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Research Article

A Novel Generalized n-Dimensions Sixtic B-Spline Function to Solving n-Dimensions Mathematical Models

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In this study, we present a novel framework for the sixtic B-spline collocation approach in n-dimensions. Building upon previous research that focused on developing B-spline functions in n-dimensions to solve mathematical models, this work represents an extension of those efforts. We provide formulations of the sixtic B-spline collocation algorithm in one-dimensional, two-dimensional, and three-dimensional settings. These structures play a crucial role in solving mathematical models across diverse fields of study. To showcase the efficacy and accuracy of the proposed method, we employ a range of test problems in two and three dimensions. These examples serve as demonstrations of the effectiveness and precision of the suggested approach.

1. Introduction

Academics across various disciplines, including physics and fluid mechanics, have made attempts to solve mathematical models in n-dimensions using analytical and approximative methods. However, many of these models pose challenges for analytical solutions, prompting researchers to explore numerical approaches. One such numerical strategy is finite differences, as exemplified in [1], which can be employed to tackle n-dimensional models. Additionally, researchers have endeavored to adapt techniques designed for one-dimensional models to handle n-dimensional problems, such as spectral methods [2, 3]. Yet, spectral methods have proven to be challenging when dealing with most nonlinear models. To address specific two-dimensional issues, Gardner et al. investigated a two-dimensional version of the bi-cubic B-spline finite element method [4]. Arora and colleagues utilized the bi-cubic B-spline collocation approach to numerically solve a second-order two-dimensional hyperbolic problem [5]. Another study by Mittal and others employed modified bi-cubic B-splines within the framework of finite elements to examine a two-dimensional diffusion problem [6]. A modified cubic B-spline differential quadrature method was employed by Elsherbeny and colleagues to investigate the 2D-Poisson equation [7]. Kutluay et al. utilized modified bi-quintic B-splines to solve the two-dimensional unsteady Burgers' equation, Poisson equation, and diffusion equations [8–10]. Further advancements were made by Raslan et al., who explored the generalization of B-spline functions in n-dimensions for solving partial differential equations. They discussed extended cubic B-splines [11], n-dimensional quadratic B-splines [12], as well as three- and four-dimensional cubic and trigonometric B-spline collocation methods [13, 14]. The n-dimensional quartic and quintic B-spline collocation methods were investigated in separate studies [15, 16]. Boundary value problems have been studied using B-spline methods in many papers [17–20]. The B-spline collocation approach has been widely employed in numerous articles to solve various mathematical models. This study contributes to the advancement of research on B-spline collocation functions by describing the sixtic B-spline collocation algorithm in n-dimensions. Additionally, a substantial number of numerical examples are provided to illustrate its application.

The following is the structure of this article: In the second section, the formulas for the sixtic B-spline are presented for use with n-dimensional space. In the fourth section, numerical examples are presented for the first time. The final part of this investigation presents the conclusion that was reached throughout.

2. N-Dimensions Sixtic B-Spline Functions

The n-dimensional sixtic B-splines are displayed for your review in this section.

2.1. Sixtic B-Spline in One Dimension [21]. Let us assume that $l \le \mathfrak{X} \le m$ and $\mathscr{G}_i(\mathfrak{X})$ are the sixtic B-splines with knots at the locations $\mathfrak{X}_i$. After that, the series of sixtic B-splines $\mathscr{G}_{-3}(\mathfrak{X}), \mathscr{G}_{-2}(\mathfrak{X}), \mathscr{G}_0(\mathfrak{X}), \ldots, \mathscr{G}_{N-1}(\mathfrak{X}), \mathscr{G}_N(\mathfrak{X}), \mathscr{G}_{N+2}(\mathfrak{X}), \mathscr{G}_{N+3}$, serves as the foundation for functions given over a range of values. The following provides the $G^N(\mathfrak{X})$ approximation to $G(\mathfrak{X})$:

$$G^N(\mathfrak{X}) = \sum_{i=-3}^{N+3} \mathscr{A}_i \mathscr{G}_i(\mathfrak{X}), \tag{1}$$

where $\mathscr{A}_i$ unknown term and $\mathscr{G}_i(\mathfrak{X})$ is a function given by

$$\mathscr{G}_i(\mathfrak{X}) = \frac{1}{h^6}\begin{cases} a_1 = (\mathfrak{X} - \mathfrak{X}_i + 3h)^6, & \mathfrak{X}_{i-3} \le \mathfrak{X} < \mathfrak{X}_{i-2}, \\ a_2 = a_1 - 7(\mathfrak{X} - \mathfrak{X}_i + 2h)^6, & \mathfrak{X}_{i-2} \le \mathfrak{X} \le \mathfrak{X}_{i-1}, \\ a_3 = a_2 + 21(\mathfrak{X} - \mathfrak{X}_i + h)^6, & \mathfrak{X}_{i-1} \le \mathfrak{X} \le \mathfrak{X}_{i}, \\ a_4 = a_3 - 35(\mathfrak{X} - \mathfrak{X}_i)^6, & \mathfrak{X}_{i} \le \mathfrak{X} \le \mathfrak{X}_{i+1}, \\ b_3 = b_2 + 21(\mathfrak{X} - \mathfrak{X}_i - 2h)^6, & \mathfrak{X}_{i+1} \le \mathfrak{X} \le \mathfrak{X}_{i+2}, \\ b_2 = b_1 - 7(\mathfrak{X} - \mathfrak{X}_i - 3h)^6, & \mathfrak{X}_{i+2} \le \mathfrak{X} \le \mathfrak{X}_{i+3}, \\ b_1 = (\mathfrak{X} - \mathfrak{X}_i - 4h)^6, & \mathfrak{X}_{i+3} \le \mathfrak{X} \le \mathfrak{X}_{i+4}, \\ 0, & \text{otherwise}. \end{cases} \tag{2}$$

We use equations (1) and (2) with substitution by collection points to find $G_i, dG_i/d\mathfrak{X}, d^2G_i/d\mathfrak{X}^2$ as follows:

$$\begin{aligned} G_i &= \frac{1}{64}\left(\mathscr{A}_{i-3} + 722\mathscr{A}_{i-2} + 10543\mathscr{A}_{i-1} + 23548\mathscr{A}_i + 10543\mathscr{A}_{i+1} + 722\mathscr{A}_{i+2} + \mathscr{A}_{i+3}\right), \\ \frac{dG_i}{d\mathfrak{X}} &= \frac{3}{16h}\left(\mathscr{A}_{i-3} + 236\mathscr{A}_{i-2} + 1445\mathscr{A}_{i-1} - 1445\mathscr{A}_{i+1} - 236\mathscr{A}_{i+2} - \mathscr{A}_{i+3}\right), \\ \frac{d^2G_i}{d\mathfrak{X}^2} &= \frac{15}{18h^2}\left(\mathscr{A}_{i-3} + 74\mathscr{A}_{i-2} + 79\mathscr{A}_{i-1} - 308\mathscr{A}_i + 79\mathscr{A}_{i+1} + 74\mathscr{A}_{i+2} + \mathscr{A}_{i+3}\right). \end{aligned} \tag{3}$$

The following theorem can be derived from the analysis presented above.

Theorem 1. *From equation (1) the approximation functions to $G_i, dG_i/d\mathfrak{X}$ and $d^2G_i/d\mathfrak{X}^2$ are given in terms of the $\mathscr{A}_i$ at equation (3).*

2.2. Two-Dimensions Sixtic B-Spline. The subsection that follows gives the formula for a two-dimensional sixtic spline function on a rectangular grid with regular rectangular finite elements on both sides. By using the knots $(\mathfrak{X}_i, \mathfrak{Y}_F)$, where $i = 0, 1, \ldots, N$ and $F = 0, 1, \ldots, M$, we can obtain $h = \Delta\mathfrak{X}$ and $k = \Delta\mathfrak{Y}$. The following gives an approximation of $G^N(\mathfrak{X}, \mathfrak{Y})$ to $G(\mathfrak{X}, \mathfrak{Y})$:

$$G^N(\mathfrak{X}, \mathfrak{Y}) = \sum_{i=-3}^{N+3}\sum_{F=-3}^{M+3} \mathscr{A}_{i,F}\mathscr{P}_{i,F}(\mathfrak{X}, \mathfrak{Y}), \tag{4}$$

where $\mathscr{A}_{i,F}$ are the amplitudes of the sixtic B-splines, $\mathscr{P}_{i,F}(\mathfrak{X}, \mathfrak{Y})$ are determined by the following formula:

$$\mathscr{P}_{i,F}(\mathfrak{X}, \mathfrak{Y}) = \mathscr{G}_i(\mathfrak{X})\mathscr{G}_F(\mathfrak{Y}). \tag{5}$$

Find the peaks on the knot $(\mathfrak{X}_i, \mathfrak{Y}_j)$ and $\mathscr{G}_i(\mathfrak{X}), \mathscr{G}_F(\mathfrak{Y})$ that have the same shape as the one-dimensional sixtic B-splines. When this occurred, the formulations of

$G_{i,\digamma}, \partial G_{i,\digamma}/\partial \mathfrak{X}, \partial G_{i,\digamma}/\partial \mathfrak{Y}$, $\partial^2 G_{i,\digamma}/\partial \mathfrak{X}^2, \partial^2 G_{i,\digamma}/\partial \mathfrak{Y}^2, \partial^2 G_{i,\digamma}/\partial \mathfrak{X}\partial \mathfrak{Y}, \ldots$ are given by

$$
\begin{aligned}
G_{i,\digamma} = \frac{1}{4096}\Big(&\mathscr{A}_{i-3,\digamma-3} + 722\mathscr{A}_{i-3,\digamma-2} + 10543\mathscr{A}_{i-3,\digamma-1} + 23548\mathscr{A}_{i-3,\digamma} + 10543\mathscr{A}_{i-3,\digamma+1}\\
&+ 722\mathscr{A}_{i-3,\digamma+2} + \mathscr{A}_{i-3,\digamma+3} + 722\mathscr{A}_{i-2,\digamma-3} + 521284\mathscr{A}_{i-2,\digamma-2} + 7612046\mathscr{A}_{i-2,\digamma-1}\\
&+ 17001656\mathscr{A}_{i-2,\digamma} + 7612046\mathscr{A}_{i-2,\digamma+1} + 521284\mathscr{A}_{i-2,\digamma+2} + 722\mathscr{A}_{i-2,\digamma+3}\\
&+ 10543\mathscr{A}_{i-1,\digamma-3} + 7612046\mathscr{A}_{i-1,\digamma-2} + 111154849\mathscr{A}_{i-1,\digamma-1} + 248266564\mathscr{A}_{i-1,\digamma}\\
&+ 111154849\mathscr{A}_{i-1,\digamma+1} + 7612046\mathscr{A}_{i-1,\digamma+2} + 10543\mathscr{A}_{i-1,\digamma+3} + 23548\mathscr{A}_{i,\digamma-3}\\
&+ 17001656\mathscr{A}_{i,\digamma-2} + 248266564\mathscr{A}_{i,\digamma-1} + 554508304\mathscr{A}_{i,\digamma} + 248266564\mathscr{A}_{i,\digamma+1}\\
&+ 17001656\mathscr{A}_{i,\digamma+2} + 23548\mathscr{A}_{i,\digamma+3} + 10543\mathscr{A}_{i+1,\digamma-3} + 7612046\mathscr{A}_{i+1,\digamma-2}\\
&+ 111154849\mathscr{A}_{i+1,\digamma-1} + 248266564\mathscr{A}_{i+1,\digamma} + 111154849\mathscr{A}_{i+1,\digamma+1} + 7612046\mathscr{A}_{i+1,\digamma+2}\\
&+ 10543\mathscr{A}_{i+1,\digamma+3} + 722\mathscr{A}_{i+2,\digamma-3} + 521284\mathscr{A}_{i+2,\digamma-2} + 7612046\mathscr{A}_{i+2,\digamma-1}\\
&+ 17001656\mathscr{A}_{i+2,\digamma} + 7612046\mathscr{A}_{i+2,\digamma+1} + 521284\mathscr{A}_{i+2,\digamma+2} + 722\mathscr{A}_{i+2,\digamma+3} + \mathscr{A}_{i+3,\digamma-3}\\
&+ 722\mathscr{A}_{i+3,\digamma-2} + 10543\mathscr{A}_{i+3,\digamma-1} + 23548\mathscr{A}_{i+3,\digamma}\\
&+ 10543\mathscr{A}_{i+3,\digamma+1} + 722\mathscr{A}_{i+3,\digamma+2} + \mathscr{A}_{i+3,\digamma+3}\Big),\\
\frac{\partial G_{i,\digamma}}{\partial \mathfrak{X}} = \frac{3}{1024h}\Big(&\mathscr{A}_{i-3,\digamma-3} + 722\mathscr{A}_{i-3,\digamma-2} + 10543\mathscr{A}_{i-3,\digamma-1} + 23548\mathscr{A}_{i-3,\digamma}\\
&+ 10543\mathscr{A}_{i-3,\digamma+1} + 722\mathscr{A}_{i-3,\digamma+2} + \mathscr{A}_{i-3,\digamma+3} + 236\mathscr{A}_{i-2,\digamma-3} + 170392\mathscr{A}_{i-2,\digamma-2}\\
&+ 2488148\mathscr{A}_{i-2,\digamma-1} + 5557328\mathscr{A}_{i-2,\digamma} + 2488148\mathscr{A}_{i-2,\digamma+1} + 170392\mathscr{A}_{i-2,\digamma+2}\\
&+ 236\mathscr{A}_{i-2,\digamma+3} + 1445\mathscr{A}_{i-1,\digamma-3} + 1043290\mathscr{A}_{i-1,\digamma-2} + 15234635\mathscr{A}_{i-1,\digamma-1}\\
&+ 34026860\mathscr{A}_{i-1,\digamma} + 15234635\mathscr{A}_{i-1,\digamma+1} + 1043290\mathscr{A}_{i-1,\digamma+2} + 1445\mathscr{A}_{i-1,\digamma+3}\\
&- 1445\mathscr{A}_{i+1,\digamma-3} - 1043290\mathscr{A}_{i+1,\digamma-2} - 15234635\mathscr{A}_{i+1,\digamma-1} - 34026860\mathscr{A}_{i+1,\digamma}\\
&- 15234635\mathscr{A}_{i+1,\digamma+1} - 1043290\mathscr{A}_{i+1,\digamma+2} - 1445\mathscr{A}_{i+1,\digamma+3} - 236\mathscr{A}_{i+2,\digamma-3}\\
&- 170392\mathscr{A}_{i+2,\digamma-2} - 2488148\mathscr{A}_{i+2,\digamma-1} - 5557328\mathscr{A}_{i+2,\digamma} - 2488148\mathscr{A}_{i+2,\digamma+1}\\
&- 170392\mathscr{A}_{i+2,\digamma+2} - 236\mathscr{A}_{i+2,\digamma+3} - \mathscr{A}_{i+3,\digamma-3} - 722\mathscr{A}_{i+3,\digamma-2} - 10543\mathscr{A}_{i+3,\digamma-1}\\
&- 23548\mathscr{A}_{i+3,\digamma} - 10543\mathscr{A}_{i+3,\digamma+1} - 722\mathscr{A}_{i+3,\digamma+2} - \mathscr{A}_{i+3,\digamma+3}\Big),
\end{aligned}
$$

$$\begin{aligned}
\frac{\partial G_{i,\digamma}}{\partial \mathfrak{Y}} = \frac{3}{1024k}\Big(&\mathscr{A}_{i-3,\digamma-3} + 236\mathscr{A}_{i-3,\digamma-2} + 1445\mathscr{A}_{i-3,\digamma-1} \\
&- 1445\mathscr{A}_{i-3,\digamma+1} - 236\mathscr{A}_{i-3,\digamma+2} - \mathscr{A}_{i-3,\digamma+3} + 722\mathscr{A}_{i-2,\digamma-3} + 170392\mathscr{A}_{i-2,\digamma-2} \\
&+ 1043290\mathscr{A}_{i-2,\digamma-1} - 1043290\mathscr{A}_{i-2,\digamma+1} - 170392\mathscr{A}_{i-2,\digamma+2} - 722\mathscr{A}_{i-2,\digamma+3} \\
&+ 10543\mathscr{A}_{i-1,\digamma-3} + 2488148\mathscr{A}_{i-1,\digamma-2} + 15234635\mathscr{A}_{i-1,\digamma-1} - 15234635\mathscr{A}_{i-1,\digamma+1} \\
&- 2488148\mathscr{A}_{i-1,\digamma+2} - 10543\mathscr{A}_{i-1,\digamma+3} + 23548\mathscr{A}_{i,\digamma-3} + 5557328\mathscr{A}_{i,\digamma-2} + 34026860\mathscr{A}_{i,\digamma-1} \\
&- 34026860\mathscr{A}_{i,\digamma+1} - 5557328\mathscr{A}_{i,\digamma+2} - 23548\mathscr{A}_{i,\digamma+3} + 10543\mathscr{A}_{i+1,\digamma-3} + 2488148\mathscr{A}_{i+1,\digamma-2} \\
&+ 15234635\mathscr{A}_{i+1,\digamma-1} - 15234635\mathscr{A}_{i+1,\digamma+1} - 2488148\mathscr{A}_{i+1,\digamma+2} - 10543\mathscr{A}_{i+1,\digamma+3} \\
&+ 722\mathscr{A}_{i+2,\digamma-3} + 170392\mathscr{A}_{i+2,\digamma-2} + 1043290\mathscr{A}_{i+2,\digamma-1} - 1043290\mathscr{A}_{i+2,\digamma+1} \\
&- 170392\mathscr{A}_{i+2,\digamma+2} - 722\mathscr{A}_{i+2,\digamma+3} + \mathscr{A}_{i+3,\digamma-3} + 236\mathscr{A}_{i+3,\digamma-2} + 1445\mathscr{A}_{i+3,\digamma-1} \\
&- 1445\mathscr{A}_{i+3,\digamma+1} - 236\mathscr{A}_{i+3,\digamma+2} - \mathscr{A}_{i+3,\digamma+3}\Big),
\end{aligned} \tag{6}$$

