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Research Article

Modern Approach in Pattern Recognition Using Circular Fermatean Fuzzy Similarity Measure for Decision Making with Practical Applications

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The circular Fermatean fuzzy (CFF) set is an advancement of the Fermatean fuzzy (FF) set and the interval-valued Fermatean fuzzy (IVFF) set which deals with uncertainty. The CFF set is represented as a circle of radius ranging from 0 to $\sqrt{2}$ with the center at the degree of association (DA) and degree of nonassociation (DNA). If multiple people are involved in making decisions, the CFF set, as an alternative to the FF and IVFF sets, can deal with ambiguity more effectively by encircling the decision values within a circle rather than taking an average. Using algorithms, a pattern can be observed computationally or visually. Machine learning algorithm utilizes pattern recognition as an instrument for identifying patterns and also similarity measure (SM) is a beneficial pattern recognition tool used to classify items, discover variations, and make future predictions for decision making. In this work, we introduce the CFF cosine and Dice similarity measures (CFFDMs and CFFSMs), and their properties are studied. Unlike traditional approaches of decision making, which emphasize a single number, the proposed CFFSMs observe the pattern over the circular region to help in dealing with uncertainty more effectively. We introduce an innovative decision-making method in the FF setting. Available bank loans and applicants' eligibility levels are represented as CFF set using their FF criteria and are taken as loan patterns and customer eligibility patterns. The loan is allocated to the applicant by measuring the CFFCSM and CFFDSM between the two patterns. Also, laptops are suggested to the customers by measuring the similarity between specification pattern and requirement pattern. The correctness and consistency of the proposed models are ensured by comparison analysis and graphical simulations of the input and similarity CFFNs.

1. Introduction

Pattern recognition is a developed but exciting field that keeps growing swiftly. It provides a platform for developments in associated disciplines notably computer vision, image processing, gene expression analysis, protein structure prediction, medical image analysis, text classification, sentiment analysis, named entity recognition, speech recognition, gesture recognition, biometric identification, neural networks, and many more.

The nearest neighbor algorithm, initiated in 1967, is the origin of pattern recognition. Statistical pattern recognition methods, machine learning algorithms, and deep learning techniques are some of the pattern recognition methods.

Relevant feature identification from the raw data is the first stage in almost all pattern recognition tasks. Features that aid in recognizing between patterns could be statistical, structural, physical, or spectral attributes. Similarities in the data are then recognized and used to produce recommendations. Hagedoorn [1] used similarity matching for pattern recognition in his PhD thesis. Julian et al. [2] modified Mitchell similarity and introduced new SM for pattern recognition. Liu [3] introduced new similarity measures between intuitionistic fuzzy sets and between elements. Yen et al. [4] applied the SM for pattern recognition in 2013. In the year 1980, Kono [5] identified a pattern recognition system that can classify 22 kinds of patterns within 4 seconds in the minicomputer.

The conventional methods of pattern recognition classify a collection of patterns into clusters based on similar characteristics. However, the majority of pattern recognition issues employ feature selection that is gathered at random, which leads to blending of pattern classes and ambiguity in object detection.

Zadeh [6] implemented fuzzy set (FS) theory in 1965, which assumes DA in [0, 1] instead of a characteristic function that maps to 0 or 1 in the crisp set. Fuzzy classification is a way of sorting elements into fuzzy sets with association functions identified by the truth value of a fuzzy propositional function. Based on the exponential area of established intervals of membership functions, Dutta et al. [7] constructed a new Dice similarity measure for fuzzy numbers. Venugopal [8] used fuzzy indicator and Masoomi [9] applied neutrosophic indicator for performance analysis. In 1986, Pal et al. [10] and in 2006 Guo et al. [11] described fuzzy pattern recognition, which provides a powerful framework for modeling and analyzing complex patterns in real-world data.

Despite the fact that FSs are useful in instances of uncertainty and have no clear boundary, they fail to represent DNA. Both DA and DNA are important for the mathematical representation of real-life situations. For that need, Atanassov [12] coined the concept of an intuitionistic fuzzy (IF) set as an extension of Zadeh's FS theory in 1983. In the IF set environment, DA and DNA take the value in $[0, 1]$, with their sum between 0 and 1. Chen [13] introduced a new SM between Atanassov's intuitionistic fuzzy sets. Liu et al. [14] examined the cosine SM between hybrid IF sets. To recognize the pattern of COVID-19, Saeed et al. [15] applied the IF tool. Ye [16–18] introduced IF and interval-valued IF cosine SM and the Dice SM based on the deducted IF sets which help in the field of pattern recognition. Nwokoro [19] developed the IF approach for predicting maternal outcomes, Zhou et al. [20] applied the generalized IF similarity operator recognition principle, and Ejegwa [21] explored the IF correlation coefficient for classification process. Zhang [22] considered the IF score function for pattern recognition and medical diagnosis.

