

Research Article

A Competitive Bilevel Programming Model for Green, CLSCs in Light of Government Incentives

Arsalan Rahmani (),¹ Meysam Hosseini (),² and Amir Sahami ()³

¹Department of Mathematics, University of Kurdistan, Sanandaj, Iran ²Department of Mathematics, University of Kurdistan, Campus of Bijar, Sanandaj, Iran ³Department of Mathematics, Faculty of Basic Science, Ilam University, Ilam, Iran

Correspondence should be addressed to Arsalan Rahmani; a.rahmani@uok.ac.ir

Received 29 September 2023; Revised 26 January 2024; Accepted 9 February 2024; Published 23 February 2024

Academic Editor: Debdas Ghosh

Copyright © 2024 Arsalan Rahmani et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The growth of world population has fueled environmental, legal, and social concerns, making governments and companies attempt to mitigate the environmental and social implications stemming from supply chain operations. The state-run Environmental Protection Agency has initially offered financial incentives (subsidies) meant to encourage supply chain managers to use cleaner technologies in order to minimize pollution. In today's competitive markets, using green technologies remains vital. In the present project, we have examined a class of closed-loop supply chain competitive facility location-routing problems. According to the framework of the competition, one of the players, called the Leader, opens its facilities first. The second player, called the Follower, makes its decision when Leader's location is known. Afterwards, each customer chooses an open facility based on some preference huff rules before returning the benefits to one of the two companies. The follower, under the influence of the leader's decisions, performs the best reaction in order to obtain the maximum capture of the market. So, a bilevel mixed-integer linear programming model is formulated. The objective function at both levels includes market capture profit, fixed and operating costs, and financial incentives. A metaheuristic quantum binary particle swarm optimization (PSO) is developed via Benders decomposition algorithm to solve the proposed model. To evaluate the convergence rate and solution quality, the method is applied to some random test instances generated in the literature. The computational results indicate that the proposed method is capable of efficiently solving the model.

1. Introduction

In facility location problem—a well-known discrete optimization—a company decides on plants and facilities to be opened from a given set, open plants to be assigned to each open facilities, and open facilities to be allocated to each customer to maximize profit or market capture. For decades, logistic network design problems that consider facility locations and product flow have been extensively studied. Recently, given the increasing pressures from environmental and social requirements, many manufacturers have adopted second-hand products and their recovery activities in the production process. Consequently, the design of reverse logistics networks have caused concern pertaining to not only economic aspects but also how such networks can affect other aspects of human life, such as environment and the sustainability of natural resources. Implementing reverse logistics operations requires setting up additional appropriate logistics infrastructure for the arising flows of used and recovered products. Physical location, plants, facilities, and transportation links need to be chosen to transfer a new product from manufacturers to end customers and convey used reverse products from customers to manufacturers or suppliers for recovery or safe disposal [1].

Furthermore, global concerns about sustainable issues have grown rapidly, leading all governmental and nongovernmental environmental officials and activists into trying to encourage companies to take into account sustainability and environmental issues in their strategic decision-making [2]. Sustainable development is responsive to the needs of current generations without being able to meet future needs [3]. As stated by Zhang and Li, [4], Mondal and Giri [5], and Zhang et al. [6], governments use the idea of encouragement or pressure to motivate companies into aiming for lower emissions through utilizing innovative technologies when manufacturing products. Such state intervention could be carried out via a federal agency.

For instance, Temur and Bolat [7] refer to two common regulatory approaches considered by the US Environmental Protection Agency. The first, standardization in the technology used in production processes, is a general rule for all manufacturers. The second is an optional mode, with a performance-based mechanism in which the manufacturers can use the technology they want. Evidently, in addition to the two frameworks mentioned, the US Environmental Protection Agency has recently used subsidies or financial incentives for manufacturers to bring new technologies into action [1]. Indeed, the government is increasing its environmental concerns by providing such financial incentives. The idea, in other words, acts the opposite way that taxation does. Jung and Feng have investigated the government's subsidy design problem for using green technologies in an evolving industry and the subsidy's impact on the environment and social welfare [8]. These ideas are not unique to a specific country or continent. Thus, besides the United States, some Asian countries, such as South Korea, China, Taiwan, Philippines, and Thailand, subsidize using electric vehicles and solar panels in supply chains [9]. Aldieri and Vinci, in [10], have provided an overview of pollution control incentives for developing countries. Moreover, Jia et al. [11] have also elaborated on the importance and potential impacts of pollution control incentives on the supply chain. In Zare et al. [12] and Zare et al. [13], there is a detailed explanation of new technologies, including environmentally friendly technologies that governments can support companies in using.

The fundamentals of the present research are based upon the work carried out by Chalmardi and Camacho-Vallejo [1], and Mondal and Giri [5]. They assume that the government subsidizes suppliers, who use green technologies to manufacture products. Chalmardi and Camacho-Vallejo [1] propose a bilevel model with the first level representing government incentives, while the second represents the producer firms. End consumers are involved in the second level of the model. There are multiple significant differences [1, 5] in the present research. The first is how to consider government incentives. It is assumed that the government offers subsidies to both competing firms at two decision levels. Additionally, the type of supply chain is taken into account; a closed-loop-supply chain (CLSC) includes all the components involved. The third is the competition over obtaining market share by two rival companies, present in our study but not in previous works. Yet the third difference makes it possible to strike a balance among subsidies for clean technology and profits via customer attraction. Therefore, this paper examines a bilevel leader-follower model, which looks into government-offered financial incentives to the CLSC managers at both levels. The maximum objective capture should be used to incorporate location decisions into leader-follower decisions. Closed-Loop Supply Chain Management (CLSCM) motivates the actors involved to integrate cleaner technologies.

The key contributions are highlighted as follows:

- (i) Proposing a new mathematical competitive bilevel model based in green sustainable CLSCM.
- (ii) Incorporating government-offered financial incentives meant to encourage two competitive firms to use green technology in the proposed model.
- (iii) Presenting a hybrid model based on quantum binary PSO and Benders decomposition method
- (iv) Generalizing the example of new test examples based on similar cases in the subject literature to evaluating the algorithm

2. Literature Review

Based on the existing literature, most studies conducted in the field of supply chain management (SCM) have dealt with multiobjective models, single-level, and supply chains without considering all layers involved. Besides, works related to competitive bilevel programming models that include sustainability issues have received comparatively less attention. Three fundamental aspects should generally be considered in sustainable decisions: economy, environment, and community. That is to say that a sustainable company is one capable of doing business with the purposes of community, environmental, and economic well-being in its operations [14]. Growing environmental concerns and their consequent pressures lead to the development and implementation of sustainable approaches during corporate decision-making. Therefore, much of recent research focuses on reducing pollution caused by SCM; such reductions can be made possible by a greater tendency toward cleaner production and technologies. As a whole, in SCM issues, measures are taken based on the location, quantity, and capacity of facilities, while also taking into account the flow of material between locations. In supply chain issues, with the main approach of sustainability (SSCM), the objective is commonly to optimize long-term economic profitability, environmental performance, and social aspects [15]. Nowadays, in light of such aspects as environmental conditions, the management of the complete consumption cycle of a product is the responsibility of the primary service provider, meaning it is not possible to fully manage a supply chain from the point of view of environmental conditions without considering how to manage the products consumed. A CLSC manager is responsible for the production and product management both. Listes and Dekker [16] are pioneers in designing Closed-Loop Supply Chain Management, followed by too much effort. A mixed-integer (MIP) biobjective problem is proposed in [17]. In this model, the objective functions maximize the responsiveness of the proposed CLSCM network while minimizing the total costs. Furthermore, a memetic algorithm is applied to solve the model. Keyvanshokooh et al. [18] have introduced an MIP model for the multiechelon, multiperiod, and multicommodity SCM, in which a dynamic pricing approach is used to solve the model. In [19], a mixed-integer nonlinear programming (MINLP) model is proposed to optimize crucial strategic and tactical decisions in the CLSCM network. A nondeterministic biobjective model is developed in [20]. The interactive fuzzy solution method is used in the newly introduced model. The same model is addressed in [21, 22]. A comprehensive review of CLSCM has been done in [23], in which a robust model is proposed for tackling uncertainties in some cost data, supply parameters, or demand elements. They propose a more complex stochastic CLSCM for a multiechelon multiproduct model. Under such a scenario, the return rate is determined by a finite (limited) set of feasible possibilities, with the demands holding unpredictable values. The ability of the multiperiod, multiproduct, and multilevel supply chain model to handle the risks involved in a buildto-order strategy is confirmed by Keyvanshokooh et al. [24]. They have proposed a profit maximization MILP CLSCM model by considering flexibility in satisfying demand and collected returns based on a specific procedure. A hybrid uncertainty CLSCM is addressed in [25]. Soleimani and Kannan [26] have studied the deterministic multilevel multiproduct periodic CLSCM for large-scale problems. A hybrid method based on genetic algorithm (GA) and particle swarm optimization (PSO) has been applied to their proposed model. Yadegari et al. [27] have considered three transportation modes and propose an integrated logistics network model. The memetic algorithm is used to solve the presented model.

The green supply chain is a slightly novel concept in CLSCM. Talaei et al. [28] have defined a problem that looks into environmental issues, such as reduced carbon dioxide emission rates. A green sustainable location-routing inventory model in CLSCM is presented in [29]. In this model, economic, social, and environmental conditions as well as effects have been reviewed. A robust nonlinear programming model has been extended by [30] which works on price, advertisement, and uncertain demands, solved by a Lagrangian relaxation algorithm.

A review of the green reverse logistic and environmental aspects has been completed in the survey paper on reverse logistic models [31]. More environmental factors in a CLSCM have been focused upon in a model by Giri et al. [32]. A disruption risk using the transshipment strategy in CLSCM has been proposed in [33].

Asghari et al. [34] plan an operational and tactical framework to promote advertising programs in a direct-sales CLSCM taking into consideration different elasticity effects. In order to impact the profitability of manufacturers positively, they propose an optimization model for pricing similar products. Soleimani et al. have developed one of the most comprehensive mathematical models on the sustainability of CLSCM [35]. In theirs, which involves three choices of remanufacturing, recycling, and disposing of the returned items, the distribution facilities act as warehouses and collection centers.

The competition is an integral part of supply chain issues. Recently, the approach was partially included in the CLSCM literature. For instance, a competitive bilevel approach to CLSCM design model is presented by Rezapour et al. [36]. A nondeterministic competitive CLSCM is designed in [37]. It considers Stackelberg's competition between two supply chains based on their returns, profits, and demands. Such competition between manufacturers and retailers in the CLSCM was proposed in [38]. The purpose of this study is to examine the impact of manufacturer fairness concerns on CLSC decisions and profits in the green economy. Fathollahi-Fard et al. [39] address a static Stackelberg game between nurses, and patients within the HHCS framework by proposing a bilevel programming model [40] develops two BiLevel Stackelberg Models (BLSMs) under special conditions in the presence of strategic customers. Optimal production and order quantities and prices are determined by a sequential game at both levels. Fathollahi-Fard et al. [41] have developed a multilevel static Stackelberg game programming model to design the location allocation of the real case study of the CLSC. Inspired by the performed work, this paper aims to define a competitive, sustainable CLSC model. In addition to environmental and sustainability issues, the competition problem between two companies is covered. Indeed, when a company intends to produce, distribute, or provide a particular service in the market, it cannot ignore other companies with similar activities. There is, therefore, definite competition among these companies as different decision-makers. Such hierarchy distinguishes bilevel programming problems from those of biobjective. In competitive SCM problems, one of the main factors to yield a company's success in gaining market share is the correct location of centers. The basic duty of competitive location models is to find the location at which the captured market share is maximized under the existing conditions. A Stackelberg game leaderfollower model is considered, proposing that location decisions should be incorporated in leader and follower decisions by the maximum objective capture in a green CLSC. In general, in the leader-follower model, a new competing facility known as the follower is expected to join the market at some point in the future, trying to maximize its market share by establishing its facility in the location. The leader's location decision affects the competitor's location decision. On the other hand, it is expected that a future competitive entrance would impact the leader's choice of site in some way. The leader's goal is to choose a site that will maximize its market share across the time horizon, including the market share that will be captured when the rival enters the market [42]. Most

competitive issues lead to the appearance of bilevel or multilevel models. These problems are entirely different from single-level ones due mostly to structure and solution methods applied.

NP-hardness of bilevel programming problems (even in the "simplest case linear") has been established in [43] and was strongly proved in [44]. Thus, solving such problems is challenging, which is why many algorithms have been proposed in the literature. Some, including [45], have provided global optimization algorithms by relaxing the inner issue in a convex manner before representing the corresponding result using required and sufficient optimality criteria [46] which suggests a global optimization method that uses bilevel nonlinear optimization to determine the flexibility test. Multiparametric programming, which can transform bilevel problem into a family of single-level optimization problems, was proposed in [47]. Thus, the authors in [45] verify some new other methods; [48, 49] and [50] particularly consider algorithms for bilevel linear programming. The cuttingplane approach to approximate the lower-level feasible region has been proposed in [51]; hence, this algorithm is applicable when the upper-level problem includes continuous variables while the lower-level problem encompasses integer variables. On the other hand, linear programming duality was used for the opposite case [52], where the lower-level problem is linear, and the upperlevel problem is an integer. In [53], the authors have developed a basic enumeration scheme to identify feasible solutions in the discrete case. Mathieu, has in [54], proposed a two-level GA-based (GABBA) method to solve bilevel programming problems (BLPPs). The same GAbased method is proposed by Hejazi et al. [55], as the results are put together for comparison with those of Gendreau et al. [56]. A developed version of GA has been proposed by Oduguwa and Roy [57], an elitist algorithm created to encourage limited asymmetric collaboration between two players to solve different classes of BLPPs within a single framework.

An applicable Lagrangian-based algorithm [58] and the metaheuristic method using quantum binary particle swarm optimization- (QBPSO-) based method [59] were used to solve a particular class of bilevel problems. Considering the overall similarity of the model structure presented in [59] and the computational results of the PSO method in [59], the method can be implemented on P_1 . However, given the difficulty in solving the generated subproblems in the PSO method, Benders heuristic decomposition algorithm is efficiently used for some of the following problems. Therefore, one of the key goals of the present study is to demonstrate whether it is possible to modify and adapt the hybrid method of QBPSO and Benders decomposition to successfully solve the proposed BLPP.

Summarizing the findings of the study, data presented in Table 1 indicate that there is no mathematical model for a competitive CLSC when government incentives are in place. The remainder of the present paper is organized to cover the following: Section 3, where the proposed problem and its related mathematical model are described. Section 4 explains a hybrid method. Section 5 incorporates the introduction of the sample test problem and computational results.

3. Problem Definition

The considered CLSC to be designed needs five echelons, including suppliers, plants, facilities, customers, and recycling centers. The basic assumptions considered in the model formulation are as follows:

- (i) Unit material costs are the same for all suppliers in supply chain.
- (ii) Recycling centers and facilities buy returned products from customers with the different and variable price offered by the two competing companies.
- (iii) All customer demands must be satisfied, and all returned products must be purchased.
- (iv) A single period and single product supply chain is taken into account.

