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Research Article

Inclusion and Neighborhood on a Multivalent q-Symmetric Function with Poisson Distribution Operators

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In this paper, by using Poisson distribution probability, some characteristics of analytic multivalent q-symmetric starlike and q-symmetric convex functions of order η are examined. Then, by utilizing the Poisson distribution and the concept of the q-analogue Salagean integral operator, the p-valent convergence polynomial was introduced. Furthermore, a number of subclasses of analytic symmetric p-valent functions linked to novel polynomials are also deduced. After that, specific coefficient constraints are determined and symmetric (δ, q)-neighborhoods for p-valent functions are defined. In relation to symmetric (δ, q)-neighborhoods of q-symmetric p-valent functions formed by Poisson distributions, this paper presents new inclusion results. In addition, a detailed discussion of certain q-symmetric inequalities of analytic functions with negative coefficients is also provided.

1. Introduction

Recently, the concept of q-calculus has attracted many researchers due to its prominent use in the development of different classes of univalent functions. Although an extensive review of the q-calculus theory was given by Jackson [1, 2], Srivastava [3] establishes a connection between the geometric nature of the univalent function and the q-derivative operator. In [4], the authors studied a subclass of biunivalent functions by using q-difference operators. Kanas et al. [5] described a symmetric operator by employing a q-derivative on a conic region, while Arif et al. [6] studied a symmetric operator to popularize the multivalent analytic functions. Moreover, the authors in [7] investigated a subclass of p-valent functions by using probability Borel distribution operators and established some properties of several normalized analytic functions. On the other hand, Khan et al. [8] discussed the q-Ruscheweyh-type derivative operator and its application to multivalent functions. Alpay et al. in [9] introduced polyanalytic functions. They studied integral representations on the quaternionic unit ball. In addition, the authors in [10] investigated some subclasses of multivalent functions associated with the q-calculus theory. However, various difference and q-difference operators are investigated for some subclasses of univalent functions; see, e.g., [11–13] and references cited therein. For $q, 0 < q < 1$, the q-analogue of the derivative of a function f is introduced by

$$D_q f(\zeta) = \frac{f(\zeta) - f(q\zeta)}{\zeta(1-q)}, \quad \zeta \neq 0, q \neq 1. \tag{1}$$

Then, the relation between the q-derivative operator D_q and the ordinary derivative operator is given by

$$\lim_{q \longrightarrow 1^-} D_q h(\zeta) = \lim_{q \longrightarrow 1^-} \frac{f(\zeta) - f(q\zeta)}{(1-q)\zeta} = f'(\zeta). \tag{2}$$

The symmetric q-calculus has been successfully applied in many areas of science associated with the geometric function theory and the quantum mechanics. Brito and Martins [14] studied the symmetric q-derivative and derived better convergence properties than the classical q-derivative. For fixed $q, 0<q<1$, the symmetric q-derivative or the q-symmetric derivative of a function f in $\mathscr{A}$ is defined by

$$\widetilde{\mathfrak{D}}_q f(\zeta)=\frac{f(q\zeta)-f\left(q^{-1}\zeta\right)}{\left(q-q^{-1}\right)\zeta}, \quad \zeta\in\Omega, \zeta\neq 0, \tag{3}$$

whereas the symmetric q-number $\widetilde{[k]}_q$ of a natural number $k\in\mathbb{N}$ is defined by [15]

$$\widetilde{[k]}_q=\begin{cases}\dfrac{q^k-q^{-k}}{q-q^{-1}}, & k\in\mathbb{N},\\ k, & q\longrightarrow 1^-.\end{cases} \tag{4}$$

Let $\mathscr{A}_q$ $(p\in\mathbb{N})$ be the class of p-valent analytic functions f defined on the open unite disk $\Omega=\{\zeta\colon |\zeta|<1\}$ such that the series form $f(\zeta)=\zeta^p+\sum_{m=1}^{\infty}a_{k+p}\zeta^{m+p}$ holds. Then, for $p=1$, the new subclass $\mathscr{A}$ of univalent functions was investigated by Graham and Kohr [16]. By simple computations, we obtain that

(i) $\widetilde{\mathfrak{D}}_q(f+g)(\zeta)=\widetilde{\mathfrak{D}}_q f(\zeta)+\widetilde{\mathfrak{D}}_q g(\zeta)$,
(ii) $\widetilde{\mathfrak{D}}_q fg(\zeta)=\widetilde{\mathfrak{D}}_q f(\zeta)g(q\zeta)+\widetilde{\mathfrak{D}}_q g(\zeta)f(q^{-1}\zeta)$,
(iii) $\widetilde{\mathfrak{D}}_q f/g(\zeta)=\widetilde{\mathfrak{D}}_q f(\zeta)g(q^{-1}\zeta)-\widetilde{\mathfrak{D}}_q g(\zeta)f(q^{-1}\zeta)/g(q\zeta)g(q^{-1}\zeta)$.

We denote by $\mathscr{U}_p$ $(p\in\mathbb{N})$ the set of all p-valent functions f which are holomorphic or analytic on a subset $\Omega=\{\zeta; |\zeta|<1\}$ in the complex plan $\mathbb{C}$ with the series form

$$f(\zeta)=\zeta^p-\sum_{m=1}^{\infty}a_{m+p}\zeta^{m+p}, \quad a_{m+p}\geq 0, \zeta\in\Omega. \tag{5}$$

Let f be given by (5) and $g(\zeta)=\zeta^p-\sum_{m=1}^{\infty}b_{m+p}\zeta^{m+p}$, $(b_{m+p}\geq 0)$. We use the following notation to denote the convolution of two functions f and g:

$$f*g(\zeta)=\zeta^p-\sum_{m=1}^{\infty}\left(a_{m+p}b_{m+p}\zeta^{m+p},\left(a_{m+p}\geq 0, b_{m+p}\right)>0, \quad \zeta\in\Omega\right). \tag{6}$$

In recent decades, inequalities and inclusions of various types of analytic functions are studied in the geometric function theory. Liu and Xu in [17] considered the concept of (h_1, h_2)-convex functions and discovered new Hermite–Hadamard type inequalities on intervals of Riemann integrable functions. The authors in [18] discussed the concept of q-derivative and studied several inequalities of analytic functions by using the generalized Sǎlǎgean differential operator. On the other hand, Lashin et al. in [19] discovered some inclusion relations of subclasses of analytic functions associated with the Pascal distribution probability.A random variable X with Poisson distribution takes the exponential probability that is given by

$$p(X=m)=\frac{k^m e^{-k}}{m!}, \tag{7}$$

where $m=0,1,2,3,\ldots$ and k is a parameter (see [20]).

The main properties of the Poisson distribution probability are in fact a limiting case of the binomial distribution. The Poisson distribution, like the binomial distribution, is a counted number of times of something happens. The observed difference is that there is no specified number of possible tries. Here is one way that it can arise. If an event happens independently and randomly over a time and the mean rate of occurrence is constant over the time, then the number of occurrences in a fixed amount of time lead to a Poisson distribution. The Poisson model is used extensively for modeling a count of data in a range of different scientific fields (see, e.g., [21]).