There have been drawbacks to the IF set, even though it has been utilized in various situations. The IF set, in particular, lacks the capacity to cope with such information when a decision maker proposes it, but its total DA and DNA are greater than 1. In this connection, Yager et al. [23] recommended the Pythagorean fuzzy (PF) set theory in 2013 by modifying the IF set perspective as well. A PF set incorporates the feature that the aggregate value of the squares of DA and DNA lies between 0 and 1, thereby enhancing the acceptance range more than the IF set. Peng [24] used various parametric SMs, and Nguyen et al. [25] developed the exponential SM. Ejegwa et al. [26] applied the PF correlation coefficient for pattern recognition, decision making [27, 28], disaster control, and diagnostic analysis [29]. Zulqarnain [30–36] applied various forms of the FS Einstein aggregation operators, and Kuppusamy et al. [37] used a bipolar PF approach and its topological properties in decision making.

The FF set developed by Senapati [38] has an extensively broad range of acceptance compared to the IF set and PF set by taking DA and DNA with the condition that the sum of cubes of DA and DNA lies between 0 and 1. Alahmadi et al. [39] used FF t-conorms for SM in pattern analysis. Deng et al. [40] applied FF distance measures. Khan et al. [41] derived a FF benchmark similarity using S-norm. Akram et al. [42, 43] discussed transportation problem under FF environment. Ejegwa applied FF composite relation for pattern recognition [44]. Also, he applied FF composite relation and FF correlation coefficient in diagnostic analysis [45, 46]. Onyeke [47] applied the modified Senapati and Yager's FF distance and its application in students' course placement in tertiary institution. Kishorekumar et al. [48] and Revathy et al. [49] introduced interval-valued picture fuzzy geometric Bonferroni mean operators and FF normalized Bonferroni mean operator. Mishra et al. [50] applied t−norm and s−conorm and Wania et al. [51] used Fermatean Probabilistic Hesitant Fuzzy Set for MCDM. Sahoo [52] and Zhou et al. [53] applied the FF SM and FF ELECTRE methods in group decision making. Xu et al. [54] and Ashraf et al. [55] applied SMs for FF pattern recognition. Demir [56] utilized correlation coefficients for interval-valued Fermatean hesitant fuzzy sets for pattern recognition. Rahim et al. [57] examined improved cosine SM and distance measures-based TOPSIS method for cubic Fermatean fuzzy sets in 2023.

The circular intuitionistic fuzzy (CIF) set, created by Atanassov [58] in 2020, is a profound enhancement to FS. Unlike an IF set, a CIF set provides each component as a circle. These are the sets in which each component of the universe has a value of DA and DNA as the center of a circle around them, and that circle has a radius of $[0, \sqrt{2}\,]$ such that the sum of DA and DNA within this circle is not more than 1. Also, in reference [59], he derived four distance measures of CIF set. Cakir et al. [60], Irem [61], and Alkan [62] applied the CIF set in decision making. Bolturk [63] discussed the difference between the IVIF set and the CIF set. As an extension of the CIF set, Olgun [64] introduced a circular Pythagorean fuzzy (CPF) set. Khan et al. [65] expanded the PF set as a circular and disc PF set.

The circular Fermatean fuzzy (CFF) set introduced by Revathy et al. [66] is an amplification of the FF and IVFF sets. The CFF set is a circle whose center is at (DA, DNA) and of radius at most $\sqrt{2}$ with the sum of cubes of DA and DNA between 0 and 1. The CFF set can address ambiguity in a better way when many different people are participating in decision making because it encircles various values within a circle instead of averaging them. We deal with high-dimensional, more complicated data patterns in today's world. It might be difficult for traditional pattern recognition techniques to analyze these patterns and find appropriate knowledge efficiently and successfully. Because of its wide range in the willingness of a decision maker, the CFF pattern recognition offers a framework to optimize decision making by considering the most efficient paths through these patterns swiftly and promptly.

