local inertia parameter

**Greek Symbols**

$k$ [Wm$^{-1}$K$^{-1}$]: Thermal conductivity  
$\phi$ [-]: Porosity parameter  
$\beta_o$ [-]: Thermal expansion parameter  
$\rho$ [ML$^{-3}$]: Mass density  
$\nu$ [M$^0$L$^2$T$^{-1}$]: Kinematic viscosity  
$\theta$: Dimensionless temperature.

**Data Availability**

No data were used to support the findings of this study.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**Authors’ Contributions**

Dur-e-Shehwar Sagheer conducted conceptualization, methodology, handled software, formal analysis, and writing of the original draft. M. Sabeel Khan and M. Asif Memon performed writing of the original draft, collected resources, data curation, investigation, visualization, and validation. R. Shehzad and Amsalu Fenta conducted project administration, writing of the review, editing, and supervision. Mohammad Alqudah conducted validation, investigation, physical interpretation, writing of the review, and editing. Nawal A. Alshehri has contributed in the final drafting of the revised version of the paper by reviewing it critically for important intellectual content, editing, and proofreading. However, he has also addressed a few points of the revision.

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