Al-Shaqsi [22] defined the convergence polynomial as follows:

$$\mathscr{P}(k,\zeta)=z+\sum_{m=2}^{\infty}\frac{k^{m-1}e^{-k}}{(m-1)!}\zeta^m, \quad \zeta\in\Omega. \tag{8}$$

We note that this polynomial has radius of convergence at infinity. Now, we introduce the following p-valent functions.

$$\mathscr{F}_p(k,\zeta)=\zeta^p-\sum_{m=1}^{\infty}\frac{k^{m+p-1}e^{-k}}{(m+p-1)!}\zeta^{m+p}, \quad \zeta\in\Omega. \tag{9}$$

We define a linear operator $\mathscr{R}_k(\zeta)\colon \mathscr{U}_p\longrightarrow\mathscr{U}_p$ as follows:

$$\mathscr{R}_k(\zeta)=\mathscr{F}(k,\zeta)*f(\zeta)=\zeta^p-\sum_{m=1}^{\infty}\frac{k^{m+p-1}e^{-k}}{(m+p-1)!}a_{m+p}\zeta^{m+p}, \quad \zeta\in\Omega. \tag{10}$$

Then, benefited from the definition of the q-symmetric derivative of a function $\mathscr{R}_k$, we establish that

$$\widetilde{\mathfrak{D}}_q \mathscr{R}_k(\zeta) = \widetilde{[p]}_q \zeta^{p-1} - \sum_{m=1}^{\infty} \widetilde{[m+p]}_q \frac{k^{m+p-1} e^{-k}}{(m+p-1)!} a_{m+p} \zeta^{m+p-1}, \quad \zeta \in \Omega. \tag{11}$$

Let $\delta > 0$. By using the idea of Salagean in [23], we will define the q-symmetric derivative of the linear Poisson distribution operator $\widetilde{\mathfrak{D}}_q^{\delta,v} \mathscr{R}_v(\zeta)$: $\mathscr{U}_p \longrightarrow \mathscr{U}_p$ as follows:

$$\begin{aligned}
\widetilde{\mathfrak{D}}_q^{\delta,0} \mathscr{R}_k(\zeta) &= \mathscr{R}_k(\zeta),\\
\widetilde{\mathfrak{D}}_q^{\delta,1} \mathscr{R}_k(\zeta) &= (1-\delta)\widetilde{\mathfrak{D}}_q^{\delta,0} \mathscr{R}_k(\zeta) + \delta \frac{\zeta}{\widetilde{[p]}_q} \widetilde{\mathfrak{D}}_q \mathscr{R}_k(\zeta)\\
&= \zeta^p - \sum_{m=1}^{\infty} \left(1-\delta+\delta \frac{\widetilde{[m+p]}_q}{\widetilde{[p]}_q}\right) \frac{k^{m+p-1} e^{-k}}{(m+p-1)!} a_{m+p} \zeta^{m+p},\\
\widetilde{\mathfrak{D}}_q^{\delta,2} \mathscr{R}_k(\zeta) &= (1-\delta)\widetilde{\mathfrak{D}}_q^{\delta,1} \mathscr{R}_k(\zeta) + \delta \frac{\zeta}{\widetilde{[p]}_q} \widetilde{\mathfrak{D}}_q \left(\widetilde{\mathfrak{D}}_q^{\delta,1} \mathscr{R}_k(\zeta)\right)\\
&= \zeta^p - \sum_{m=1}^{\infty} \left(1-\delta+\delta \frac{\widetilde{[m+p]}_q}{\widetilde{[p]}_q}\right)^2 \frac{k^{m+p-1} e^{-k}}{(m+p-1)!} a_{m+p} \zeta^{m+p},\\
&\vdots\\
\widetilde{\mathfrak{D}}_q^{\delta,v} \mathscr{R}_k(\zeta) &= (1-\delta)\widetilde{\mathfrak{D}}_q^{\delta,v-1} \mathscr{R}_k(\zeta) + \delta \frac{\zeta}{\widetilde{[p]}_q} \widetilde{\mathfrak{D}}_q \left(\widetilde{\mathfrak{D}}_q^{\delta,v-1} \mathscr{R}_k(\zeta)\right)\\
&= \zeta^p - \sum_{m=1}^{\infty} \left(1-\delta+\delta \frac{\widetilde{[m+p]}_q}{\widetilde{[p]}_q}\right)^v \frac{k^{m+p-1} e^{-k}}{(m+p-1)!} a_{m+p} \zeta^{m+p},\\
&\qquad (v, k \in \mathbb{N} \cup \{0\}, 0 < q < 1, 0 \le \delta \le 1).
\end{aligned} \tag{12}$$

By simple notation, we write

$$\widetilde{\mathfrak{D}}_q^{\delta,v} \mathscr{R}_k(\zeta) = \zeta^p - \sum_{m=1}^{\infty} \Lambda_{q,m}^{k,\delta,v} a_{m+p} \zeta^{m+p} \quad (v, k \in \mathbb{N} \cup \{0\}, 0 < q < 1, 0 \le \delta \le 1), \tag{13}$$

where

$$\Lambda_{q,m}^{k,\delta,v} = \left(1-\delta+\delta \frac{\widetilde{[m+p]}_q}{\widetilde{[p]}_q}\right)^v \frac{k^{m+p-1} e^{-k}}{(m+p-1)!}. \tag{14}$$

We now state the following necessary definitions.

Definition 1. Let $0 < q < 1$, $p \in \mathbb{N}$, $0 \le \eta < \widetilde{[p]}_q$, and $f \in \mathscr{A}_p$. Then, the class $\widetilde{S}_{q,k}^{\delta,v}(p,\eta)$ of p-valent symmetric q-starlike functions of order η with Poisson distribution is defined by

$$\widetilde{S}_{q,k}^{\delta,v}(p,\eta) = \left\{ f \in \mathscr{A}_p \colon Re\left(\frac{\zeta \widetilde{\mathfrak{D}}_q \left(\widetilde{\mathfrak{D}}_q^{\delta,v} \mathscr{R}_k(\zeta)\right)}{\widetilde{\mathfrak{D}}_q^{\delta,v} \mathscr{R}_k(\zeta)} \right) > \eta, \quad \zeta \in \Omega \right\}. \tag{15}$$

Note that if $f \in \mathscr{U}_p$, then the calls $\widetilde{S}_{q,k}^{\delta,v}(p,\eta)$ is denoted by $\widetilde{TS}_{q,k}^{\delta,v}(p,\eta)$. If $p = 1$ and $\eta = 0$, we have $\widetilde{TS}_{q,k}^{\delta,v}(1,0) = \widetilde{S}_{q,k}^{\delta,v}$.

Example 1. In this example, we show that the set $\widetilde{S}_{q,k}^{\delta,v}(p,\eta)$ is nonempty.

We set $\alpha = \eta / \widetilde{[p]}_q$. Since $Re(1 + (1-2\alpha)z/(1-z)) > \alpha$, we can find a function $f(z)$ such that

$$\frac{\zeta \widetilde{\mathfrak{D}}_q \left(\widetilde{\mathfrak{D}}_q^{\delta,v} \mathscr{R}_k(\zeta)\right)}{\widetilde{[p]}_q \widetilde{\mathfrak{D}}_q^{\delta,v} \mathscr{R}_k(\zeta)} = \frac{1 + (1-2\alpha)z}{1-z}. \tag{16}$$

By using the assertion (13), we can write

$$\frac{\widetilde{[p]}_q\zeta^p+\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}\,[\widetilde{m+p}]_q a_{m+p}\zeta^{m+p}}{\widetilde{[p]}_q\zeta^p+\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}\,\widetilde{[p]}_q a_{m+p}\zeta^{m+p}}=\frac{1+(1-2\alpha)z}{1-z}, \tag{17}$$

or equivalently

$$\frac{\widetilde{[p]}_q+\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}\,[\widetilde{m+p}]_q a_{m+p}\zeta^{m}}{\widetilde{[p]}_q+\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}\,\widetilde{[p]}_q a_{m+p}\zeta^{m}}=1+\sum_{m=1}^{\infty}2(1-\alpha)z^m. \tag{18}$$