The decision makers' DA and DNA were addressed by specific values in all of the previous sets, such as fuzzy, IF, PF, and FF. In our CFF set, the decision maker values are determined by the circles. Hence, the decision will be same if the decision makers DA and DNA lie in the circles.

Bank loans are advantageous way that people can utilize to achieve various kinds of intended goals. Loan allocation that is automated can speed up decisions for borrowers, minimize the human burden for lenders, and optimize the lending process. Lenders must, however, ensure their automated systems maintain to legal as well as moral norms, and borrowers need to review the details of any loan offer thoroughly before accepting it. In this present work, we introduce CFF cosine and Dice similarity (CFFCSM and CFFDSM); also, their properties are investigated. The CFF set utilizing the applicants' FF criteria represents the available bank loans and the applicants' eligibility level that are taken as loan patterns and customer eligibility patterns. The applicant is allocated a loan based on the evaluation of the two patterns' CFFCSM and CFFDSM measurements. Additionally, customers are provided laptop suggestions based on how closely their expectations and need patterns coincide.

The key factors that encouraged us to do the present study are as follows:

(i) To deal with uncertainty, the FF sets are used in several fields, such as control system engineering, image processing, power engineering, industrial automation, robotics, consumer electronics, and optimization. If multiple decision makers are involved in the process of decision making, the average of their decision value is taken into account.

(ii) Through the CFF set environment, the decision values can be represented by circles which will assist to handle the uncertainty more effectively since the circles will express the decision makers' opinion in a region instead of a particular value.

(iii) In the majority of decision-making aggregation methods, if one of the decision values has 0 in either DNA or DA, the aggregated value becomes zero. This can be avoided by using similarity measures instead of aggregation.

(iv) The CFF similarity measure will be a better tool for decision making than FF precisely because of its computational structure.

The contribution of this present work is given below:

(i) The cosine and Dice similarity measures are defined in the CFF environment and applied for decision making in bank loan allocation and laptop suggestion using pattern recognition.

(ii) Set of FF decision values has been transformed into the CFFN and their graphical representation is demonstrated. The consistency of the proposed method is represented by graphical representation which is the special feature of the work.

(iii) Also, the comparative analysis with the existing methods and the statistical analysis with the help of SPSS software 27 are done for the proposed decision making.

The article is organised as follows. Section 2 gives the preliminary. Section 3 describes the circular Fermatean fuzzy similarity measure. Section 4 explains the application of the CFFS in pattern recognition and analyzes and portrays the obtained result. Section 5 states the limitations, and finally Section 6 ends up with the conclusion and future work.

The nomenclature used in the current research is listed in Table 1.

2. Preliminaries

We now present some preliminary definitions and notions necessary to apprehend our findings.

Definition 1 (see [38]). A set $\mathscr{F} = \{\langle x, \alpha_F(x), \beta_F(x)\rangle : x \in X\}$ in the universe of discourse X is called Fermatean fuzzy (FF) set if $0 \leq (\alpha_F(x))^3 + (\beta_F(x))^3 \leq 1$ where $\alpha_F\colon X \longrightarrow [0,1]$ is the DA of x in F, $\beta_F\colon X \longrightarrow [0,1]$ is the DNA of x in F, and $\pi = \sqrt[3]{1 - (\alpha_F(x))^3 - (\beta_F(x))^3}$ is the DI of x in F. The components of the FF set are taken as the FF number (FFN) and it is represented by $\mathscr{F} = (\alpha_F, \beta_F)$ whose complement is $\mathscr{F}^c = (\beta_F, \alpha_F)$.