In this system (Figure 1), the priority of distribution and planning is as follows: a raw material is delivered from the supplier to the plant, a primary product from the plant to the facility, a primary product from the facility to the customer, a reverse product from the customer to the facility, a usable reverse product from the facility to the plant, a useless reverse product from the facility to the recycling center, and ultimately a renewed material from the recycling center to the supplier. All purchased raw materials are made in the factories, and all reverted goods are first sent back to the facilities for product recovery, where some are recovered and sent back to the factories as a main good. Other checked reverse products are then sent back from the facilities to the recycling centers, where they are renewed as useless raw materials. Afterwards, they are shipped back to the suppliers. Forward and recovered products, indicating that the renewed from the facilities are the same as the primary in terms of customer satisfaction, could meet the demand.

In supply chains, the environmental effect of all factors is measured against harmful greenhouse gas (GHG) emissions, such as CO_2 , CFCs, and NOx. However, the opened plants, facilities, and recycling centers have environmental effects, which are taken into account. In this regard, governments with their responsibilities for environmental protection, encourage supply chain managers to fight off ecological damage. Transporting products throughout the network, opening plants, facilities, and recycling centers, as well as manufacturing at plants and recycling at recycling centers, cause and contribute to these contaminants. Against such a backdrop, the present study looks into governmentgranted financial incentives to persuade firms into using cleaner technologies across plants and recycling centers. In order to promote sustainable networks, the governments

	Solution method		Hybrid method	Genetic-based algorithm	1	Simulated annealing	1	Memetic algorithm	ε -constraint approach	Hybrid metaheuristic	Lagrangean relaxation	Modified projection	Combining a heuristic and exact	Heuristic and exact	Karush-Kuhn-Tucker approach	Hybrid QBPSO and Benders decomposition
	Preference	rule	Ι			I	I							I	>	>
ture.	Competition	issue	I		I	I	>		I			>		I	>	>
1: A comprehensive review of the sustainable CLSC literature.	Government Competition Preference incentives issue rule					>	>							I	Ι	>
sustainable			I	I	>	I	Ι	Ι	I				>	I	>	Ι
review of the	Sustainability aspects	litilevel Economic Environmental Social	>	>	>	>	>	I	>	>	>	I	I	I	Ι	>
mprehensive	Sustai	Economic I			I	>	I	>		>	>	>	>	>	I	>
TABLE 1: A CO			I	>		Ι		>	>	>	I	>	I	>	Ι	>
TAI	Model type	Multiobjective Mu	~	>	>	I	I	>	>	>				I		Ι
	Model structure	Bilevel stackelberg	I			>	>					>	>		>	>
		WILP MINLP		I	Ι	Ι	>	Ι	>	I	>	>	>	Ι	>	I
	Model structur	MILP	>	>	>	>	>	>	I	I		I	I	>	>	>
	Reference		[2]	[3]	[13]	[1]	[5]	[15]	[28]	[27]	[28]	[36]	[39]	[41]	[40]	The present work

TABLE 1: A comprehensive review of the sustainable CLSC literature.

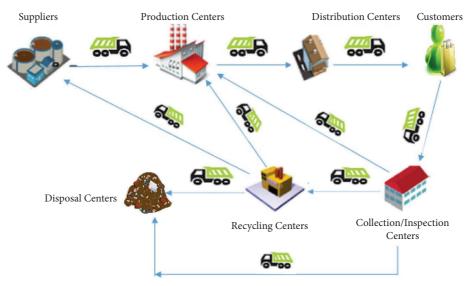


FIGURE 1: Closed-loop supply chain network.

make those offers to supply chain managers. As an incentive, there is a limited budget available. Within such a competitive market, all players get to have a say, including the leader company, the follower company, and the customers. It is convenient to summarize this decision-making into the following process: In the first stage, the leader company decides on the location of plants, facilities, and recycling centers with types of technologies (offers to protect the environment), and fixes the prices for the reverse product. In the second, taking the leader company's decision into account, the follower company decides on the location of the plants, facilities, and recycling centers (with technologies) to be opened for new and reverse products. Government incentives often lead both leaders and followers to act in the direction of reducing environmental pollution risks. Finally, in the third stage, each consumer enjoys the option to meet his or her demands using the goods of both businesses, purchasing new items and selling old ones to generate revenue for both companies. The decision-making problem for the leader company pertains to choices that yield the highest profit, with the knowledge that the follower company is in a push to maximize profit and attract customers. As a rule of thumb, it is assumed that when the leader company makes decision, it does have precise information on the types of facilities the follower company could offer, i.e., a detailed forecast is provided by the follower or for each decision from the leader which generated the best response. Moreover, it is assumed that both companies recognize the fact that the preferences of all customers are based on the distance, price, and the facilities' attractiveness (e.g., good services, spacious centers, restaurants, and accessible parking lots). Indeed, changes in those varying preferences could affect the companies' capture.

3.1. Attractiveness and Distance. At any given time, customers do prefer to walk into facilities with higher attractiveness and shorter distance.

3.2. Price. Customers or sellers usually prefer to sell their products to the highest bids. This preference, indeed, is related to the firms, rather than facilities; however, the previous preference has to do with the facilities.

Let $i, i' \in I$ be the index of potential facilities, with $j \in J$ standing as the index for demand points; d_{ij} is the distance from facility *i* to demand point *j*, σ^2 is a related distance function, qr_{ij}^L is the price at which the leader purchases the products from customers, and σ^3 is a related price function. Incorporating the attractiveness of facility to capture demand point, A_i is the attractiveness of potential facility *i*, σ^1 is a related attractiveness function of facility *i*, and $\rho_{ii}^L(\rho_{ii}^{\prime L})$ is the attractiveness of facility i for demand point j for new (reverse) products defined as $\rho_{ii}^L = (\sigma^1(A_i)/\sigma^2(d_{ii}))$ and $\rho'_{ij}^{L} = (\sigma^1(A_i) + \sigma^3(qr_{ij}^L)/\sigma^2(d_{ij}))$. The rule could then be affected by the following linear order notation. Indeed, the preferences are determined by a linear order \prec_i on *I*; relation $i' \prec_i i$ for any $i, i' \in I$ means that if two facilities of i and i' are opened, customer j prefers facility i. Moreover, for all $i, i' \in I$, relation $i' \prec i$ means that either $i' \prec_j i$ or i' = i. It is assumed that σ^1 and σ^3 are linear functions, while σ^2 is a quadratic function. All other symbols used are given in Appendix A. The mathematical model of the competitive green CLSC with government incentive is addressed in the following (refer to P_1):

$$P_{1}: Z = \max \sum_{j} h_{j}q_{j} \sum_{i} (1 - x_{ij}^{F}) - \sum_{i} \sum_{j} qr_{ij}^{L}x_{ij}^{'L} - \sum_{p} \sum_{t} cy_{pt}y_{pt}^{L}$$

$$-\sum_{i} cx_{i}x_{i}^{L} - \sum_{k} \sum_{t} cz_{kt}z_{pt}^{L} - \sum_{s} F_{s}e_{s}^{L}$$

$$-\sum_{s} \sum_{p} cu_{sp}u_{sp}^{L} - \sum_{p} \sum_{i} cv_{pi}v_{pi}^{L} - \sum_{i} \sum_{j} cx_{ij}q_{j}x_{ij}^{L} - \sum_{i} \sum_{k} cz_{ik}w_{ik}^{'L}$$

$$-\sum_{i} \sum_{p} cz_{ip}w_{ip}^{L} - \sum_{k} \sum_{s} cs_{ks}o_{ks}^{L} - \sum_{p} \sum_{t} cg_{pt}g_{pt}^{L}$$

$$-\sum_{i} \sum_{j} cr_{i}^{L}r_{i}x_{ij}^{'L} - \sum_{k} \sum_{r'} cg_{kt}'g_{kt'}^{'L} - \sum_{i} \sum_{j} pr_{ij}^{L}w_{ij}^{L} + \sum_{i} \sum_{p} p\gamma_{ip}^{L}w_{ip}^{L}$$

$$+\sum_{k} \sum_{s} p'\gamma_{ks}^{L}o_{ks}^{L} - \sum_{s} \sum_{p} cbu_{sp}u_{sp}^{L} - \sum_{s} \sum_{p} Ecu_{sp}u_{sp}^{L}$$

$$-\sum_{p} \sum_{i} Ecv_{pi}(v_{pi}^{L} + w_{ip}^{L}) - \sum_{i} \sum_{j} Ecx_{ij}q_{j}x_{ij}^{L} - \sum_{i} \sum_{k} Ecz_{ik}w_{ik}^{L}$$

$$-\sum_{k} \sum_{s} Ecs_{ks}\sigma_{ks}^{L} - \sum_{p} \sum_{t} Ecy_{pt}y_{pt}^{L} - \sum_{k} \sum_{t'} Ecz_{kt'}z_{kt'}^{L}$$

$$\sum_{s} u_{sp}^{L} + \sum_{i} w_{ip}^{L} = \sum_{t} g_{pt}^{L} \quad \forall p,$$
(2)

$$\sum_{t} g_{pt}^{L} = \sum_{i} v_{pi}^{L} \quad \forall p,$$
(3)

$$\sum_{p} v_{pi}^{L} \ge \sum_{j} x_{ij}^{L} q_{j} \quad \forall i,$$
(4)

$$\sum_{p} u_{sp}^{L} \le \operatorname{Cap}_{s} e_{s}^{L} \quad \forall s,$$
(5)

$$g_{\rm pt}^L \le {\rm Cap}_{\rm pt} y_{\rm pt}^L \quad \forall \, p, t, \tag{6}$$

$$\sum_{p} v_{pi}^{L} \le \operatorname{Cap}_{i} x_{i}^{L} \quad \forall i,$$
(7)

$$\sum_{j} r_{j} x_{ij}^{\prime} \leq \operatorname{Cap}_{i}^{\prime} x_{i}^{L} \quad \forall i,$$
(8)

$$\sum_{t} y_{pt}^{L} \le 1 \quad \forall p,$$
(9)

$$\sum_{t'} z_{kt'}^L \le 1 \quad \forall k, \tag{10}$$

$$x_{ij}^{L} \le x_{i}^{L} \quad \forall \, i, \, j, \tag{11}$$

$$x_i^L + \sum_{i < ji} x_{ij}^L \le 1 \quad \forall i, j,$$
(12)

$$\sum_{i \in I} x_{ij}^L = 1 \quad \forall j, \tag{13}$$

$$\mathbf{x}_{ij}^{'L} \le \mathbf{x}_i^L \quad \forall \, i, \, j, \tag{14}$$

$$x_{i}^{L} + \sum_{i' < ji} x_{i'j}^{'L} \le 1 \quad \forall i, j,$$
(15)

$$w_{ik}^{L} + w_{ip}^{L} = \sum_{j} q_{j}^{r} \mathbf{x}_{ij}^{'L} \quad \forall i, p, k,$$

$$(16)$$

$$w_{ip}^{L} = \sum_{j} \gamma r_{j} \mathbf{x}_{ij}^{'L} \quad \forall \, i, \, p,$$
(17)

$$w_{ik}^{'L} = \sum_{j} (1 - \gamma) r_j x_{ij}^{'L} \quad \forall i, k,$$
(18)

$$g_{kt}^{'L'} \leq \operatorname{Cap}_{kt} z_{kt}^{L}, \quad \forall k, t',$$
(19)

$$g_{kt'}^{'L} = \sum_{i} w_{ik}^{'L} \quad \forall k, t',$$
 (20)

$$\sum_{t'} g_{kt'}^{'L} = \sum_{s} o_{ks}^{L} \quad \forall k,$$
(21)

$$\sum_{i} x_{i}^{L} \le X_{\max}, \tag{22}$$

$$\sum_{p} \sum_{t} y_{pt}^{L} \le Y_{max},$$
(23)

$$\sum_{k} \sum_{t'} z_{kt'}^{L} \leq Z_{\max},$$
(24)

$$\sum_{p} \sum_{t} \lambda_{pt} y_{pt}^{L} + \sum_{k} \sum_{t'} \eta_{kt'} z_{kt'}^{L} \le B,$$
(25)

$$e_{s}^{L}, y_{pt}^{L}, x_{i}^{L}, x_{ij}^{L}, x_{ik}^{'L}, z_{kt}^{L}, \in \{0, 1\}, u_{sp}^{L}, g_{pt}^{L}, g_{kt}^{'L}, v_{pi}^{L}, qr_{ij}^{L}, o_{ks}^{L}, w_{ip}^{L}, w_{ik}^{'L} \ge 0.$$

$$(26)$$

Equation (1) presents the leader's objective function, with the first statement showing the leader's market capture amount. The second phrase demonstrates the cost of purchased returned products from the customers by facilities, while the third, fourth, and fifth terms represent fixed opening costs of plants, facilities, and recycling centers, respectively. The sixth one deals with long-term contract costs with suppliers. The transportation costs from suppliers to plants, plants to facilities, facilities to customers, and facilities to recycling centers, facilities to plants, and recycling centers to suppliers are exhibited in terms seven to twelve, respectively. Thirteen to sixteen are the manufacturing operating costs, as well as those of separating, recovering, and recycling. The seventeenth term shows the profits of returning the reverse products from facilities to plants, and the eighteenth pertains to the achieved profit by selling reverse products to suppliers from recycling centers. The nineteen concerns the costs of purchasing raw materials from suppliers by plants. The next seven summations measure the environmental effect linked to the shipment of material from suppliers to plants, plants to facilities (or facilities to plants), facilities to customers, facilities to recycling centers, and recycling centers to suppliers beside the environmental effect related to the manufacturing of the products in plants and recycling the reverse products in the recycling centers, respectively. The twenty-seventh and twenty-eighth phrases indicate the costs of contamination caused by manufacturing and remanufacturing of the products at plants and recycling centers, respectively. Finally, the last two terms represent the financial incentives obtained by the supply chain manager thanks to using the installed cleaner technologies at plants and recycling centers, respectively.