This, indeed, implies that

$$\begin{aligned}\widetilde{[p]}_q+\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}\,[\widetilde{m+p}]_q a_{m+p}\zeta^{m}=\widetilde{[p]}_q&+\left(2(1-\alpha)+\Lambda_{q,1}^{k,\delta,\nu}\widetilde{[p]}_q a_{1+p}\right)z\\&+\left(2(1-\alpha)+2(1-\alpha)\Lambda_{q,1}^{k,\delta,\nu}\widetilde{[p]}_q a_{1+p}+\Lambda_{q,2}^{k,\delta,\nu}\widetilde{[p]}_q a_{2+p}\right)z^2\\&+\left(2(1-\alpha)+2(1-\alpha)\Lambda_{q,1}^{k,\delta,\nu}\widetilde{[p]}_q a_{1+p}+2(1-\alpha)\Lambda_{q,2}^{k,\delta,\nu}\widetilde{[p]}_q a_{2+p}+\Lambda_{q,3}^{k,\delta,\nu}\widetilde{[p]}_q a_{3+p}\right)z^3+\ldots.\end{aligned} \tag{19}$$

Therefore, we can obtain the coefficient of function $f(z)$ as follows:

$$\begin{aligned}a_{p+1}&=\frac{2(1-\alpha)}{\Lambda_{q,1}^{k,\delta,\nu}\left([\widetilde{p+1}]_q-\widetilde{[p]}_q\right)},\\a_{p+2}&=\frac{2(1-\alpha)}{\Lambda_{q,2}^{k,\delta,\nu}\left([\widetilde{p+2}]_q-\widetilde{[p]}_q\right)}\left(1+\frac{2(1-\alpha)\widetilde{[p]}_q}{[\widetilde{p+1}]_q-\widetilde{[q]}_q}\right),\\a_{p+m}&=\frac{2(1-\alpha)}{\Lambda_{q,m}^{k,\delta,\nu}\left([\widetilde{p+m}]_q-\widetilde{[p]}_q\right)}\prod_{i=1}^{m-1}\left(1+\frac{2(1-\alpha)\widetilde{[p]}_q}{[\widetilde{p+i}]_q-\widetilde{[q]}_q}\right),\quad m=3,4,5,\ldots.\end{aligned} \tag{20}$$

Hence, the set $\widetilde{S}_{q,k}^{\delta,\nu}(p,\eta)$ is nonempty.

Definition 2. Let $0<q<1$, $p\in\mathbb{N}$, $0\le\eta<\widetilde{[p]}_q$, and $f\in\mathscr{A}_p$. Then, the class $\widetilde{C}_{q,k}^{\delta,\nu}(p,\eta)$ of p-valent symmetric q-convex functions of order η with Poisson distribution is defined by

$$\widetilde{C}_{q,k}^{\delta,\nu}(p,\eta)=\left\{f\in\mathscr{A}_p:\operatorname{Re}\left(1+\frac{\widetilde{\mathfrak{D}}_q\left(\zeta\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)\right)\right)}{\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)\right)}\right)>\eta,\quad\zeta\in\Omega\right\}. \tag{21}$$

Note that if $f\in\mathscr{U}_p$, then the calls $\widetilde{C}_{q,k}^{\delta,\nu}(p,\eta)$ is denoted by $\widetilde{TC}_{q,k}^{\delta,\nu}(p,\eta)$. If $p=1$ and $\eta=0$, we have $\widetilde{C}_{q,k}^{\delta,\nu}(1,0)=\widetilde{C}_{q,k}^{\delta,\nu}$.

Example 2. In this example, we show that the set $\widetilde{C}_{q,k}^{\delta,\nu}(p,\eta)$ is nonempty.

We set $\alpha=\eta/\widetilde{[p]}_q$. Since $\operatorname{Re}(1+(1-2\alpha)z/(1-z))>\alpha$, we can find a function $f(z)$ such that

$$\frac{\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)\right)+\widetilde{\mathfrak{D}}_q\left(\zeta\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)\right)\right)}{\widetilde{[p]}_q\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)\right)}=\frac{1+(1-2\alpha)z}{1-z}. \tag{22}$$

By using the assertion (13), we can write

$$\frac{\left(\widetilde{[p]}_q+\widetilde{[p]}_q^2\right)\zeta^{p-1}+\sum_{m=1}^{\infty}\left(\widetilde{[m+p]}_q+\widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,v}a_{m+p}\zeta^{m+p-1}}{\widetilde{[p]}_q^2\zeta^{p-1}+\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,v}\widetilde{[p+m]}_q\widetilde{[p]}_q a_{m+p}\zeta^{m+p-1}}=\frac{1+(1-2\alpha)z}{1-z}. \tag{23}$$

or equivalently

$$\frac{\left(\widetilde{[p]}_q+\widetilde{[p]}_q^2\right)+\sum_{m=1}^{\infty}\left(\widetilde{[m+p]}_q+\widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,v}a_{m+p}\zeta^{m}}{\widetilde{[p]}_q^2+\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,v}\widetilde{[p+m]}_q\widetilde{[p]}_q a_{m+p}\zeta^{m}}=1+\sum_{m=1}^{\infty}2(1-\alpha)z^m. \tag{24}$$

This, indeed, implies that

$$\begin{aligned}
&\left(\widetilde{[p]}_q+\widetilde{[p]}_q^2\right)+\sum_{m=1}^{\infty}\left(\widetilde{[m+p]}_q+\widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,v}a_{m+p}\zeta^{m}\\
&\quad=\widetilde{[p]}_q+\widetilde{[p]}_q^2+\left(2(1-\alpha)+\widetilde{[p]}_q\widetilde{[p+1]}_q\Lambda_{q,1}^{k,\delta,v}a_{p+1}\right)z\\
&\qquad+\left(2(1-\alpha)+2(1-\alpha)\widetilde{[p]}_q\widetilde{[p+1]}_q\Lambda_{q,1}^{k,\delta,v}a_{1+p}+\widetilde{[p]}_q\widetilde{[p+2]}_q\Lambda_{q,2}^{k,\delta,v}a_{2+p}\right)z^2\\
&\qquad+\left(2(1-\alpha)+2(1-\alpha)\widetilde{[p]}_q\widetilde{[p+1]}_q\Lambda_{q,1}^{k,\delta,v}a_{1+p}+2(1-\alpha)\widetilde{[p]}_q\widetilde{[p+2]}_q\Lambda_{q,2}^{k,\delta,v}a_{2+p}+\widetilde{[p]}_q\widetilde{[p+3]}_q\Lambda_{q,3}^{k,\delta,v}a_{3+p}\right)z^3+\ldots.
\end{aligned} \tag{25}$$

Therefore, we can obtain the coefficient of function $f(z)$ as follows.

$$\begin{aligned}
a_{p+1}&=\frac{2(1-\alpha)}{\Lambda_{q,1}^{k,\delta,v}\widetilde{[p+1]}_q\left(1+\widetilde{[p+1]}_q-\widetilde{[p]}_q\right)},\\
a_{p+2}&=\frac{2(1-\alpha)}{\Lambda_{q,2}^{k,\delta,v}\widetilde{[p+2]}_q\left(1+\widetilde{[p+2]}_q-\widetilde{[p]}_q\right)}\left(1+\frac{2(1-\alpha)\widetilde{[p]}_q}{1+\widetilde{[p+1]}_q-\widetilde{[q]}_q}\right),\\
&\vdots\\
a_{p+m}&=\frac{2(1-\alpha)}{\Lambda_{q,m}^{k,\delta,v}\widetilde{[p+m]}_q\left(1+\widetilde{[p+m]}_q-\widetilde{[p]}_q\right)}\prod_{i=1}^{m-1}\left(1+\frac{2(1-\alpha)\widetilde{[p]}_q}{1+\widetilde{[p+i]}_q-\widetilde{[q]}_q}\right),\quad m=3,4,5,\ldots
\end{aligned} \tag{26}$$

Hence, the set $\widetilde{C}_{q,k}^{\delta,v}(p,\eta)$ is nonempty.

For $p=1$, the subclasses $\widetilde{S}_q^*(\eta)$, resp., $\widetilde{C}_q(\eta)$, of symmetric q-starlike functions of order η and the class of symmetric q-convex functions of order η were thoroughly discussed in [24].