$$\begin{aligned}
\frac{\partial^2 G_{i,\digamma}}{\partial \mathfrak{X}^2} = \frac{15}{512h^2}\Big(&\mathscr{A}_{i-3,\digamma-3} + 722\mathscr{A}_{i-3,\digamma-2} + 10543\mathscr{A}_{i-3,\digamma-1} + 23548\mathscr{A}_{i-3,\digamma} \\
&+ 10543\mathscr{A}_{i-3,\digamma+1} + 722\mathscr{A}_{i-3,\digamma+2} + \mathscr{A}_{i-3,\digamma+3} + 74\mathscr{A}_{i-2,\digamma-3} + 53428\mathscr{A}_{i-2,\digamma-2} \\
&+ 780182\mathscr{A}_{i-2,\digamma-1} + 1742552\mathscr{A}_{i-2,\digamma} + 780182\mathscr{A}_{i-2,\digamma+1} + 53428\mathscr{A}_{i-2,\digamma+2} \\
&+ 74\mathscr{A}_{i-2,\digamma+3} + 79\mathscr{A}_{i-1,\digamma-3} + 57038\mathscr{A}_{i-1,\digamma-2} + 832897\mathscr{A}_{i-1,\digamma-1} + 1860292\mathscr{A}_{i-1,\digamma} \\
&+ 832897\mathscr{A}_{i-1,\digamma+1} + 57038\mathscr{A}_{i-1,\digamma+2} + 79\mathscr{A}_{i-1,\digamma+3} - 308\mathscr{A}_{i,\digamma-3} - 222376\mathscr{A}_{i,\digamma-2} \\
&- 3247244\mathscr{A}_{i,\digamma-1} - 7252784\mathscr{A}_{i,\digamma} - 3247244\mathscr{A}_{i,\digamma+1} - 222376\mathscr{A}_{i,\digamma+2} - 308\mathscr{A}_{i,\digamma+3} \\
&+ 79\mathscr{A}_{i+1,\digamma-3} + 57038\mathscr{A}_{i+1,\digamma-2} + 832897\mathscr{A}_{i+1,\digamma-1} + 1860292\mathscr{A}_{i+1,\digamma} + 832897\mathscr{A}_{i+1,\digamma+1} \\
&+ 57038\mathscr{A}_{i+1,\digamma+2} + 79\mathscr{A}_{i+1,\digamma+3} + 74\mathscr{A}_{i+2,\digamma-3} + 53428\mathscr{A}_{i+2,\digamma-2} + 780182\mathscr{A}_{i+2,\digamma-1} \\
&+ 1742552\mathscr{A}_{i+2,\digamma} + 780182\mathscr{A}_{i+2,\digamma+1} + 53428\mathscr{A}_{i+2,\digamma+2} + 74\mathscr{A}_{i+2,\digamma+3} + \mathscr{A}_{i+3,\digamma-3} \\
&+ 722\mathscr{A}_{i+3,\digamma-2} + 10543\mathscr{A}_{i+3,\digamma-1} + 23548\mathscr{A}_{i+3,\digamma} + 10543\mathscr{A}_{i+3,\digamma+1} \\
&+ 722\mathscr{A}_{i+3,\digamma+2} + \mathscr{A}_{i+3,\digamma+3}\Big),
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial^2 G_{i,\digamma}}{\partial \mathfrak{Y}^2} = \frac{15}{512k^2}\big(&\mathscr{A}_{i-3,\digamma-3} + 74\mathscr{A}_{i-3,\digamma-2} + 79\mathscr{A}_{i-3,\digamma-1} - 308\mathscr{A}_{i-3,\digamma} + 79\mathscr{A}_{i-3,\digamma+1}\\
&+ 74\mathscr{A}_{i-3,\digamma+2} + \mathscr{A}_{i-3,\digamma+3} + 722\mathscr{A}_{i-2,\digamma-3} + 53428\mathscr{A}_{i-2,\digamma-2} + 57038\mathscr{A}_{i-2,\digamma-1}\\
&- 222376\mathscr{A}_{i-2,\digamma} + 57038\mathscr{A}_{i-2,\digamma+1} + 53428\mathscr{A}_{i-2,\digamma+2} + 722\mathscr{A}_{i-2,\digamma+3} + 10543\mathscr{A}_{i-1,\digamma-3}\\
&+ 780182\mathscr{A}_{i-1,\digamma-2} + 832897\mathscr{A}_{i-1,\digamma-1} - 3247244\mathscr{A}_{i-1,\digamma} + 832897\mathscr{A}_{i-1,\digamma+1}\\
&+ 780182\mathscr{A}_{i-1,\digamma+2} + 10543\mathscr{A}_{i-1,\digamma+3} + 23548\mathscr{A}_{i,\digamma-3} + 1742552\mathscr{A}_{i,\digamma-2}\\
&+ 1860292\mathscr{A}_{i,\digamma-1} - 7252784\mathscr{A}_{i,\digamma} + 1860292\mathscr{A}_{i,\digamma+1} + 1742552\mathscr{A}_{i,\digamma+2} + 23548\mathscr{A}_{i,\digamma+3}\\
&+ 10543\mathscr{A}_{i+1,\digamma-3} + 780182\mathscr{A}_{i+1,\digamma-2} + 832897\mathscr{A}_{i+1,\digamma-1} - 3247244\mathscr{A}_{i+1,\digamma}\\
&+ 832897\mathscr{A}_{i+1,\digamma+1} + 780182\mathscr{A}_{i+1,\digamma+2} + 10543\mathscr{A}_{i+1,\digamma+3} + 722\mathscr{A}_{i+2,\digamma-3}\\
&+ 53428\mathscr{A}_{i+2,\digamma-2} + 57038\mathscr{A}_{i+2,\digamma-1} - 222376\mathscr{A}_{i+2,\digamma} + 57038\mathscr{A}_{i+2,\digamma+1}\\
&+ 53428\mathscr{A}_{i+2,\digamma+2} + 722\mathscr{A}_{i+2,\digamma+3} + \mathscr{A}_{i+3,\digamma-3} + 74\mathscr{A}_{i+3,\digamma-2} + 79\mathscr{A}_{i+3,\digamma-1}\\
&- 308\mathscr{A}_{i+3,\digamma} + 79\mathscr{A}_{i+3,\digamma+1} + 74\mathscr{A}_{i+3,\digamma+2} + \mathscr{A}_{i+3,\digamma+3}\big),\\
&\vdots
\end{aligned}
\tag{7}
$$

The following theorem can be derived from the analysis presented above.

Theorem 2. *From equation (4) the approximation functions to* $G_{i,\digamma}, \partial G_{i,\digamma}/\partial\mathfrak{X}, \partial G_{i,\digamma}/\partial\mathfrak{Y}, \partial^2 G_{i,\digamma}/\partial\mathfrak{X}^2, \partial^2 G_{i,\digamma}/\partial\mathfrak{Y}^2, \partial^2 G_{i,\digamma}/\partial\mathfrak{X}\partial\mathfrak{Y}, \ldots$ *are given in terms of the* $\mathscr{A}_{i,\digamma}$ *at equations (5) and (6).*

2.3. Sixtic B-Spline in the Three-Dimensions. We now have the sixtic B-spline in three approximations on a structure that is divided into a small number of side components. The knots determine the values $h = \Delta\mathfrak{X}$, $k = \Delta\mathfrak{Y}$, and $q = \Delta\mathfrak{Z}$. If $i = 0, 1, \ldots, N$, $\digamma = 0, 1, \ldots, M$ and $\wp = 0, 1, \ldots, R$, then $(\mathfrak{X}_i, \mathfrak{Y}_\digamma, \mathfrak{Z}_\wp)$ can be interpolated in terms of piece-wise sixtic B-splines. If the equation $G(\mathfrak{X}, \mathfrak{Y}, \mathfrak{Z})$ is a function of $\mathfrak{X}$, $\mathfrak{Y}$, and $\mathfrak{Z}$, it can be demonstrated that the expression $G^N(\mathfrak{X}, \mathfrak{Y}, \mathfrak{Z})$ has the following unique approximation:

$$
G^N(\mathfrak{X}, \mathfrak{Y}, \mathfrak{Z}) = \sum_{i=-3}^{N+3}\sum_{\digamma=-3}^{M+3}\sum_{\wp=-3}^{R+3}\mathscr{A}_{i,\digamma,\wp}B_{i,\digamma,\wp}(\mathfrak{X}, \mathfrak{Y}, \mathfrak{Z}), \tag{8}
$$

where $\mathscr{A}_{i,\digamma,\wp}$ are the sixtic B-spline amplitudes $B_{i,\digamma,\wp}(\mathfrak{X}, \mathfrak{Y}, \mathfrak{Z})$ supplied by

$$
B_{i,\digamma,\wp}(\mathfrak{X}, \mathfrak{Y}, \mathfrak{Z}) = \mathscr{G}_i(\mathfrak{X})\mathscr{G}_\digamma(\mathfrak{Y})\mathscr{G}_\wp(\mathfrak{Z}). \tag{9}
$$

In addition, the shapes of $\mathscr{G}_i(\mathfrak{X}), \mathscr{G}_\digamma(\mathfrak{Y})$ and $\mathscr{G}_\wp(\mathfrak{Z})$ are the same as those of sixtic B-splines when seen in one dimension. The compositions of $G_{i,\digamma,\wp}, \partial G_{i,\digamma,\wp}/\partial\mathfrak{X}, \partial G_{i,\digamma,\wp}/\partial\mathfrak{Y}, \partial G_{i,\digamma,\wp}/\partial\mathfrak{Z}, \partial^2 G_{i,\digamma,\wp}/\partial\mathfrak{X}^2, \partial^2 G_{i,\digamma,\wp}/\partial\mathfrak{Y}^2, \partial^2 G_{i,\digamma,\wp}/\partial\mathfrak{Z}^2, \partial^2 G_{i,\digamma,\wp}/\partial\mathfrak{X}\partial\mathfrak{Y}, \partial^2 G_{i,\digamma,\wp}/\partial\mathfrak{X}\partial\mathfrak{Z}, \ldots$, are given in terms of the $\mathscr{A}_{i,\digamma,\wp}$ by

$$
\begin{aligned}
G_{i,\digamma,\wp} = \frac{1}{262144}\big(&\mathscr{A}_{i-3,\digamma-3,\wp-3} + 722\mathscr{A}_{i-3,\digamma-3,\wp-2} + 10543\mathscr{A}_{i-3,\digamma-3,\wp-1} + 23548\mathscr{A}_{i-3,\digamma-3,\wp}\\
&+ 10543\mathscr{A}_{i-3,\digamma-3,\wp+1} + 722\mathscr{A}_{i-3,\digamma-3,\wp+2} + \mathscr{A}_{i-3,\digamma-3,\wp+3} + 722\mathscr{A}_{i-3,\digamma-2,\wp-3}
\end{aligned}
$$

$$
\begin{aligned}
&+521284\mathscr{A}_{i-3,\digamma-2,\wp-2}+7612046\mathscr{A}_{i-3,\digamma-2,\wp-1}+17001656\mathscr{A}_{i-3,\digamma-2,\wp}\\
&+7612046\mathscr{A}_{i-3,\digamma-2,\wp+1}+521284\mathscr{A}_{i-3,\digamma-2,\wp+2}+722\mathscr{A}_{i-3,\digamma-2,\wp+3}+10543\mathscr{A}_{i-3,\digamma-1,\wp-3}\\
&+7612046\mathscr{A}_{i-3,\digamma-1,\wp-2}+111154849\mathscr{A}_{i-3,\digamma-1,\wp-1}+248266564\mathscr{A}_{i-3,\digamma-1,\wp}\\
&+111154849\mathscr{A}_{i-3,\digamma-1,\wp+1}+7612046\mathscr{A}_{i-3,\digamma-1,\wp+2}+10543\mathscr{A}_{i-3,\digamma-1,\wp+3}\\
&+23548\mathscr{A}_{i-3,\digamma,\wp-3}+17001656\mathscr{A}_{i-3,\digamma,\wp-2}+248266564\mathscr{A}_{i-3,\digamma,\wp-1}\\
&+554508304\mathscr{A}_{i-3,\digamma,\wp}+248266564\mathscr{A}_{i-3,\digamma,\wp+1}+17001656\mathscr{A}_{i-3,\digamma,\wp+2}\\
&+23548\mathscr{A}_{i-3,\digamma,\wp+3}+10543\mathscr{A}_{i-3,\digamma+1,\wp-3}+7612046\mathscr{A}_{i-3,\digamma+1,\wp-2}+111154849\mathscr{A}_{i-3,\digamma+1,\wp-1}\\
&+248266564\mathscr{A}_{i-3,\digamma+1,\wp}+111154849\mathscr{A}_{i-3,\digamma+1,\wp+1}+7612046\mathscr{A}_{i-3,\digamma+1,\wp+2}\\
&+10543\mathscr{A}_{i-3,\digamma+1,\wp+3}+722\mathscr{A}_{i-3,\digamma+2,\wp-3}+521284\mathscr{A}_{i-3,\digamma+2,\wp-2}+7612046\mathscr{A}_{i-3,\digamma+2,\wp-1}\\
&+17001656\mathscr{A}_{i-3,\digamma+2,\wp}+7612046\mathscr{A}_{i-3,\digamma+2,\wp+1}+521284\mathscr{A}_{i-3,\digamma+2,\wp+2}\\
&+722\mathscr{A}_{i-3,\digamma+2,\wp+3}+\mathscr{A}_{i-3,\digamma+3,\wp-3}+722\mathscr{A}_{i-3,\digamma+3,\wp-2}+10543\mathscr{A}_{i-3,\digamma+3,\wp-1}\\
&+23548\mathscr{A}_{i-3,\digamma+3,\wp}+10543\mathscr{A}_{i-3,\digamma+3,\wp+1}+722\mathscr{A}_{i-3,\digamma+3,\wp+2}+\mathscr{A}_{i-3,\digamma+3,\wp+3}\\
&+722\mathscr{A}_{i-2,\digamma-3,\wp-3}+521284\mathscr{A}_{i-2,\digamma-3,\wp-2}+7612046\mathscr{A}_{i-2,\digamma-3,\wp-1}\\
&+17001656\mathscr{A}_{i-2,\digamma-3,\wp}+7612046\mathscr{A}_{i-2,\digamma-3,\wp+1}+521284\mathscr{A}_{i-2,\digamma-3,\wp+2}\\
&+722\mathscr{A}_{i-2,\digamma-3,\wp+3}+521284\mathscr{A}_{i-2,\digamma-2,\wp-3}+376367048\mathscr{A}_{i-2,\digamma-2,\wp-2}\\
&+5495897212\mathscr{A}_{i-2,\digamma-2,\wp-1}+12275195632\mathscr{A}_{i-2,\digamma-2,\wp}+5495897212\mathscr{A}_{i-2,\digamma-2,\wp+1}\\
&+376367048\mathscr{A}_{i-2,\digamma-2,\wp+2}+521284\mathscr{A}_{i-2,\digamma-2,\wp+3}+7612046\mathscr{A}_{i-2,\digamma-1,\wp-3}\\
&+5495897212\mathscr{A}_{i-2,\digamma-1,\wp-2}+80253800978\mathscr{A}_{i-2,\digamma-1,\wp-1}+179248459208\mathscr{A}_{i-2,\digamma-1,\wp}\\
&+80253800978\mathscr{A}_{i-2,\digamma-1,\wp+1}+5495897212\mathscr{A}_{i-2,\digamma-1,\wp+2}+7612046\mathscr{A}_{i-2,\digamma-1,\wp+3}\\
&+17001656\mathscr{A}_{i-2,\digamma,\wp-3}+12275195632\mathscr{A}_{i-2,\digamma,\wp-2}+179248459208\mathscr{A}_{i-2,\digamma,\wp-1}\\
&+400354995488\mathscr{A}_{i-2,\digamma,\wp}+179248459208\mathscr{A}_{i-2,\digamma,\wp+1}+12275195632\mathscr{A}_{i-2,\digamma,\wp+2}\\
&+17001656\mathscr{A}_{i-2,\digamma,\wp+3}+7612046\mathscr{A}_{i-2,\digamma+1,\wp-3}+5495897212\mathscr{A}_{i-2,\digamma+1,\wp-2}\\
&+80253800978\mathscr{A}_{i-2,\digamma+1,\wp-1}+179248459208\mathscr{A}_{i-2,\digamma+1,\wp}+80253800978\mathscr{A}_{i-2,\digamma+1,\wp+1}\\
&+5495897212\mathscr{A}_{i-2,\digamma+1,\wp+2}+7612046\mathscr{A}_{i-2,\digamma+1,\wp+3}+521284\mathscr{A}_{i-2,\digamma+2,\wp-3}\\
&+376367048\mathscr{A}_{i-2,\digamma+2,\wp-2}+5495897212\mathscr{A}_{i-2,\digamma+2,\wp-1}+12275195632\mathscr{A}_{i-2,\digamma+2,\wp}
\end{aligned}
$$

$$
\begin{aligned}
&+ 376367048\mathscr{A}_{i-2,\digamma+2,\wp+2} + 521284\mathscr{A}_{i-2,\digamma+2,\wp+3} + 722\mathscr{A}_{i-2,\digamma+3,\wp-3} + 521284\mathscr{A}_{i-2,\digamma+3,\wp-2}\\
&+ 7612046\mathscr{A}_{i-2,\digamma+3,\wp-1} + 17001656\mathscr{A}_{i-2,\digamma+3,\wp} + 7612046\mathscr{A}_{i-2,\digamma+3,\wp+1}\\
&+ 521284\mathscr{A}_{i-2,\digamma+3,\wp+2} + 722\mathscr{A}_{i-2,\digamma+3,\wp+3} + 10543\mathscr{A}_{i-1,\digamma-3,\wp-3} + 7612046\mathscr{A}_{i-1,\digamma-3,\wp-2}\\
&+ 111154849\mathscr{A}_{i-1,\digamma-3,\wp-1} + 248266564\mathscr{A}_{i-1,\digamma-3,\wp} + 111154849\mathscr{A}_{i-1,\digamma-3,\wp+1}\\
&+ 7612046\mathscr{A}_{i-1,\digamma-3,\wp+2} + 10543\mathscr{A}_{i-1,\digamma-3,\wp+3} + 7612046\mathscr{A}_{i-1,\digamma-2,\wp-3}\\
&+ 5495897212\mathscr{A}_{i-1,\digamma-2,\wp-2} + 80253800978\mathscr{A}_{i-1,\digamma-2,\wp-1} + 179248459208\mathscr{A}_{i-1,\digamma-2,\wp}\\
&+ 80253800978\mathscr{A}_{i-1,\digamma-2,\wp+1} + 5495897212\mathscr{A}_{i-1,\digamma-2,\wp+2} + 7612046\mathscr{A}_{i-1,\digamma-2,\wp+3}\\
&+ 111154849\mathscr{A}_{i-1,\digamma-1,\wp-3} + 80253800978\mathscr{A}_{i-1,\digamma-1,\wp-2} + 1171905573007\mathscr{A}_{i-1,\digamma-1,\wp-1}\\
&+ 2617474384252\mathscr{A}_{i-1,\digamma-1,\wp} + 1171905573007\mathscr{A}_{i-1,\digamma-1,\wp+1} + 80253800978\mathscr{A}_{i-1,\digamma-1,\wp+2}\\
&+ 111154849\mathscr{A}_{i-1,\digamma-1,\wp+3} + 248266564\mathscr{A}_{i-1,\digamma,\wp-3} + 5495897212\mathscr{A}_{i-2,\digamma+2,\wp+1}\\
&+ 179248459208\mathscr{A}_{i-1,\digamma,\wp-2} + 2617474384252\mathscr{A}_{i-1,\digamma,\wp-1} + 5846181049072\mathscr{A}_{i-1,\digamma,\wp}\\
&+ 2617474384252\mathscr{A}_{i-1,\digamma,\wp+1} + 179248459208\mathscr{A}_{i-1,\digamma,\wp+2} + 248266564\mathscr{A}_{i-1,\digamma,\wp+3}\\
&+ 111154849\mathscr{A}_{i-1,\digamma+1,\wp-3} + 80253800978\mathscr{A}_{i-1,\digamma+1,\wp-2} + 1171905573007\mathscr{A}_{i-1,\digamma+1,\wp-1}\\
&+ 2617474384252\mathscr{A}_{i-1,\digamma+1,\wp} + 1171905573007\mathscr{A}_{i-1,\digamma+1,\wp+1} + 80253800978\mathscr{A}_{i-1,\digamma+1,\wp+2}\\
&+ 111154849\mathscr{A}_{i-1,\digamma+1,\wp+3} + 7612046\mathscr{A}_{i-1,\digamma+2,\wp-3} + 5495897212\mathscr{A}_{i-1,\digamma+2,\wp-2}\\
&+ 80253800978\mathscr{A}_{i-1,\digamma+2,\wp-1} + 179248459208\mathscr{A}_{i-1,\digamma+2,\wp} + 80253800978\mathscr{A}_{i-1,\digamma+2,\wp+1}\\
&+ 5495897212\mathscr{A}_{i-1,\digamma+2,\wp+2} + 7612046\mathscr{A}_{i-1,\digamma+2,\wp+3} + 10543\mathscr{A}_{i-1,\digamma+3,\wp-3}\\
&+ 7612046\mathscr{A}_{i-1,\digamma+3,\wp-2} + 111154849\mathscr{A}_{i-1,\digamma+3,\wp-1} + 248266564\mathscr{A}_{i-1,\digamma+3,\wp}\\
&+ 111154849\mathscr{A}_{i-1,\digamma+3,\wp+1} + 7612046\mathscr{A}_{i-1,\digamma+3,\wp+2} + 10543\mathscr{A}_{i-1,\digamma+3,\wp+3} + 23548\mathscr{A}_{i,\digamma-3,\wp-3}\\
&+ 17001656\mathscr{A}_{i,\digamma-3,\wp-2} + 248266564\mathscr{A}_{i,\digamma-3,\wp-1} + 554508304\mathscr{A}_{i,\digamma-3,\wp}\\
&+ 248266564\mathscr{A}_{i,\digamma-3,\wp+1} + 17001656\mathscr{A}_{i,\digamma-3,\wp+2} + 23548\mathscr{A}_{i,\digamma-3,\wp+3}\\
&+ 17001656\mathscr{A}_{i,\digamma-2,\wp-3} + 12275195632\mathscr{A}_{i,\digamma-2,\wp-2} + 179248459208\mathscr{A}_{i,\digamma-2,\wp-1}\\
&+ 400354995488\mathscr{A}_{i,\digamma-2,\wp} + 179248459208\mathscr{A}_{i,\digamma-2,\wp+1} + 12275195632\mathscr{A}_{i,\digamma-2,\wp+2}\\
&+ 17001656\mathscr{A}_{i,\digamma-2,\wp+3} + 248266564\mathscr{A}_{i,\digamma-1,\wp-3} + 179248459208\mathscr{A}_{i,\digamma-1,\wp-2}\\
&+ 2617474384252\mathscr{A}_{i,\digamma-1,\wp-1} + 5846181049072\mathscr{A}_{i,\digamma-1,\wp} + 2617474384252\mathscr{A}_{i,\digamma-1,\wp+1}
\end{aligned}
$$