Constraint (2) ensures all raw materials shipped from suppliers, and reverse products from facilities to plants to be used during the manufacturing process. Constraint (3) guarantees the shipment of all manufactured products from plants to facilities. Constraint (4) shows that every single customer demand must be satisfied. Constraint (5) expresses the maximum limit of products purchased from suppliers by opened plants. Constraint (6) shows the limitation of the manufacturing capacity at an opened plant using a certain type of technology. Constraints (7) and (8) express the capacity limit of open facilities for products shipped from open plants and reverse products from customers, respectively. Constraints (9) and (10) guarantee a unique type of technology to be used in an opened plant and recycling center, respectively. Constraint (11) asserts that each customer can only select an installed facility. Inequality (12) means that only one open facility can be chosen based upon selection rule \prec_i on *I*. Constraints (13) stipulate that each customer is assigned to exactly one leader's facility at first. Constraints (14) and (15) are the same as constraints (10) and (11) for reverse products. Constraint (16) guarantees all purchased reverse products to be shipped from facilities to plants and recycling centers. Constraints (17) and (18) show the reverse shipped products from facilities to plants and recycling centers, respectively. The recycling capacity cap for a recycling facility that has been operational and is using a certain technology is shown in constraint (19). All reverse products must be transferred from facilities to recycling centers in order to be employed in the recycling process, according to constraint (20). Constraint (21) guarantees the shipment of all recycled products from recycling centers to suppliers. Constraints (22-24) state a maximum number of opened facilities, plants, and recycling centers, respectively. Constraint (25) is the budget at the government's disposal to provide financial incentives to the plants and recycling centers in exchange for their use of cleaner technologies. Constraint (26) indicates the type of variables linked to the follower's decision, which the followers' problem is stated as follows:

$$\max \sum_{j} h_{j}q_{j} \sum_{i} x_{ij}^{F} - \sum_{i} \sum_{j} qr_{ij}^{F} x_{ij}^{F} - \sum_{p} \sum_{t} cy_{pt}y_{pt}^{F}$$

$$- \sum_{i} cx_{i}x_{i}^{F} - \sum_{k} \sum_{t} cz_{kt}z_{pt}^{F} - \sum_{s} F_{s}e_{s}^{F}$$

$$- \sum_{s} \sum_{p} cu_{sp}u_{sp}^{F} - \sum_{p} \sum_{i} cv_{pi}v_{pi}^{F} - \sum_{i} \sum_{j} cx_{ij}q_{j}x_{ij}^{F} - \sum_{i} \sum_{k} cz_{ik}w_{ik}^{F}$$

$$- \sum_{i} \sum_{p} cz_{ip}w_{ip}^{F} - \sum_{k} \sum_{s} cs_{ks}o_{ks}^{F} - \sum_{p} \sum_{t} cg_{pt}g_{pt}^{F}$$

$$- \sum_{i} \sum_{j} cr_{i}^{T}r_{i}x_{ij}^{F} - \sum_{k} \sum_{t'} cg_{kt}'g_{kt'}^{F} - \sum_{i} \sum_{j} pr_{ij}^{L}w_{ij}^{F} + \sum_{i} \sum_{p} p\gamma_{ip}^{L}w_{ip}^{F}$$

$$+ \sum_{k} \sum_{s} p'\gamma_{ks}o_{ks}^{F} - \sum_{s} \sum_{p} cbu_{sp}u_{sp}^{F} - \sum_{s} \sum_{p} Ecu_{sp}u_{sp}^{F}$$

$$- \sum_{p} \sum_{i} Ecv_{pi}(v_{pi}^{F} + w_{ip}^{F}) - \sum_{i} \sum_{j} Ecx_{ij}q_{j}x_{ij}^{F} - \sum_{i} \sum_{k} Ecz_{ik}w_{ik}^{F}$$

$$- \sum_{k} \sum_{s} Ecs_{ks}o_{ks}^{F} - \sum_{p} \sum_{t} Ecy_{pt}y_{pt}^{F} - \sum_{k} \sum_{t'} k_{pt}Y_{pt}^{F} + \sum_{k} \sum_{t'} \eta_{kt'}z_{kt'}^{F},$$

$$\sum_{s} u_{sp}^{F} + \sum_{i} w_{ip}^{F} = \sum_{t} g_{pt}^{F} \quad \forall p,$$
(28)

$$\sum_{t} g_{pt}^{F} = \sum_{i} v_{pi}^{F} \quad \forall p,$$
(29)

$$\sum_{i} v_{pi}^{F} \ge \sum_{j} x_{ij}^{F} q_{j} \quad \forall i,$$
(30)

$$\sum_{p} u_{sp}^{F} \le \operatorname{Cap}_{s} e_{s}^{F} \quad \forall s,$$
(31)

$$g_{\rm pt}^F \le y_{\rm pt}^F {\rm Cap}_{\rm pt} \quad \forall \ p, t, \tag{32}$$

Journal of Mathematics

$$\sum_{p} v_{pi}^{F} \le \operatorname{Cap}_{i} x_{i}^{F} \quad \forall i,$$
(33)

$$\sum_{j} r_{j} x_{ij}^{'F} \leq \operatorname{Cap}_{i}' x_{i}^{F} \quad \forall i,$$
(34)

$$\sum_{t} y_{pt}^{L} + \sum_{t} y_{pt}^{F} \le 1 \quad \forall p,$$
(35)

$$\sum_{t'} z_{kt'}^{L} + \sum_{t'} z_{kt'}^{F} \le 1 \quad \forall k,$$
(36)

$$x_{ij}^F \le x_i^F \quad \forall \, i, \, j, \tag{37}$$

$$x_{i}^{L} + x_{i}^{F} + \sum_{i' < ji} x_{i'j}^{F} \le 1 \forall i, j$$
(38)

$$\sum_{i \in I} x_{ij}^F \le 1 \quad \forall j,$$
(39)

$$x_{ij}^{'F} \le x_i^F \quad \forall \, i, \, j, \tag{40}$$

$$x_{i}^{L} + x_{i}^{F} + \sum_{i' < ji} x_{i'j}^{'F} \le 1 \quad \forall i, j,$$
(41)

$$w_{ik}^{F} + w_{ip}^{F} = \sum_{j} q_{j}^{r} x_{ij}^{'F} \quad \forall i, p, k,$$
(42)

$$w_{ip}^{F} = \sum_{j} \gamma r_{j} x_{ij}^{'F} \quad \forall i, p,$$
(43)

$$x_{ik}^{'F} = \sum_{j} (1 - \gamma) r_j x_{ij}^{'F} \quad \forall i, k,$$
(44)

$$g_{kt'}^{'F} \leq z_{kt'}^{F} \operatorname{Cap}_{kt'} \quad \forall \, k, t',$$
(45)

$$g_{kt'}^{'F} = \sum_{i} w_{ik}^{'F}$$
(46)

$$\sum_{t'} g_{kt'}^{'F} = \sum_{s} o_{ks}^{F} \quad \forall k,$$
(47)

$$\sum_{i} x_i^F \le X'_{\max},\tag{48}$$

$$\sum_{p} \sum_{t} y_{pt}^{F} \le Y'_{max},\tag{49}$$

$$\sum_{k} \sum_{t'} z_{kt'}^F \le Z'_{\max},\tag{50}$$

$$\sum_{p} \sum_{t} \lambda_{pt} y_{pt}^{F} + \sum_{k} \sum_{t'} \eta_{kt'} z_{kt'}^{F} \le B^{F},$$
(51)

$$e_{s}^{F}, y_{pt}^{F}, x_{i}^{F}, x_{ij}^{F}, w_{ij}^{F}, z_{kt'}^{F}, \in \{0, 1\}, u_{sp}^{F}, g_{pt}^{F}, g_{kt'}^{F}, v_{pi}^{F}, q_{ij}^{F}, o_{ks}^{F}, w_{ip}^{F}, w_{ik}^{F} \ge 0.$$

$$(52)$$

The maximum total utility of the follower is stated as function (27). Constraints (28)–(52) can be considered in a fashion similar to the leader for the follower. Constraints (38)–(41) guarantee the selection of an open facility by a customer according to the given rule; moreover, it shows that if a facility is opened by the leader, it cannot be opened by the follower.

Evidently, the above bilevel model involves the following hierarchy:

$$P_{2}: \max_{x} Z_{1} = c^{1}x + d^{1}y$$

s.t. $A^{1}x + B^{1}y \le g^{1}$
 $x \in X$
 $\max_{y} c^{2}x + d^{2}y$
s.t. $A^{2}x + B^{2}y \le g^{2}$
 $y \in Y_{2}$
(53)

where x and y are the upper-level and lower-level variables, respectively, $c^1x + d^1y$, $c^2x + d^2y$ are upper-level and lowerlevel objective functions, and $A^1x + B^1y \le g^1$ and $A^2x + B^2y \le g^2$ are known as upper-level and lower-level constraints, respectively. In the section to follow, a hybrid metaheuristic algorithm is applied to solve the model.

4. Solution Method

To solve bilevel problems and their extensions, many algorithms have been discussed, including exact and heuristic algorithms. The metaheuristic hybrid algorithm presented here functions better than other ones in the literature. It can also be easily implemented by varying software. To properly implement the algorithm, the structure of model, and its features need to be clearly understood. This model is expected to be characterized by the following properties:

- (a) The constraints of the upper-level problem do not have any lower-level variables.
- (b) Any feasible solution to the upper-level problem leads to a feasible solution to the lower-level one.
- (c) The lower-level problem includes only the binary location variables of the upper-level problem.

Such features pave the way for the QBPSO method to be used, details of which will be discussed further.

QBPSO is comprehensively applied to solve IP problems. The key contribution of the present paper is the way it adapts the algorithm with the Benders decomposition algorithm to solve the efficiency of the bilevel problem in question. An overview of both methods is given below and how to implement it will be explained later.

4.1. Preliminaries. The particle swarm optimization (PSO) method is a population-based algorithm initially proposed by Kennedy and Eberhart [60], in which the social behavior of a flock of birds is simulated, as the birds (particles) show the candidate solutions to the problem, flying via the search space to find the optimal solution. In each iteration, the particles move to the desired level by following their best solutions before the best global position is discovered by each particle in the swarm.

Should *K* be assumed as the dimension of the search space, then for the *i*th particle, the current position and velocity vectors are represented as $X_i = (x_i^1, ..., x_i^j, ..., x_i^K)$ and $V_i = (v_i^1, ..., v_i^j, ..., v_i^K)$. Let *P* best_i = $(P \text{ best}_i^1, ..., P \text{ best}_i^J, ..., P \text{ best}_i^K)$ show the best position of the *i*th particle and *g* best = $(g \text{ best}^1, ..., g \text{ best}^J, ..., g \text{ best}^K)$ be the group's best position recorded so far. The following equations will demonstrate how to update the velocity and position respectively of *i*th particle at *t*th iteration to t + 1th iteration:

$$V_{i}(t+1) = wV_{i}(t) + c_{1}r_{1}((P \operatorname{best}_{i}(t)) - X_{i}(t) + c_{2}r_{2}(G\operatorname{best}(t)) - X_{i}(t))$$

$$X_{i}(t+1) = X_{i}(t) + V_{i}(t+1),$$
(54)

where w is the inertia weight and c_1 and c_2 are acceleration coefficients representing the degree of belief in the particle's own experience and the whole swarm experience, respectively. Then, r_1 and r_2 are random values within a range of [0, 1].

4.1.1. *Binary PSO (BPSO)*. Kennedy and Eberhart [60] have proposed the binary version of PSO (BPSO). In theirs, each particle is represented by a string of 0 and 1. Particle velocity is related to the probability with which a slight flip can occur.

Remarkably, the updating equation for velocity remains unchanged while the equation of updating position changes as shown in the following:

$$x_i^j(t) = \begin{cases} 1, & \text{if } u_i^j(t) > S(v_i^j(t)), \\ 0, & \text{otherwise,} \end{cases}$$
(55)

where $u_i^j(t)$ is a random number belonging to [0,1], and $S(v_i^j(t))$ is the velocity value sigmoid function $(1/(1 + \exp(-v_i^j(t))))$ that specifies the probability of j^{th} bit of the i^{th} particle, i.e., $x_i^j(t)$ being 0 or 1 at the t^{th} iteration.

4.1.2. Quantum Binary PSO (QBPSO). An example of a successful theory is QBPSO proposed by Mirhassani et al. as a "mechanism of biological evolution" and "simulation of things [59]. One of the major drawbacks of BPSO is being unintelligent $S(v_i^j)$, making it unable to lead the particles into the promising region of the search space. Furthermore, parameter selection is difficult. QBPSO, by contrast, overcomes such challenges. In QBPSO, instead of a sigmoid function, a population of quantum particle vector Q(t), which is based on the quantum bit, is used. Remarkably, the smallest unit that carries information is known as a quantum bit or "qubit," which can be either "1" or "0" or in any extraordinary position. Hence, the quantum particle population is expressed as $Q(t) = [Q_1(t), ..., Q_i(t), ..., Q_N(t)]$, where $Q_i(t) = [q_i^1(t), ..., q_i^j(t), ..., q_i^K(t)]$ and $0 \le q_i^j(t) \le 1, i = 1, 2, ..., N, j = 1, 2, ..., K$ represent the probability of the *j*th bit for the *i*th particle to take zero value at the *t*th iteration.

In QBPSO, a quantum particle vector $Q_i(t) = [q_i^1(t), ..., q_i^j(t), ..., q_i^K(t)]$ updates the position of particles by the following rule:

$$x_i^j(t) = \begin{cases} 1, & \text{if } u_i^j(t) > q_i^j(t), \\ 0, & \text{otherwise.} \end{cases}$$
(56)

Thus, the following rules are used to help update the quantum particle vector:

$$Q_{\text{groupbest}}(t) = \alpha * G \operatorname{best}(t) + \beta * (1 - G \operatorname{best}(t)),$$

$$Q_{\text{selfbest},i}(t) = \alpha * P \operatorname{best}_{i}(t) + \beta * (1 - P \operatorname{best}_{i}(t)),$$

$$Q_{i}(t+1) = w * Q_{i}(t) + c_{1} * Q_{\text{selfbest},i}(t) + c_{2} * Q_{\text{groupbest}}(t),$$
(57)

in which α, β are control parameters satisfied in relations $\alpha + \beta = 1, \ 0 \le \alpha, \beta \le 1$. Additionally, these parameters are tuned to control the degree of Q_i . w, c_1, c_2 are PSO parameters and satisfied in relations $w + c_1 + c_2 = 1, \ 0 \le w, c_1, c_2 \le 1$. The parameters represent the degree of belief in oneself, local maximum, and global maximum, respectively. The general framework of QBPSO for the proposed bilevel model is described as follows.

4.1.3. Benders Decomposition. To verify the Benders decomposition, consider the following problem:

$$\min Z = c_1 x + c_2 y$$

s.t. Ax + By = b (58)
 $x \ge 0, y \in Y.$

The above problem is decomposed in the following Benders subproblem (SP):

$$\min Z(x) = c_1 x$$

s.t. $Ax = b - B\tilde{y}$ (59)
 $x \ge 0$,

whose dual is

$$\max Z(u) = (b - B\tilde{y})u$$
s.t. $uA \le c_1$,
(60)

where *u* is the dual value corresponding to constraint $x \ge 0$. Based on the dual SP mentioned above, the Master Benders (MP) problem is

$$\min c_2 y + \alpha$$

s.t. $(b - By)u \le \alpha \quad \forall u \in P \subseteq \mho$
 $(b - By)u \le 0 \quad \forall u \in B \subseteq \mho$
 $y \in Y$, (61)

where P and B are subsets of extreme points and extreme rays of \mathcal{O} , respectively, and \mathcal{O} is defined by constraint $uA \le c_1$. Benders showed that this algorithm below finds an optimal solution after a finite number of steps, or proves that none exists Algorithm 1.

In the given algorithm, UB stands for the upper bound and LB stands for the lower bound. The constant ε represents the user-defined optimality gap.

4.1.4. Benders Decomposition with Pareto-Optimal. In this section, we will try to show how to create the strongest possible cut.