In [25], the authors defined the neighborhood of function $f(\zeta)=\zeta$. Then, Altinta et al. in [26] introduced the n-neighborhood for a starlike function with negative coefficient. Moreover, the authors of [27–29] investigated some neighborhood properties for a subclass of complex analytic functions. Thus, similar to the previous result, we can define the symmetric (β, q)-neighborhood of analytic function as follows.

Definition 3. Let $\beta > 0$ and

$$\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{G}_k(\zeta) = \zeta^p - \sum_{m=2}^{\infty} \Lambda_{q,m}^{k,\delta,v} b_{m+p} \zeta^{m+p}. \tag{27}$$

The symmetric (β, q)-neighborhood of a function $\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta), \beta > 0$, is defined as

$$\widetilde{\mathcal{N}}_{\beta,q,k}^{\delta,v}\left(\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta)\right) = \left\{\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{G}_k(\zeta) : \sum_{m=1}^{\infty} [\widetilde{m+p}]_q \Lambda_{q,m}^{k,\delta,v} \left|a_{m+p} - b_{m+p}\right| \leq \beta\right\}. \tag{28}$$

Then, it follows from (28) that if $h(\zeta) = \zeta^p, p \in \mathbb{N}$, then

$$\widetilde{\mathcal{N}}_{\beta,q,k}^{\delta,v}(h) = \left\{\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{G}_k(\zeta) : \sum_{m=1}^{\infty} [\widetilde{m+p}]_q \Lambda_{q,m}^{k,\delta,v} \left|b_{m+p}\right| \leq \beta\right\}. \tag{29}$$

Our object, in this present paper, is to investigate various properties and characteristics of symmetric analytic p-valent functions belonging to the class $\widetilde{\mathcal{K}}_{q,k}^{\delta,v}(p,\mu,\eta)$ of all functions $\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta)$ such that the following inequality holds.

$$\operatorname{Re}\left\{\frac{\zeta\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta)\right) + \mu\zeta\widetilde{\mathfrak{D}}_q\left(\zeta\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta)\right)\right)}{(1-\mu)\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta) + \mu\zeta\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta)\right)}\right\} > \eta, \quad \zeta \in \Omega, 0 \leq \eta < \widetilde{[p]}_q. \tag{30}$$

Certainly, we, in terms of the simpler classes $\widetilde{\mathrm{TS}}_{q,k}^{\delta,v}(p,\eta)$ and $\widetilde{\mathrm{TC}}_{q,k}^{\delta,v}(p,\eta)$, respectively, have

$$\widetilde{\mathcal{K}}_{q,k}^{\delta,v}(p,0,\eta) = \widetilde{\mathrm{TS}}_{q,k}^{\delta,v}(p,\eta) \quad \text{and} \quad \widetilde{\mathcal{K}}_{q,k}^{\delta,v}(p,1,\eta) = \widetilde{\mathrm{TC}}_{q,k}^{\delta,v}(p,\eta). \tag{31}$$

Also, we denote by $\widetilde{\mathcal{M}}_{q,k}^{\delta,v}(p,\mu,\eta)$ the set of all functions $\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta)$ satisfying the inequality

$$\operatorname{Re}\left\{\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta)\right) + \mu\widetilde{\mathfrak{D}}_q\left(\zeta\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta)\right)\right)\right\} > 0. \tag{32}$$

Then, we establish several inclusion relationships involving symmetric (q,δ)-neighborhoods of analytic p-valent functions belonging to such subclasses in details.

2. A Set of Coefficient Inequalities

In this section, we aim to obtain coefficient inequalities for functions in the subclasses $\widetilde{\mathcal{K}}_q(p,\mu,\eta)$ and $\widetilde{\mathcal{R}}_q(p,\mu,\eta)$.

Theorem 4. *Let $0 < q < 1$, $0 \leq \delta \leq 1$, $p, k, v \in \mathbb{N}$, $0 \leq \mu \leq \widetilde{[p]}_q$, and $\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta)$ be given by (13). Then, $\widetilde{\mathfrak{D}}_q^{\delta,v} \mathcal{R}_k(\zeta) \in \widetilde{\mathcal{K}}_{p,k}^{\delta,v}(p,\mu,\eta)$ if and only if*

$$\begin{gathered}\sum_{m=1}^{\infty}\left(-\eta(1-\mu) + (1-\mu\eta)[\widetilde{m+p}]_q + \mu[\widetilde{m+p}]_q^2\right)\Lambda_{q,m}^{k,\delta,v} a_{m+p} \leq \phi_p,\\ \left(0 \leq \eta < 1, 0 \leq \mu \leq 1, \left(1-\mu\phi_p\right)\widetilde{[p]}_q + \mu\widetilde{[p]}_q^2 > \phi_p(1-\mu), \quad \zeta \in \Omega,\right),\end{gathered} \tag{33}$$

where $\phi_p = -\eta(1-\mu) + (1-\eta\mu)\widetilde{[p]}_q + \mu\widetilde{[p]}_q^2$ and $\Lambda_{q,m}^{k,\delta,v}$ is given by (14). The result is also sharp.

Proof. Suppose that $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)\in\widetilde{\mathscr{K}}_{p,k}^{\delta,\nu}(p,\mu,\eta)$. Then, we have

$$\operatorname{Re}\left\{\frac{\left(\widetilde{[p]}_q+\mu\widetilde{[p]}_q^2\right)\zeta^p-\sum_{m=1}^{\infty}\left(\widetilde{[m+p]}_q+\mu\widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}a_{m+p}\zeta^{m+p}}{\left(1-\mu+\mu\widetilde{[p]}_q\right)\zeta^p-\sum_{k=1}^{\infty}\left(1-\mu+\mu\widetilde{[m+p]}_q\right)\Lambda_{q,m}^{k,\delta,\nu}a_{m+p}\zeta^{m+p}}\right\}>\eta,$$
$$\text{where}\quad 0\le\eta<1,0\le\mu\le1,\left(1-\mu\phi_p\right)\widetilde{[p]}_q+\mu\widetilde{[p]}_q^2>\phi_p(1-\mu),\quad \zeta\in\Omega. \tag{34}$$

By choosing ζ to be real and $\zeta\longrightarrow1^-$, we obtain

$$\frac{\left(\widetilde{[p]}_q+\mu\widetilde{[p]}_q^2\right)-\sum_{m=1}^{\infty}\widetilde{[m+p]}_q+\mu\widetilde{[m+p]}_q^2\Lambda_{q,m}^{k,\delta,\nu}a_{m+p}}{\left(1-\mu+\mu\widetilde{[p]}_q\right)-\sum_{m=1}^{\infty}\left(1-\mu+\mu\widetilde{[m+p]}_q\right)\Lambda_{q,m}^{k,\delta,\nu}a_{m+p}}>\eta,$$
$$\left(0\le\eta<1,0\le\mu\le1,\left(1-\mu\phi_p\right)\widetilde{[p]}_q+\mu\widetilde{[p]}_q^2>\phi_p(1-\mu),\quad \zeta\in\Omega\right). \tag{35}$$

Or, equivalently, we get

$$-\sum_{m=1}^{\infty}\left(-\eta(1-\mu)+(\mu\eta-1)\widetilde{[m+p]}_q+\mu\widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}a_{m+p}>\eta(1-\mu)-(1-\mu\eta)\widetilde{[p]}_q-\mu\widetilde{[p]}_q^2,$$
$$\left(0\le\eta<1,0\le\mu\le1,\left(1-\mu\phi_p\right)\widetilde{[p]}_q+\mu\widetilde{[p]}_q^2>\phi_p(1-\mu),\quad \zeta\in\Omega\right). \tag{36}$$