$$
\begin{aligned}
&+ 179248459208\mathscr{A}_{i,\digamma-1,\wp+2} + 248266564\mathscr{A}_{i,\digamma-1,\wp+3} + 554508304\mathscr{A}_{i,\digamma,\wp-3} \\
&+ 400354995488\mathscr{A}_{i,\digamma,\wp-2} + 5846181049072\mathscr{A}_{i,\digamma,\wp-1} + 13057561542592\mathscr{A}_{i,\digamma,\wp} \\
&+ 5846181049072\mathscr{A}_{i,\digamma,\wp+1} + 400354995488\mathscr{A}_{i,\digamma,\wp+2} + 554508304\mathscr{A}_{i,\digamma,\wp+3} \\
&+ 248266564\mathscr{A}_{i,\digamma+1,\wp-3} + 179248459208\mathscr{A}_{i,\digamma+1,\wp-2} + 2617474384252\mathscr{A}_{i,\digamma+1,\wp-1} \\
&+ 5846181049072\mathscr{A}_{i,\digamma+1,\wp} + 2617474384252\mathscr{A}_{i,\digamma+1,\wp+1} + 179248459208\mathscr{A}_{i,\digamma+1,\wp+2} \\
&+ 248266564\mathscr{A}_{i,\digamma+1,\wp+3} + 17001656\mathscr{A}_{i,\digamma+2,\wp-3} + 12275195632\mathscr{A}_{i,\digamma+2,\wp-2} \\
&+ 179248459208\mathscr{A}_{i,\digamma+2,\wp-1} + 400354995488\mathscr{A}_{i,\digamma+2,\wp} + 179248459208\mathscr{A}_{i,\digamma+2,\wp+1} \\
&+ 12275195632\mathscr{A}_{i,\digamma+2,\wp+2} + 17001656\mathscr{A}_{i,\digamma+2,\wp+3} + 23548\mathscr{A}_{i,\digamma+3,\wp-3} \\
&+ 17001656\mathscr{A}_{i,\digamma+3,\wp-2} + 248266564\mathscr{A}_{i,\digamma+3,\wp-1} + 554508304\mathscr{A}_{i,\digamma+3,\wp} \\
&+ 248266564\mathscr{A}_{i,\digamma+3,\wp+1} + 17001656\mathscr{A}_{i,\digamma+3,\wp+2} + 23548\mathscr{A}_{i,\digamma+3,\wp+3} + 10543\mathscr{A}_{i+1,\digamma-3,\wp-3} \\
&+ 7612046\mathscr{A}_{i+1,\digamma-3,\wp-2} + 111154849\mathscr{A}_{i+1,\digamma-3,\wp-1} + 248266564\mathscr{A}_{i+1,\digamma-3,\wp} \\
&+ 111154849\mathscr{A}_{i+1,\digamma-3,\wp+1} + 7612046\mathscr{A}_{i+1,\digamma-3,\wp+2} + 10543\mathscr{A}_{i+1,\digamma-3,\wp+3} \\
&+ 7612046\mathscr{A}_{i+1,\digamma-2,\wp-3} + 5495897212\mathscr{A}_{i+1,\digamma-2,\wp-2} + 80253800978\mathscr{A}_{i+1,\digamma-2,\wp-1} \\
&+ 179248459208\mathscr{A}_{i+1,\digamma-2,\wp} + 80253800978\mathscr{A}_{i+1,\digamma-2,\wp+1} + 5495897212\mathscr{A}_{i+1,\digamma-2,\wp+2} \\
&+ 7612046\mathscr{A}_{i+1,\digamma-2,\wp+3} + 111154849\mathscr{A}_{i+1,\digamma-1,\wp-3} + 80253800978\mathscr{A}_{i+1,\digamma-1,\wp-2} \\
&+ 1171905573007\mathscr{A}_{i+1,\digamma-1,\wp-1} + 2617474384252\mathscr{A}_{i+1,\digamma-1,\wp} + 1171905573007\mathscr{A}_{i+1,\digamma-1,\wp+1} \\
&+ 80253800978\mathscr{A}_{i+1,\digamma-1,\wp+2} + 111154849\mathscr{A}_{i+1,\digamma-1,\wp+3} + 248266564\mathscr{A}_{i+1,\digamma,\wp-3} \\
&+ 179248459208\mathscr{A}_{i+1,\digamma,\wp-2} + 2617474384252\mathscr{A}_{i+1,\digamma,\wp-1} + 5846181049072\mathscr{A}_{i+1,\digamma,\wp} \\
&+ 2617474384252\mathscr{A}_{i+1,\digamma,\wp+1} + 179248459208\mathscr{A}_{i+1,\digamma,\wp+2} + 248266564\mathscr{A}_{i+1,\digamma,\wp+3} \\
&+ 111154849\mathscr{A}_{i+1,\digamma+1,\wp-3} + 80253800978\mathscr{A}_{i+1,\digamma+1,\wp-2} + 1171905573007\mathscr{A}_{i+1,\digamma+1,\wp-1} \\
&+ 2617474384252\mathscr{A}_{i+1,\digamma+1,\wp} + 1171905573007\mathscr{A}_{i+1,\digamma+1,\wp+1} + 80253800978\mathscr{A}_{i+1,\digamma+1,\wp+2} \\
&+ 111154849\mathscr{A}_{i+1,\digamma+1,\wp+3} + 7612046\mathscr{A}_{i+1,\digamma+2,\wp-3} + 5495897212\mathscr{A}_{i+1,\digamma+2,\wp-2} \\
&+ 80253800978\mathscr{A}_{i+1,\digamma+2,\wp-1} + 179248459208\mathscr{A}_{i+1,\digamma+2,\wp} + 80253800978\mathscr{A}_{i+1,\digamma+2,\wp+1} \\
&+ 5495897212\mathscr{A}_{i+1,\digamma+2,\wp+2} + 7612046\mathscr{A}_{i+1,\digamma+2,\wp+3} + 10543\mathscr{A}_{i+1,\digamma+3,\wp-3} \\
&+ 7612046\mathscr{A}_{i+1,\digamma+3,\wp-2} + 111154849\mathscr{A}_{i+1,\digamma+3,\wp-1} + 248266564\mathscr{A}_{i+1,\digamma+3,\wp}
\end{aligned}
$$

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\begin{aligned}
&+ 111154849\mathscr{A}_{i+1,\digamma+3,\wp+1} + 7612046\mathscr{A}_{i+1,\digamma+3,\wp+2} + 10543\mathscr{A}_{i+1,\digamma+3,\wp+3}\\
&+ 722\mathscr{A}_{i+2,\digamma-3,\wp-3} + 521284\mathscr{A}_{i+2,\digamma-3,\wp-2} + 7612046\mathscr{A}_{i+2,\digamma-3,\wp-1} + 17001656\mathscr{A}_{i+2,\digamma-3,\wp}\\
&+ 7612046\mathscr{A}_{i+2,\digamma-3,\wp+1} + 521284\mathscr{A}_{i+2,\digamma-3,\wp+2} + 722\mathscr{A}_{i+2,\digamma-3,\wp+3}\\
&+ 521284\mathscr{A}_{i+2,\digamma-2,\wp-3} + 376367048\mathscr{A}_{i+2,\digamma-2,\wp-2} + 5495897212\mathscr{A}_{i+2,\digamma-2,\wp-1}\\
&+ 12275195632\mathscr{A}_{i+2,\digamma-2,\wp} + 5495897212\mathscr{A}_{i+2,\digamma-2,\wp+1} + 376367048\mathscr{A}_{i+2,\digamma-2,\wp+2}\\
&+ 521284\mathscr{A}_{i+2,\digamma-2,\wp+3} + 7612046\mathscr{A}_{i+2,\digamma-1,\wp-3} + 5495897212\mathscr{A}_{i+2,\digamma-1,\wp-2}\\
&+ 80253800978\mathscr{A}_{i+2,\digamma-1,\wp-1} + 179248459208\mathscr{A}_{i+2,\digamma-1,\wp} + 80253800978\mathscr{A}_{i+2,\digamma-1,\wp+1}\\
&+ 5495897212\mathscr{A}_{i+2,\digamma-1,\wp+2} + 7612046\mathscr{A}_{i+2,\digamma-1,\wp+3} + 17001656\mathscr{A}_{i+2,\digamma,\wp-3}\\
&+ 12275195632\mathscr{A}_{i+2,\digamma,\wp-2} + 179248459208\mathscr{A}_{i+2,\digamma,\wp-1} + 400354995488\mathscr{A}_{i+2,\digamma,\wp}\\
&+ 179248459208\mathscr{A}_{i+2,\digamma,\wp+1} + 12275195632\mathscr{A}_{i+2,\digamma,\wp+2} + 17001656\mathscr{A}_{i+2,\digamma,\wp+3}\\
&+ 7612046\mathscr{A}_{i+2,\digamma+1,\wp-3} + 5495897212\mathscr{A}_{i+2,\digamma+1,\wp-2} + 80253800978\mathscr{A}_{i+2,\digamma+1,\wp-1}\\
&+ 179248459208\mathscr{A}_{i+2,\digamma+1,\wp} + 80253800978\mathscr{A}_{i+2,\digamma+1,\wp+1} + 5495897212\mathscr{A}_{i+2,\digamma+1,\wp+2}\\
&+ 7612046\mathscr{A}_{i+2,\digamma+1,\wp+3} + 521284\mathscr{A}_{i+2,\digamma+2,\wp-3} + 376367048\mathscr{A}_{i+2,\digamma+2,\wp-2}\\
&+ 5495897212\mathscr{A}_{i+2,\digamma+2,\wp-1} + 12275195632\mathscr{A}_{i+2,\digamma+2,\wp} + 5495897212\mathscr{A}_{i+2,\digamma+2,\wp+1}\\
&+ 376367048\mathscr{A}_{i+2,\digamma+2,\wp+2} + 521284\mathscr{A}_{i+2,\digamma+2,\wp+3} + 722\mathscr{A}_{i+2,\digamma+3,\wp-3}\\
&+ 521284\mathscr{A}_{i+2,\digamma+3,\wp-2} + 7612046\mathscr{A}_{i+2,\digamma+3,\wp-1} + 17001656\mathscr{A}_{i+2,\digamma+3,\wp}\\
&+ 7612046\mathscr{A}_{i+2,\digamma+3,\wp+1} + 521284\mathscr{A}_{i+2,\digamma+3,\wp+2} + 722\mathscr{A}_{i+2,\digamma+3,\wp+3} + \mathscr{A}_{i+3,\digamma-3,\wp-3}\\
&+ 722\mathscr{A}_{i+3,\digamma-3,\wp-2} + 10543\mathscr{A}_{i+3,\digamma-3,\wp-1} + 23548\mathscr{A}_{i+3,\digamma-3,\wp} + 10543\mathscr{A}_{i+3,\digamma-3,\wp+1}\\
&+ 722\mathscr{A}_{i+3,\digamma-3,\wp+2} + \mathscr{A}_{i+3,\digamma-3,\wp+3} + 722\mathscr{A}_{i+3,\digamma-2,\wp-3} + 521284\mathscr{A}_{i+3,\digamma-2,\wp-2}\\
&+ 7612046\mathscr{A}_{i+3,\digamma-2,\wp-1} + 17001656\mathscr{A}_{i+3,\digamma-2,\wp} + 7612046\mathscr{A}_{i+3,\digamma-2,\wp+1}\\
&+ 521284\mathscr{A}_{i+3,\digamma-2,\wp+2} + 722\mathscr{A}_{i+3,\digamma-2,\wp+3} + 10543\mathscr{A}_{i+3,\digamma-1,\wp-3}\\
&+ 7612046\mathscr{A}_{i+3,\digamma-1,\wp-2} + 111154849\mathscr{A}_{i+3,\digamma-1,\wp-1} + 248266564\mathscr{A}_{i+3,\digamma-1,\wp}\\
&+ 111154849\mathscr{A}_{i+3,\digamma-1,\wp+1} + 7612046\mathscr{A}_{i+3,\digamma-1,\wp+2} + 10543\mathscr{A}_{i+3,\digamma-1,\wp+3}\\
&+ 23548\mathscr{A}_{i+3,\digamma,\wp-3} + 17001656\mathscr{A}_{i+3,\digamma,\wp-2} + 248266564\mathscr{A}_{i+3,\digamma,\wp-1}\\
&+ 554508304\mathscr{A}_{i+3,\digamma,\wp} + 248266564\mathscr{A}_{i+3,\digamma,\wp+1} + 17001656\mathscr{A}_{i+3,\digamma,\wp+2}
\end{aligned}
$$

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\begin{aligned}
&+ 23548\mathscr{A}_{i+3,\digamma,\wp+3} + 10543\mathscr{A}_{i+3,\digamma+1,\wp-3} + 7612046\mathscr{A}_{i+3,\digamma+1,\wp-2}\\
&+ 111154849\mathscr{A}_{i+3,\digamma+1,\wp-1} + 248266564\mathscr{A}_{i+3,\digamma+1,\wp} + 111154849\mathscr{A}_{i+3,\digamma+1,\wp+1}\\
&+ 7612046\mathscr{A}_{i+3,\digamma+1,\wp+2} + 10543\mathscr{A}_{i+3,\digamma+1,\wp+3} + 722\mathscr{A}_{i+3,\digamma+2,\wp-3}\\
&+ 521284\mathscr{A}_{i+3,\digamma+2,\wp-2} + 7612046\mathscr{A}_{i+3,\digamma+2,\wp-1} + 17001656\mathscr{A}_{i+3,\digamma+2,\wp}\\
&+ 7612046\mathscr{A}_{i+3,\digamma+2,\wp+1} + 521284\mathscr{A}_{i+3,\digamma+2,\wp+2} + 722\mathscr{A}_{i+3,\digamma+2,\wp+3} + \mathscr{A}_{i+3,\digamma+3,\wp-3}\\
&+ 722\mathscr{A}_{i+3,\digamma+3,\wp-2} + 10543\mathscr{A}_{i+3,\digamma+3,\wp-1} + 23548\mathscr{A}_{i+3,\digamma+3,\wp} + 10543\mathscr{A}_{i+3,\digamma+3,\wp+1}\\
&+ 722\mathscr{A}_{i+3,\digamma+3,\wp+2} + \mathscr{A}_{i+3,\digamma+3,\wp+3}\Big),
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
\frac{\partial G_{i,\digamma,\wp}}{\partial \mathfrak{X}} = \frac{3}{65536h}\Big(&\mathscr{A}_{i-3,\digamma-3,\wp-3} + 722\mathscr{A}_{i-3,\digamma-3,\wp-2} + 10543\mathscr{A}_{i-3,\digamma-3,\wp-1}\\
&+ 23548\mathscr{A}_{i-3,\digamma-3,\wp} + 10543\mathscr{A}_{i-3,\digamma-3,\wp+1} + 722\mathscr{A}_{i-3,\digamma-3,\wp+2} + \mathscr{A}_{i-3,\digamma-3,\wp+3}\\
&+ 722\mathscr{A}_{i-3,\digamma-2,\wp-3} + 521284\mathscr{A}_{i-3,\digamma-2,\wp-2} + 7612046\mathscr{A}_{i-3,\digamma-2,\wp-1}\\
&+ 17001656\mathscr{A}_{i-3,\digamma-2,\wp} + 7612046\mathscr{A}_{i-3,\digamma-2,\wp+1} + 521284\mathscr{A}_{i-3,\digamma-2,\wp+2}\\
&+ 722\mathscr{A}_{i-3,\digamma-2,\wp+3} + 10543\mathscr{A}_{i-3,\digamma-1,\wp-3} + 7612046\mathscr{A}_{i-3,\digamma-1,\wp-2}\\
&+ 111154849\mathscr{A}_{i-3,\digamma-1,\wp-1} + 248266564\mathscr{A}_{i-3,\digamma-1,\wp} + 111154849\mathscr{A}_{i-3,\digamma-1,\wp+1}\\
&+ 7612046\mathscr{A}_{i-3,\digamma-1,\wp+2} + 10543\mathscr{A}_{i-3,\digamma-1,\wp+3} + 23548\mathscr{A}_{i-3,\digamma,\wp-3}\\
&+ 17001656\mathscr{A}_{i-3,\digamma,\wp-2} + 248266564\mathscr{A}_{i-3,\digamma,\wp-1} + 554508304\mathscr{A}_{i-3,\digamma,\wp}\\
&+ 248266564\mathscr{A}_{i-3,\digamma,\wp+1} + 17001656\mathscr{A}_{i-3,\digamma,\wp+2} + 23548\mathscr{A}_{i-3,\digamma,\wp+3}\\
&+ 10543\mathscr{A}_{i-3,\digamma+1,\wp-3} + 7612046\mathscr{A}_{i-3,\digamma+1,\wp-2} + 111154849\mathscr{A}_{i-3,\digamma+1,\wp-1}\\
&+ 248266564\mathscr{A}_{i-3,\digamma+1,\wp} + 111154849\mathscr{A}_{i-3,\digamma+1,\wp+1} + 7612046\mathscr{A}_{i-3,\digamma+1,\wp+2}\\
&+ 10543\mathscr{A}_{i-3,\digamma+1,\wp+3} + 722\mathscr{A}_{i-3,\digamma+2,\wp-3} + 521284\mathscr{A}_{i-3,\digamma+2,\wp-2} + 7612046\mathscr{A}_{i-3,\digamma+2,\wp-1}\\
&+ 17001656\mathscr{A}_{i-3,\digamma+2,\wp} + 7612046\mathscr{A}_{i-3,\digamma+2,\wp+1} + 521284\mathscr{A}_{i-3,\digamma+2,\wp+2}\\
&+ 722\mathscr{A}_{i-3,\digamma+2,\wp+3} + \mathscr{A}_{i-3,\digamma+3,\wp-3} + 722\mathscr{A}_{i-3,\digamma+3,\wp-2} + 10543\mathscr{A}_{i-3,\digamma+3,\wp-1}\\
&+ 23548\mathscr{A}_{i-3,\digamma+3,\wp} + 10543\mathscr{A}_{i-3,\digamma+3,\wp+1} + 722\mathscr{A}_{i-3,\digamma+3,\wp+2} + \mathscr{A}_{i-3,\digamma+3,\wp+3}\\
&+ 236\mathscr{A}_{i-2,\digamma-3,\wp-3} + 170392\mathscr{A}_{i-2,\digamma-3,\wp-2} + 2488148\mathscr{A}_{i-2,\digamma-3,\wp-1}\\
&+ 5557328\mathscr{A}_{i-2,\digamma-3,\wp} + 2488148\mathscr{A}_{i-2,\digamma-3,\wp+1} + 170392\mathscr{A}_{i-2,\digamma-3,\wp+2}\\
&+ 236\mathscr{A}_{i-2,\digamma-3,\wp+3} + 170392\mathscr{A}_{i-2,\digamma-2,\wp-3} + 123023024\mathscr{A}_{i-2,\digamma-2,\wp-2}\\
&+ 1796442856\mathscr{A}_{i-2,\digamma-2,\wp-1} + 4012390816\mathscr{A}_{i-2,\digamma-2,\wp} + 1796442856\mathscr{A}_{i-2,\digamma-2,\wp+1}\\
&+ 123023024\mathscr{A}_{i-2,\digamma-2,\wp+2} + 170392\mathscr{A}_{i-2,\digamma-2,\wp+3} + 2488148\mathscr{A}_{i-2,\digamma-1,\wp-3}\\
&+ 1796442856\mathscr{A}_{i-2,\digamma-1,\wp-2} + 26232544364\mathscr{A}_{i-2,\digamma-1,\wp-1} + 58590909104\mathscr{A}_{i-2,\digamma-1,\wp}\\
&+ 26232544364\mathscr{A}_{i-2,\digamma-1,\wp+1} + 1796442856\mathscr{A}_{i-2,\digamma-1,\wp+2} + 2488148\mathscr{A}_{i-2,\digamma-1,\wp+3}\\
&+ 5557328\mathscr{A}_{i-2,\digamma,\wp-3} + 4012390816\mathscr{A}_{i-2,\digamma,\wp-2} + 58590909104\mathscr{A}_{i-2,\digamma,\wp-1}\\
&+ 130863959744\mathscr{A}_{i-2,\digamma,\wp} + 58590909104\mathscr{A}_{i-2,\digamma,\wp+1} + 4012390816\mathscr{A}_{i-2,\digamma,\wp+2}
\end{aligned}
$$