A cut $(b - By)u \le \alpha$ corresponding to $u_1 \subseteq U$, dominates that corresponding to $u_2 \subseteq U$, if

$$(b - By)u_2 \le (b - By)u_1 \quad \forall y \in Y.$$
(62)

We say that cut is Pareto-optimal cut if it is not dominated by any other cut. Also, the point u corresponding to this intersection is called Pareto-optimal.

Now, let ri(S) and S^c be the relative interior and the convex hull of the set $S \subseteq R^k$, then every point $y \in ri(Y^c)$ is called the principal or core point of *Y*. The following theorems help us to create a Pareto-optimal cut.

Theorem 1. Let $y_0 \in ri(Y^c)$ and \tilde{u} be the optimal solution of the dual SP, then the optimal solution u_0 of the following problem is Pareto-optimal.

$P \leftarrow B \leftarrow \emptyset, UB \leftarrow \infty.$
MP: Solve the relaxed MP.
if infeasible then return infeasible.
SP: Use \tilde{y} to solve the dual SP.
if dual SP unbounded
then $B \leftarrow B \cup \{\tilde{u}\}$, goto MP.
$\mathbf{P} \leftarrow \mathbf{P} \cup \{\widetilde{u}\}, LB \leftarrow c_2 \widetilde{y} + \alpha,$
$UB \leftarrow \min\{UB, c_2y + Z(\tilde{u})\}.$
if UB – LB > ε then goto MP.
return \tilde{x} (the dual of \tilde{u}) and \tilde{y} .

ALGORITHM 1: Benders algorithm.

$$\max Z(u) = (b - By_0)u$$

s.t. $uA \le c_1$ (63)
 $(b - B\tilde{y})u = Z(\tilde{u}).$

Proof. Conversely, suppose that u_0 is not Pareto-optimal; that is, there is a u_1 , such that $\forall y \in Y$: $(b - By)u_0 \le (b - By)u_1$. Now, let $\omega \in Y^c$, so there exist $y_1, y_2, ..., y_n \in Y, \lambda_1, \lambda_2, ..., \lambda_n$ with $0 \le \lambda_i \le 1$ such that $\omega = \sum_{i=1}^n \lambda_i y_i$, so for every $\omega \in Y^c$, $(b - B\omega)u_0 \le (b - B\omega)u_1$.

And so, $y = \tilde{y}$ that u_1 must be the optimal solution of the Z(u), i.e., $u_1 \subseteq \mho(\tilde{y})$. It implies that $(b - By_0)u_1 = (b - By_0)u_0$, and since u_1 dominates u_0 , there is at least one $\tilde{y} \in Y$ such that $(b - B\tilde{y})u_0 < (b - B\tilde{y})u_1$. On the other hand, $y_0 \in ri(Y^c)$ and this means that there exists a scalar $\theta > 0$, in which $\omega = \theta y_0 + (1 - \theta) \tilde{y}$ is belong to the Y^c. Then, by multiplying θ the equation $(b - By_0)u_1 = (b - By_0)u_0$ and multiplying $(b - B\tilde{y})u_0 < (b - B\tilde{y})u_1$ by $(1 - \theta)$ and adding them gives $(b - B\omega)u_0 > (b - B\omega)u_1$. However, this inequality contradicts the hypothesis and shows that our assumption that u_0 is not Pareto-optimal was wrong. This completes the argument. П

Theorem 2. With the same hypothesis as in of Theorem 1, the following problem is also Pareto-optimal.

Proof. The proof is almost identical to Theorem 1 proof.

$$\max Z(u) = (b - By_0)u$$

s.t. $uA \le c_1$. (64)

The above theorems help the Benders master problem make progress towards the best solution from the first iteration. This is shown by the following steps Algorithm 2: \Box

4.2. QBPSO for Solving the Proposed BLPP. The QBPSO algorithm [61] is adopted to solve mixed-integer linear BLPPs, where calculating upper and lower levels iteratively makes it possible to get closer to the optimal solution step by step. Representation of each solution is done through location variables, and other variables can be determined from these binary variables by solving some of the following subproblems. Therefore, *X* and *Y* are used for the upper and lower level location variables, respectively (Algorithm 3).

The pseudocode of the algorithm is given by Figure 2 in Appendix B. Based on properties (a) and (b), the follower's optimal solution always exists for each fixed X_r . Furthermore, feasible solutions are generated for the leader in the initialization phase, a feature which is preserved during the algorithm implementation.

4.3. Benders Decomposition Phase. What remains is how to solve the lower-level problem, which is an NP-hard problem warranting an efficient solution. By examining the follower model, it is found that this submodel is a suitable case for the Benders decomposition algorithm to be used. The model does generally have a set of binary variables (i.e., $e_s^F, y_{\rm pt}^F, x_i^F, x_{ij}^F, x_{ij}^{F}, z_{kt'}^F)$, and when these variables are assumed constant, a simple linear model is obtained. Based on the follower's objective function and the type of constraints, the x_i^F variable can be considered continuous $0 \le x_i^F \le 1$. Hence, the variables e_s^F , y_{pt}^F , x_{ij}^F , $x_{ij}^{'F}$, $z_{kt}^{'}$ are binary, while the variables $x_i^F, u_{sp}^F, g_{pt}^F, g_{kt}^{'F}, v_{pi}^F, qr_{ij}^F, \sigma_{ks}^F, w_{ip}^F, w_{ik}^{'F}$ are continuous. Therefore, in the decomposition algorithm, one subproblem contains continuous variables and constant values of binary ones (27, 28, 29, 30, 31, 32, 33, 36, 37, 39, 40, 41, 42, 43, 44, 45, and 46); the other subproblem, referred to as the master problem, consists of binary, and constraint variables (34, 35, 38, 47, 48, and 49). An optimization constraint at each iteration of the Benders algorithm is obviously added to each master problem.

$P \leftarrow B \leftarrow \emptyset, UB \leftarrow \infty.$
Pareto: Find a point y_0 .
Use y_0 to solve the independent problem of theorem 2.
$P \leftarrow P \cup \{\tilde{u}\}$
MP: Solve the relaxed MP.
if infeasible then return infeasible.
SP: Use \tilde{y} to solve the dual SP.
if dual SP unbounded
then $B \leftarrow B \cup \{\tilde{u}\}$, goto Pareto.
$P \leftarrow P \cup \{\widetilde{u}\}, LB \leftarrow c_2 \widetilde{y} + \alpha,$
$UB \leftarrow \min \{UB, c_2 y + Z(\tilde{u})\}.$
if $UB - LB > \varepsilon$ then goto Pareto.
return \tilde{x} (the dual of \tilde{u}) and \tilde{y} .

ALGORITHM 2: Benders algorithm with Pareto-optimal cut.

Step1: (Initialization)

1.1. Set the value parameters of the QBPSO algorithm and population sizes N, Q_i .

1.2. By equation $x_i^j(t)$, randomly generate the initialized value of the leader's decision variables $X_r = (e_s^{\text{Lr}}, y_{\text{pt}}^{\text{Lr}}, z_i^{\text{Lr}}, z_{\text{tr}}^{\text{Lr}})$ for any

particle; next, in the lower level, the follower's problem is solved by fixing X_r . Let $Y_r^* = (e_s^{\text{Fr}}, y_{\text{pt}}^{\text{Fr}}, x_i^{\text{Fr}}, z_{\text{kt}}^{\text{Fr}})^*$ (also $(x_{ij}^{\text{Fr}}, x_{ij}^{\text{Fr}}, g_{\text{pt}}^{\text{Fr}}, g_{\text{pt}}^{\text{Fr}}, g_{\text{pt}}^{\text{Fr}}, g_{\text{kt}}^{\text{Fr}})^*$) be the optimal solution to the follower problem. Then, the leader problem is solved by fixing X_r and optimal values of the lower level (regarding X_r) in the upper-level problem, while the fitness value of the current position of the r^{th} particle (X_r, Y_r^*) is calculated using $F(X_r, Y_r^*)$, where F is the leader's Let $Y_r^* = (e_s^{\text{Fr}}, y_p^{\text{Fr}}, z_{\text{kt}}^{\text{Fr}}, z_{\text{kt}}^{\text{Fr}})^*$ (also $(X_r^{\text{Fr}}, x_{ij}^{\text{Fr}}, z_{\text{kt}}^{\text{Fr}}, y_{pl}^{\text{Fr}}, z_{kt}^{\text{Fr}})^*$) be the optimal solution to the follower problem. Then, the leader problem is solved by fixing X_r and optimal values of the lower level (regarding $X_r)$ in the upper-level problem, while the fitness value of the current position of the r^{th} particle (X_r, Y_r^*) is calculated using $F(X_r, Y_r^*)$, where F is the leader's Let $Y_r^* = (e_s^{\text{Fr}}, y_r^{\text{Fr}}, z_{\text{kt}})^*$. objective function. Consider the personal best position of the r^{th} particle $P_{best_r} = (X_{best_r}, Y_{best_r})$ equal to (X_r, Y_r^*) and set the global best position Gbest = (Xbest, Ybest).

Step 2: (Updating X_r) Change Q_r and update X_r according to the QBPSO algorithm.

Step 3: (Obtaining the follower's reaction and determining the particle's fitness) by fixing new X_r in the lower-level problem, obtain the follower's optimal solution Y_r^* . Afterwards, in the upper-level problem, the position and fitness value of $F(X_r, Y_r^*)$ is calculated by fixing (X_r, Y_r^*) .

Step 4: (Updating the personal best position) If the fitness value of (X_r, Y_r^*) is better than that of $Pbest_i = (Xbest_i, Ybest_i)$, update Pbest_i.

Step 5: (Updating the global best position) after comparing the fitness value of Gbest with that of all personal best positions, Gbest is updated with the global best position Gbest = (Xgbest, Ygbest).

Step 6: (Termination) Go to step 2 until the stopping criterion is met.

ALGORITHM 3: Adopted QBPSO algorithm.

Pseudo-code of OBPSO. Input: Set, Maxiter, MaxCPU, (initial value of the QBPSO parameters) Output: the best solution 1: Initialize the binary leader's decision variables population X_{i} (i = 1, 2...N) regarding to related equation $x_i^j(t)$. 2: Solve the follower's problem solved to obtain Y_r^* (with the fixed value of X_r via benders decomposition) 3. Solve the leader's problem by fixing X_r and Y_r^* in the upper level to obtain $F(X_r, Y_r^*)$. 3. Calculating the personal best position of the r^{th} particle $Pbest_r = (Xbest_r, Ybest_r)$ equal to (X_{r}, Y_{r}^{*}) 4. Calculating the best position Gbest = (Xbest, Ybest). 5. While $\tau \leq Maxiter$ and CPUTime $\leq MaxCPU$, do 6. (Updating X_r) Change Q_r and update X_r according to the QBPSO algorithm. 7. (Obtaining the follower's reaction Y_{*}^{*}) Solve the follower problem with the fixed value of new X_r by benders decomposition algorithm. 8. (Determining the particle's fitness $F(X_r, Y_r^*)$) Solve the leader problem with the fixed values of X_r and Y_r^* . 9. (Updating the personal best position) if $F(X_r, Y_r^*) > F(Xbest_r, Ybest_r)$ then 10. $F(X_r, Y_r^*) = F(Xbest_r, Ybest_r)$ and $(X_r, Y_r^*) = (Xbest_r, Ybest_r)$ 11. End if 12. Updating the global best position *Gbest* = (*Xgbest*, *Ygbest*) 13: end while

14. Return the best value

FIGURE 2: Pseudocode of the QBPSO algorithm.

4.3.1. Benders Subproblem. For a given variable $(\hat{e}_s^F, \hat{y}_{pt}^F, \hat{x}_{ij}^F, \hat{z}_{kt}^F)$, the Benders subproblem SP(1) is written as follows:

$$SP(1): \min \sum_{i} cx_{i}x_{i}^{F} + \sum_{s} \sum_{p} cu_{sp}u_{sp}^{F} + \sum_{p} \sum_{i} cv_{pi}v_{pi}^{F} + \sum_{i} \sum_{k} cz_{ik}w_{ik}^{F} + \sum_{i} \sum_{p} cz_{ip}w_{ip}^{F} + \sum_{k} \sum_{s} cs_{ks}o_{ks}^{F}$$

$$+ \sum_{p} \sum_{t} cg_{pt}g_{pt}^{F} + \sum_{k} \sum_{t'} cg_{kt'}g'_{kt'}^{F} + \sum_{i} \sum_{j} pr_{ij}^{L}w_{ij}^{F} - \sum_{i} \sum_{p} p\gamma_{ip}^{L}w_{ip}^{F} - \sum_{k} \sum_{s} p'\gamma_{ks}o_{ks}^{F} + \sum_{s} \sum_{p} cbu_{sp}u_{sp}^{F}$$

$$+ \sum_{s} \sum_{p} Ecu_{sp}u_{sp}^{F} + \sum_{p} \sum_{i} Ecv_{pi}(v_{pi}^{F} + w_{ip}^{F}) + \sum_{i} \sum_{k} Ecz_{ik}w_{ik}^{F} + \sum_{k} \sum_{s} Ecs_{ks}o_{ks}^{F} + \sum_{k} \sum_{t'} Ecz_{kt'}w_{kt'}^{F},$$

$$\sum_{s} u_{sp}^{F} + \sum_{i} w_{ip}^{F} = \sum_{t} g_{pt}^{F} \forall p,$$

$$\sum_{t} g_{pt}^{F} = \sum_{i} v_{pi}^{F} \quad \forall p,$$

$$\begin{split} \sum_{i} v_{pi}^{F} &\geq \sum_{j} \widehat{x}_{ij}^{F} q_{j} \quad \forall i, \\ &\sum_{p} u_{sp}^{F} \leq \operatorname{Cap}_{s} \widehat{e}_{s}^{F} \quad \forall s, \\ &g_{pi}^{F} \leq \widehat{y}_{pi}^{F} \operatorname{Cap}_{pi} \quad \forall p, t, \\ &\sum_{p} v_{pi}^{F} \leq \operatorname{Cap}_{i} x_{i}^{F} \quad \forall i, \\ &\sum_{j} r_{j} \widehat{x}_{ij}^{F} \leq \operatorname{Cap}_{i} x_{i}^{F} \quad \forall i, \\ &\widehat{x}_{ij}^{F} \leq x_{i}^{F} \quad \forall i, j, \\ &x_{i}^{L} + x_{i}^{F} + \sum_{i \leq j} \widehat{x}_{ij}^{F} \leq 1 \quad \forall i, j, \\ &x_{i}^{L} + x_{i}^{F} + \sum_{i \leq j} \widehat{x}_{ij}^{F} \leq 1 \quad \forall i, j, \\ &w_{ip}^{F} = \sum_{j} \gamma r_{j} \widehat{x}_{ij}^{F} \quad \forall i, p, \\ &w_{ip}^{F} = \sum_{j} (1 - \gamma) r_{j} \widehat{x}_{ij}^{F} \quad \forall i, k, \\ &g_{kt}^{F} \leq \widehat{z}_{kt}^{F} \operatorname{Cap}_{kt} \quad \forall k, t', \\ &g_{kt}^{F} = \sum_{i} w_{ik}^{F} \quad \forall k, t', \\ &\sum_{i} g_{kt}^{F} = \sum_{i} o_{ks}^{F} \forall k, t', \\ &\sum_{i} x_{i}^{F} \leq x_{max}, \\ &x_{i}^{F} \in [0, 1], u_{sp}^{F}, g_{pt}^{F}, g_{kt}^{F}, v_{pi}^{F}, g_{ij}^{F}, w_{ij}^{F}, w_{ij}^{F} \geq 0. \end{split}$$