Hence, we reach to assertion (33) of Theorem 4. Conversely, suppose that inequality (33) holds true and $\zeta\in\partial\Omega=\{\zeta\in\mathbb{C},|\zeta|=1\}$. Then, from definition (5), we find that

$$\left|\frac{\zeta\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)\right)+\mu\zeta\widetilde{\mathfrak{D}}_q\left(\zeta\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)\right)\right)}{(1-\mu)\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)\right)+\mu\zeta\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)\right)}-\phi_p\right|$$
$$\le\frac{\left(-\phi_p(1-\mu)+\left(1-\mu\phi_p\right)\widetilde{[p]}_q+\mu\widetilde{[p]}_q^2\right)|\zeta^p|}{\left(1-\mu+\mu\widetilde{[p]}_q\right)|\zeta^p|-\sum_{m=1}^{\infty}\left(1-\mu+\mu\widetilde{[m+p]}_q\right)\Lambda_{q,m}^{k,\delta,\nu}a_{m+p}|\zeta^{m+p}|}$$
$$+\frac{\sum_{m=1}^{\infty}\left(-\phi_p(1-\mu)+\left(1-\mu\phi_p\right)\widetilde{[p+m]}_q+\mu\widetilde{[p+m]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}a_{m+p}|\zeta^{m+p}|}{\left(1-\mu+\mu\widetilde{[p]}_q\right)|\zeta^p|-\sum_{m=1}^{\infty}\left(1-\mu+\mu\widetilde{[m+p]}_q\right)\Lambda_{q,m}^{k,\delta,\nu}a_{m+p}|\zeta^{m+p}|}$$
$$\le\phi_p-\eta, \tag{37}$$

where $0\le\eta<1,0\le\mu\le1$, $(1-\mu\phi_p)\widetilde{[p]}_q+\mu\widetilde{[p]}_q^2>\phi_p$ $(1-\mu),z\in\partial\Omega$. Thus, inequality (33) satisfies. Hence, by the maximum modulus theorem, we conclude $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)\in\widetilde{\mathscr{K}}_{p,k}^{\delta,\nu}(p,\mu,\eta)$.

Finally, we note that inequality (33) of Theorem 4 is sharp. Therefore, the extremal function is given by

$$\varphi(\zeta) = \zeta^p - \frac{-\eta(1-\mu) + (1-\eta\mu)\widetilde{[p]}_q + \mu\widetilde{[p]}_q^2}{\left(-\eta(1-\mu) + (1-\eta\mu)\widetilde{[p+m]}_q + \mu\widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}}\zeta^{m+p}$$
$$\left(0 \le \eta < 1, 0 \le \mu \le \widetilde{[p]}_q, \left(1-\mu\phi_p\right)\widetilde{[p]}_q + \mu\widetilde{[p]}_q^2 > \phi_p(1-\mu), \quad \zeta \in \Omega\right). \tag{38}$$

By using the abovementioned theorem, we can easily obtain the following result. □

Corollary 5. *Let $0 < q < 1$, $0 \le \delta \le 1$, $p, k, \nu \in \mathbb{N}$, $0 \le \eta \le \widetilde{[p]}_q$, and $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)$ be given by (13). Then, $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta) \in \widetilde{TS}_{q,k}^{\delta,\nu}(p,\eta)$ if and only if*

$$\sum_{m=1}^{\infty}\left(-\eta + \widetilde{[m+p]}_q\right)\Lambda_{q,m}^{k,\delta,\nu}a_{m+p}\zeta^{m+p} \le -\eta + \widetilde{[p]}_q, \quad \zeta \in \Omega, \tag{39}$$

where $\Lambda_{q,m}^{k,\delta,\nu}$ is given by (14). The result is sharp for the function

$$\varphi(\zeta) = \zeta^p - \frac{-\eta + \widetilde{[p]}_q}{\left(-\eta + \widetilde{[m+p]}_q\right)\Lambda_{q,m}^{k,\delta,\nu}}\zeta^{m+p}, \quad \zeta \in \Omega. \tag{40}$$

Corollary 6. *Let $0 < q < 1$, $0 < \delta \le 1$, $k, \nu \in \mathbb{N}$, and $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)$ be given by 1.3. Then, $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta) \in \widetilde{TS}_{q,k}^{\delta,\nu}$ if and only if*

$$\sum_{m=1}^{\infty}\left(\widetilde{[m+1]}_q\right)\Lambda_{q,m}^{k,\delta,\nu}a_{m+1} \le 1, \quad \zeta \in \Omega. \tag{41}$$

The result is sharp for the function.

$$\varphi(\zeta) = \zeta - \frac{1}{\widetilde{[m+1]}_q\Lambda_{q,m}^{k,\delta,\nu}}\zeta^{m+1}, \quad \zeta \in \Omega. \tag{42}$$

Corollary 7. *Let $0 < q < 1$, $0 \le \delta \le 1$, $p, k, \nu \in \mathbb{N}$, $0 \le \eta \le \widetilde{[p]}_q$, and $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)$ be given by (13). Then, $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta) \in \widetilde{TC}_{q,k}^{\delta,\nu}(p,\eta)$ if and only if*

$$\sum_{m=1}^{\infty}\left((1-\eta)\widetilde{[m+p]}_q + \widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}a_{m+p} \le (1-\eta)\widetilde{[p]}_q + \widetilde{[p]}_q^2, \quad \zeta \in \Omega. \tag{43}$$

The result is sharp for the function.

$$\varphi(\zeta) = \zeta^p - \frac{(1-\eta)\widetilde{[p]}_q + \widetilde{[p]}_q^2}{\left((1-\eta)\widetilde{[m+p]}_q + \widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}}\zeta^{m+p}, \quad \zeta \in \Omega. \tag{44}$$

Corollary 8. *Let $0 < q < 1$, $0 < \delta \le 1$, $k, \nu \in \mathbb{N}$, and $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta)$ be given by (13). Then, $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathscr{R}_k(\zeta) \in \widetilde{TC}_{q,k}^{\delta,\nu}$ if and only if*

$$\sum_{m=1}^{\infty}\left(\widetilde{[m+1]}_q + \widetilde{[m+1]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}a_{m+p} \le 2, \quad \zeta \in \Omega. \tag{45}$$

The result is sharp for the function.

$$\varphi(\zeta) = \zeta - \frac{2}{\left(\widetilde{[m+1]}_q + \widetilde{[m+1]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}}\zeta^{m+1}, \quad \zeta \in \Omega. \tag{46}$$

Theorem 9. *Let* $0 < q < 1, 0 \leq \delta \leq 1$, $p, k, \nu \in \mathbb{N}$, $0 \leq \eta < \widetilde{[p]}_q$, *and* $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)$ *be given by (13). Then,* $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta) \in \widetilde{\mathcal{M}}_{q,k}^{\delta,\nu}(p,\mu,\eta)$ *if and only if*

$$\sum_{m=1}^{\infty}\left(\widetilde{[m+p]}_q + \mu\widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}a_{k+p} \leq \widetilde{[p]}_q + \mu\widetilde{[p]}_q^2 - \eta. \tag{47}$$

The result is sharp for a function φ given by

$$\varphi(\zeta) = \zeta^p - \frac{\widetilde{[p]}_q + \mu\widetilde{[p]}_q - \eta}{\left(\widetilde{[m+p]}_q + \mu\widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}}\zeta^{m+p}, \quad \zeta \in \Omega. \tag{48}$$

Proof. Suppose that $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta) \in \widetilde{\mathcal{M}}_{q,k}^{\delta,\nu}(p,\mu,\eta)$. Then, we have

$$\left(\widetilde{[p]}_q + \mu\widetilde{[p]}_q^2\right)\zeta^p - \sum_{m=1}^{\infty}\left(\widetilde{[m+p]}_q + \mu\widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}a_{k+p}\zeta^{k+p} \geq \eta. \tag{49}$$