$$
\begin{aligned}
&+ 5557328\mathscr{A}_{i-2,\digamma,\wp+3} + 2488148\mathscr{A}_{i-2,\digamma+1,\wp-3} + 1796442856\mathscr{A}_{i-2,\digamma+1,\wp-2}\\
&+ 26232544364\mathscr{A}_{i-2,\digamma+1,\wp-1} + 58590909104\mathscr{A}_{i-2,\digamma+1,\wp} + 26232544364\mathscr{A}_{i-2,\digamma+1,\wp+1}\\
&+ 1796442856\mathscr{A}_{i-2,\digamma+1,\wp+2} + 2488148\mathscr{A}_{i-2,\digamma+1,\wp+3} + 170392\mathscr{A}_{i-2,\digamma+2,\wp-3}\\
&+ 123023024\mathscr{A}_{i-2,\digamma+2,\wp-2} + 1796442856\mathscr{A}_{i-2,\digamma+2,\wp-1} + 4012390816\mathscr{A}_{i-2,\digamma+2,\wp}\\
&+ 1796442856\mathscr{A}_{i-2,\digamma+2,\wp+1} + 123023024\mathscr{A}_{i-2,\digamma+2,\wp+2} + 170392\mathscr{A}_{i-2,\digamma+2,\wp+3}\\
&+ 236\mathscr{A}_{i-2,\digamma+3,\wp-3} + 170392\mathscr{A}_{i-2,\digamma+3,\wp-2} + 2488148\mathscr{A}_{i-2,\digamma+3,\wp-1}\\
&+ 5557328\mathscr{A}_{i-2,\digamma+3,\wp} + 2488148\mathscr{A}_{i-2,\digamma+3,\wp+1} + 170392\mathscr{A}_{i-2,\digamma+3,\wp+2}\\
&+ 236\mathscr{A}_{i-2,\digamma+3,\wp+3} + 1445\mathscr{A}_{i-1,\digamma-3,\wp-3} + 1043290\mathscr{A}_{i-1,\digamma-3,\wp-2} + 15234635\mathscr{A}_{i-1,\digamma-3,\wp-1}\\
&+ 34026860\mathscr{A}_{i-1,\digamma-3,\wp} + 15234635\mathscr{A}_{i-1,\digamma-3,\wp+1} + 1043290\mathscr{A}_{i-1,\digamma-3,\wp+2}\\
&+ 1445\mathscr{A}_{i-1,\digamma-3,\wp+3} + 1043290\mathscr{A}_{i-1,\digamma-2,\wp-3} + 753255380\mathscr{A}_{i-1,\digamma-2,\wp-2}\\
&+ 10999406470\mathscr{A}_{i-1,\digamma-2,\wp-1} + 24567392920\mathscr{A}_{i-1,\digamma-2,\wp} + 10999406470\mathscr{A}_{i-1,\digamma-2,\wp+1}\\
&+ 753255380\mathscr{A}_{i-1,\digamma-2,\wp+2} + 1043290\mathscr{A}_{i-1,\digamma-2,\wp+3} + 15234635\mathscr{A}_{i-1,\digamma-1,\wp-3}\\
&+ 10999406470\mathscr{A}_{i-1,\digamma-1,\wp-2} + 160618756805\mathscr{A}_{i-1,\digamma-1,\wp-1} + 358745184980\mathscr{A}_{i-1,\digamma-1,\wp}\\
&+ 160618756805\mathscr{A}_{i-1,\digamma-1,\wp+1} + 10999406470\mathscr{A}_{i-1,\digamma-1,\wp+2} + 15234635\mathscr{A}_{i-1,\digamma-1,\wp+3}\\
&+ 801264499280\mathscr{A}_{i-1,\digamma,\wp} + 358745184980\mathscr{A}_{i-1,\digamma,\wp+1} + 24567392920\mathscr{A}_{i-1,\digamma,\wp+2}\\
&+ 34026860\mathscr{A}_{i-1,\digamma,\wp+3} + 15234635\mathscr{A}_{i-1,\digamma+1,\wp-3} + 10999406470\mathscr{A}_{i-1,\digamma+1,\wp-2}\\
&+ 160618756805\mathscr{A}_{i-1,\digamma+1,\wp-1} + 358745184980\mathscr{A}_{i-1,\digamma+1,\wp} + 160618756805\mathscr{A}_{i-1,\digamma+1,\wp+1}\\
&+ 10999406470\mathscr{A}_{i-1,\digamma+1,\wp+2} + 15234635\mathscr{A}_{i-1,\digamma+1,\wp+3} + 1043290\mathscr{A}_{i-1,\digamma+2,\wp-3}\\
&+ 10999406470\mathscr{A}_{i-1,\digamma+2,\wp-1} + 24567392920\mathscr{A}_{i-1,\digamma+2,\wp} + 10999406470\mathscr{A}_{i-1,\digamma+2,\wp+1}\\
&+ 753255380\mathscr{A}_{i-1,\digamma+2,\wp+2} + 1043290\mathscr{A}_{i-1,\digamma+2,\wp+3} + 1445\mathscr{A}_{i-1,\digamma+3,\wp-3}\\
&+ 1043290\mathscr{A}_{i-1,\digamma+3,\wp-2} + 15234635\mathscr{A}_{i-1,\digamma+3,\wp-1} + 34026860\mathscr{A}_{i-1,\digamma+3,\wp}\\
&+ 15234635\mathscr{A}_{i-1,\digamma+3,\wp+1} + 1043290\mathscr{A}_{i-1,\digamma+3,\wp+2} + 1445\mathscr{A}_{i-1,\digamma+3,\wp+3} - 1445\mathscr{A}_{i+1,\digamma-3,\wp-3}\\
&- 1043290\mathscr{A}_{i+1,\digamma-3,\wp-2} - 15234635\mathscr{A}_{i+1,\digamma-3,\wp-1} - 34026860\mathscr{A}_{i+1,\digamma-3,\wp}\\
&- 15234635\mathscr{A}_{i+1,\digamma-3,\wp+1} - 1043290\mathscr{A}_{i+1,\digamma-3,\wp+2} - 1445\mathscr{A}_{i+1,\digamma-3,\wp+3}\\
&+ 753255380\mathscr{A}_{i-1,\digamma+2,\wp-2} - 1043290\mathscr{A}_{i+1,\digamma-2,\wp-3} - 753255380\mathscr{A}_{i+1,\digamma-2,\wp-2}
\end{aligned}
$$

$$
\begin{aligned}
&- 10999406470\mathscr{A}_{i+1,\digamma-2,\wp-1} - 24567392920\mathscr{A}_{i+1,\digamma-2,\wp} - 10999406470\mathscr{A}_{i+1,\digamma-2,\wp+1}\\
&- 753255380\mathscr{A}_{i+1,\digamma-2,\wp+2} - 1043290\mathscr{A}_{i+1,\digamma-2,\wp+3} - 15234635\mathscr{A}_{i+1,\digamma-1,\wp-3}\\
&- 10999406470\mathscr{A}_{i+1,\digamma-1,\wp-2} - 160618756805\mathscr{A}_{i+1,\digamma-1,\wp-1} - 358745184980\mathscr{A}_{i+1,\digamma-1,\wp}\\
&- 160618756805\mathscr{A}_{i+1,\digamma-1,\wp+1} - 10999406470\mathscr{A}_{i+1,\digamma-1,\wp+2} - 15234635\mathscr{A}_{i+1,\digamma-1,\wp+3}\\
&- 34026860\mathscr{A}_{i+1,\digamma,\wp-3} - 24567392920\mathscr{A}_{i+1,\digamma,\wp-2} - 358745184980\mathscr{A}_{i+1,\digamma,\wp-1}\\
&- 801264499280\mathscr{A}_{i+1,\digamma,\wp} - 358745184980\mathscr{A}_{i+1,\digamma,\wp+1} - 24567392920\mathscr{A}_{i+1,\digamma,\wp+2}\\
&- 34026860\mathscr{A}_{i+1,\digamma,\wp+3} - 15234635\mathscr{A}_{i+1,\digamma+1,\wp-3} - 10999406470\mathscr{A}_{i+1,\digamma+1,\wp-2}\\
&- 160618756805\mathscr{A}_{i+1,\digamma+1,\wp-1} - 358745184980\mathscr{A}_{i+1,\digamma+1,\wp} - 160618756805\mathscr{A}_{i+1,\digamma+1,\wp+1}\\
&- 10999406470\mathscr{A}_{i+1,\digamma+1,\wp+2} - 15234635\mathscr{A}_{i+1,\digamma+1,\wp+3} - 1043290\mathscr{A}_{i+1,\digamma+2,\wp-3}\\
&- 10999406470\mathscr{A}_{i+1,\digamma+2,\wp+1} - 753255380\mathscr{A}_{i+1,\digamma+2,\wp+2} - 1043290\mathscr{A}_{i+1,\digamma+2,\wp+3}\\
&- 1445\mathscr{A}_{i+1,\digamma+3,\wp-3} - 1043290\mathscr{A}_{i+1,\digamma+3,\wp-2} - 15234635\mathscr{A}_{i+1,\digamma+3,\wp-1}\\
&- 34026860\mathscr{A}_{i+1,\digamma+3,\wp} - 15234635\mathscr{A}_{i+1,\digamma+3,\wp+1} - 1043290\mathscr{A}_{i+1,\digamma+3,\wp+2} - 1445\mathscr{A}_{i+1,\digamma+3,\wp+3}\\
&- 236\mathscr{A}_{i+2,\digamma-3,\wp-3} - 170392\mathscr{A}_{i+2,\digamma-3,\wp-2} - 2488148\mathscr{A}_{i+2,\digamma-3,\wp-1} - 5557328\mathscr{A}_{i+2,\digamma-3,\wp}\\
&- 2488148\mathscr{A}_{i+2,\digamma-3,\wp+1} - 170392\mathscr{A}_{i+2,\digamma-3,\wp+2} - 236\mathscr{A}_{i+2,\digamma-3,\wp+3} - 170392\mathscr{A}_{i+2,\digamma-2,\wp-3}\\
&- 123023024\mathscr{A}_{i+2,\digamma-2,\wp-2} - 1796442856\mathscr{A}_{i+2,\digamma-2,\wp-1} - 4012390816\mathscr{A}_{i+2,\digamma-2,\wp}\\
&- 2488148\mathscr{A}_{i+2,\digamma-1,\wp-3} - 1796442856\mathscr{A}_{i+2,\digamma-1,\wp-2} - 26232544364\mathscr{A}_{i+2,\digamma-1,\wp-1}\\
&- 58590909104\mathscr{A}_{i+2,\digamma-1,\wp} - 26232544364\mathscr{A}_{i+2,\digamma-1,\wp+1} - 1796442856\mathscr{A}_{i+2,\digamma-1,\wp+2}\\
&- 2488148\mathscr{A}_{i+2,\digamma-1,\wp+3} - 5557328\mathscr{A}_{i+2,\digamma,\wp-3} - 4012390816\mathscr{A}_{i+2,\digamma,\wp-2}\\
&- 58590909104\mathscr{A}_{i+2,\digamma,\wp-1} - 130863959744\mathscr{A}_{i+2,\digamma,\wp} - 58590909104\mathscr{A}_{i+2,\digamma,\wp+1}\\
&- 4012390816\mathscr{A}_{i+2,\digamma,\wp+2} - 5557328\mathscr{A}_{i+2,\digamma,\wp+3} - 2488148\mathscr{A}_{i+2,\digamma+1,\wp-3}\\
&- 26232544364\mathscr{A}_{i+2,\digamma+1,\wp+1} - 1796442856\mathscr{A}_{i+2,\digamma+1,\wp+2} - 2488148\mathscr{A}_{i+2,\digamma+1,\wp+3}\\
&- 170392\mathscr{A}_{i+2,\digamma+2,\wp-3} - 123023024\mathscr{A}_{i+2,\digamma+2,\wp-2} - 1796442856\mathscr{A}_{i+2,\digamma+2,\wp-1}\\
&- 4012390816\mathscr{A}_{i+2,\digamma+2,\wp} - 1796442856\mathscr{A}_{i+2,\digamma+2,\wp+1} - 123023024\mathscr{A}_{i+2,\digamma+2,\wp+2}\\
&- 170392\mathscr{A}_{i+2,\digamma+2,\wp+3} - 236\mathscr{A}_{i+2,\digamma+3,\wp-3} - 170392\mathscr{A}_{i+2,\digamma+3,\wp-2} - 2488148\mathscr{A}_{i+2,\digamma+3,\wp-1}\\
&- 5557328\mathscr{A}_{i+2,\digamma+3,\wp} - 2488148\mathscr{A}_{i+2,\digamma+3,\wp+1} - 170392\mathscr{A}_{i+2,\digamma+3,\wp+2}
\end{aligned}
$$

$$
\begin{aligned}
&-236\mathscr{A}_{i+2,\digamma+3,\wp+3}-\mathscr{A}_{i+3,\digamma-3,\wp-3}-722\mathscr{A}_{i+3,\digamma-3,\wp-2}-10543\mathscr{A}_{i+3,\digamma-3,\wp-1}\\
&-23548\mathscr{A}_{i+3,\digamma-3,\wp}-10543\mathscr{A}_{i+3,\digamma-3,\wp+1}-722\mathscr{A}_{i+3,\digamma-3,\wp+2}-\mathscr{A}_{i+3,\digamma-3,\wp+3}\\
&-722\mathscr{A}_{i+3,\digamma-2,\wp-3}-521284\mathscr{A}_{i+3,\digamma-2,\wp-2}-7612046\mathscr{A}_{i+3,\digamma-2,\wp-1}\\
&-17001656\mathscr{A}_{i+3,\digamma-2,\wp}-7612046\mathscr{A}_{i+3,\digamma-2,\wp+1}-521284\mathscr{A}_{i+3,\digamma-2,\wp+2}-722\mathscr{A}_{i+3,\digamma-2,\wp+3}\\
&-10543\mathscr{A}_{i+3,\digamma-1,\wp-3}-7612046\mathscr{A}_{i+3,\digamma-1,\wp-2}-111154849\mathscr{A}_{i+3,\digamma-1,\wp-1}\\
&-248266564\mathscr{A}_{i+3,\digamma-1,\wp}-111154849\mathscr{A}_{i+3,\digamma-1,\wp+1}-7612046\mathscr{A}_{i+3,\digamma-1,\wp+2}\\
&-10543\mathscr{A}_{i+3,\digamma-1,\wp+3}-23548\mathscr{A}_{i+3,\digamma,\wp-3}-17001656\mathscr{A}_{i+3,\digamma,\wp-2}-248266564\mathscr{A}_{i+3,\digamma,\wp-1}\\
&-17001656\mathscr{A}_{i+3,\digamma+2,\wp}-7612046\mathscr{A}_{i+3,\digamma+2,\wp+1}-521284\mathscr{A}_{i+3,\digamma+2,\wp+2}-722\mathscr{A}_{i+3,\digamma+2,\wp+3}\\
&-\mathscr{A}_{i+3,\digamma+3,\wp-3}-722\mathscr{A}_{i+3,\digamma+3,\wp-2}-10543\mathscr{A}_{i+3,\digamma+3,\wp-1}-23548\mathscr{A}_{i+3,\digamma+3,\wp}\\
&-10543\mathscr{A}_{i+3,\digamma+3,\wp+1}-722\mathscr{A}_{i+3,\digamma+3,\wp+2}-\mathscr{A}_{i+3,\digamma+3,\wp+3}\Big),\\
&\vdots\,.
\end{aligned}
\tag{11}
$$

The following theorem can be derived from the analysis presented above.