Definition 2 (see [66]). A circular Fermatean fuzzy (CFF) set $c\mathscr{F}$, is stated as $c\mathscr{F} = \{\langle \iota, \mu_{c\mathscr{F}}(\iota), \nu_{c\mathscr{F}}(\iota); \rho_{c\mathscr{F}}\rangle : \iota \in \mathscr{I}\}$ in the space of discussion $\mathscr{I}$ is a circle with center at DA and DNA, $\mu_{c\mathscr{F}}, \nu_{c\mathscr{F}}\colon \mathscr{I} \longrightarrow [0,1]$ and of radius $\rho_{c\mathscr{F}} \in [0, \sqrt{2}]$ such that $0 \leq (\mu_{c\mathscr{F}}(\iota))^3 + (\nu_{c\mathscr{F}}(\iota))^3 \leq 1$. $\pi_{c\mathscr{F}}(\iota) = \sqrt[3]{1 - \mu_{c\mathscr{F}}(\iota))^3 - (\nu_{c\mathscr{F}}(\iota))^3}$ is the DI ι in $c\mathscr{F}$ Z $c\mathscr{F}$. The component of CFF set $c\mathscr{F} = (\mu_{c\mathscr{F}}, \nu_{c\mathscr{F}}; \rho_{c\mathscr{F}})$ is called circular Fermatean fuzzy number (CFFN) and its complement is $c\mathscr{F}^c = \{\langle \iota, \nu_{c\mathscr{F}}(\iota), \mu_{c\mathscr{F}}(\iota); \rho_{c\mathscr{F}}\rangle : \iota \in \mathscr{I}\}$.

Proposition 3. *Let* $\mathscr{F}_i = \{\langle \mu_{f_{i,1}}, \nu_{f_{i,1}}\rangle, \langle \mu_{f_{i,2}}, \nu_{f_{i,2}}\rangle, \ldots, \langle \mu_{f_{i,k_i}}, \nu_{f_{i,k_i}}\rangle\}$ *be collection of FFNs. Then* $c\mathscr{F}_i = \{\langle \iota_i, \mu_{c\mathscr{F}}(\iota_i), \nu_{c\mathscr{F}}(\iota_i); \rho_{c\mathscr{F}i}\rangle : \iota_i \in \mathscr{I}\}$ *are CFF sets, where*

Table 1: Nomenclature and expansion.

Nomenclature	Expansion
DA	Degree of association (notation: $\mu_{c\mathcal{F}}$)
DNA	Degree of nonassociation (notation: $\nu_{c\mathcal{F}}$)
DI	Degree of indeterminacy (notation: $\pi_{c\mathcal{F}}$)
FS	Fuzzy set
IF	Intuitionistic fuzzy
PF	Pythagorean fuzzy
IVFF	Interval-valued Fermatean fuzzy
FF	Fermatean fuzzy
FFN	Fermatean fuzzy number (notation: $\mathcal{F} = \langle \mu_{\mathcal{F}}, \nu_{\mathcal{F}} \rangle$)
CIF	Circular intuitionistic fuzzy (notation: $c\mathcal{F} = \langle \mu_{c\mathcal{F}}, \nu_{c\mathcal{F}}; \rho_{cF} \rangle$)
CPF	Circular Pythagorean fuzzy
CFF	Circular Fermatean fuzzy
CFFN	Circular Fermatean fuzzy number
MCDM	Multicriteria decision making
SM	Similarity measure
FFSM	Fermatean fuzzy similarity measure
S_{FF}	Fermatean fuzzy similarity
CFFSM	Circular Fermatean fuzzy similarity measure
CFFCSM	Circular Fermatean fuzzy cosine similarity measure
CFFDSM	Circular Fermatean fuzzy Dice similarity measure
PL	Personal loan
JL	Jewelry loan
ML	Mortgage loan
NL	No loan
RI	Regular income
CS	CIBIL score
JA	Jewelry availability
AV	Asset value
GL	Gaming laptop
HSL	Home and student laptop
BL	Business laptop
MB	MacBook

$$\begin{aligned}\langle \mu_{c\mathcal{F}}(\iota_i), \nu_{c\mathcal{F}}(\iota_i)\rangle &= \left\langle \sqrt[3]{\frac{\sum_{j=1}^{k_i}\mu_{f_{i,j}}^3}{k_i}}, \sqrt[3]{\frac{\sum_{j=1}^{k_i}\nu_{f_{i,j}}^3}{k_i}} \right\rangle \text{ and} \\ \rho_{c\mathcal{F}_i} &= \min\left\{ \max_{1\le j\le k_i} \sqrt{\left(\mu_{c\mathcal{F}_i}(\iota_i) - \mu_{f_{i,j}}\right)^2 + \left(\nu_{c\mathcal{F}_i}(\iota_i) - \nu_{f_{i,j}}\right)^2}, \sqrt{2} \right\}.\end{aligned} \tag{1}$$