$$\tag{65}$$

Given the structure of SP(1), the model certainly serves as a feasible solution (since SP(1) is a minimization problem, and the included variables are bounded from below). The SP(1) subproblem can be solved and broken down into a linear subproblem based on two sets of variables $x_i^F, u_{sp}^F, g_{pt}^F, v_{pi}^F, w_{ip}^F$ and $g_{kt}^{'F}, o_{ks}^F, w_{ik}^{'F}$. Assuming that $\theta^1, ..., \theta^{17}$ are dual variables of constraints 27, 28, 29, 30, 31, 32, 33, 36, 37, 39, 40, 41, 42, 43, 44, 45, and 46, respectively, then the following model is the dual of the Bands subproblem shown in SPD ($\theta \mid \hat{e}, \hat{y}, \hat{x}, \hat{x'}, \hat{z}$):

$$SPD\left(\theta \mid \hat{e}, \hat{y}, \hat{x}, \hat{x'}, \hat{z}\right)$$

$$\max \sum_{i} \theta_{i}^{3} \left(\sum_{j} \hat{x}_{ij}^{F} q_{j}\right) + \sum_{s} \theta_{s}^{4} \left(Cap_{s}\hat{e}_{s}^{F}\right) + \sum_{p} \sum_{t} \theta_{p,t}^{5} \left(\hat{y}_{pt}^{F}Cap_{pt}\right) + \sum_{i} \theta_{i}^{7} \left(\sum_{j} r_{j}\hat{x}_{ij}^{F}\right)$$

$$+ \sum_{i} \sum_{j} \theta_{ij}^{8} \left(\hat{x}_{ij}^{F}\right) + \sum_{i} \sum_{j} \theta_{ij}^{9} \left(1 - \sum_{i' < j^{i}} \hat{x}_{i'j}^{F}\right) + \sum_{i} \sum_{j} \theta_{ij}^{10} \left(\hat{x}_{ij}^{'F}\right) + \sum_{i} \sum_{j} \theta_{ij}^{11} \left(1 - \sum_{i' < j^{i}} \hat{x}_{i'j}^{F}\right)$$

$$+ \sum_{i} \sum_{p} \theta_{ip}^{12} \left(\sum_{j} \gamma r_{j} \hat{x}_{ij}^{'F}\right) + \sum_{i} \sum_{k} \theta_{ik}^{13} \left(\sum_{j} (1 - \gamma) r_{j} \hat{x}_{ij}^{'F}\right) + \sum_{k} \sum_{t'} \theta_{kt'}^{14} \left(\hat{z}_{kt'}^{F}Cap_{kt'}\right) + \theta^{17} X'_{max}$$

$$s.t. \left(\theta^{1}, ..., \theta^{17}\right) \in \Delta,$$

$$(69)$$

in which Δ set is a polyhedron created from the solution space of the dual problem. The solution to problem SPD $(\theta | \hat{e}, \hat{y}, \hat{x}, \hat{x}', \hat{z})$ can be used to generate the feasible cut constraint (which is the optimal cut) in the Benders master problem. 4.3.2. Benders Master Subproblem. D(SPD) set can be viewed as the extreme point of the dual problem $SPD(\theta^1, ..., \theta^{17} | \hat{e}, \hat{y}, \hat{x}, \hat{x'}, \hat{z})$. The Benders master problem (MP) could, as a result, be written in the following form:

$$\begin{aligned}
\text{MP: max} \quad \sum_{j} h_{j}q_{j}\sum_{i} x_{ij}^{F} - \sum_{p}\sum_{t} c\mathbf{y}_{p}\mathbf{y}_{pt}^{F} - \sum_{k}\sum_{t} c\mathbf{z}_{kt'}\mathbf{z}_{pt'}^{F} - \sum_{s}F_{s}\mathbf{e}_{s}^{F} - \sum_{i}\sum_{j} c\mathbf{x}_{ij}q_{j}\mathbf{x}_{ij}^{F} \\
- \sum_{i}\sum_{j} c\mathbf{r}_{i}^{L}\mathbf{r}_{i}\mathbf{x}_{ij}^{T} - \sum_{i}\sum_{j} Ec\mathbf{x}_{ij}q_{j}\mathbf{x}_{ij}^{F} - \sum_{p}\sum_{t} Ec\mathbf{y}_{p}\mathbf{y}_{pt}^{F} + \sum_{p}\sum_{t}\lambda_{pt}\mathbf{y}_{pt}^{F} + \sum_{k}\sum_{i}\eta_{ki'}\mathbf{z}_{ki'}^{F} - \pi, \\
\sum_{i}y_{pt}^{L} + \sum_{i}y_{pt}^{F} \leq 1 \quad \forall p, \\
\sum_{i}z_{ki'}^{L} + \sum_{t}z_{ki'}^{F} \leq 1 \quad \forall k, \\
\sum_{i\in I}x_{ij}^{F} \leq 1 \quad \forall j, \\
\sum_{p}\sum_{t}y_{pt}^{F} \leq Y_{max}', \\
\sum_{k}\sum_{i'}x_{i}^{F} \leq Z_{max}', \\
\sum_{p}\sum_{t}\lambda_{pt}\mathbf{y}_{pt}^{F} + \sum_{k}\sum_{i'}\eta_{ki'}\mathbf{z}_{ki'}^{E} \leq B^{F}, \\
\sum_{p}\theta_{i}^{3}\left(\sum_{j}\tilde{x}_{ij}^{F}q_{j}\right) + \sum_{s}\theta_{s}^{4}(Cap_{s}\hat{e}_{s}^{F}) + \sum_{p}\sum_{t}\theta_{j}^{5}(\tilde{y}_{pt}^{F}Cap_{pt}) + \sum_{i}\theta_{i}^{2}\left(\sum_{j}r_{j}\tilde{x}_{ij}^{F}\right) + \sum_{i}\sum_{j}\theta_{ij}^{4}(\tilde{x}_{ij}^{F}) \\
+ \sum_{i}\sum_{j}\theta_{ij}^{4}\left(1 - \sum_{i$$

The master problem is now transformed into a problem with binary variables e_s^F , y_{pt}^F , x_{ij}^F , z_{kt}^F , z_{kt}^F , and a continuous variable π . Using double values obtained from the Benders subproblem, a generated cut (51) is incorporated into the master problem in each iteration. As the Benders subproblem is always bounded and feasible, the dual subproblem is similarly feasible and bounded as well based on the strong dual theorem. Therefore, the added constraint in each iteration is an optimal cut constraint and merely eliminates the nonoptimal ones. In order to examine the constraints added to the MP model based on extreme points D (SPD), the branch-and-cut idea is proposed. The MP problem is solved before solving the subproblem to reach the optimal one in the classical implementation of the Benders

method, meaning that in the MP problem, similar solutions are unnecessarily remet. Therefore, the problem is solved in a branch and boundary tree as [62]. In each potential solution of the search tree, the subproblem is solved, and the optimal generated cut is added if needed.

4.3.3. Pareto-Optimal Cut Generation. Reducing the number of iterations is of significant value in speeding up the implementation of the Benders method. In this section, a cutting Pareto-optimum solution method, introduced by Magnanti and Wang [63], is presented. Let Z be a set of feasible solutions to the master problem, MP, and opt (P) be the optimal solution to a problem P, then define the function $C(e_s^F, y_{pt}^F, x_{ij}^F, z_{kt}^F, \theta^1, ..., \theta^{17})$ as follows:

$$C\left(e_{s}^{F}, y_{pt}^{F}, x_{ij}^{F}, z_{kt'}^{F}, \theta^{1}, ..., \theta^{17}\right) = \sum_{i} \theta_{i}^{3} \left(\sum_{j} x_{ij}^{F} q_{j}\right) + \sum_{s} \theta_{s}^{4} (\operatorname{Cap}_{s} e_{s}^{F}) + \sum_{p} \sum_{t} \theta_{p,t}^{5} (y_{pt}^{F} \operatorname{Cap}_{pt}) + \sum_{t} \sum_{j} \theta_{ij}^{10} \left(\sum_{j} r_{j} x_{ij}^{F}\right) + \sum_{i} \sum_{j} \theta_{ij}^{8} (x_{ij}^{F}) + \sum_{i} \sum_{j} \theta_{ij}^{9} \left(1 - \sum_{i' < j} x_{i'j}^{F}\right) + \sum_{i} \sum_{j} \theta_{ij}^{10} \left(x_{ij}^{'F}\right) + \sum_{i} \sum_{j} \theta_{ij}^{11} \left(1 - \sum_{i' < j} x_{i'j}^{'F}\right) + \sum_{i} \sum_{p} \theta_{ip}^{12} \left(\sum_{j} \gamma r_{j} x_{ij}^{'F}\right) + \sum_{i} \sum_{k} \theta_{ik}^{13} \left(\sum_{j} (1 - \gamma) r_{j} x_{ij}^{'F}\right) + \sum_{k} \sum_{t'} \theta_{kt'}^{14} (z_{kt'}^{F} \operatorname{Cap}_{kt'}) + \theta^{17} X'_{max}.$$

$$(71)$$

Magnanti and Wang introduced the concept of dominant cut within the Benders method. The cut generated by the dual solution $(\hat{\theta}^1, ..., \hat{\theta}^{17})$ is said to be dominant over the cut generated by the dual solution $(\tilde{\theta}^1, ..., \tilde{\theta}^{17})$ if and only if

$$C\left(e_{s}^{F}, y_{pt}^{F}, x_{ij}^{F}, x_{ij}^{F}, z_{kt'}^{F}, \tilde{\theta}^{1}, ..., \tilde{\theta}^{17}\right) \leq C\left(e_{s}^{F}, y_{pt}^{F}, x_{ij}^{F}, z_{kt'}^{F}, \tilde{\theta}^{1}, ..., \tilde{\theta}^{17}\right) \forall \left(e_{s}^{F}, y_{pt}^{F}, x_{ij}^{F}, z_{kt'}^{F}\right) \in \mathbb{Z}.$$
(72)

Furthermore, inequality, at least at one point, is strict. This is called an optimal-Pareto cut if no more dominant cut is available. Additionally, inequality, at least at one point, is strict. This is called an optimal-Pareto cut if no more dominant cut is available. To obtain the inferred optimal-Pareto cut, solving the following scheduling problem is needed:

$$\begin{aligned} \mathsf{MW}\left(\theta^{1},...,\theta^{17} \mid \widehat{e}_{s}^{F}, \widehat{y}_{\mathsf{pt}}^{F}, \widehat{x}_{\mathsf{ij}}^{F}, \widehat{z}_{\mathsf{kt}'}^{F}, \overline{e}_{s}^{F}, \overline{y}_{\mathsf{pt}}^{F}, \overline{x}_{\mathsf{ij}}^{F}, \overline{z}_{\mathsf{kt}'}^{F}\right) & \text{max} \quad C\left(\overline{e}_{s}^{F}, \overline{y}_{\mathsf{pt}}^{F}, \overline{x}_{\mathsf{ij}}^{F}, \overline{z}_{\mathsf{kt}'}^{F}, \theta^{1}, ..., \theta^{17}\right) \\ & \text{s.t.} \left(\theta^{1},...,\theta^{17}\right) \in \Delta \\ C\left(\widehat{e}_{s}^{F}, \widehat{y}_{\mathsf{pt}}^{F}, \widehat{x}_{\mathsf{ij}}^{F}, \widehat{z}_{\mathsf{kt}'}^{F}, \theta^{1}, ..., \theta^{17}\right) & = \operatorname{opt}\left(\operatorname{SPD}\left(\theta^{1},...,\theta^{17} \mid \widehat{e}_{s}^{F}, \widehat{y}_{\mathsf{pt}}^{F}, \widehat{x}_{\mathsf{ij}}^{F}, \widehat{z}_{\mathsf{kt}'}^{F}\right)\right), \end{aligned}$$
(73)

where the polyhedron Δ is generated from the solution space of dual problem SPD; also, $(\overline{e}_s^F, \overline{y}_{pt}^F, \overline{x}_{ij}^F, \overline{x}_{ij}^F, \overline{z}_{kt}^F) \in \operatorname{ri}(Z^c)$ is an interior point of the convex hull of Z, named a central point.

For the first value of the central point, one can employ a heuristic method, leading in turn to a feasible solution. Besides, for the iterations to come, the mean of the certain current point and the active solution to the MP problem is used. Finally, the following steps are taken to perform the Benders technique:

Step 1: Consider the start point $(\hat{e}_s^F, \hat{y}_{pt}^F, \hat{x}_{ij}^F, \hat{z}_{kt'}^{'F}, \hat{z}_{kt'}^{F})$ and interior point $(\overline{e}_s^F, \overline{y}_{pt}^F, \overline{x}_{ij}^F, \overline{x}_{kt'}^{'F}, \overline{z}_{kt'}^{F}) \in ri(Z^c)$ for the beginning.

Step 2: Solve the MW model based on $(\hat{e}_s^F, \hat{y}_{pt}^F, \hat{x}_{ij}^F, \hat{z}_{kt'}^F, \hat{z}_{kt'}^F)$ and $(\bar{e}_s^F, \bar{y}_{pt}^F, \bar{x}_{ij}^F, \bar{z}_{kt'}^F, \bar{z}_{kt'}^F) \in ri(Z^c)$, and generate the optimal-Pareto cut.

Step 3: Add the Pareto cut produced in step 2 to the MW, before solving it with the newly added cut.

Step 4: If the difference in the solution of the main Benders model in two consecutive iterations (current and previous) is below a given value ε or the number of iterations is equal to or greater than a defined boundary, then the algorithm ends. Otherwise, go to the second stage.

Step 5: The last optimal solution of the subproblem determines the value of continuous variables, and it is the last solution to the MW which determines the value of integer variables.

The flowchart (Figure 3) of the proposed method is given below. Also, the pseudocode of the algorithms is given by Figure 4 in Appendix B.

5. Computational Results

This section presents the computational experiments used to assess the performance of the proposed hybrid algorithm in solving the model, while also offering some sensitive analyses. To evaluate the efficiency of the method in terms of convergence rate and solution quality, the method is applied to random-test instances generated to assess the algorithms' performance. Computational experimentation is conducted over a set of 30 instances with varying scales.