Theorem 3. *From equation (7), the approximation formulas to* $G_{i,\digamma,\wp}, \partial G_{i,\digamma,\wp}/\partial\mathfrak{X}, \partial G_{i,\digamma,\wp}/\partial\mathfrak{Y}, \partial G_{i,\digamma,\wp}/\partial\mathfrak{Z}, \partial^2 G_{i,\digamma,\wp}/\partial\mathfrak{X}^2, \partial^2 G_{i,\digamma,\wp}/\partial\mathfrak{Y}^2, \partial^2 G_{i,\digamma,\wp}/\partial\mathfrak{Z}^2, \partial^2 G_{i,\digamma,\wp}/\partial\mathfrak{X}\partial\mathfrak{Y}, \partial^2 G_{i,\digamma,\wp}/\partial\mathfrak{X}\partial\mathfrak{Z}, \ldots,$ *are given in terms of the* $\mathscr{A}_{i,\digamma,\wp}$ *at equations (8) and (9). You can find some other approximate formulas in Appendix.*

3. The Numerical Outcomes

It is time to evaluate the accuracy and efficiency of this strategy, which was created by displaying its constructs in n-dimensions. This will make it possible to comprehend the method's operation better. This section presents a number of numerical examples that cover a range of dimensions in order to show the validity of this methodology. We give a selection of the collected figures in addition to contrasting our results with those that have previously been attained. The Mathematica 12.1 software suite was used to create each example, which was then executed on a typical machine (Intel(R) core(TM) i7-351U, CPU@1.90 Hz 2.40 GHz). It is crucial to remember this.

3.1. The First Test Problem: [3, 11]. Take into consideration the problem in two dimensions using the structure below:

$$
\mathscr{H}_{\mathfrak{X}\mathfrak{X}}(\mathfrak{X},\mathfrak{Y})+\mathscr{H}_{\mathfrak{Y}\mathfrak{Y}}(\mathfrak{X},\mathfrak{Y})-\sin(\pi\mathfrak{X})\sin(\pi\mathfrak{Y})=0,\quad \mathfrak{X},\mathfrak{Y}\in[l,m].
\tag{12}
$$

The following is the precise approach that should be taken to solve that problem:

$$
\mathscr{H}(\mathfrak{X},\mathfrak{Y})=-\frac{\sin(\pi\mathfrak{X})\sin(\pi\mathfrak{Y})}{2\pi^2}.
\tag{13}
$$

In order to solve the third problem, we take the boundary conditions in the following form:

$$
\begin{aligned}
\mathscr{H}(l,\mathfrak{Y})&=\mathscr{H}(\mathfrak{X},l)=\alpha,\\
\mathscr{H}(m,\mathfrak{Y})&=\mathscr{H}(\mathfrak{X},m)=\beta.
\end{aligned}
\tag{14}
$$

By making the replacement from equations (6) and (7) into equations (12) and (14), we are able to acquire the numerical findings that are presented in the following table.

Table 1 shows the numerical results we reached by applying the method to the problem. These results were extracted using 15×15 mesh grid points at $\mathfrak{Y}=0.4$. It seems to us that the results we have reached are largely acceptable, and this appears before us through the stability of the absolute error value, which indicates that this generalization has borne fruit. To further confirm the good results we have achieved, we have given some two- and three-dimensional shapes to illustrate the comparison between the numerical and exact solutions, the shape of the absolute error, and the extent of its stability, and this appears in Figures 1–3.

Table 1: Numerical and exact results for the first problem at $\mathfrak{Y} = 0.4, \mathfrak{X}, \mathfrak{Y} \in [0, 1]$.

$\mathfrak{X}$	Num.	Ex	Abs. error
0.2	−0.028321	−0.02832	3.85278×10^{-10}
0.4	−0.045822	−0.04582	6.23393×10^{-10}
0.6	−0.045822	−0.04582	6.23393×10^{-10}
0.8	−0.028320	−0.02832	3.85278×10^{-10}

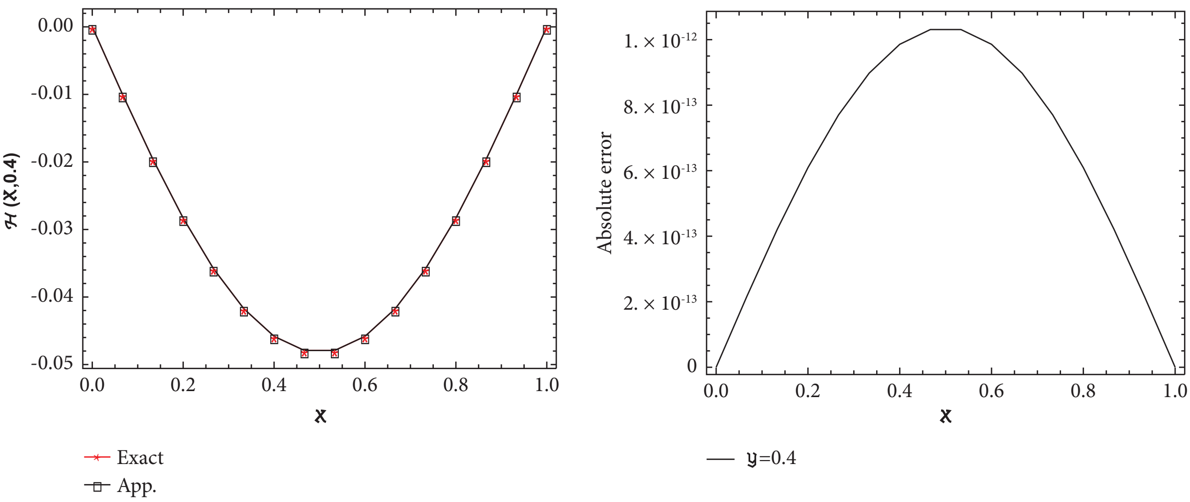

Figure 1: 2D figure showing the numerical and exact solutions and the absolute error at $\mathfrak{Y} = 0.4$.

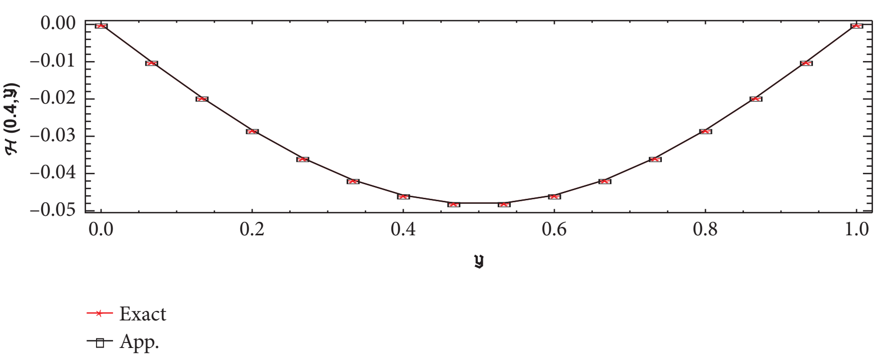

Figure 2: 2D figure showing the numerical and exact solutions at $x = 0.4$.

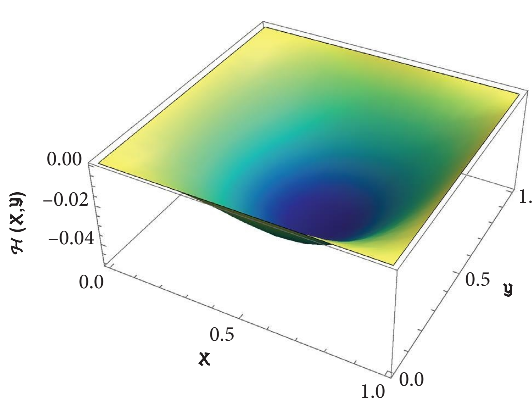

Figure 3: 3D figure showing the numerical results for the first test problem.

Table 2: Compare our results with some other results through the absolute error values.

Our method	QBSM [11]	MCBDQM [7]	DQM [22]	H. wavelet [2]	SCA [3]
$3.852E-10$	$3.72E-5$	$2.11E-5$	$1.62E-4$	$3.08E-4$	$3.08E-4$

We compare the results of the suggested method to those from several other methods, including the Haar wavelet method [2], the wavelet method [3], the modified cubic B-spline differential quadrature method [7], the n-dimensions quadratic B-splines [11], and the spline-based DQM [22]. All of these are shown in Table 2 and use 15×15 mesh grid points.

3.2. The Second Test Problem. Consider the following formulation of the nonlinear issue in two dimensions:

$$\mathcal{H}_{\mathfrak{X}\mathfrak{X}}(\mathfrak{X},\mathfrak{Y}) + \exp\left(\mathfrak{X}^2\right)\mathcal{H}_{\mathfrak{Y}\mathfrak{Y}}(\mathfrak{X},\mathfrak{Y}) + \sin(\mathcal{H}(\mathfrak{X},\mathfrak{Y})) - f(\mathfrak{X},\mathfrak{Y}) = 0, \quad \mathfrak{X},\mathfrak{Y}\in[l,m], \tag{15}$$

where

$$\begin{aligned} f(\mathfrak{X},\mathfrak{Y}) = {} & e^{\mathfrak{X}^2}\Big(-\left(1-e^{1-\mathfrak{X}}\right)\mathfrak{X}^2\left(1-e^{1-\mathfrak{Y}}\right)\sin(\mathfrak{X}\mathfrak{Y}) - \left(1-e^{1-\mathfrak{X}}\right)e^{1-\mathfrak{Y}}\sin(\mathfrak{X}\mathfrak{Y}) + 2\left(1-e^{1-\mathfrak{X}}\right)xe^{1-\mathfrak{Y}}\cos(\mathfrak{X}\mathfrak{Y})\Big) \\ & - \left(1-e^{1-\mathfrak{X}}\right)\left(1-e^{1-\mathfrak{Y}}\right)\mathfrak{Y}^2\sin(\mathfrak{X}\mathfrak{Y}) - e^{1-\mathfrak{X}}\left(1-e^{1-\mathfrak{Y}}\right)\sin(\mathfrak{X}\mathfrak{Y}) + \sin\Big(\left(1-e^{1-\mathfrak{X}}\right)\left(1-e^{1-\mathfrak{Y}}\right)\sin(\mathfrak{X}\mathfrak{Y})\Big) \\ & + 2e^{1-\mathfrak{X}}\left(1-e^{1-\mathfrak{Y}}\right)y\cos(\mathfrak{X}\mathfrak{Y}). \end{aligned} \tag{16}$$

The following is the precise approach that should be taken to solve that problem:

$$\mathcal{H}(\mathfrak{X},\mathfrak{Y}) = \sin(\mathfrak{Y}\mathfrak{X})(1-\exp(1-\mathfrak{X}))(1-\exp(1-\mathfrak{Y})). \tag{17}$$

In order to solve the third problem, we take the boundary conditions in the following form:

$$\begin{aligned} \mathcal{H}(l,\mathfrak{Y}) &= \mathcal{H}(\mathfrak{X},l) = \alpha, \\ \mathcal{H}(m,\mathfrak{Y}) &= \mathcal{H}(\mathfrak{X},m) = \beta. \end{aligned} \tag{18}$$

By making the substitution from equations (6)–(7) into equations (15) with (18), we are able to obtain the numerical results that are presented in the following table.

Table 3 presents the results that were obtained using the two-dimensional spherical B-spline method for the value of 50×50. When it comes to the results, we may safely assume that they meet our expectations. The numerical values for the equation $\mathfrak{Y}=0.5$ are displayed in Figures 4 and 5, and they are accurate. A graph of the numerical findings is shown in three dimensions in Figure 6.

3.3. The Third Test Problem: [11, 12]. Consider the following formulation for the second test problem in three dimensions:

$$\mathcal{H}_{\mathfrak{X}\mathfrak{X}}(\mathfrak{X},\mathfrak{Y},\mathfrak{Z}) + \mathcal{H}_{\mathfrak{Y}\mathfrak{Y}}(\mathfrak{X},\mathfrak{Y},\mathfrak{Z}) + \mathcal{H}_{\mathfrak{Z}\mathfrak{Z}}(\mathfrak{X},\mathfrak{Y},\mathfrak{Z}) - f(\mathfrak{X},\mathfrak{Y}) = 0, \quad \mathfrak{X},\mathfrak{Y},\mathfrak{Z}\in[l,m], \tag{19}$$

where

$$f(\mathfrak{X},\mathfrak{Y}) = \mathfrak{X}\mathfrak{Y}\mathfrak{Z}\left(e^{\mathfrak{X}+\mathfrak{Y}+\mathfrak{Z}}\right)(3\mathfrak{Y}\mathfrak{X}\mathfrak{Z} + \mathfrak{Y}\mathfrak{X} + \mathfrak{Z}\mathfrak{X} - 5\mathfrak{X} + \mathfrak{Z}\mathfrak{Y} - 5\mathfrak{Y} - 5\mathfrak{Z} + 9). \tag{20}$$

The following is an example of a specific solution to the issue that was discussed:

$$\mathcal{H}(\mathfrak{X},\mathfrak{Y},\mathfrak{Z}) = \left(\mathfrak{X}-\mathfrak{X}^2\right)\left(\mathfrak{Y}-\mathfrak{Y}^2\right)\left(\mathfrak{Z}-\mathfrak{Z}^2\right)e^{\mathfrak{X}+\mathfrak{Y}+\mathfrak{Z}}. \tag{21}$$

In order to solve the fourth problem, we take the boundary conditions in the following form:

$$\begin{aligned} \mathcal{H}(l,\mathfrak{Y},\mathfrak{Z}) &= \mathcal{H}(\mathfrak{X},l,\mathfrak{Z}) = \mathcal{H}(\mathfrak{X},\mathfrak{Y},l) = \alpha, \\ \mathcal{H}(m,\mathfrak{Y},\mathfrak{Z}) &= \mathcal{H}(\mathfrak{X},m,\mathfrak{Z}) = \mathcal{H}(\mathfrak{X},\mathfrak{Y},m) = \beta. \end{aligned} \tag{22}$$

By making the substitution from equations (10) and (11) into equations (19) and (22), we are able to obtain the numerical findings that are presented in the following table.

Table 4 presents a comparison of the results that we achieved with those that were acquired by utilizing the quadratic B-spline technique and the cubic B-spline approach with meshes of 20×20. When it comes to the findings, we may deduce from what we have seen that they can be trusted, which is good news. At the point where $\mathfrak{Y}=\mathfrak{Z}=0.5$, the numerical findings with exact answers are displayed in Figure 7. A graph of the numerical data presented in three dimensions is shown in Figure 8.

3.4. The Fourth Test Problem: [3, 14]. In three dimensions, we approach the test problem using the following form:

$$\mathcal{H}_{\mathfrak{X}\mathfrak{X}}(\mathfrak{X},\mathfrak{Y},\mathfrak{Z}) + \mathcal{H}_{\mathfrak{Y}\mathfrak{Y}}(\mathfrak{X},\mathfrak{Y},\mathfrak{Z}) + \mathcal{H}_{\mathfrak{Z}\mathfrak{Z}}(\mathfrak{X},\mathfrak{Y},\mathfrak{Z}) - \sin(\pi\mathfrak{X})\sin(\pi\mathfrak{Y})\sin(\pi\mathfrak{Z}) = 0, \quad \mathfrak{X},\mathfrak{Y},\mathfrak{Z}\in[l,m]. \tag{23}$$

The following is the methodical course of action that ought to be done in order to resolve the problem:

$$\mathcal{H}(\mathfrak{X},\mathfrak{Y},\mathfrak{Z}) = -\frac{\sin(\pi\mathfrak{X})\sin(\pi\mathfrak{Y})\sin(\pi\mathfrak{Z})}{2\pi^2}. \tag{24}$$

In order to solve the third problem, we take the boundary conditions in the following form:

Table 3: Numerical and exact results for the first problem at $\mathfrak{Y} = 0.5$, $\mathfrak{X}, \mathfrak{Y} \in [0, 1]$.

$\mathfrak{X}$	Num.	Ex	Abs. error
0.2	0.079328	0.079371	4.28168×10^{-5}
0.4	0.105922	0.105956	3.39476×10^{-5}
0.6	0.094261	0.094287	2.61754×10^{-5}
0.8	0.055914	0.055931	1.71553×10^{-5}

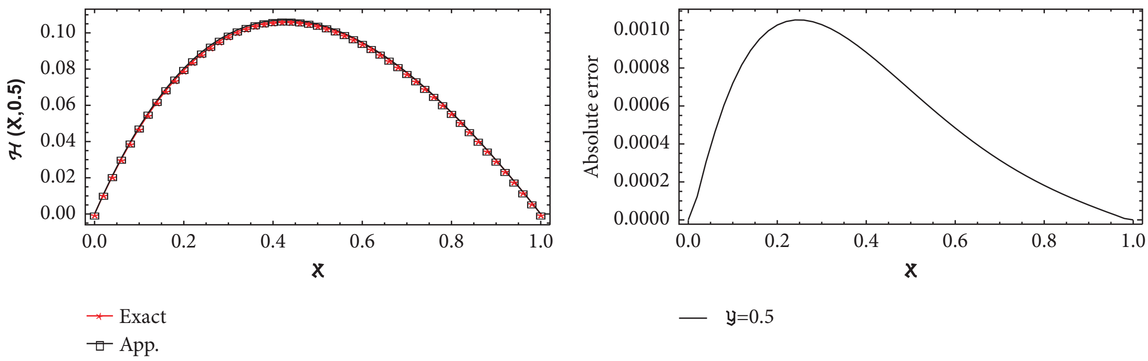

Figure 4: 2D figure showing the numerical and exact solutions and the absolute error at $\mathfrak{Y} = 0.5$.

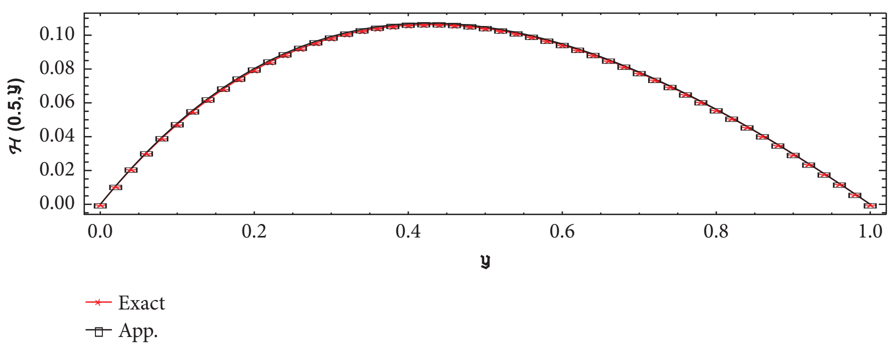

Figure 5: 2D figure showing the numerical solutions at $x = 0.5$.

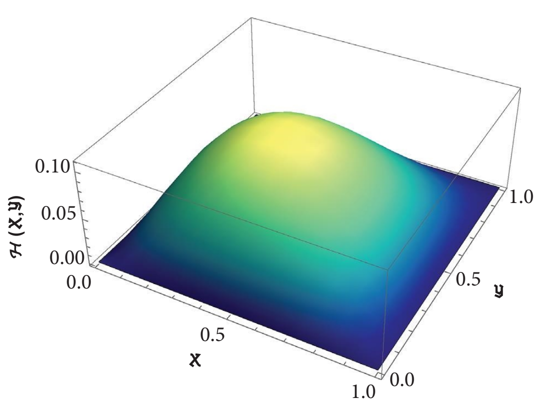

Figure 6: 3D figure showing the numerical results for the second test problem.

$$\begin{aligned} \mathscr{H}(l, \mathfrak{Y}, \mathfrak{Z}) &= \mathscr{H}(\mathfrak{X}, l, \mathfrak{Z}) = \mathscr{H}(\mathfrak{X}, \mathfrak{Y}, l) = \alpha, \\ \mathscr{H}(m, \mathfrak{Y}, \mathfrak{Z}) &= \mathscr{H}(\mathfrak{X}, m, \mathfrak{Z}) = \mathscr{H}(\mathfrak{X}, \mathfrak{Y}, m) = \beta. \end{aligned} \tag{25}$$

By making the substitution from equations (10) and (11) into equations (23) and (25), we are able to obtain the numerical findings that are presented in the following table.