Proof. Consider

$$\begin{aligned} 0 \le \mu_{c\mathcal{F}}^3(\iota_i) + \nu_{c\mathcal{F}}^3(\iota_i) &= \frac{\sum_{j=1}^{k_i}\mu_{f_{i,j}}^3}{k_i} + \frac{\sum_{j=1}^{k_i}\nu_{f_{i,j}}^3}{k_i} \\ &= \frac{\sum_{j=1}^{k_i}\mu_{f_{i,j}}^3 + \sum_{j=1}^{k_i}\nu_{f_{i,j}}^3}{k_i} \\ &\le \frac{\sum_{j=1}^{k_i}1}{k_i} = 1. \end{aligned} \tag{2}$$

Meanwhile, $0 \le \rho_{c\mathcal{F}_i} \le \sqrt{2}, \forall i$. Consequently, $c\mathcal{F} = \left\{\langle \iota, \mu_{c\mathcal{F}_i}(\iota_i), \nu_{c\mathcal{F}}(\iota_i); \rho_{c\mathcal{F}_i}\rangle \colon \iota_i \in \mathcal{I}\right\}$ is a CFF set.

Example 1. If $\mathcal{F}_1 = \{\langle 0.7, 0.5\rangle, \langle 0.65, 0.55\rangle, \langle 0.8, 0.6\rangle, \langle 0.4, 0.7\rangle\}$, $\mathcal{F}_2 = \{\langle 0.1, 0.3\rangle, \langle 0.2, 0.25\rangle, \langle 0.15, 0.35\rangle, \langle 0.25, 0.4\rangle\}$ are the collection of FFNs, then the CFFN are $\mathrm{CF}_1 = (0.67, 0.60; 0.29)$ and $\mathrm{CF}_2 = (0.19, 0.33; 0.09)$. The visualization of this transformation is represented in Figure 1.

The black color points are the FFN. The circles CF_1 and CF_2 are drawn with the radius and center calculated by Proposition 3. Since the sum of cubes of DA and DNA is between 0 and 1, the circles are drawn only in the acceptance region. These circles represent the decision makers' opinion.

Remark 4. All FF sets can be viewed as a CFF set because each FF set possesses the structure $c\mathcal{F} = \{\langle \iota, \mu_{c\mathcal{F}}(\iota), \nu_{c\mathcal{F}}(\iota); 0\rangle \iota \in \mathcal{I}\} = \mathcal{F}$. That is, every FF set is a CFF set with radius 0.

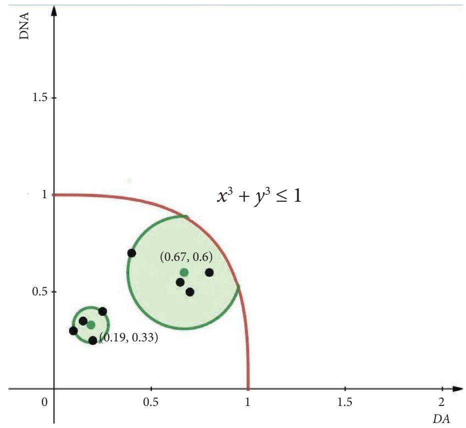


FIGURE 1: Representation of CFFN.

3. Circular Fermatean Fuzzy Similarity Measure

In this section, we bring in CFF cosine similarity and CFF Dice similarity measures.

Definition 5. Let $c\mathcal{F} = \langle \mu_{c\mathcal{F}}, \nu_{c\mathcal{F}}; \rho_{cF} \rangle$, $c\mathcal{G} = \langle \mu_{c\mathcal{G}}, \nu_{c\mathcal{G}}; \rho_{cG} \rangle$ be two CFFNs. The CFFCSM and CFFDSM between $c\mathcal{F}$ and $c\mathcal{G}$ are defined as

$$\mathrm{CFFCSM}(c\mathcal{F}, c\mathcal{G}) = \frac{1}{2}\left(\frac{\mu_{c\mathcal{F}}^3 \mu_{c\mathcal{G}}^3 + \nu_{c\mathcal{F}}^3 \nu_{c\mathcal{G}}^3}{\sqrt[3]{\mu_{c\mathcal{F}}^6 + \nu_{c\mathcal{F}}^6}\sqrt[3]{\mu_{c\mathcal{G}}^6 + \nu_{c\mathcal{G}}^6}} + 1 - \frac{|\rho_{cF} - \rho_{cG}|}{\sqrt{2}}\right), \tag{3}$$

$$\mathrm{CFFDSM}(c\mathcal{F}, c\mathcal{G}) = \frac{1}{2}\left(\frac{2\left(\mu_{c\mathcal{F}}^3 \mu_{c\mathcal{G}}^3 + \nu_{c\mathcal{F}}^3 \nu_{c\mathcal{G}}^3\right)}{\left(\mu_{c\mathcal{F}}^6 + \nu_{c\mathcal{F}}^6\right) + \left(\mu_{c\mathcal{G}}^6 + \nu_{c\mathcal{G}}^6\right)} + 1 - \frac{|\rho_{cF} - \rho_{cG}|}{\sqrt{2}}\right). \tag{4}$$