By choosing ζ to be real and $\zeta \longrightarrow 1^-$, we obtain

$$\widetilde{[p]}_q + \mu\widetilde{[p]}_q^2 - \sum_{m=1}^{\infty}\left(\widetilde{[m+p]}_q + \mu\widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}a_{k+p} \geq \eta. \tag{50}$$

Hence, we reach to assertion (47). Conversely, suppose that (47) holds true and $\zeta \in \partial(\Omega) = \{\zeta \in \mathbb{C}, |\zeta| = 1\}$. Then, from definition (5), we find that

$$\begin{aligned}
&\left|\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)\right) + \mu\widetilde{\mathfrak{D}}_q\left(\zeta\widetilde{\mathfrak{D}}_q\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)\right)\right) - \widetilde{[p]}_q - \mu\widetilde{[p]}_q^2 + \eta\right| \\
&\quad\leq \left(\widetilde{[p]}_q + \mu\widetilde{[p]}_q^2\right)|\zeta|^p + \sum_{m=1}^{\infty}\left(\widetilde{[m+p]}_q + \mu\widetilde{[m+p]}_q^2\right)\Lambda_{q,m}^{k,\delta,\nu}a_{k+p}|\zeta|^{k+p} - \widetilde{[p]}_q - \mu\widetilde{[p]}_q^2 + \eta \\
&\quad\leq \widetilde{[p]}_q + \mu\widetilde{[p]}_q^2,
\end{aligned} \tag{51}$$

where $0 < q < 1, 0 \leq \delta \leq 1$, $p, k, \nu \in \mathbb{N}$, $0 \leq \eta < \widetilde{[p]}_q$. Thus, inequality (47) satisfies. Hence, by the maximum modulus theorem, we conclude $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta) \in \widetilde{\mathcal{M}}_{q,k}^{\delta,\nu}(p,\mu,\eta)$. □

Theorem 10. *Let* $0 < q < 1, 0 \leq \delta \leq 1$, $p, k, \nu \in \mathbb{N}$, $0 \leq \mu \leq \widetilde{[p]}_q$, *and* $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)$ *expressed in (13) be in the class* $\widetilde{\mathcal{K}}_{q,k}^{\delta,\nu}(p,\eta,\mu)$. *Then,*

$$\sum_{m=1}^{\infty} \Lambda_{q,m}^{k,\delta,\nu} a_{m+p} \leq \frac{\phi_p}{\phi_{m+p}} \tag{52}$$

and

$$\sum_{m=1}^{\infty} [\widetilde{k+p}]_q \Lambda_{q,m}^{k,\delta,\nu} a_{m+p} \leq \frac{\phi_p [\widetilde{m+p}]_q}{\phi_{m+p}}, \tag{53}$$

where ϕ_{m+p} *is defined by*

$$\phi_{m+p} = \left(-\eta(1-\mu) + (1-\eta\mu)[\widetilde{m+p}]_q + \mu[\widetilde{m+p}]_q^2\right), \quad m = 0, 1, 2, \ldots. \tag{54}$$

Proof. From inequality (33) of Theorem 4, we can easily see that

$$\begin{aligned}
&\left(-\eta(1-\mu) + (1-\eta\mu)[\widetilde{m+p}]_q + \mu[\widetilde{m+p}]_q^2\right) \sum_{m=1}^{\infty} \Lambda_{q,m}^{k,\delta,\nu} a_{m+p} \\
&\leq \sum_{m=1}^{\infty} \left(-\eta(1-\mu) + (1-\eta\mu)[\widetilde{m+p}]_q + \mu[\widetilde{m+p}]_q^2\right) \Lambda_{q,m}^{k,\delta,\nu} a_{m+p} \\
&\leq -\eta(1-\mu) + (1-\eta\mu)[\widetilde{p}]_q + \mu[\widetilde{p}]_q^2,
\end{aligned} \tag{55}$$

which immediately yields the first inequality (52) of Theorem 10.

By using inequality (33), we also have

$$-\eta(1-\mu) \sum_{m=1}^{\infty} \Lambda_{q,m}^{k,\delta,\nu} a_{m+p} + \left((1-\eta\mu) + \mu[\widetilde{m+p}]_q\right) \sum_{m=1}^{\infty} [\widetilde{m+p}]_q \Lambda_{q,m}^{k,\delta,\nu} a_{m+p} \leq \sum_{m=1}^{\infty} \phi_{m+p} a_{m+p} \leq \phi_p. \tag{56}$$

Next, we obtain

$$\begin{aligned}
&\left((1-\eta\mu) + \mu[m+p]_q\right) \sum_{m=1}^{\infty} [\widetilde{m+p}]_q \Lambda_{q,m}^{k,\delta,\nu} a_{m+p} \leq \phi_p + \eta(1-\mu) \sum_{m=1}^{\infty} \Lambda_{q,m}^{k,\delta,\nu} a_{m+p} \\
&= \phi_p + \eta(1-\mu) \frac{\phi_p}{\phi_{m+p}}.
\end{aligned} \tag{57}$$

This yields

$$\frac{\left(\phi_{m+p} + \eta(1-\mu)\right)}{[m+p]_q} \sum_{m=1}^{\infty} [\widetilde{m+p}] \Lambda_{q,m}^{k,\delta,\nu} a_{m+p} \leq \frac{\phi_p\left(\phi_{m+p} + \eta(1-\mu)\right)}{\phi_{m+p}}. \tag{58}$$

Hence, we obtain the desired inequality (53).

By using the abovementioned theorem, we can easily obtain the following result. □

Corollary 11. *Let* $0<q<1$, $0\le\delta\le1$, $p,k,v\in\mathbb{N}$, $0\le\mu\le\widetilde{[p]}_q$, *and* $\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)$ *given by (13) be in the class* $\widetilde{TS}_{q,k}^{\delta,v}(p,\eta)$. *Then, we have*

$$\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,v}a_{m+p}\le\frac{-\eta+\widetilde{[p]}_q}{-\eta+\widetilde{[m+p]}_q},\text{ and}$$
$$\sum_{m=1}^{\infty}\widetilde{[m+p]}_q\Lambda_{q,m}^{k,\delta,v}a_{m+p}\le\frac{\left(-\eta+\widetilde{[p]}_q\right)\widetilde{[m+p]}_q}{-\eta+\widetilde{[m+p]}_q}. \quad (59)$$

Corollary 12. *Let* $0<q<1$, $0\le\delta\le1$, $p,k,v\in\mathbb{N}$, $0\le\mu\le\widetilde{[p]}_q$, *and* $\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)$ *given by (13) be in the class* $\widetilde{TC}_{q,k}^{\delta,v}(p,\eta)$. *Then, we have*

$$\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,v}a_{m+p}\le\frac{(1-\eta)\widetilde{[p]}_q+\widetilde{[p]}_q^2}{(1-\eta)\widetilde{[m+p]}_q+\widetilde{[m+p]}_q^2},\text{ and}$$
$$\sum_{m=1}^{\infty}\widetilde{[m+p]}_q\Lambda_{q,m}^{k,\delta,v}a_{m+p}\le\frac{\left((1-\eta)\widetilde{[p]}_q+\widetilde{[p]}_q^2\right)\widetilde{[m+p]}_q}{(1-\eta)\widetilde{[m+p]}_q+\widetilde{[m+p]}_q^2}. \quad (60)$$

Similarly, we can obtain a new theorem as follows.