These instances are generated based on the real-case instances presented in [2, 64]. The size of each instance is reported in Table 2. The instances are denoted as Ins L (S, P, T, T', I, J, K), in which L accounts for the instance number, and S, P, T, T', I, J, and K represent the number of suppliers, plants, producing technology, recycling

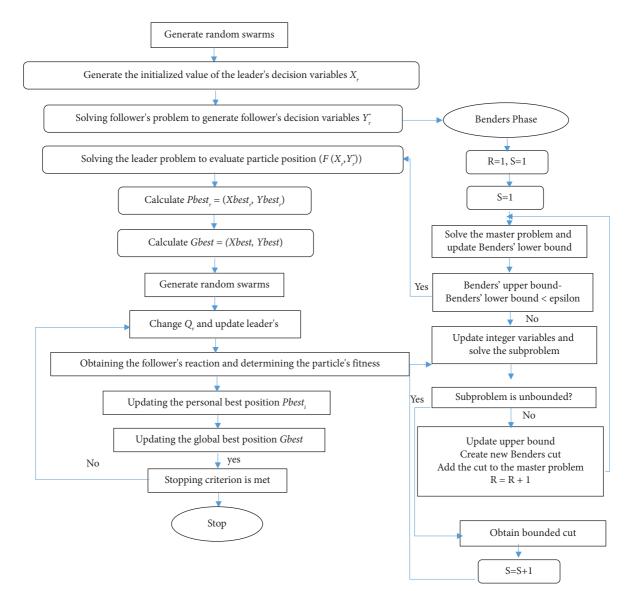


FIGURE 3: Flowchart of the hybrid PSO-benders decomposition method.

technology, distribution centers, and customers, as well as the number of recycling centers, respectively.

5.1. Parameter Setting. One of the most practical techniques to determine the values of parameters in metaheuristic algorithms for the purpose of retaining optimum performance is the Taguchi method. Hence, to reach the optimal or nearoptimal solution in this step, the effect of three parameters c_1, c_2 and α on the QBPSO efficiency and capability is provided. Remarkably enough, based on the relations $\alpha + \beta = 1$ and $w + c_1 + c_2 = 1$, setting the values of parameters c_1, c_2 , and α suffices. For these factors, five levels are considered in Table 3. As such, the Taguchi L_{25} orthogonal array is selected (due to the number of parameters and their selected levels). Therefore, 25 experiments should be performed using a combination of levels for each parameter based on L_{25} .

The medium size of the instance (5, 10, 10, 10, 12, 30, 10) is used to calibrate the parameters of the proposed hybrid-based heuristic algorithm. Twenty-five particles are regarded as the population size, and the maximum iteration number of 100 is used for the stopping criterion. For each set of parameters, ten runs of algorithm are considered. The average value of profit for the leader as the signal-to-noise (S/N) ratio is performed with the best-known value (S/N) being used when it comes to more efficient performance.

Consequently, to identify the best optimal combination of the levels of parameters, the mean value of S/N is computed for each single level. The results are presented in Figures 5–7. Pseudo-code of Benders decomposition of lower level. 1. Initial feasible integer solution $LB = -\infty$ $UB = \infty$ S = R = 02. While $(UB - LB) \ge \varepsilon$ Do 3. Solve sub-problem 4. If the sub-problem is unbounded, then 5. Get unbounded ray 6. Add a feasibility cut to the master problem 7. S = S + 18. Elif 9. Get extreme point θ 10. Add an optimality cut 11. R = R + 1 $UB \coloneqq \min\{UB, \sum_{i} \theta_{i}^{3}\left(\sum_{j} \hat{x}_{ij}^{F} q_{j}\right) + \sum_{s} \theta_{s}^{4}\left(Cap_{s} \hat{e}_{s}^{F}\right) + \sum_{p} \sum_{t} \theta_{p,t}^{5}\left(\hat{y}_{pt}^{F} Cap_{pt}\right) + \sum_{i} \theta_{i}^{7}\left(\sum_{j} r_{j} \hat{x}_{ij}^{F}\right) + \sum_{i} \sum_{j} \theta_{ij}^{8}\left(\hat{x}_{ij}^{F}\right)$ $12. + \sum_{i} \sum_{j} \theta_{ij}^{9} \left(1 - \sum_{i' \ , i} \hat{x}_{i'j}^{F} \right) + \sum_{i} \sum_{j} \theta_{ij}^{10} \left(\hat{x}_{ij'}^{\prime F} \right) + \sum_{i} \sum_{j} \theta_{ij}^{11} \left(1 - \sum_{i' \ , i} \hat{x}_{i'j}^{\prime F} \right) + \sum_{i} \sum_{p} \theta_{ip}^{12} \left(\sum_{j} \gamma r_{j} \hat{x}_{ij}^{\prime F} \right)$ $+\sum_{i}\sum_{k}\theta_{ik}^{13}\left(\sum_{i}(1-\gamma)r_{j}\hat{x}_{ij}^{\prime F}\right)+\sum_{k}\sum_{i'}\theta_{ki'}^{14}\left(\hat{z}_{ki'}^{F}Cap_{ki'}\right)+\theta^{17}X_{\max}^{\prime}\}$ 13. End if 14. Solve the master problem 15. $LB = \overline{Z} / / \text{Result of the master problem}$ 16. End while Fı

IGURE 4: Pseudocode of the benders de	lecomposition algorithm.
---------------------------------------	--------------------------

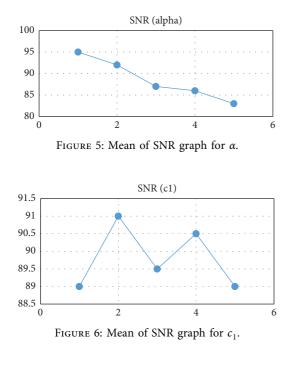
Small-scale sizes	Medium-scale size	Large-scale size
Ins1 (2,3,4,4,3,10,3)	Ins11 (5,5,5,5,10,25,5)	Ins21 (7,10,10,10,15,35,10)
Ins2 (2,4,4,4,3,12,4)	Ins12 (5,5,5,5,12,25,5)	Ins22 (7,12,12,12,15,35,12)
Ins3 (2,3,4,4,5,12,3)	Ins13 (5,6,6,6,12,25,6)	Ins23 (8,12,12,12,15,38,12)
Ins4 (2,3,4,4,5,15,3)	Ins14 (5,7,7,7,12,25,7)	Ins24 (8,12,12,12,18,40,12)
Ins5 (3,4,4,4,7,15,4)	Ins15 (5,8,8,8,12,25,8)	Ins25 (8,15,10,10,20,40,15)
Ins6 (3,4,5,5,8,15,4)	Ins16 (5,8,8,8,12,28,8)	Ins26 (8,17,10,10,20,42,17)
Ins7 (3,4,5,5,8,20,4)	Ins17 (5,10,10,10,12,30,8)	Ins27 (8,17,10,10,22,45,17)
Ins8 (3,4,5,5,10,20,5)	Ins18 (5,10,10,10,12,30,10)	Ins28 (8,17,10,10,25,50,17)
Ins9 (3,5,5,5,10,22,5)	Ins19 (5,10,10,10,15,30,10)	Ins29 (8,18,10,10,27,52,18)
Ins10 (3,5,5,5,10,25,5)	Ins20 (5,10,10,10,15,32,10)	Ins30 (8,20,10,10,30,55,20)

Table	3:	The	levels	of	parameters.
-------	----	-----	--------	----	-------------

Danamatan			Levels		
Parameter	1	2	3	4	5
α	0.15	0.35	0.55	0.75	0.95
<i>c</i> ₁	0.1	0.2	0.3	0.4	0.5
<i>c</i> ₂	0.1	0.2	0.3	0.4	0.5

The results from the above figures show that the effects of the parameter α on QBPSO performance to reach the optimal value of a leader's profit are more significant than the

two remaining parameters c_1, c_2 . However, the optimum level of parameter c_1, c_2 , and α stand at $c_1(2) = 0.2, c_2(3) = 0.3$, and $\alpha(1) = 0.15$.



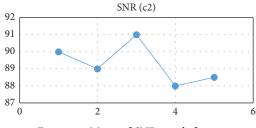


FIGURE 7: Mean of SNR graph for c_2 .

Table 4 reports the analysis of variance to determine the most effective parameters in QBPSO performance. The column *percentage contribution* shows the significance of parameter effects on the PSO-based method performance. It can be clearly deduced that α is an important and unique parameter in this method, with a 98.3% share in yield.

Finally, based on the above analysis, a proposed hybrid algorithm with an optimal combination of parameters is used to solve the problem being studied.

5.2. Evaluation of the Proposed Algorithm. All generated instances are carried out by the proposed hybrid algorithm on a PC Pentium Intel Core i5 with 4 GB of RAM. The algorithms are coded in AIMMS 3.12 optimization software with solver CPLEX12. For the most efficient performance of the hybrid method, different swarm sizes are brought in from N = 10 to 100 to be allocated to each group of instances. The most efficient performance for the algorithm (based on the highest objective function quality) is within 20 to 55. Since the samples are randomly generated, the optimal solution remains unknown. Thus, the bilevel parametric optimization toolbox (B-POP) [65], Lagrangean relaxation-based method [58], and hybrid method based on the genetic

algorithm [58] are applied to show the efficiency of the proposed algorithm. Moreover, for each data test, 10 independent runs are performed due to the randomness involved in the hybrid algorithm, from which the best and average Gaps of the leader's objective function are selected and presented. For readability, a relative percentile Gap is applied as the following formula:

$$Gap = \frac{\left(F^{H}\left(F^{LR} \text{ or } F^{G}\right) - F\right)}{F} \times 100, \tag{74}$$

where F^H , F^{LR} , F^G denotes the best values of the solution vielded by the proposed hybrid, Lagrangean relaxation (upper bound), and genetic-based algorithms, respectively, for the leader's profit. F denotes the best lower bound value solution achieved by the B-POP toolbox. For the termination condition, the maximum CPU time allowed (in seconds) is used. The results obtained for small and medium scales are given in Tables 5 and 6. The first column of the tables demonstrates the swarm size N (also applied to a number of the genes in the genetic-based method). The third and fourth columns show the best and average gaps between the proposed method and the B-POP toolbox, respectively. The fifth and sixth columns illustrate the best and average gaps between the Lagrangean relation algorithm and the B-POP toolbox, respectively. The seventh and eighth columns show the best and average gaps between the genetic-based method and the B-POP toolbox, respectively. The maximum CPU time allowed (rounded in seconds) for termination of the algorithms is reported in the next adjacent columns. The time is raised to 7200 sec to catch up with the growing size of the problem.

For small-size tests, the B-POP is able to find the optimal solution in a reasonable time span. This, however, is not possible for medium-sized tests. Therefore, the most workable solution is reported with a time limit of 7200 seconds.

Based on Table 5, the proposed hybrid method can obtain the optimal solution for all small-size cases, except for Ins7 and 10, and the best optimality gap ranges are between 0 and 0.5, proof of the efficiency of the proposed algorithm for an optimal solution for those instances.

For the same medium cases in particular, e.g., Ins17 to Ins20, the optimal solution is not available by the B-POP within the given time constraints. In most of these cases, the proposed algorithm either introduces a better solution or at least finds a solution equal to that of the feasible bound provided by the B-POP. This is evidently confirmed by nonpositive gaps. However, the gap between the solution obtained by the B-POP and the objective value of the hybrid solution remains below 2%.

Based on a general analogy between the proposed method and Lagrangian and genetic-based methods, the QBPSO-Benders works relatively faster, and the geneticbased algorithm is speedier than the QBPSO-Benders. Hence, the solutions obtained from the proposed method work better than those from the other two. Generally, the time required increases as expected but is still reasonable for this category of problems. For large-scale tests, the B-POP is

	TABLE 4. The analysis of variance.								
Parameters	Sum of squares	Degrees of freedom	Mean squares	Percentage contribution (%)					
α	467.56	4	115.64	98.29					
c_1	1.72	4	0.61	0.58					
c_2	3.61	4	1.23	1.13					
Total	472.89	12		100					

TABLE 4. The analysis of variance

TABLE 5: Results for the small size.

Prob. type	Ν	Best gap (H)	Ave gap (H)	Best gap (LR)	Ave gap (LR)	Best gap (G)	Ave gap (G)	Ave time (H)	Ave time (LR)	Ave time (G)
Ins1	20	0	0.32	0	0.25	0	0.48	12	25	8
Ins2	20	0	0.65	0	0.79	0	1.19	14	29	11
Ins3	20	0	0.38	0.12	1.44	0.3	1.58	14	28	12
Ins4	20	0	0.45	1.1	2.17	0.57	2.94	17	38	16
Ins5	20	0	0.8	0.54	1.06	1.13	3.84	25	47	20
Ins6	20	0	1.52	1.42	2.43	0.8	2.91	28	64	22
Ins7	20	0.5	1.4	0.87	2.39	1.18	3.11	42	96	35
Ins8	25	0	1.03	1.93	3.21	1.14	4.94	70	162	57
Ins9	25	0	0.67	1.15	2.94	1.63	4.10	85	198	63
Ins10	25	0.4	1.13	1.06	2.57	0.71	3.37	93	373	72

TABLE 6: Results for the medium size.

Prob. type	Ν	Best gap (H)	Ave gap (H)	Best gap (LR)	Ave gap (LR)	Best gap (G)	Ave gap (G)	Ave time (H)	Ave time (LR)	Ave time (G)
Ins11	25	0	2.45	1.45	3.53	1.78	4.45	104	447	86
Ins12	25	0	1.87	1.77	2.29	1.13	2.87	110	615	95
Ins13	25	-0.39	1.05	2.50	4.15	2.41	5.01	133	739	105
Ins14	30	-0.42	1.14	3.26	5.84	2.02	5.72	320	1026	197
Ins15	30	0	3.21	1.85	5.37	2.46	6.16	415	1618	275
Ins16	30	0	2.58	1.34	3.43	2.51	5.49	440	1750	348
Ins17	40	-0.71	3.13	-0.49	4.16	0.05	3.26	1000	3204	703
Ins18	40	-1.11	2.48	-0.72	5.83	-0.1	1.22	1000	3107	803
Ins19	40	-1.75	-0.07	-1.08	1.64	-0.62	1.84	1000	3320	812
Ins20	40	—	—	—	—	—	_	1050	3503	880

incapable of finding any feasible solutions in a reasonable time of 7200 seconds. Therefore, only a comparison (Table 7) is made between the three methods. The average gap (Gap = $(F^H - F^{LR}(F^G))/F^{LR}(F^G) \times 100)$ between these three methods and the execution time of the methods have also been incorporated and reported.

The results in Table 7 underline the superiority and quality of the solution obtained from QBPSO-based method. In all instances, the proposed algorithm stops earlier than the specified time, while the Lagrangian-based method fails to yield better solutions despite spending longer time. The genetic algorithm, however, comes to an end highly quickly. This is explained by the fact that the suggested solution employs the Benders decomposition methodology, which takes longer to execute. Instead, the outcome is a superior, more intelligent answer. Figure 8 compares the time solutions of the methods.

To study the convergence of the proposed method, a medium-size instance (5,8,8,8,12,28,8) is applied. The convergence characteristics are shown in Figure 9.