Table 5 presents the results that were obtained while applying the sixtic B-spline method in three dimensions while using a mesh 15×15. In terms of what can be observed, it seems as though the outcomes are satisfactory. The precise numerical findings are displayed in Figure 9, which is based on the equation $\mathfrak{Y} = \mathfrak{Z} = 0.5$. A graph of the numerical findings is shown in three dimensions in Figure 10.

Table 4: Numerical and exact results for the first problem at $\mathfrak{Z} = \mathfrak{Y} = 0.5$, $\mathfrak{X}, \mathfrak{Y}, \mathfrak{Z} \in [0, 1]$.

$\mathfrak{X}$	Num. results	Ex. results	Abs. error	Quadratic B-spline method [11]	Cubic B-spline method [12]
0.1	0.016868	0.016898	3.00566×10^{-5}	3.2494×10^{-5}	3.48009×10^{-5}
0.2	0.033142	0.033201	5.91862×10^{-5}	6.4994×10^{-5}	7.05722×10^{-5}
0.4	0.060716	0.060828	1.11868×10^{-4}	1.2707×10^{-4}	1.41846×10^{-4}
0.6	0.074136	0.074295	1.59332×10^{-4}	1.9233×10^{-4}	2.24718×10^{-4}
0.8	0.060271	0.060496	2.25047×10^{-4}	3.0463×10^{-4}	3.82276×10^{-4}
0.9	0.050423	0.037608	2.27691×10^{-4}	4.1116×10^{-4}	5.34402×10^{-4}

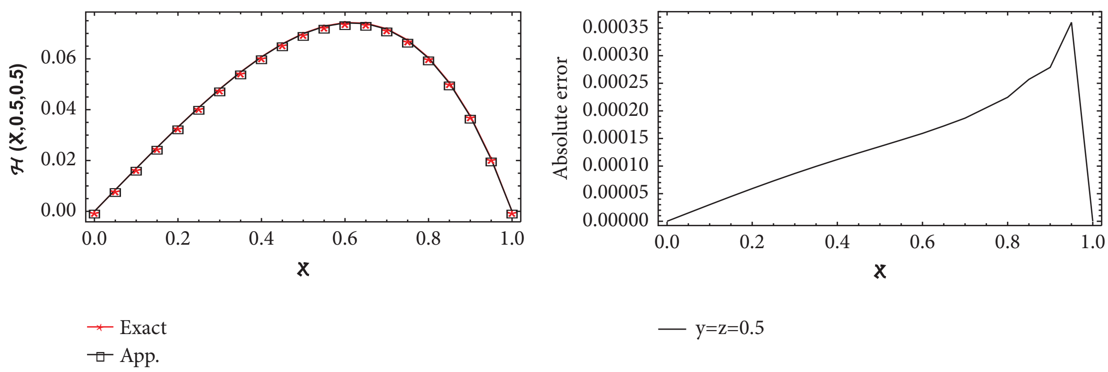

Figure 7: 2D figure showing the numerical and exact solutions and the absolute error at $\mathfrak{Y} = \mathfrak{Z} = 0.5$ for the third test problem.

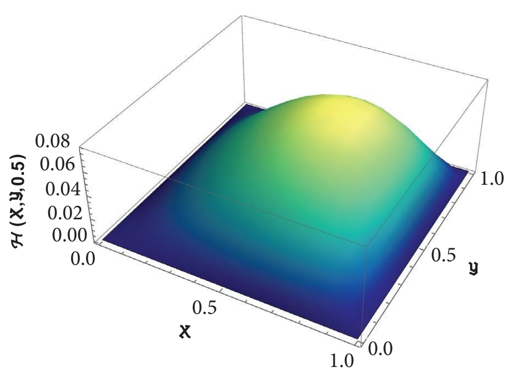

Figure 8: 3D figure showing the numerical results for the third test problem.

Table 5: Compare our results with some other results through the absolute error values at $\mathfrak{Y} = \mathfrak{Z} = 0.5$, $\mathfrak{X}, \mathfrak{Y}, \mathfrak{Z} \in [0, 1]$.

$\mathfrak{X}$	Num.	Exact	Abs. error	L_∞ of our method	L_∞ [14]	L_∞ [3]
0.2	−0.01985	−0.01985	4.793×10^{-11}	8.155×10^{-11}	4.966×10^{-5}	8.922×10^{-4}
0.4	−0.03212	−0.03212	7.756×10^{-11}	—	—	—
0.6	−0.03212	−0.03210	7.756×10^{-11}	—	—	—
0.8	−0.01985	−0.01985	4.793×10^{-11}	—	—	—

3.5. Discussion. This work demonstrates the challenges researchers face when dealing with mathematical models of multiple dimensions. The paper introduces new forms that experienced users can utilize to achieve successful results. A generalization of the shape of a sixtic spline in two and three dimensions is provided, enabling the solution of mathematical models and examples with a fifth or lower order. The effectiveness and quality of the method are supported by numerical results and error rate analysis in the third section. Four numerical examples of mathematical models in two and three dimensions are presented. While concerns about the cost and time required by this method

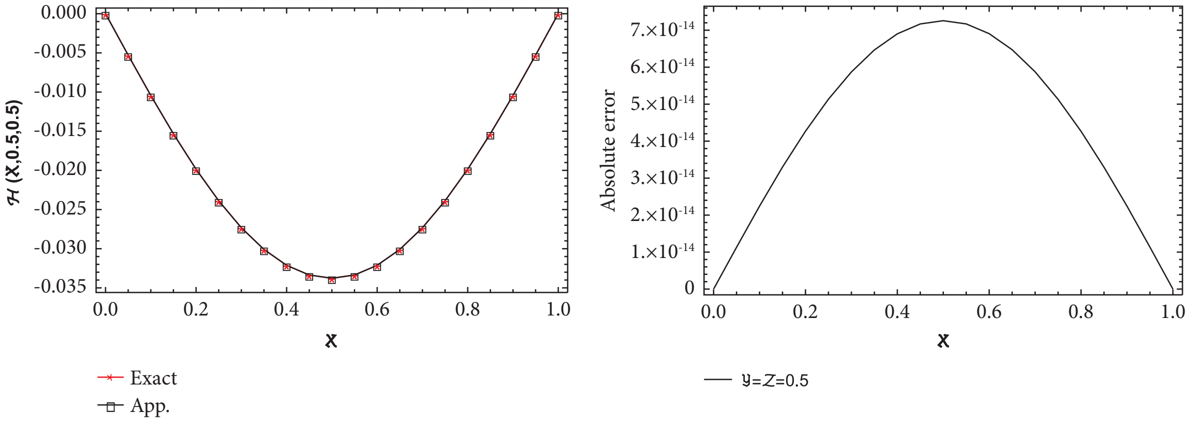

FIGURE 9: 2D figure showing the numerical and exact solutions and the absolute error at $\mathfrak{Y} = \mathfrak{Z} = 0.5$ for the fourth test problem.

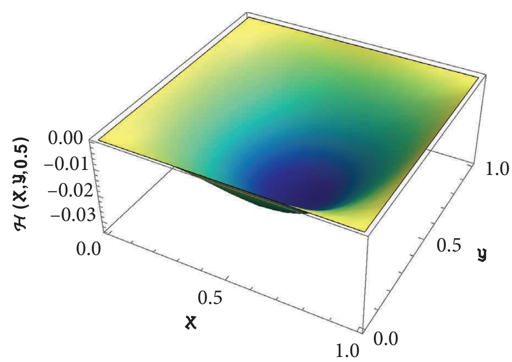

FIGURE 10: 3D figure showing the numerical results for the fourth test problem.

are acknowledged, the focus is currently on finding a solution. Future efforts will explore approaches to minimize time and effort. It should be noted that many researchers may require additional assistance to effectively utilize these highly effective features.

4. Conclusion

This investigation addresses the challenges researchers face when working with mathematical models in multiple dimensions. Our study aims to contribute significantly to solving these problems. The importance of this research topic has generated great anticipation among academics. Through conversations with researchers who have explored solutions to partial differential equations in one, two, and three dimensions, we gained insight into the difficulties encountered as the dimension increases. The researchers presented their respective findings on partial differential equations in these dimensions, highlighting the need to extend the original one-dimensional B-spline approach. It became evident that a single additional dimension would not suffice. To assess the accuracy and usefulness of the developed systems, we utilized numerical examples across various dimensions. By comparing the numerical results of the derived functions to the actual solutions, we were able to evaluate their precision. Based on this evaluation, significant progress has been made in addressing problems involving partial differential equations in multiple dimensions. As part of our ongoing research, we aim to generalize additional B-spline forms to serve as solutions to differential equations in n-dimensions. This will enable the creation of more intricate and precise models.

Appendix

$$\frac{\partial G_{i,\digamma,\wp}}{\partial \mathfrak{Y}} = \frac{3}{65536k}\Big(\mathscr{A}_{i-3,\digamma-3,\wp-3} + 722\mathscr{A}_{i-3,\digamma-3,\wp-2} + 10543\mathscr{A}_{i-3,\digamma-3,\wp-1}$$
$$+ 23548\mathscr{A}_{i-3,\digamma-3,\wp} + 10543\mathscr{A}_{i-3,\digamma-3,\wp+1} + 722\mathscr{A}_{i-3,\digamma-3,\wp+2}$$

$$
\begin{aligned}
&+ \mathscr{A}_{i-3,\digamma-3,\wp+3} + 236\mathscr{A}_{i-3,\digamma-2,\wp-3} + 170392\mathscr{A}_{i-3,\digamma-2,\wp-2} + 2488148\mathscr{A}_{i-3,\digamma-2,\wp-1} \\
&+ 5557328\mathscr{A}_{i-3,\digamma-2,\wp} + 2488148\mathscr{A}_{i-3,\digamma-2,\wp+1} \\
&+ 170392\mathscr{A}_{i-3,\digamma-2,\wp+2} + 236\mathscr{A}_{i-3,\digamma-2,\wp+3} + 1445\mathscr{A}_{i-3,\digamma-1,\wp-3} \\
&+ 1043290\mathscr{A}_{i-3,\digamma-1,\wp-2} + 15234635\mathscr{A}_{i-3,\digamma-1,\wp-1} \\
&+ 34026860\mathscr{A}_{i-3,\digamma-1,\wp} + 15234635\mathscr{A}_{i-3,\digamma-1,\wp+1} \\
&+ 1043290\mathscr{A}_{i-3,\digamma-1,\wp+2} + 1445\mathscr{A}_{i-3,\digamma-1,\wp+3} - 1445\mathscr{A}_{i-3,\digamma+1,\wp-3} \\
&- 1043290\mathscr{A}_{i-3,\digamma+1,\wp-2} - 15234635\mathscr{A}_{i-3,\digamma+1,\wp-1} \\
&- 34026860\mathscr{A}_{i-3,\digamma+1,\wp} - 15234635\mathscr{A}_{i-3,\digamma+1,\wp+1} \\
&- 1043290\mathscr{A}_{i-3,\digamma+1,\wp+2} - 1445\mathscr{A}_{i-3,\digamma+1,\wp+3} - 236\mathscr{A}_{i-3,\digamma+2,\wp-3} \\
&- 170392\mathscr{A}_{i-3,\digamma+2,\wp-2} - 2488148\mathscr{A}_{i-3,\digamma+2,\wp-1} \\
&- 5557328\mathscr{A}_{i-3,\digamma+2,\wp} - 2488148\mathscr{A}_{i-3,\digamma+2,\wp+1} \\
&- 170392\mathscr{A}_{i-3,\digamma+2,\wp+2} - 236\mathscr{A}_{i-3,\digamma+2,\wp+3} - \mathscr{A}_{i-3,\digamma+3,\wp-3} - 722\mathscr{A}_{i-3,\digamma+3,\wp-2} \\
&- 10543\mathscr{A}_{i-3,\digamma+3,\wp-1} - 23548\mathscr{A}_{i-3,\digamma+3,\wp} - 10543\mathscr{A}_{i-3,\digamma+3,\wp+1} \\
&- 722\mathscr{A}_{i-3,\digamma+3,\wp+2} - \mathscr{A}_{i-3,\digamma+3,\wp+3} + 722\mathscr{A}_{i-2,\digamma-3,\wp-3} + 521284\mathscr{A}_{i-2,\digamma-3,\wp-2} \\
&+ 7612046\mathscr{A}_{i-2,\digamma-3,\wp-1} + 17001656\mathscr{A}_{i-2,\digamma-3,\wp} \\
&+ 7612046\mathscr{A}_{i-2,\digamma-3,\wp+1} + 521284\mathscr{A}_{i-2,\digamma-3,\wp+2} \\
&+ 722\mathscr{A}_{i-2,\digamma-3,\wp+3} + 170392\mathscr{A}_{i-2,\digamma-2,\wp-3} + 123023024\mathscr{A}_{i-2,\digamma-2,\wp-2} \\
&+ 1796442856\mathscr{A}_{i-2,\digamma-2,\wp-1} + 4012390816\mathscr{A}_{i-2,\digamma-2,\wp} \\
&+ 1796442856\mathscr{A}_{i-2,\digamma-2,\wp+1} + 123023024\mathscr{A}_{i-2,\digamma-2,\wp+2} \\
&+ 170392\mathscr{A}_{i-2,\digamma-2,\wp+3} + 1043290\mathscr{A}_{i-2,\digamma-1,\wp-3} \\
&+ 753255380\mathscr{A}_{i-2,\digamma-1,\wp-2} + 10999406470\mathscr{A}_{i-2,\digamma-1,\wp-1} \\
&+ 24567392920\mathscr{A}_{i-2,\digamma-1,\wp} + 10999406470\mathscr{A}_{i-2,\digamma-1,\wp+1} \\
&+ 753255380\mathscr{A}_{i-2,\digamma-1,\wp+2} + 1043290\mathscr{A}_{i-2,\digamma-1,\wp+3} \\
&- 1043290\mathscr{A}_{i-2,\digamma+1,\wp-3} - 753255380\mathscr{A}_{i-2,\digamma+1,\wp-2} \\
&- 10999406470\mathscr{A}_{i-2,\digamma+1,\wp-1} - 24567392920\mathscr{A}_{i-2,\digamma+1,\wp}
\end{aligned}
$$

$$\begin{aligned}
&- 10999406470\mathscr{A}_{i-2,\digamma+1,\wp+1} - 753255380\mathscr{A}_{i-2,\digamma+1,\wp+2}\\
&- 1043290\mathscr{A}_{i-2,\digamma+1,\wp+3} - 170392\mathscr{A}_{i-2,\digamma+2,\wp-3}\\
&- 123023024\mathscr{A}_{i-2,\digamma+2,\wp-2} - 1796442856\mathscr{A}_{i-2,\digamma+2,\wp-1}\\
&- 4012390816\mathscr{A}_{i-2,\digamma+2,\wp} - 1796442856\mathscr{A}_{i-2,\digamma+2,\wp+1}\\
&- 123023024\mathscr{A}_{i-2,\digamma+2,\wp+2} - 170392\mathscr{A}_{i-2,\digamma+2,\wp+3}\\
&- 722\mathscr{A}_{i-2,\digamma+3,\wp-3} - 521284\mathscr{A}_{i-2,\digamma+3,\wp-2} - 7612046\mathscr{A}_{i-2,\digamma+3,\wp-1}\\
&- 17001656\mathscr{A}_{i-2,\digamma+3,\wp} - 7612046\mathscr{A}_{i-2,\digamma+3,\wp+1}\\
&- 521284\mathscr{A}_{i-2,\digamma+3,\wp+2} - 722\mathscr{A}_{i-2,\digamma+3,\wp+3} + 10543\mathscr{A}_{i-1,\digamma-3,\wp-3}\\
&+ 7612046\mathscr{A}_{i-1,\digamma-3,\wp-2} + 111154849\mathscr{A}_{i-1,\digamma-3,\wp-1}\\
&+ 248266564\mathscr{A}_{i-1,\digamma-3,\wp} + 111154849\mathscr{A}_{i-1,\digamma-3,\wp+1}\\
&+ 7612046\mathscr{A}_{i-1,\digamma-3,\wp+2} + 10543\mathscr{A}_{i-1,\digamma-3,\wp+3}\\
&+ 2488148\mathscr{A}_{i-1,\digamma-2,\wp-3} + 1796442856\mathscr{A}_{i-1,\digamma-2,\wp-2}\\
&+ 26232544364\mathscr{A}_{i-1,\digamma-2,\wp-1} + 58590909104\mathscr{A}_{i-1,\digamma-2,\wp}\\
&+ 26232544364\mathscr{A}_{i-1,\digamma-2,\wp+1} + 1796442856\mathscr{A}_{i-1,\digamma-2,\wp+2}\\
&+ 2488148\mathscr{A}_{i-1,\digamma-2,\wp+3} + 15234635\mathscr{A}_{i-1,\digamma-1,\wp-3}\\
&+ 10999406470\mathscr{A}_{i-1,\digamma-1,\wp-2} + 160618756805\mathscr{A}_{i-1,\digamma-1,\wp-1}\\
&+ 358745184980\mathscr{A}_{i-1,\digamma-1,\wp} + 160618756805\mathscr{A}_{i-1,\digamma-1,\wp+1}\\
&+ 10999406470\mathscr{A}_{i-1,\digamma-1,\wp+2} + 15234635\mathscr{A}_{i-1,\digamma-1,\wp+3}\\
&- 15234635\mathscr{A}_{i-1,\digamma+1,\wp-3} - 10999406470\mathscr{A}_{i-1,\digamma+1,\wp-2}\\
&- 160618756805\mathscr{A}_{i-1,\digamma+1,\wp-1} - 358745184980\mathscr{A}_{i-1,\digamma+1,\wp}\\
&- 160618756805\mathscr{A}_{i-1,\digamma+1,\wp+1} - 10999406470\mathscr{A}_{i-1,\digamma+1,\wp+2}\\
&- 15234635\mathscr{A}_{i-1,\digamma+1,\wp+3} - 2488148\mathscr{A}_{i-1,\digamma+2,\wp-3}\\
&- 1796442856\mathscr{A}_{i-1,\digamma+2,\wp-2} - 26232544364\mathscr{A}_{i-1,\digamma+2,\wp-1}\\
&- 58590909104\mathscr{A}_{i-1,\digamma+2,\wp} - 26232544364\mathscr{A}_{i-1,\digamma+2,\wp+1}\\
&- 1796442856\mathscr{A}_{i-1,\digamma+2,\wp+2} - 2488148\mathscr{A}_{i-1,\digamma+2,\wp+3}
\end{aligned}$$