Theorem 13. *Let* $0<q<1$, $0\le\delta\le1$, $p,k,v\in\mathbb{N}$, $0\le\mu\le\widetilde{[p]}_q$, *and* $\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)$ *given by 1.3 be in the class* $\widetilde{\mathscr{M}}_{q,k}^{\delta,v}(p,\mu,\eta)$. *Then, we have*

$$\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,v}a_{m+p}\le\frac{\widetilde{[p]}_q+\mu\widetilde{[p]}_q^2-\eta}{\widetilde{[m+p]}_q+\mu\widetilde{[m+p]}_q^2},\text{ and}$$
$$\sum_{m=1}^{\infty}\widetilde{[m+p]}_q\Lambda_{q,m}^{k,\delta,v}a_{k+p}\le\frac{\widetilde{[p]}_q+\mu\widetilde{[p]}_q^2-\eta}{\mu\left(1+\widetilde{[m+p]}_q\right)}. \quad (61)$$

Theorem 14. *Let* $0<q<1$, $0\le\delta\le1$, $p,k,v\in\mathbb{N}\ 0\le\eta<\widetilde{[p]}_q$, $0\le\mu\le1$, *and* $\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)$ *be a function given by (13). If* $\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\in\widetilde{\mathscr{K}}_{q,k}^{\delta,v}(p,\mu,\eta)$, *then we have*

$$r^p-r^{p+1}\frac{\phi_p}{\phi_{m+p}}\le\left|\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\right|\le r^p+r^{p+1}\frac{\phi_p}{\phi_{m+p}}, \quad (62)$$

where ϕ_{m+p} *is given by (54).*

Proof. Since $\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\in\widetilde{\mathscr{K}}_{q,k}^{\delta,v}(p,\mu,\eta)$, we have the following inequality:

$$\sum_{m=2}^{\infty}a_k\le\frac{\phi_p}{\phi_{m+p}}. \quad (63)$$

Moreover, since the function $\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)$ is given by (13), we obtain

$$|\zeta|^p-|\zeta|^{p+1}\sum_{m=2}^{\infty}\Lambda_{q,m}^{k,\delta,v}a_m|\zeta|^{m-1}\le\left|\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\right|\le|\zeta|^p+|\zeta|^{p+1}\sum_{m=2}^{\infty}\Lambda_{q,m}^{k,\delta,v}a_m|\zeta|^{m-1}. \quad (64)$$

Consequently, we have

$$|\zeta|^p-|\zeta|^{p+1}\sum_{m=2}^{\infty}\Lambda_{q,m}^{k,\delta,v}a_m\le\left|\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\right|\le|\zeta|^p+|\zeta|^{p+1}\sum_{m=2}^{\infty}\Lambda_{q,m}^{k,\delta,v}a_m. \quad (65)$$

Hence, from the assertion (63), we derive the desired inequality (62).

By using the abovementioned theorem, we can easily obtain the following result. □

Corollary 15. *Let* $0<q<1$, $0\le\delta\le1$, $p,k,v\in\mathbb{N}\ 0\le\eta<\widetilde{[p]}_q$, $0\le\mu\le1$, *and* $\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)$ *be a function given by (13). If* $\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\in\widetilde{TS}_{q,k}^{\delta,v}$, *then we have*

$$r^p-r^{p+1}\frac{1}{\widetilde{[m+1]}_q}\le\left|\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\right|\le r^p+r^{p+1}\frac{1}{\widetilde{[m+p]}_q}. \quad (66)$$

Corollary 16. *Let* $0<q<1$, $0\le\delta\le1$, $p,k,v\in\mathbb{N}\ 0\le\eta<\widetilde{[p]}_q$, $0\le\mu\le1$, *and* $\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)$ *be a function given by (13). If* $\widetilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\in\widetilde{TC}_{q,k}^{\delta,v}$, *then we have*

$$r^p - r^{p+1}\frac{2}{[m+1]_q + [m+1]_q^2} \le \left|\tilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\right| \le r^p + r^{p+1}\frac{2}{[m+1]_q + [m+1]_q^2}. \tag{67}$$

Alike to the proof applied to Theorem 14, we can establish the following result.

Theorem 17. *Let $0<q<1$, $0\le\delta\le1$, $p,k,v\in\mathbb{N}$ $0\le\eta<\widetilde{[p]}_q$, $0\le\mu\le1$, and $\tilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)$ be a function given by (13). If $\tilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\in\tilde{\mathscr{K}}_{q,k}^{\delta,v}(p,\mu,\eta)$, then we have*

$$r^p - r^{p+1}\frac{\phi_p\,\widetilde{[m+p]}_q}{\phi_{m+p}} \le \left|\zeta\tilde{D}_q\left(\tilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k\right)\right| \le r^p + r^{p+1}\frac{\phi_p\,\widetilde{[m+p]}_q}{\phi_{m+p}}, \tag{68}$$

where ϕ_{k+p} is given by (54).

By using the abovementioned theorem, we can easily obtain the following result.

Corollary 18. *Let $0<q<1$, $0\le\delta\le1$, $p,k,v\in\mathbb{N}$ $0\le\eta<\widetilde{[p]}_q$, $0\le\mu\le1$, and $\tilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)$ be a function given by (13). If $\tilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\in\widetilde{TS}_{q,k}^{\delta,v}(p,\eta)$, then we have*

$$r^p - r^{p+1}\frac{\left(-\eta+\widetilde{[p]}_q\right)\widetilde{[m+p]}_q}{-\eta+\widetilde{[m+p]}_q} \le \left|\zeta\tilde{D}_q\left(\tilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k\right)\right| \le r^p + r^{p+1}\frac{\left(-\eta+\widetilde{[p]}_q\right)\widetilde{[m+p]}_q}{-\eta+\widetilde{[m+p]}_q}. \tag{69}$$

Corollary 19. *Let $0<q<1$, $0\le\delta\le1$, $p,k,v\in\mathbb{N}$ $0\le\eta<\widetilde{[p]}_q$, $0\le\mu\le1$, and $\tilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)$ be a function given by (13). If $\tilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)\in\widetilde{TC}_{q,k}^{\delta,v}$, then we have*

$$r^p - r^{p+1}\frac{2}{1+\widetilde{[m+1]}_q} \le \left|\zeta\tilde{D}_q\left(\tilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k\right)\right| \le r^p + r^{p+1}\frac{2}{1+\widetilde{[m+1]}_q}. \tag{70}$$

3. Inclusion Results Involving the Symmetric (n,δ)-Neighborhoods

In this section, we determine the neighborhood properties for each of the following function classes:

$$\tilde{\mathscr{K}}_{q,k}^{\delta,v}(p,\eta,\mu),\ \widetilde{\mathscr{M}}_{q,k}^{\delta,v}(p,\eta,\mu),\ \widetilde{\mathrm{TS}}_{q,k}^{\delta,v}(p,\eta), \text{and } \widetilde{\mathrm{TC}}_{q,k}^{\delta,v}(p,\eta). \tag{71}$$

Theorem 20. *Let $0<q<1$, $0\le\delta\le1$, $p,k,v\in\mathbb{N}$, and $0\le\mu\le\widetilde{[p]}_q$. If $\tilde{\mathfrak{D}}_q^{\delta,v}\mathscr{R}_k(\zeta)$ is given by (13) belongs to the class $\tilde{\mathscr{K}}_{q,k}^{\delta,v}(p,\eta,\mu)$, then we have*

$$\tilde{\mathscr{K}}_{q,k}^{\delta,v}(p,\eta,\mu)\subset\tilde{\mathscr{N}}_{\beta,q,k}^{\delta,v}(h), \tag{72}$$

where $h(\zeta)=\zeta^p,\zeta\in\Omega$ and

$$\beta := \frac{\phi_p\,\widetilde{[m+p]}_q}{\phi_{m+p}}, \tag{73}$$

where ϕ_{m+p} is given by (54).