In this certain instance, 443 seconds are totally needed to obtain an upper-bound value of 1.83×10^{-8} by the Lagrangian-based algorithm, and in 187 seconds, the genetic-based algorithm is stopped with the objective value 1.63×10^{-8} , while the proposed QBPSO-based algorithm is able to obtain 1.7×10^{-8} in 202 seconds, a manifest indication that the QBPSO-based algorithm is a relatively fast, reliable, and efficient method.

5.3. Sensitive Analysis. Let us now examine the effects of financial incentives in an example, for which Ins10 is taken. The instance is solved by the hybrid algorithm, with the results showing that the leader uses technology 4 in plant 2 and technology 3 at recycling center 2 for the purpose of securing financial incentives. The related environmental impact and financial incentives of the technologies in the plant and recycling centers are given in Tables 8-11, respectively:

Although opening and manufacturing costs remain too high, plant 2 with technology 4 and recycling center 2 with technology 3 (which the relevant values highlighted in Tables

Journal of Mathematics

Prob. type	Ν	Ave gap (LR)	Ave gap (G)	Ave time (H)	Ave time (LR)	Ave time (G)				
Ins21	40	-2.01	3.28	1300	4433	931				
Ins22	45	-1.36	4.10	1750	4580	1276				
Ins23	45	-0.91	2.45	2000	5953	1531				
Ins24	45	-2.38	5.54	2150	6209	1639				
Ins25	50	-3.01	4.70	3000	7200	1882				
Ins26	50	-2.64	5.28	3100	7200	2133				
Ins27	50	-3.55	3.72	3300	7200	2223				
Ins28	50	-3.03	6.18	3500	7200	2534				
Ins29	55	-5.35	8.21	4305	7200	2901				
Ins30	55	-5.88	7.84	4920	7200	3120				

TABLE 7: Results for the large size.

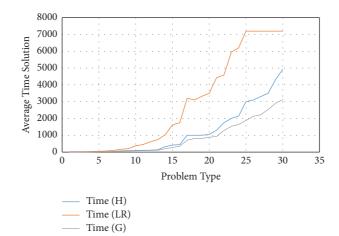


FIGURE 8: Comparison of the time solutions.

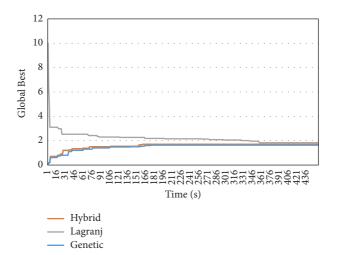


FIGURE 9: Convergence characteristics of the hybrid method.

Plants		Technology				
1	1,324,251	1,210,259	1,001,280	1,011,145	1,132,385	
2	2,205,764	971,011	1,170,582	843,868	908,028	
3	2,047,886	999,257	1,312,687	1,079,444	1,670,673	
4	1,127,193	985,624	1,351,648	824,154	1,440,499	
5	1,443,283	1,006,654	1,427,250	1,412,013	1,910,309	

Recycling center	Technology				
1	449,911	568,967	454,881	541,543	512,538
2	505,023	455,331	467,885	553,317	473,843
3	467,706	549,534	538,545	486,452	495,552
4	498,213	504,099	508,864	528,415	481,231
5	512,833	436,604	532,062	481,310	490,553

TABLE 9: Parameter $Ecz_{kt'}$ related to technologies.

TABLE 10: Parameter λ_{pt} related to financial incentives of technologies.

Plants			Technology		
1	5,543,423	6,196,259	5,506,980	5,097,565	5,516,657
2	4,321,218	5,906,65	6,207,582	6,21,809	4,004,216
3	6,323,356	5,689,598	5,742,870	5,307,405	6,698,417
4	4,406,429	4,608,386	5,453,438	4,760,002	5,104,525
5	5,453,498	5,761,697	4,341,876	5,120,003	5,508,064

TABLE 11: Parameter $\eta_{kt'}$ related to financial incentives of technologies.

Recycling center			Technology		
1	2,298,002	2,874,610	2,501,129	2,475,558	1,155,769
2	1,431,540	2,619,047	2,590,583	1,901,253	2,583,015
3	2,809,621	2,056,593	1,805,080	1,139,651	2,407,293
4	1,904,002	2,650,009	2,841,053	1,990,241	2,243,574
5	1,254,337	1,403,513	2,650,790	1,396,547	2,005,374

TABLE 12: The follower's market share loss due to halved financial incentives.

Prob. type	Loss (%) (fixed $\lambda_{\rm pt}$ and $\eta_{\rm kt'}$)	$Loss(\%)(\lambda_{pt}/2)$	$Loss(\%)(\eta_{kt'}/2)$
Ins1	74.416	75.909	74.485
Ins2	58.309	58.374	58.333
Ins3	48.416	49.557	48.037
Ins4	55.638	56.440	56.105
Ins5	58.043	60.181	59.542
Ins6	62.906	63.508	63.409
Ins7	58.573	59.007	58.730
Ins8	45.889	46.372	46.221
Ins9	39.226	40.583	39.378
Ins10	68.380	70.116	69.055
Ins11	51.329	51.550	51.526
Ins12	60.534	61.338	61.097
Ins13	51.333	54.275	52.542
Ins14	64.677	64.905	64.810
Ins15	49.002	50.335	49.366
Ins16	58.600	59.608	58.992
Ins17	53.812	53.880	53.812
Ins18	60.079	61.406	60.152
Ins19	57.544	59.265	58.390
Ins20	60.951	61.432	61.277

8–11) bring along less environmental impacts. These cleaner technologies stand a higher chance of receiving government incentives.

Now, to show the effectiveness of financial incentives, the leader's advantages and the follower's disadvantages are quantified when the financial incentives are fixed for the leader. To this end, it is demonstrated how the benefits to the leader and follower vary if the financial incentives to the leader are halved, i.e., $\lambda_{\rm pt}/2$ and $\eta_{\rm kt'}/2$, respectively. Furthermore, Table 12 reported the percentages of follower losses of market share stemming from the changes in the financial incentives.

Table 12 shows that followers bag more market share when government incentives to the leader are lowered. In fact, with the reduction of financial incentives, the leader, as the first decision-maker, will enjoy less power in choosing the facility. Rather than incentives, it is the costs that remain the key deciding factor in choosing facilities with technologies. Therefore, some plants, facilities, and technologies which are cost effective with government incentives, may no longer stay so after incentives are slashed.

Yet the same conclusion cannot be stated for incentives HC_i and LT_i , as in the test instances, the effect of HC_i proves greater than LT_i .

6. Conclusions

The present study models bilevel programming on a competitive, CLSC network design problem. In the proposed model, the environmental impacts brought about by establishing or opening plants, recycling centers, and facilities, as well as the emissions for which product manufacturing is responsible, recycling (in different centers), and transporting, are all looked into. Assume that two rival companies produce a commodity simultaneously and they decide to open plants, recycling centers, and facilities. The best open facilities will satisfy customer demand based on each customer's huff rule preferences. It is presumed that with its financial incentives, the government aims to encourage competitive firms to bring in cleaner technologies. This new model, which represents one of the main innovations of the article, can be used in many areas that benefit from government incentives. Another innovation of the article refers to the problem solution method, which is a heuristic method.

Since bilevel problems are inherently Np-hard, to solve the proposed model, a metaheuristic algorithm based on the quantum PSO method is adopted, for which several singlelevel subproblems, some still NP-hard, are solved. Based on the structure of these subproblems, a Benders decomposition method is used, while a new rule for cut generation is implemented at Benders decomposition steps.

To evaluate the algorithm, we try to generalize the example of new test examples based on similar cases in the subject literature. So, thirty instances with varying sizes are considered.

Computational experiments demonstrate that the proposed method is a fast and efficient algorithm to help solve instances when the appropriate parameters are employed. The results of the computational experiments show that financial incentives positively affect the use of cleaner technologies in the design of supply chain networks, mitigating environmental damage.

Therefore, the proposed model can be used as a fresh approach to CLSC sustainability, promising significant contribution to tackling environmental implications.

As a novel and innovative strategy, financial incentives can be variable. A three-level model is needed where the government considers financial incentives at the first level, and delivers them to the competing companies, at the other two. Moreover, uncertainty could arise with regard to some parameters, such as the demand of retailers or the amount of financial incentives required to deploy technologies in a factory, leading to a more serious issue. Lastly, the Lagrangian method or even the branch and bound method can be brought forward to implement metaheuristic methods, such as the PSO.

Appendix

A. The Symbols Used in the Paper

The used sets, indices, and parameters in the model formulation are as follows:

Sets and Indexes:

- S The set of suppliers $(s \in S)$
- *P* The set of plants $(p \in P)$
- T The set of technologies $(t \in T)$
- *L* An index used to show the leader company
- *I* The set of potential locations for facilities $(i \in I)$
- *J* The set of customers $(j \in J)$
- *K* The set of recycling centers $(k \in K)$
- F An index used to show the follower company

Parameters:

 q_j The amount of demand related to the *j* th customer h_j The amount of profit earned from each share unit obtained from the *j* th customer

 cu_{sp} The unit cost of conveying the raw material from supplier *s* to plant *p*

 cv_{pi} The unit cost of conveying the raw material from plant *p* to facility *i*

 Ecv_{pi} Environmental impact linked to the distribution of the raw material from plant *p* to facility *i*

 cx_{ij} The unit cost of conveying the product from facility *i* to customer *j*

 Ecx_{ij} Environmental impact linked to the distribution of the products by facility *i* to customer *j*

 cbu_{sp} The cost of buying the raw material from supplier *s* for plant *p*

 cg_{pt} The manufacturing cost of the raw material in plant *p* by technology *t*

 $cg'_{kt'}$ The manufacturing cost of the recycled product at recycling center k with technology t'

 cr_i The operating cost of the unit reverse products in facility *i*

 cx_i The fixed operating cost of opening facility *i* Ecu_{sp} Environmental impact linked to the distribution

of primary products by supplier s in plant p

 F_p The fixed operating cost of opening plant p

 F_s The fixed cost of inking contract with supplier s CE_{pt} The fixed operating cost of technology t at plant p

 cy_{pt} The fixed operating cost of plant p with technology $t(cy_{pt} = F_p + CE_{pt})$

 Ecy_{pt} Environmental impact related to opening plant p with technology t

 r_i The reverse rate of products from customer j

 $\lambda_{\rm pt}$ The financial incentives in the case of opening plant p with technology t

 γ The percentage at which the reverse products can be recovered at a facility for shipment to the plant

 $pr_{ii}^{L}(pr_{ii}^{F})$ The cost of purchasing the reverse products from customer i with facility i by the leader (follower) firm

 $p\gamma_{ip}^{L}(p\gamma_{ip}^{F})$ The price realized by the leader (follower) firm for selling reverse products by facility *i* to plant l cz_{ip} The cost of conveying reverse products from facility *i* to plant *p*

czik The cost of conveying reverse products from facility i to recycling center k

Ecz_{ik} Environmental impact linked to conveying reverse products from facility *i* to recycling center *k*

 $CEz_{tt'}$ The fixed operating cost of technology t' at recycling center k

 $cz_{kt'}$ The fixed operating cost of recycling center k with technology $t'(cz_{kt'} = F_k + CEz_{kt'})$

 $Ecz_{kt'}$ Environmental impact related to technology t' at recycling center k

 $\eta_{\rm kt'}$ The financial incentives in the case of opening recycling center k with technology t

 $c\gamma_{\rm tt'}$ The per-unit recovering cost of reverse products at recycling center k with technology t

 $p' \gamma_{ks}^L (p' \gamma_{ks}^F)$ The price realized by the leader (follower) firm for selling reverse products to supplier s by recycling center k

 $qr_{ii}^L(qr_{ii}^F)$ The per-unit price realized by the leader for purchasing reverse products from customer j at facility *i* by the leader (follower)

csks The cost of conveying reverse products from recycling center k to supplier s

Ecs_{ks} Environmental impact related to the distribution of reverse products from recycling center k to supplier s

EM_{pt} The per-unit cost of emission caused by manufacturing at plant p with technology t

EM_{1+'} The per-unit cost of emission caused by remanufacturing at recycling center k with technology

 Cap_s The capacity of supplier s to handle primary products

 Cap_{pt} The capacity of plant p with technology t to handle primary products

 Ca_i The capacity of facility *i* to handle primary products

 $\operatorname{Cad}_{dt'}$ The capacity of recycling center d with technology t' to handle primary products

cbu_{sp} The cost of purchasing each unit from supplier s by plant p

 $B^{L}(B^{F})$ The maximum financial incentives realized by the government for the leader (follower)

 A_i The attractiveness parameter of facility *i* d_{ij} The distance of facility *i* to customer *j*

 X_{\max}

 $(Y_{\text{max}}, Z_{\text{max}})$ Maximum number of opened facilities (plants, recycling centers)

Decision variables:

 $e_s^L(e_s^F)$ 1 if supplier s is selected for offering the services by the leader (follower); otherwise, 0

 $u_{sp}^{L}(u_{sp}^{F})$ Quantity of the forward products shipped from supplier s to plant p by the leader (follower) firm $g_{pt}^{L}(g_{pt}^{F})$ Amount of products manufactured at plant p with technology t by the leader (follower)

 $g_{kt}^{'L}(g_{kt}^{'F})$ Amount of products recycled at recycling center k with technology t' by the leader (follower) $y_{pt}^L(y_{pt}^F)$ 1 if plant p is used with technology t by the leader (follower); otherwise, 0

 $v_{pi}^L(v_{pi}^F)$ Quantity of the forward products shipped from plant *p* to facility *i* by the leader (follower)

 $x_i^L(x_i^F)$ 1 if the location i is open by the leader (follower) as a facility; otherwise, 0

 $x_{ii}^{L}(x_{ii}^{F})$ 1 if customer j chooses facility i used by the leader (follower); otherwise, 0

 $\mathbf{x}_{ii}^{'L}(\mathbf{x}_{ii}^{'F})$ 1 if customer j chooses facility i (for reverse products) used by the leader (follower); otherwise, 0 $z_{kt'}^{L}(z_{kt'}^{F})$ 1 if recycling center k is used with technology t' by the leader (follower); otherwise, 0

 $o_{ks}^{L}(o_{ks}^{F})$ Quantity of the reverse products shipped from recycling center k to supplier s by the leader (follower)

 $w_{ip}^{L}(w_{ip}^{F})$ Quantity of the reverse products shipped from facility *i* to plant *p* by the leader (follower)

 $w_{ii}^L(w_{ii}^F)$ Quantity of the reverse products shipped from customer *j* to plant *i* by the leader (follower) $w_{ik}^{T}(w_{ik}^{F})$ Quantity of the reverse products shipped from facility i to recycling center k by the leader (follower)

B. The Pseudo-Code of QBPSO and Benders **Decomposition Algorithms**

Pseudocode of the algorithms given in Figures 2 and 4.