$$
\begin{aligned}
&- 10543\mathscr{A}_{i-1,\digamma+3,\wp-3} - 7612046\mathscr{A}_{i-1,\digamma+3,\wp-2} \\
&- 111154849\mathscr{A}_{i-1,\digamma+3,\wp-1} - 248266564\mathscr{A}_{i-1,\digamma+3,\wp} \\
&- 111154849\mathscr{A}_{i-1,\digamma+3,\wp+1} - 7612046\mathscr{A}_{i-1,\digamma+3,\wp+2} \\
&- 10543\mathscr{A}_{i-1,\digamma+3,\wp+3} + 23548\mathscr{A}_{i,\digamma-3,\wp-3} \\
&+ 17001656\mathscr{A}_{i,\digamma-3,\wp-2} + 248266564\mathscr{A}_{i,\digamma-3,\wp-1} \\
&+ 554508304\mathscr{A}_{i,\digamma-3,\wp} + 248266564\mathscr{A}_{i,\digamma-3,\wp+1} \\
&+ 17001656\mathscr{A}_{i,\digamma-3,\wp+2} + 23548\mathscr{A}_{i,\digamma-3,\wp+3} \\
&+ 5557328\mathscr{A}_{i,\digamma-2,\wp-3} + 4012390816\mathscr{A}_{i,\digamma-2,\wp-2} \\
&+ 58590909104\mathscr{A}_{i,\digamma-2,\wp-1} + 130863959744\mathscr{A}_{i,\digamma-2,\wp} \\
&+ 58590909104\mathscr{A}_{i,\digamma-2,\wp+1} + 4012390816\mathscr{A}_{i,\digamma-2,\wp+2} \\
&+ 5557328\mathscr{A}_{i,\digamma-2,\wp+3} + 34026860\mathscr{A}_{i,\digamma-1,\wp-3} \\
&+ 24567392920\mathscr{A}_{i,\digamma-1,\wp-2} + 358745184980\mathscr{A}_{i,\digamma-1,\wp-1} \\
&+ 801264499280\mathscr{A}_{i,\digamma-1,\wp} + 358745184980\mathscr{A}_{i,\digamma-1,\wp+1} \\
&+ 24567392920\mathscr{A}_{i,\digamma-1,\wp+2} + 34026860\mathscr{A}_{i,\digamma-1,\wp+3} \\
&- 34026860\mathscr{A}_{i,\digamma+1,\wp-3} - 24567392920\mathscr{A}_{i,\digamma+1,\wp-2} \\
&- 358745184980\mathscr{A}_{i,\digamma+1,\wp-1} - 801264499280\mathscr{A}_{i,\digamma+1,\wp} \\
&- 358745184980\mathscr{A}_{i,\digamma+1,\wp+1} - 24567392920\mathscr{A}_{i,\digamma+1,\wp+2} \\
&- 34026860\mathscr{A}_{i,\digamma+1,\wp+3} - 5557328\mathscr{A}_{i,\digamma+2,\wp-3} \\
&- 4012390816\mathscr{A}_{i,\digamma+2,\wp-2} - 58590909104\mathscr{A}_{i,\digamma+2,\wp-1} \\
&- 130863959744\mathscr{A}_{i,\digamma+2,\wp} - 58590909104\mathscr{A}_{i,\digamma+2,\wp+1} \\
&- 4012390816\mathscr{A}_{i,\digamma+2,\wp+2} - 5557328\mathscr{A}_{i,\digamma+2,\wp+3} \\
&- 23548\mathscr{A}_{i,\digamma+3,\wp-3} - 17001656\mathscr{A}_{i,\digamma+3,\wp-2} - 10999406470\mathscr{A}_{i+1,\digamma+1,\wp+2} \\
&- 248266564\mathscr{A}_{i,\digamma+3,\wp-1} - 554508304\mathscr{A}_{i,\digamma+3,\wp} \\
&- 248266564\mathscr{A}_{i,\digamma+3,\wp+1} - 17001656\mathscr{A}_{i,\digamma+3,\wp+2} \\
&- 23548\mathscr{A}_{i,\digamma+3,\wp+3} + 10543\mathscr{A}_{i+1,\digamma-3,\wp-3} + 7612046\mathscr{A}_{i+1,\digamma-3,\wp-2}
\end{aligned}
$$

$$
\begin{aligned}
&+ 111154849\mathscr{A}_{i+1,\digamma-3,\wp-1} + 248266564\mathscr{A}_{i+1,\digamma-3,\wp}\\
&+ 111154849\mathscr{A}_{i+1,\digamma-3,\wp+1} + 7612046\mathscr{A}_{i+1,\digamma-3,\wp+2}\\
&+ 10543\mathscr{A}_{i+1,\digamma-3,\wp+3} + 2488148\mathscr{A}_{i+1,\digamma-2,\wp-3}\\
&+ 1796442856\mathscr{A}_{i+1,\digamma-2,\wp-2} + 26232544364\mathscr{A}_{i+1,\digamma-2,\wp-1}\\
&+ 58590909104\mathscr{A}_{i+1,\digamma-2,\wp} + 26232544364\mathscr{A}_{i+1,\digamma-2,\wp+1}\\
&+ 1796442856\mathscr{A}_{i+1,\digamma-2,\wp+2} + 2488148\mathscr{A}_{i+1,\digamma-2,\wp+3}\\
&+ 15234635\mathscr{A}_{i+1,\digamma-1,\wp-3} + 10999406470\mathscr{A}_{i+1,\digamma-1,\wp-2}\\
&+ 160618756805\mathscr{A}_{i+1,\digamma-1,\wp-1} + 358745184980\mathscr{A}_{i+1,\digamma-1,\wp}\\
&+ 160618756805\mathscr{A}_{i+1,\digamma-1,\wp+1} + 10999406470\mathscr{A}_{i+1,\digamma-1,\wp+2}\\
&+ 15234635\mathscr{A}_{i+1,\digamma-1,\wp+3} - 15234635\mathscr{A}_{i+1,\digamma+1,\wp-3}\\
&- 10999406470\mathscr{A}_{i+1,\digamma+1,\wp-2} - 160618756805\mathscr{A}_{i+1,\digamma+1,\wp-1}\\
&- 358745184980\mathscr{A}_{i+1,\digamma+1,\wp} - 160618756805\mathscr{A}_{i+1,\digamma+1,\wp+1}\\
&- 15234635\mathscr{A}_{i+1,\digamma+1,\wp+3} - 2488148\mathscr{A}_{i+1,\digamma+2,\wp-3}\\
&- 1796442856\mathscr{A}_{i+1,\digamma+2,\wp-2} - 26232544364\mathscr{A}_{i+1,\digamma+2,\wp-1}\\
&- 58590909104\mathscr{A}_{i+1,\digamma+2,\wp} - 26232544364\mathscr{A}_{i+1,\digamma+2,\wp+1}\\
&- 1796442856\mathscr{A}_{i+1,\digamma+2,\wp+2} - 2488148\mathscr{A}_{i+1,\digamma+2,\wp+3}\\
&- 10543\mathscr{A}_{i+1,\digamma+3,\wp-3} - 7612046\mathscr{A}_{i+1,\digamma+3,\wp-2} - 111154849\mathscr{A}_{i+1,\digamma+3,\wp-1}\\
&- 248266564\mathscr{A}_{i+1,\digamma+3,\wp} - 111154849\mathscr{A}_{i+1,\digamma+3,\wp+1}\\
&- 7612046\mathscr{A}_{i+1,\digamma+3,\wp+2} - 10543\mathscr{A}_{i+1,\digamma+3,\wp+3} + 722\mathscr{A}_{i+2,\digamma-3,\wp-3}\\
&+ 521284\mathscr{A}_{i+2,\digamma-3,\wp-2} + 7612046\mathscr{A}_{i+2,\digamma-3,\wp-1}\\
&+ 17001656\mathscr{A}_{i+2,\digamma-3,\wp} + 7612046\mathscr{A}_{i+2,\digamma-3,\wp+1}\\
&+ 521284\mathscr{A}_{i+2,\digamma-3,\wp+2} + 722\mathscr{A}_{i+2,\digamma-3,\wp+3} + 170392\mathscr{A}_{i+2,\digamma-2,\wp-3}\\
&+ 123023024\mathscr{A}_{i+2,\digamma-2,\wp-2} + 1796442856\mathscr{A}_{i+2,\digamma-2,\wp-1}\\
&+ 4012390816\mathscr{A}_{i+2,\digamma-2,\wp} + 1796442856\mathscr{A}_{i+2,\digamma-2,\wp+1}\\
&+ 123023024\mathscr{A}_{i+2,\digamma-2,\wp+2} + 170392\mathscr{A}_{i+2,\digamma-2,\wp+3}
\end{aligned}
$$

$$
\begin{aligned}
&+ 1043290\mathscr{A}_{i+2,\digamma-1,\wp-3} + 753255380\mathscr{A}_{i+2,\digamma-1,\wp-2} \\
&+ 10999406470\mathscr{A}_{i+2,\digamma-1,\wp-1} + 24567392920\mathscr{A}_{i+2,\digamma-1,\wp} \\
&+ 10999406470\mathscr{A}_{i+2,\digamma-1,\wp+1} + 753255380\mathscr{A}_{i+2,\digamma-1,\wp+2} \\
&+ 1043290\mathscr{A}_{i+2,\digamma-1,\wp+3} - 1043290\mathscr{A}_{i+2,\digamma+1,\wp-3} \\
&- 753255380\mathscr{A}_{i+2,\digamma+1,\wp-2} - 10999406470\mathscr{A}_{i+2,\digamma+1,\wp-1} \\
&- 24567392920\mathscr{A}_{i+2,\digamma+1,\wp} - 10999406470\mathscr{A}_{i+2,\digamma+1,\wp+1} \\
&- 753255380\mathscr{A}_{i+2,\digamma+1,\wp+2} - 1043290\mathscr{A}_{i+2,\digamma+1,\wp+3} \\
&- 170392\mathscr{A}_{i+2,\digamma+2,\wp-3} - 123023024\mathscr{A}_{i+2,\digamma+2,\wp-2} \\
&- 1796442856\mathscr{A}_{i+2,\digamma+2,\wp-1} - 4012390816\mathscr{A}_{i+2,\digamma+2,\wp} \\
&- 1796442856\mathscr{A}_{i+2,\digamma+2,\wp+1} - 123023024\mathscr{A}_{i+2,\digamma+2,\wp+2} \\
&- 170392\mathscr{A}_{i+2,\digamma+2,\wp+3} - 722\mathscr{A}_{i+2,\digamma+3,\wp-3} - 521284\mathscr{A}_{i+2,\digamma+3,\wp-2} \\
&- 7612046\mathscr{A}_{i+2,\digamma+3,\wp-1} - 17001656\mathscr{A}_{i+2,\digamma+3,\wp} \\
&- 7612046\mathscr{A}_{i+2,\digamma+3,\wp+1} - 521284\mathscr{A}_{i+2,\digamma+3,\wp+2} - 722\mathscr{A}_{i+2,\digamma+3,\wp+3} \\
&+ \mathscr{A}_{i+3,\digamma-3,\wp-3} + 722\mathscr{A}_{i+3,\digamma-3,\wp-2} + 10543\mathscr{A}_{i+3,\digamma-3,\wp-1} + 23548\mathscr{A}_{i+3,\digamma-3,\wp} \\
&+ 10543\mathscr{A}_{i+3,\digamma-3,\wp+1} + 722\mathscr{A}_{i+3,\digamma-3,\wp+2} + \mathscr{A}_{i+3,\digamma-3,\wp+3} \\
&+ 236\mathscr{A}_{i+3,\digamma-2,\wp-3} + 170392\mathscr{A}_{i+3,\digamma-2,\wp-2} + 2488148\mathscr{A}_{i+3,\digamma-2,\wp-1} \\
&+ 5557328\mathscr{A}_{i+3,\digamma-2,\wp} + 2488148\mathscr{A}_{i+3,\digamma-2,\wp+1} \\
&+ 170392\mathscr{A}_{i+3,\digamma-2,\wp+2} + 236\mathscr{A}_{i+3,\digamma-2,\wp+3} + 1445\mathscr{A}_{i+3,\digamma-1,\wp-3} \\
&+ 1043290\mathscr{A}_{i+3,\Theta-1,\wp-2} + 15234635\mathscr{A}_{i+3,\Theta-1,\wp-1} \\
&+ 34026860\mathscr{A}_{i+3,\Theta-1,\wp} + 15234635\mathscr{A}_{i+3,\Theta-1,\wp+1} \\
&+ 1043290\mathscr{A}_{i+3,\Theta-1,\wp+2} + 1445\mathscr{A}_{i+3,\Theta-1,\wp+3} - 1445\mathscr{A}_{i+3,\digamma+1,\wp-3} \\
&- 1043290\mathscr{A}_{i+3,\digamma+1,\wp-2} - 15234635\mathscr{A}_{i+3,\digamma+1,\wp-1} \\
&- 34026860\mathscr{A}_{i+3,\digamma+1,\wp} - 15234635\mathscr{A}_{i+3,\digamma+1,\wp+1} \\
&- 1043290\mathscr{A}_{i+3,\digamma+1,\wp+2} - 1445\mathscr{A}_{i+3,\digamma+1,\wp+3} - 236\mathscr{A}_{i+3,\digamma+2,\wp-3} \\
&- 170392\mathscr{A}_{i+3,\digamma+2,\wp-2} - 2488148\mathscr{A}_{i+3,\digamma+2,\wp-1}
\end{aligned}
$$

$$
\begin{aligned}
&- 5557328\mathscr{A}_{i+3,\digamma+2,\wp} - 2488148\mathscr{A}_{i+3,\digamma+2,\wp+1}\\
&- 170392\mathscr{A}_{i+3,\digamma+2,\wp+2} - 236\mathscr{A}_{i+3,\digamma+2,\wp+3} - \mathscr{A}_{i+3,\digamma+3,\wp-3}\\
&- 722\mathscr{A}_{i+3,\digamma+3,\wp-2} - 10543\mathscr{A}_{i+3,\digamma+3,\wp-1} - 23548\mathscr{A}_{i+3,\digamma+3,\wp}\\
&-10543\mathscr{A}_{i+3,\digamma+3,\wp+1} - 722\mathscr{A}_{i+3,\digamma+3,\wp+2} - \mathscr{A}_{i+3,\digamma+3,\wp+3}\Big).
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial G_{i,\digamma,\wp}}{\partial \mathfrak{Z}} = \frac{3}{65536q}\Big(&\mathscr{A}_{i-3,\digamma-3,\wp-3} + 236\mathscr{A}_{i-3,\digamma-3,\wp-2}\\
&+ 1445\mathscr{A}_{i-3,\digamma-3,\wp-1} - 1445\mathscr{A}_{i-3,\digamma-3,\wp+1} - 236\mathscr{A}_{i-3,\digamma-3,\wp+2}\\
&- \mathscr{A}_{i-3,\digamma-3,\wp+3} + 722\mathscr{A}_{i-3,\digamma-2,\wp-3} + 170392\mathscr{A}_{i-3,\digamma-2,\wp-2}\\
&+ 1043290\mathscr{A}_{i-3,\digamma-2,\wp-1} - 1043290\mathscr{A}_{i-3,\digamma-2,\wp+1}\\
&- 170392\mathscr{A}_{i-3,\digamma-2,\wp+2} - 722\mathscr{A}_{i-3,\digamma-2,\wp+3} + 10543\mathscr{A}_{i-3,\digamma-1,\wp-3}\\
&+ 2488148\mathscr{A}_{i-3,\digamma-1,\wp-2} + 15234635\mathscr{A}_{i-3,\digamma-1,\wp-1}\\
&- 15234635\mathscr{A}_{i-3,\digamma-1,\wp+1} - 2488148\mathscr{A}_{i-3,\digamma-1,\wp+2}\\
&- 10543\mathscr{A}_{i-3,\digamma-1,\wp+3} + 23548\mathscr{A}_{i-3,\digamma,\wp-3} + 5557328\mathscr{A}_{i-3,\digamma,\wp-2}\\
&+ 34026860\mathscr{A}_{i-3,\digamma,\wp-1} - 34026860\mathscr{A}_{i-3,\digamma,\wp+1} - 5557328\mathscr{A}_{i-3,\digamma,\wp+2}\\
&- 23548\mathscr{A}_{i-3,\digamma,\wp+3} + 10543\mathscr{A}_{i-3,\digamma+1,\wp-3} + 2488148\mathscr{A}_{i-3,\digamma+1,\wp-2}\\
&+ 15234635\mathscr{A}_{i-3,\digamma+1,\wp-1} - 15234635\mathscr{A}_{i-3,\digamma+1,\wp+1}\\
&- 2488148\mathscr{A}_{i-3,\digamma+1,\wp+2} - 10543\mathscr{A}_{i-3,\digamma+1,\wp+3} + 722\mathscr{A}_{i-3,\digamma+2,\wp-3}\\
&+ 170392\mathscr{A}_{i-3,\digamma+2,\wp-2} + 1043290\mathscr{A}_{i-3,\digamma+2,\wp-1}\\
&- 1043290\mathscr{A}_{i-3,\digamma+2,\wp+1} - 170392\mathscr{A}_{i-3,\digamma+2,\wp+2} - 722\mathscr{A}_{i-3,\digamma+2,\wp+3}\\
&+ \mathscr{A}_{i-3,\digamma+3,\wp-3} + 236\mathscr{A}_{i-3,\digamma+3,\wp-2} + 1445\mathscr{A}_{i-3,\digamma+3,\wp-1}\\
&- 1445\mathscr{A}_{i-3,\digamma+3,\wp+1} - 236\mathscr{A}_{i-3,\digamma+3,\wp+2} - \mathscr{A}_{i-3,\digamma+3,\wp+3}\\
&+ 722\mathscr{A}_{i-2,\digamma-3,\wp-3} + 170392\mathscr{A}_{i-2,\digamma-3,\wp-2} + 1043290\mathscr{A}_{i-2,\digamma-3,\wp-1}\\
&- 1043290\mathscr{A}_{i-2,\digamma-3,\wp+1} - 170392\mathscr{A}_{i-2,\digamma-3,\wp+2} - 722\mathscr{A}_{i-2,\digamma-3,\wp+3}\\
&+ 521284\mathscr{A}_{i-2,\digamma-2,\wp-3} + 123023024\mathscr{A}_{i-2,\digamma-2,\wp-2}\\
&+ 753255380\mathscr{A}_{i-2,\digamma-2,\wp-1} - 753255380\mathscr{A}_{i-2,\digamma-2,\wp+1}\\
&- 123023024\mathscr{A}_{i-2,\digamma-2,\wp+2} - 521284\mathscr{A}_{i-2,\digamma-2,\wp+3}\\
&+ 7612046\mathscr{A}_{i-2,\digamma-1,\wp-3} + 1796442856\mathscr{A}_{i-2,\digamma-1,\wp-2}
\end{aligned}
$$