Proof. Assertion (72) would follow easily from the definition of $\tilde{\mathscr{N}}_{\delta,q}(h,f)$, which is given by (29) when g is replaced by f and the second assertion (52) of Theorem 10. □

Theorem 21. *Let* $0<q<1$, $0\leq\delta\leq 1$, $p,k,\nu\in\mathbb{N}$, *and* $0\leq\mu\leq\widetilde{[p]}_q$. *If* $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)$ *given by (13) belongs to the class* $\widetilde{\mathcal{M}}_{q,k}^{\delta,\nu}(p,\eta,\mu)$, *then we have*

$$\widetilde{\mathcal{M}}_{q,k}^{\delta,\nu}(p,\eta,\mu)\subset\widetilde{\mathcal{N}}_{\beta,q,k}^{\delta,\nu}(h), \tag{74}$$

where $h(\zeta)=\zeta^p,\zeta\in\Omega$ *and*

$$\beta:=\frac{\widetilde{[p]}_q+\mu\widetilde{[p]}_q^2-\eta}{\mu\left(1+[\widetilde{m+p}]_q\right)}. \tag{75}$$

Putting $\mu=0$ in Theorem 20 leads to the following corollaries.

Corollary 22. *Let* $0<q<1$, $0\leq\delta\leq 1$, *and* $p,k,\nu\in\mathbb{N}$. *If* $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)$ *given by (13) belongs to the class* $\widetilde{TS}_{q,k}^{\delta,\nu}(p,\eta)$, *then we have*

$$\widetilde{\mathrm{TS}}_{q,k}^{\delta,\nu}(p,\eta)\subset\widetilde{\mathcal{N}}_{\beta,q,k}^{\delta,\nu}(h), \tag{76}$$

where $h(\zeta)=\zeta^p,\zeta\in\Omega$ *and*

$$\beta:=\frac{\left(-\eta+\widetilde{[p]}_q\right)\left([\widetilde{m+p}]_q\right)}{-\eta+[\widetilde{m+p}]_q}. \tag{77}$$

Putting $\mu=1$ in Theorem 20 leads to the following corollary.

Corollary 23. *Let* $0<q<1$, $0\leq\delta\leq 1$, *and* $p,k,\nu\in\mathbb{N}$. *If* $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)$ *given by (13) belongs to the class* $\widetilde{TC}_{q,k}^{\delta,\nu}(p,\eta)$, *then*

$$\widetilde{TC}_{q,k}^{\delta,\nu}(p,\eta)\subset\widetilde{\mathcal{N}}_{\beta,q,k}^{\delta,\nu}(h), \tag{78}$$

where $h(\zeta)=\zeta^p,\zeta\in\Omega$ *and*

$$\beta:=\frac{(1-\eta)\widetilde{[p]}_q+\widetilde{[p]}_q^2}{1-\eta+[\widetilde{m+p}]_q}. \tag{79}$$

We now state the following necessary definitions.

Definition 24. A function $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)$ is said to be in the class $\widetilde{\mathrm{TS}}_{q,k}^{\delta,\nu}(p,\gamma,\mu)$ if there exists $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{G}_k(\zeta)\in\widetilde{\mathcal{M}}_{q,k}^{\delta,\nu}(p,\eta,\mu)$ such that

$$\left|\frac{\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)}{\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{G}_k(\zeta)}-1\right|<\gamma, \tag{80}$$

where $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{G}_k(\zeta)$ is given by (27).

Theorem 25. *If* $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{G}_k(\zeta)\in\widetilde{\mathcal{M}}_{q,k}^{\delta,\nu}(p,\eta,\mu)$ *and*

$$\gamma=1-\frac{\delta}{[1+p]_q}\frac{\phi_{m+p}}{\phi_{m+p}-\phi_p}, \tag{81}$$

then we have

$$\widetilde{\mathcal{N}}_{\beta,q,k}^{\delta,\nu}\left(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)\right)\subseteq\widetilde{\mathrm{TS}}_{q,k}^{\delta,\nu}(p,\gamma,\mu), \tag{82}$$

where ϕ_{m+p} *is given by (54) and* $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)$ *is given by (27).*

Proof. Since $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)\in\widetilde{\mathcal{N}}_{\beta,q,k}^{\delta,\nu}(\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{G}_k(\zeta))$, then by using assertion (27), we derive

$$\sum_{m=1}^{\infty}[\widetilde{m+p}]_q\Lambda_{q,m}^{k,\delta,\nu}\left|a_{m+p}-b_{m+p}\right|\leq\beta. \tag{83}$$

Since $\widetilde{[n]}_q$ is a nondecreasing sequence, we infer that

$$\sum_{m=1}^{\infty}[\widetilde{1+p}]_q\Lambda_{q,m}^{k,\delta,\nu}\left|a_{m+p}-b_{m+p}\right|\leq\sum_{m=1}^{\infty}[\widetilde{m+p}]_q\Lambda_{q,m}^{k,\delta,\nu}\left|a_{m+p}-b_{m+p}\right|. \tag{84}$$

This implies that

$$[\widetilde{1+p}]_q\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}\left|a_{m+p}-b_{m+p}\right|\leq\sum_{m=1}^{\infty}[\widetilde{m+p}]_q\Lambda_{q,m}^{k,\delta,\nu}\left|a_{m+p}-b_{m+p}\right|. \tag{85}$$

By applying inequality (83), we get

$$\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}\left|a_{m+p}-b_{m+p}\right|\leq\frac{\beta}{[\widetilde{1+p}]_q},\qquad(0\leq q<1,\beta>0). \tag{86}$$

Hence, under the assumption that the function $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{G}_k(\zeta)$ belongs to the class $\widetilde{\mathcal{K}}_{q,k}^{\delta,\nu}(p,\eta,\mu)$, given by (27), we, by using the above inequality, have

$$\sum_{m=2}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}b_m \leq \frac{\phi_p}{\phi_{m+p}}. \tag{87}$$

In view of the (5) and (27), we write

$$\begin{aligned}\left|\frac{\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)}{\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{G}_k(\zeta)}-1\right| &= \left|\frac{\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}\left(a_{m+p}-b_{m+p}\right)\zeta^{m+p-1}}{1+\sum_{k=2}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}b_{m+p}\zeta^{m+p-1}}\right| \leq \left|\frac{\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}\left(a_{m+p}-b_{m+p}\right)}{1-\sum_{m=1}^{\infty}\Lambda_{q,m}^{k,\delta,\nu}b_{m+p}}\right| \\ &\leq \frac{\beta}{[1+p]_q}\left(\frac{1}{1-\sum_{m=1}^{\infty}\left|\Lambda_{q,m}^{k,\delta,\nu}b_{m+p}\right|}\right) \\ &\leq \frac{\beta}{[1+p]_q}\frac{\phi_{m+p}}{\phi_{m+p}-\phi_p}.\end{aligned} \tag{88}$$

Now, by setting

$$\gamma \leq 1-\frac{\beta}{[1+p]_q}\frac{\phi_{m+p}}{\phi_{m+p}-\phi_p}, \tag{89}$$

we, in view of Definition 24 and using the inequality (88), obtain that $\widetilde{\mathfrak{D}}_q^{\delta,\nu}\mathcal{R}_k(\zeta)\in\widetilde{TS}_{q,k}^{\delta,\nu}(p,\beta,\mu)$.

This completes the proof of Theorem 25. □

4. Concluding Remarks

In this paper, a new generalization of the linear operator which is a continuous bridge between the Poisson distribution probability and the geometric function theory has been considered. The new discussed linear operator is defined by the generalized power series, q-Sălăgean integral operator, and properties of the symmetric q-derivative. The suggestion of this operator was applied in extending the geometric function theory. Then, several subclasses of p-valent functions with negative coefficients were introduced by using symmetric q-derivatives. Furthermore, symmetric (δ,q)-neighborhoods of p-valent functions are defined. Moreover, coefficient bounds for such given functions are derived. In addition, several inclusion relations for each function in the new subclasses are also obtained. Therefore, the results obtained in this research could be further employed for discussing fractional symmetric q-derivatives to generalize certain results involving univalent functions.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The authors declare they have no conflicts of interest.

Authors' Contributions

The authors contributed equally and significantly in writing this paper. All the authors have read and approved the final manuscript.

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