Data Availability

The data used to support the findings of the study are not available.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

- [1] M. Kaboli Chalmardi and J. F. Camacho-Vallejo, "A bi-level programming model for sustainable supply chain network design that considers incentives for using cleaner technologies," Journal of Cleaner Production, vol. 213, pp. 1035-1050, 2019.
- [2] K. Govindan, A. Jafarian, and V. Nourbakhsh, "Bi-objective integrating sustainable order allocation and sustainable supply chain network strategic design with stochastic demand using a novel robust hybrid multi-objective metaheuristic," Computers & Operations Research, vol. 62, pp. 112-130, 2015.

- [3] Y. Huang, K. Wang, T. Zhang, and C. Pang, "Green supply chain coordination with greenhouse gases emissions management: a game-theoretic approach," *Journal of Cleaner Production*, vol. 112, pp. 2004–2014, 2016.
- [4] W. Zhang and G. Li, "Environmental decentralization, environmental protection investment, and green technology innovation," *Environmental Science and Pollution Research*, vol. 29, no. 9, pp. 12740–12755, 2020.
- [5] C. Mondal and B. C. Giri, "Investigating strategies of a green closed-loop supply chain for substitutable products under government subsidy," *Journal of Industrial and Production Engineering*, vol. 39, no. 4, pp. 253–276, 2022.
- [6] Y. Zhang, J. Wang, Y. Xue, and J. Yang, "Impact of environmental regulations on green technological innovative behavior: an empirical study in China," *Journal of Cleaner Production*, vol. 188, pp. 763–773, 2018.
- [7] G. T. Temur and B. Bolat, "Evaluating efforts to build sustainable WEEE reverse logistics network design: comparison of regulatory and non-regulatory approaches," *International Journal of Sustainable Engineering*, vol. 10, no. 6, pp. 358–383, 2017.
- [8] S. H. Jung and T. Feng, "Government subsidies for green technology development under uncertainty," *European Journal of Operational Research*, vol. 286, no. 2, pp. 726–739, 2020.
- [9] A. Blackman and W. Harrington, The Use of Economic Incentives in Developing Countries: Lessons from International Experience with Industrial Air Pollution, Routledge, Oxfordshire, England, UK, 2018.
- [10] L. Aldieri and C. P. Vinci, "Green economy and sustainable development: the economic impact of innovation on employment," *Sustainability*, vol. 10, pp. 35–41, 2018.
- [11] F. Jia, L. Zuluaga-Cardona, A. Bailey, and X. Rueda, "Sustainable supply chain management in developing countries: an analysis of the literature," *Journal of Cleaner Production*, vol. 189, pp. 263–278, 2018.
- [12] S. Zare, A. R. Tavakolpour-Saleh, A. Aghahosseini, M. H. Sangdani, and R. Mirshekari, "Design and optimization of Stirling engines using soft computing methods: a review," *Applied Energy*, vol. 283, 2021.
- [13] S. Zare, M. makki, M. Rasooli, H. S. Alavi, and A. R. Tavakolpour-Saleh, "Assessment of a diaphragm thermoacoustic Stirling engine using the energy standpoint and genetic algorithm," *International Journal of Energy and Environmental Engineering*, vol. 14, no. 4, pp. 743–755, 2023.
- [14] J. Butler, "Cannibals with forks: the triple bottom line of 21st century business," *R* & *D* Management, vol. 31, p. 104, 2001.
- [15] A. Chaabane, A. Ramudhin, and M. Paquet, "Design of sustainable supply chains under the emission trading scheme," *International Journal of Production Economics*, vol. 135, no. 1, pp. 37-49, 2012.
- [16] O. Listeş and R. Dekker, "A stochastic approach to a case study for product recovery network design," *European Journal* of Operational Research, vol. 160, no. 1, pp. 268–287, 2005.
- [17] M. S. Pishvaee, R. Z. Farahani, and W. Dullaert, "A memetic algorithm for bi-objective integrated forward/reverse logistics network design," *Computers & Operations Research*, vol. 37, no. 6, pp. 31100–31112, 2010.
- [18] E. Keyvanshokooh, M. Fattahi, S. Seyed-Hosseini, and R. Tavakkoli-Moghaddam, "A dynamic pricing approach for returned products in integrated forward/reverse logistics network design," *Applied Mathematical Modelling*, vol. 37, no. 24, pp. 10182–10202, 2013.

- [19] E. Özceylan, T. Paksoy, and T. Bektaş, "Modeling and optimizing the integrated problem of closed-loop supply chain network design and disassembly line balancing," *Transportation Research Part E: Logistics and Transportation Review*, vol. 61, pp. 142–164, 2014.
- [20] M. S. Pishvaee and S. A. Torabi, "A possibilistic programming approach for closed loop supply chain network design under uncertainty," *Fuzzy Sets and Systems*, vol. 161, no. 20, pp. 2668–2683, 2010.
- [21] S. H. Amin and G. Zhang, "A multi-objective facility location model for closed-loop supply chain network under uncertain demand and return," *Applied Mathematical Modelling*, vol. 37, no. 6, pp. 4165–4176, 2013.
- [22] B. Vahdani and M. Mohammadi, "A bi-objective intervalstochastic robust optimization model for designing closed loop supply chain network with multi-priority queuing system," *International Journal of Production Economics*, vol. 170, pp. 67–87, 2015.
- [23] S. Zokaee, A. Jabbarzadeh, B. Fahimnia, and S. J. Sadjadi, "Robust supply chain network design: an optimization model with real world application," *Annals of Operations Research*, vol. 257, no. 1-2, pp. 15–44, 2017.
- [24] E. Keyvanshokooh, S. M. Ryan, and E. Kabir, "Hybrid robust and stochastic optimization for closed-loop supply chain network design using accelerated Benders decomposition," *European Journal of Operational Research*, vol. 249, no. 1, pp. 76–92, 2016.
- [25] M. Farrokh, A. Azar, G. Jandaghi, and E. Ahmadi, "A novel robust fuzzy stochastic programming for closed loop supply chain network design under hybrid uncertainty," *Fuzzy Sets* and Systems, vol. 341, pp. 69–91, 2018.
- [26] H. Soleimani and G. Kannan, "A hybrid particle swarm optimization and genetic algorithm for closed-loop supply chain network design in large-scale networks," *Applied Mathematical Modelling*, vol. 39, no. 14, pp. 3990–4012, 2015.
- [27] E. Yadegari, H. Najmi, M. Ghomi-Avili, and M. Zand, "A flexible integrated forward/reverse logistics model with random path-based memetic algorithm," *Iranian Journal of Management Studies*, vol. 8, pp. 287–313, 2015.
- [28] M. Talaei, B. Farhang Moghaddam, M. S. Pishvaee, A. Bozorgi-Amiri, and S. Gholamnejad, "A robust fuzzy optimization model for carbon-efficient closed-loop supply chain network design problem: a numerical illustration in electronics industry," *Journal of Cleaner Production*, vol. 113, pp. 662–673, 2016.
- [29] M. Zhalechian, R. Tavakkoli-Moghaddam, B. Zahiri, and M. Mohammadi, "Sustainable design of a closed-loop location-routing-inventory supply chain network under mixed uncertainty," *Transportation Research Part E: Logistics and Transportation Review*, vol. 89, pp. 182–214, 2016.
- [30] B. Khorshidvand, H. Soleimani, S. Sibdari, and M. Mehdi Seyyed Esfahani, "A hybrid modeling approach for green and sustainable closed-loop supply chain considering price, advertisement and uncertain demands," *Computers & Industrial Engineering*, vol. 157, 2021.
- [31] E. Bazan, M. Y. Jaber, and S. Zanoni, "A review of mathematical inventory models for reverse logistics and the future of its modeling: an environmental perspective," *Applied Mathematical Modelling*, vol. 40, no. 5-6, pp. 4151–4178, 2016.
- [32] B. Giri, A. Chakraborty, and T. Maiti, "Pricing and return product collection decisions in a closed-loop supply chain with dual-channel in both forward and reverse logistics," *Journal of Manufacturing Systems*, vol. 42, pp. 104–123, 2017.

- [33] A. Jabbarzadeh, M. Haughton, and A. Khosrojerdi, "Closedloop supply chain network design under disruption risks: a robust approach with real world application," *Computers & Industrial Engineering*, vol. 116, pp. 178–191, 2018.
- [34] M. Asghari, H. Afshari, S. M. J. Mirzapour Al-e-hashem, A. M. Fathollahi-Fard, and M. A. Dulebenets, "Pricing and advertising decisions in a direct-sales closed-loop supply chain," *Computers & Industrial Engineering*, vol. 171, 2022.
- [35] H. Soleimani, P. Chhetri, A. M. Fathollahi-Fard, S. M. J. Mirzapour Al-e-Hashem, and S. Shahparvari, "Sustainable closed-loop supply chain with energy efficiency: Lagrangian relaxation, reformulations and heuristics," *Annals* of Operations Research, vol. 318, no. 1, pp. 531–556, 2022.
- [36] S. Rezapour, R. Z. Farahani, B. Fahimnia, K. Govindan, and Y. Mansouri, "Competitive closed-loop supply chain network design with price-dependent demands," *Journal of Cleaner Production*, vol. 93, pp. 251–272, 2015.
- [37] H. Fallah, H. Eskandari, and M. S. Pishvaee, "Competitive closed-loop supply chain network design under uncertainty," *Journal of Manufacturing Systems*, vol. 37, pp. 649–661, 2015.
- [38] J. Jian, B. Li, N. Zhang, and J. Su, "Decision-making and coordination of green closed-loop supply chain with fairness concern," *Journal of Cleaner Production*, vol. 298, 2021.
- [39] A. M. Fathollahi-Fard, M. T. Hajiaghaei-Keshteli, R. Tavakkoli-Moghaddam, and N. R. Smith, "Bi-level programming for home health care supply chain considering outsourcing," *Journal of Industrial Information Integration*, vol. 25, 2022.
- [40] M. Kaviyani-Charati, S. H. Ghodsypour, and M. Hajiagh-Keshteli, "Impact of adopting quick response and agility on supply chain competition with strategic customer behavior," *Scientia Iranica*, vol. 29, pp. 387–411, 2022.
- [41] A. M. Fathollahi-Fard, M. Hajiaghaei-Keshteli, and S. A. Mirjalili, "Hybrid optimizers to solve a tri-level programming model for a tire closed-loop supply chain network design problem," *Applied Soft Computing*, vol. 70, pp. 701– 722, 2018.
- [42] S. L. Hakimi, "Locations with spatial interactions: competitive locations and games," *Discrete Location Theory*, pp. 439–478, 1990.
- [43] R. G. Jeroslow, "The polynomial hierarchy and a simple model for competitive analysis," *Mathematical Programming*, vol. 32, no. 2, pp. 146–164, 1985.
- [44] P. Hansen, B. Jaumard, and G. Savard, "New branch-andbound rules for linear bilevel programming," *SIAM Journal on Scientific and Statistical Computing*, vol. 13, no. 5, pp. 1194– 1217, 1992.
- [45] Z. H. Gumus and C. A. Floudas, "Global optimization of mixed-integer bilevel programming problems," *Computational Management Science*, vol. 2, no. 3, pp. 181–212, 2005.
- [46] C. A. Floudas, P. M. Pardalos, C. S. Adjiman et al., Handbook of Test Problems in Local and Global Optimization, Springer nature, Berlin, Germany, 1999.
- [47] E. N. Pistikopoulos, M. C. Georgiadis, and V. Dua, "Multiparametric programming," *Weinheim:Wiley VCH*, vol. 1, 2007.
- [48] L. Vicente, Bilevel Programming: Introduction, History and Overview, Springer, Boston, MA, 2001.
- [49] B. Colson, P. Marcotte, and G. Savard, "Bilevel programming: a survey," 4OR, vol. 3, no. 2, pp. 87–107, 2005.
- [50] L. F. Domínguez and E. N. Pistikopoulos, "Multiparametric programming based algorithms for pure integer and mixedinteger bilevel programming problems," *Computers & Chemical Engineering*, vol. 34, no. 12, pp. 2097–2106, 2010.

- [51] D. Dempe, "Discrete bilevel optimization problems," Technical Report D-04109, Academia, San Francisco, CA, USA, 2001.
- [52] U. P. Wen and Y. H. Yang, "Algorithms for solving the mixed integer two-level linear programming problem," *Computers & Operations Research*, vol. 17, no. 2, pp. 133–142, 1990.
- [53] J. F. Bard and J. T. Moore, "A branch and bound algorithm for the bilevel programming problem," *SIAM Journal on Scientific* and Statistical Computing, vol. 11, no. 2, pp. 281–292, 1990.
- [54] R. Mathieu, L. Pittard, and G. Anandalingam, "Genetic algorithm based approach to bi-level linear programming," *RAIRO-Operations Research*, vol. 28, no. 1, pp. 1–21, 1994.
- [55] S. R. Hejazi, A. Memariani, G. Jahanshahloo, and M. M. Sepehri, "Linear bi-level programming solution by genetic algorithm," *Computers & Operations Research*, vol. 29, no. 13, pp. 1913–1925, 2002.
- [56] M. Gendreau, P. Marcotte, and G. Savard, "A hybrid Tabuascent algorithm for the linear bilevel programming problem," *Journal of Global Optimization*, vol. 8, no. 3, pp. 217– 233, 1996.
- [57] V. Oduguwa and R. Roy, "Bi-level Optimization Using Genetic Algorithm," in *Proceedings of the IEEE International Conference on Artifitial Intellegence Systems*, Divnomorskoe, Russia, September 2002.
- [58] A. Rahmani and S. A. Mirhassani, "Lagrangean relaxationbased algorithm for bi-level problems," *Optimization Methods* and Software, vol. 30, pp. 1–14, 2015.
- [59] S. A. Mirhassani, S. Raeisi, and A. Rahmani, "Quantum binary particle swarm optimization-based algorithm for solving a class of bi-level competitive facility location problems," *Optimization Methods and Software*, vol. 30, no. 4, pp. 756– 768, 2015.
- [60] J. Kennedy and R. C. Eberhart, "A discrete binary version of the particle swarm algorithm," in *Proceedings of the 1997 IEEE International conference on systems, man, and cybernetics. Computational cybernetics and simulation*, pp. 4104–4108, Orlando, FL, USA, October 1997.
- [61] A. Rahmani and M. Hosseini, "A time-dependent green location-routing problem with variable speed of vehicles," *Opsearch*, vol. 59, no. 3, pp. 945–973, 2022.
- [62] M. M. Fazel-Zarandi, O. Berman, and J. C. Beck, "Solving a stochastic facility location/fleet management problem with logic-based Benders' decomposition," *IIE Transactions*, vol. 45, no. 8, pp. 896–911, 2013.
- [63] T. Magnanti and R. Wong, "Accelerating Benders decomposition: algorithmic enhancement and model selection criteria," *Operations Research*, vol. 29, no. 3, pp. 464–484, 1981.
- [64] O. Dukkanci, B. Y. Kara, and T. Bektaş, "The green locationrouting problem," *Computers & Operations Research*, vol. 105, pp. 187–202, 2019.
- [65] S. Avraamidou and E. N. Pistikopoulos, "B-POP: Bi-level parametric optimization toolbox," *Computers & Chemical Engineering*, vol. 122, pp. 193–202, 2019.