$$+ 10999406470\mathscr{A}_{i-2,\digamma-1,\wp-1} - 10999406470\mathscr{A}_{i-2,\digamma-1,\wp+1}$$
$$- 1796442856\mathscr{A}_{i-2,\digamma-1,\wp+2} - 7612046\mathscr{A}_{i-2,\digamma-1,\wp+3}$$
$$+ 17001656\mathscr{A}_{i-2,\digamma,\wp-3} + 4012390816\mathscr{A}_{i-2,\digamma,\wp-2}$$
$$+ 24567392920\mathscr{A}_{i-2,\digamma,\wp-1} - 24567392920\mathscr{A}_{i-2,\digamma,\wp+1}$$
$$- 4012390816\mathscr{A}_{i-2,\digamma,\wp+2} - 17001656\mathscr{A}_{i-2,\digamma,\wp+3}$$
$$+ 7612046\mathscr{A}_{i-2,\digamma+1,\wp-3} + 1796442856\mathscr{A}_{i-2,\digamma+1,\wp-2}$$
$$+ 10999406470\mathscr{A}_{i-2,\digamma+1,\wp-1} - 10999406470\mathscr{A}_{i-2,\digamma+1,\wp+1}$$
$$- 1796442856\mathscr{A}_{i-2,\digamma+1,\wp+2} - 7612046\mathscr{A}_{i-2,\digamma+1,\wp+3}$$
$$+ 521284\mathscr{A}_{i-2,\digamma+2,\wp-3} + 123023024\mathscr{A}_{i-2,\digamma+2,\wp-2}$$
$$+ 753255380\mathscr{A}_{i-2,\digamma+2,\wp-1} - 753255380\mathscr{A}_{i-2,\digamma+2,\wp+1}$$
$$- 123023024\mathscr{A}_{i-2,\digamma+2,\wp+2} - 521284\mathscr{A}_{i-2,\digamma+2,\wp+3}$$
$$+ 722\mathscr{A}_{i-2,\digamma+3,\wp-3} + 170392\mathscr{A}_{i-2,\digamma+3,\wp-2} + 1043290\mathscr{A}_{i-2,\digamma+3,\wp-1}$$
$$- 1043290\mathscr{A}_{i-2,\digamma+3,\wp+1} - 170392\mathscr{A}_{i-2,\digamma+3,\wp+2}$$
$$- 722\mathscr{A}_{i-2,\digamma+3,\wp+3} + 10543\mathscr{A}_{i-1,\digamma-3,\wp-3} + 2488148\mathscr{A}_{i-1,\digamma-3,\wp-2}$$
$$+ 15234635\mathscr{A}_{i-1,\digamma-3,\wp-1} - 15234635\mathscr{A}_{i-1,\digamma-3,\wp+1}$$
$$- 2488148\mathscr{A}_{i-1,\digamma-3,\wp+2} - 10543\mathscr{A}_{i-1,\digamma-3,\wp+3}$$
$$+ 7612046\mathscr{A}_{i-1,\digamma-2,\wp-3} + 1796442856\mathscr{A}_{i-1,\digamma-2,\wp-2}$$
$$+ 10999406470\mathscr{A}_{i-1,\digamma-2,\wp-1} - 10999406470\mathscr{A}_{i-1,\digamma-2,\wp+1}$$
$$- 1796442856\mathscr{A}_{i-1,\digamma-2,\wp+2} - 7612046\mathscr{A}_{i-1,\digamma-2,\wp+3}$$
$$+ 111154849\mathscr{A}_{i-1,\digamma-1,\wp-3} + 26232544364\mathscr{A}_{i-1,\digamma-1,\wp-2}$$
$$+ 160618756805\mathscr{A}_{i-1,\digamma-1,\wp-1} - 160618756805\mathscr{A}_{i-1,\digamma-1,\wp+1}$$
$$- 26232544364\mathscr{A}_{i-1,\digamma-1,\wp+2} - 111154849\mathscr{A}_{i-1,\digamma-1,\wp+3}$$
$$+ 248266564\mathscr{A}_{i-1,\digamma,\wp-3} + 58590909104\mathscr{A}_{i-1,\digamma,\wp-2}$$
$$+ 358745184980\mathscr{A}_{i-1,\digamma,\wp-1} - 358745184980\mathscr{A}_{i-1,\digamma,\wp+1}$$
$$- 58590909104\mathscr{A}_{i-1,\digamma,\wp+2} - 248266564\mathscr{A}_{i-1,\digamma,\wp+3}$$

$$+ 111154849\mathscr{A}_{i-1,\digamma+1,\wp-3} + 26232544364\mathscr{A}_{i-1,\digamma+1,\wp-2}$$
$$+ 160618756805\mathscr{A}_{i-1,\digamma+1,\wp-1} - 160618756805\mathscr{A}_{i-1,\digamma+1,\wp+1}$$
$$- 26232544364\mathscr{A}_{i-1,\digamma+1,\wp+2} - 111154849\mathscr{A}_{i-1,\digamma+1,\wp+3}$$
$$+ 7612046\mathscr{A}_{i-1,\digamma+2,\wp-3} + 1796442856\mathscr{A}_{i-1,\digamma+2,\wp-2}$$
$$+ 10999406470\mathscr{A}_{i-1,\digamma+2,\wp-1} - 10999406470\mathscr{A}_{i-1,\digamma+2,\wp+1}$$
$$- 1796442856\mathscr{A}_{i-1,\digamma+2,\wp+2} - 7612046\mathscr{A}_{i-1,\digamma+2,\wp+3}$$
$$+ 10543\mathscr{A}_{i-1,\digamma+3,\wp-3} + 2488148\mathscr{A}_{i-1,\digamma+3,\wp-2}$$
$$+ 15234635\mathscr{A}_{i-1,\digamma+3,\wp-1} - 15234635\mathscr{A}_{i-1,\digamma+3,\wp+1}$$
$$- 2488148\mathscr{A}_{i-1,\digamma+3,\wp+2} - 10543\mathscr{A}_{i-1,\digamma+3,\wp+3} + 23548\mathscr{A}_{i,\digamma-3,\wp-3}$$
$$+ 5557328\mathscr{A}_{i,\digamma-3,\wp-2} + 34026860\mathscr{A}_{i,\digamma-3,\wp-1}$$
$$- 34026860\mathscr{A}_{i,\digamma-3,\wp+1} - 5557328\mathscr{A}_{i,\digamma-3,\wp+2}$$
$$- 23548\mathscr{A}_{i,\digamma-3,\wp+3} + 17001656\mathscr{A}_{i,\digamma-2,\wp-3}$$
$$+ 4012390816\mathscr{A}_{i,\digamma-2,\wp-2} + 24567392920\mathscr{A}_{i,\digamma-2,\wp-1}$$
$$- 24567392920\mathscr{A}_{i,\digamma-2,\wp+1} - 4012390816\mathscr{A}_{i,\digamma-2,\wp+2}$$
$$- 17001656\mathscr{A}_{i,\digamma-2,\wp+3} + 248266564\mathscr{A}_{i,\digamma-1,\wp-3}$$
$$+ 58590909104\mathscr{A}_{i,\digamma-1,\wp-2} + 358745184980\mathscr{A}_{i,\digamma-1,\wp-1}$$
$$- 358745184980\mathscr{A}_{i,\digamma-1,\wp+1} - 58590909104\mathscr{A}_{i,\digamma-1,\wp+2}$$
$$- 248266564\mathscr{A}_{i,\digamma-1,\wp+3} + 554508304\mathscr{A}_{i,\digamma,\wp-3}$$
$$+ 130863959744\mathscr{A}_{i,\digamma,\wp-2} + 801264499280\mathscr{A}_{i,\digamma,\wp-1}$$
$$- 801264499280\mathscr{A}_{i,\digamma,\wp+1} - 130863959744\mathscr{A}_{i,\digamma,\wp+2}$$
$$- 554508304\mathscr{A}_{i,\digamma,\wp+3} + 248266564\mathscr{A}_{i,\digamma+1,\wp-3}$$
$$+ 58590909104\mathscr{A}_{i,\digamma+1,\wp-2} + 358745184980\mathscr{A}_{i,\digamma+1,\wp-1}$$
$$- 358745184980\mathscr{A}_{i,\digamma+1,\wp+1} - 58590909104\mathscr{A}_{i,\digamma+1,\wp+2}$$
$$- 248266564\mathscr{A}_{i,\digamma+1,\wp+3} + 17001656\mathscr{A}_{i,\digamma+2,\wp-3}$$
$$+ 4012390816\mathscr{A}_{i,\digamma+2,\wp-2} + 24567392920\mathscr{A}_{i,\digamma+2,\wp-1}$$

$$
\begin{aligned}
&- 24567392920\mathscr{A}_{i,\digamma+2,\wp+1} - 4012390816\mathscr{A}_{i,\digamma+2,\wp+2}\\
&- 17001656\mathscr{A}_{i,\digamma+2,\wp+3} + 23548\mathscr{A}_{i,\digamma+3,\wp-3}\\
&+ 5557328\mathscr{A}_{i,\digamma+3,\wp-2} + 34026860\mathscr{A}_{i,\digamma+3,\wp-1}\\
&- 34026860\mathscr{A}_{i,\digamma+3,\wp+1} - 5557328\mathscr{A}_{i,\digamma+3,\wp+2}\\
&- 23548\mathscr{A}_{i,\digamma+3,\wp+3} + 10543\mathscr{A}_{i+1,\digamma-3,\wp-3} + 2488148\mathscr{A}_{i+1,\digamma-3,\wp-2}\\
&+ 15234635\mathscr{A}_{i+1,\digamma-3,\wp-1} - 15234635\mathscr{A}_{i+1,\digamma-3,\wp+1}\\
&- 2488148\mathscr{A}_{i+1,\digamma-3,\wp+2} - 10543\mathscr{A}_{i+1,\digamma-3,\wp+3}\\
&+ 7612046\mathscr{A}_{i+1,\digamma-2,\wp-3} + 1796442856\mathscr{A}_{i+1,\digamma-2,\wp-2}\\
&+ 10999406470\mathscr{A}_{i+1,\digamma-2,\wp-1} - 10999406470\mathscr{A}_{i+1,\digamma-2,\wp+1}\\
&- 1796442856\mathscr{A}_{i+1,\digamma-2,\wp+2} - 7612046\mathscr{A}_{i+1,\digamma-2,\wp+3}\\
&+ 111154849\mathscr{A}_{i+1,\digamma-1,\wp-3} + 26232544364\mathscr{A}_{i+1,\digamma-1,\wp-2}\\
&+ 160618756805\mathscr{A}_{i+1,\digamma-1,\wp-1} - 160618756805\mathscr{A}_{i+1,\digamma-1,\wp+1}\\
&- 26232544364\mathscr{A}_{i+1,\digamma-1,\wp+2} - 111154849\mathscr{A}_{i+1,\digamma-1,\wp+3}\\
&+ 248266564\mathscr{A}_{i+1,\digamma,\wp-3} + 58590909104\mathscr{A}_{i+1,\digamma,\wp-2}\\
&+ 358745184980\mathscr{A}_{i+1,\digamma,\wp-1} - 358745184980\mathscr{A}_{i+1,\digamma,\wp+1}\\
&- 58590909104\mathscr{A}_{i+1,\digamma,\wp+2} - 248266564\mathscr{A}_{i+1,\digamma,\wp+3}\\
&+ 111154849\mathscr{A}_{i+1,\digamma+1,\wp-3} + 26232544364\mathscr{A}_{i+1,\digamma+1,\wp-2}\\
&+ 160618756805\mathscr{A}_{i+1,\digamma+1,\wp-1} - 160618756805\mathscr{A}_{i+1,\digamma+1,\wp+1}\\
&- 26232544364\mathscr{A}_{i+1,\digamma+1,\wp+2} - 111154849\mathscr{A}_{i+1,\digamma+1,\wp+3}\\
&+ 7612046\mathscr{A}_{i+1,\digamma+2,\wp-3} + 1796442856\mathscr{A}_{i+1,\digamma+2,\wp-2}\\
&+ 10999406470\mathscr{A}_{i+1,\digamma+2,\wp-1} - 10999406470\mathscr{A}_{i+1,\digamma+2,\wp+1}\\
&- 1796442856\mathscr{A}_{i+1,\digamma+2,\wp+2} - 7612046\mathscr{A}_{i+1,\digamma+2,\wp+3}\\
&+ 10543\mathscr{A}_{i+1,\digamma+3,\wp-3} + 2488148\mathscr{A}_{i+1,\digamma+3,\wp-2}\\
&+ 15234635\mathscr{A}_{i+1,\digamma+3,\wp-1} - 15234635\mathscr{A}_{i+1,\digamma+3,\wp+1}\\
&- 2488148\mathscr{A}_{i+1,\digamma+3,\wp+2} - 10543\mathscr{A}_{i+1,\digamma+3,\wp+3} + 722\mathscr{A}_{i+2,\digamma-3,\wp-3}
\end{aligned}
$$

$$
\begin{aligned}
&+ 170392\mathscr{A}_{i+2,\digamma-3,\wp-2} + 1043290\mathscr{A}_{i+2,\digamma-3,\wp-1}\\
&- 1043290\mathscr{A}_{i+2,\digamma-3,\wp+1} - 170392\mathscr{A}_{i+2,\digamma-3,\wp+2} - 722\mathscr{A}_{i+2,\digamma-3,\wp+3}\\
&+ 521284\mathscr{A}_{i+2,\digamma-2,\wp-3} + 123023024\mathscr{A}_{i+2,\digamma-2,\wp-2}\\
&+ 753255380\mathscr{A}_{i+2,\digamma-2,\wp-1} - 753255380\mathscr{A}_{i+2,\digamma-2,\wp+1}\\
&- 123023024\mathscr{A}_{i+2,\digamma-2,\wp+2} - 521284\mathscr{A}_{i+2,\digamma-2,\wp+3}\\
&+ 7612046\mathscr{A}_{i+2,\digamma-1,\wp-3} + 1796442856\mathscr{A}_{i+2,\digamma-1,\wp-2}\\
&+ 10999406470\mathscr{A}_{i+2,\digamma-1,\wp-1} - 10999406470\mathscr{A}_{i+2,\digamma-1,\wp+1}\\
&- 1796442856\mathscr{A}_{i+2,\digamma-1,\wp+2} - 7612046\mathscr{A}_{i+2,\digamma-1,\wp+3}\\
&+ 17001656\mathscr{A}_{i+2,\digamma,\wp-3} + 4012390816\mathscr{A}_{i+2,\digamma,\wp-2}\\
&+ 24567392920\mathscr{A}_{i+2,\digamma,\wp-1} - 24567392920\mathscr{A}_{i+2,\digamma,\wp+1}\\
&- 4012390816\mathscr{A}_{i+2,\digamma,\wp+2} - 17001656\mathscr{A}_{i+2,\digamma,\wp+3}\\
&+ 7612046\mathscr{A}_{i+2,\digamma+1,\wp-3} + 1796442856\mathscr{A}_{i+2,\digamma+1,\wp-2}\\
&+ 10999406470\mathscr{A}_{i+2,\digamma+1,\wp-1} - 10999406470\mathscr{A}_{i+2,\digamma+1,\wp+1}\\
&- 1796442856\mathscr{A}_{i+2,\digamma+1,\wp+2} - 7612046\mathscr{A}_{i+2,\digamma+1,\wp+3}\\
&+ 521284\mathscr{A}_{i+2,\digamma+2,\wp-3} + 123023024\mathscr{A}_{i+2,\digamma+2,\wp-2}\\
&+ 753255380\mathscr{A}_{i+2,\digamma+2,\wp-1} - 753255380\mathscr{A}_{i+2,\digamma+2,\wp+1}\\
&- 123023024\mathscr{A}_{i+2,\digamma+2,\wp+2} - 521284\mathscr{A}_{i+2,\digamma+2,\wp+3}\\
&+ 722\mathscr{A}_{i+2,\digamma+3,\wp-3} + 170392\mathscr{A}_{i+2,\digamma+3,\wp-2}\\
&+ 1043290\mathscr{A}_{i+2,\digamma+3,\wp-1} - 1043290\mathscr{A}_{i+2,\digamma+3,\wp+1}\\
&- 170392\mathscr{A}_{i+2,\digamma+3,\wp+2} - 722\mathscr{A}_{i+2,\digamma+3,\wp+3} + \mathscr{A}_{i+3,\digamma-3,\wp-3}\\
&+ 1445\mathscr{A}_{i+3,\digamma-3,\wp-1} - 1445\mathscr{A}_{i+3,\digamma-3,\wp+1} - 236\mathscr{A}_{i+3,\digamma-3,\wp+2}\\
&- \mathscr{A}_{i+3,\digamma-3,\wp+3} + 722\mathscr{A}_{i+3,\digamma-2,\wp-3} + 170392\mathscr{A}_{i+3,\digamma-2,\wp-2}\\
&+ 1043290\mathscr{A}_{i+3,\digamma-2,\wp-1} - 1043290\mathscr{A}_{i+3,\digamma-2,\wp+1}\\
&- 170392\mathscr{A}_{i+3,\digamma-2,\wp+2} - 722\mathscr{A}_{i+3,\digamma-2,\wp+3} + 236\mathscr{A}_{i+3,\digamma-3,\wp-2}
\end{aligned}
$$

$$
\begin{aligned}
&+ 10543\mathscr{A}_{i+3,\digamma-1,\wp-3} + 2488148\mathscr{A}_{i+3,\digamma-1,\wp-2}\\
&+ 15234635\mathscr{A}_{i+3,\digamma-1,\wp-1} - 15234635\mathscr{A}_{i+3,\digamma-1,\wp+1}\\
&\quad 2488148\mathscr{A}_{i+3,\digamma-1,\wp+2} - 10543\mathscr{A}_{i+3,\digamma-1,\wp+3}\\
&+ 23548\mathscr{A}_{i+3,\digamma,\wp-3} + 5557328\mathscr{A}_{i+3,\digamma,\wp-2}\\
&+ 34026860\mathscr{A}_{i+3,\digamma,\wp-1} - 34026860\mathscr{A}_{i+3,\digamma,\wp+1}\\
&- 5557328\mathscr{A}_{i+3,\digamma,\wp+2} - 23548\mathscr{A}_{i+3,\digamma,\wp+3}\\
&+ 10543\mathscr{A}_{i+3,\digamma+1,\wp-3} + 2488148\mathscr{A}_{i+3,\digamma+1,\wp-2}\\
&+ 15234635\mathscr{A}_{i+3,\digamma+1,\wp-1} - 15234635\mathscr{A}_{i+3,\digamma+1,\wp+1}\\
&- 2488148\mathscr{A}_{i+3,\digamma+1,\wp+2} - 10543\mathscr{A}_{i+3,\digamma+1,\wp+3} + 722\mathscr{A}_{i+3,\digamma+2,\wp-3}\\
&+ 170392\mathscr{A}_{i+3,\digamma+2,\wp-2} + 1043290\mathscr{A}_{i+3,\digamma+2,\wp-1}\\
&- 1043290\mathscr{A}_{i+3,\digamma+2,\wp+1} - 170392\mathscr{A}_{i+3,\digamma+2,\wp+2}\\
&- 722\mathscr{A}_{i+3,\digamma+2,\wp+3} + \mathscr{A}_{i+3,\digamma+3,\wp-3} + 236\mathscr{A}_{i+3,\digamma+3,\wp-2}\\
&+ 1445\mathscr{A}_{i+3,\digamma+3,\wp-1} - 1445\mathscr{A}_{i+3,\digamma+3,\wp+1}\\
&- 236\mathscr{A}_{i+3,\digamma+3,\wp+2} - \mathscr{A}_{i+3,\digamma+3,\wp+3}\Big),\\
&\vdots .
\end{aligned}
\tag{A.1}
$$

Data Availability

Data sharing is not relevant to this topic because the current study did not produce any datasets for analysis and did not evaluate any datasets that were previously collected.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

The authors state that the research was carried out in conjunction with one another and with equal accountability. The final text was reviewed and approved by all of the authors.

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