Proposition 6. *If $c\mathcal{F} = \langle \mu_{c\mathcal{F}}, \nu_{c\mathcal{F}}; \rho_{cF} \rangle$, $c\mathcal{G} = \langle \mu_{c\mathcal{G}}, \nu_{c\mathcal{G}}; \rho_{cG} \rangle$ are two CFFNs, then $0 \leq \mathrm{CFFCSM}(c\mathcal{F}, c\mathcal{G}) \leq 1$.*

Proof. For the CFFN, we have $0 \leq \mu_{c\mathcal{G}}, \nu_{c\mathcal{G}} \leq 1$ with $0 \leq \mu_{c\mathcal{G}}^3 + \nu_{c\mathcal{G}}^3 \leq 1$.

Since $\rho \in [0, \sqrt{2}\,]$, we have $0 \leq |\rho_{cF} - \rho_{cG}|/\sqrt{2} \leq 1$.

This implies that $0 \leq (1/2)\left(\left(\mu_{c\mathcal{F}}^3 \mu_{c\mathcal{G}}^3 + \nu_{c\mathcal{F}}^3 \nu_{c\mathcal{G}}^3 / \sqrt[3]{\mu_{c\mathcal{F}}^6 + \nu_{c\mathcal{F}}^6}\sqrt[3]{\mu_{c\mathcal{G}}^6 + \nu_{c\mathcal{G}}^6}\right) + 1 - \left(|\rho_{cF} - \rho_{cG}|/\sqrt{2}\,\right)\right) \leq 1$.

Hence, we get $0 \leq \mathrm{CFFCSM}(c\mathcal{F}, c\mathcal{G}) \leq 1$. □

Proposition 7. *If $c\mathcal{F} = \langle \mu_{c\mathcal{F}}, \nu_{c\mathcal{F}}; \rho_{cF} \rangle$, $c\mathcal{G} = \langle \mu_{c\mathcal{G}}, \nu_{c\mathcal{G}}; \rho_{cG} \rangle$ are two CFFNs, then $\mathrm{CFFCSM}(c\mathcal{F}, c\mathcal{G}) = \mathrm{CFFCSM}(c\mathcal{G}, c\mathcal{F})$.*

Proof. We have

$$\begin{aligned}
\mathrm{CFFCSM}(c\mathcal{F}, c\mathcal{G}) &= \frac{1}{2}\left(\frac{\mu_{c\mathcal{F}}^3 \mu_{c\mathcal{G}}^3 + \nu_{c\mathcal{F}}^3 \nu_{c\mathcal{G}}^3}{\sqrt[3]{\mu_{c\mathcal{F}}^6 + \nu_{c\mathcal{F}}^6}\sqrt[3]{\mu_{c\mathcal{G}}^6 + \nu_{c\mathcal{G}}^6}} + 1 - \frac{|\rho_{cF} - \rho_{cG}|}{\sqrt{2}}\right) \\
&= \frac{1}{2}\left(\frac{\mu_{c\mathcal{G}}^3 \mu_{c\mathcal{F}}^3 + \nu_{c\mathcal{G}}^3 \nu_{c\mathcal{F}}^3}{\sqrt[3]{\mu_{c\mathcal{G}}^6 + \nu_{c\mathcal{G}}^6}\sqrt[3]{\mu_{c\mathcal{F}}^6 + \nu_{c\mathcal{F}}^6}} + 1 - \frac{|\rho_{cG} - \rho_{cF}|}{\sqrt{2}}\right) \\
&= \mathrm{CFFCSM}(c\mathcal{G}, c\mathcal{F}).
\end{aligned} \tag{5}$$

Proposition 8. *If $c\mathcal{F} = \langle \mu_{c\mathcal{F}}, \nu_{c\mathcal{F}}; \rho_{cF}\rangle$, $c\mathcal{G} = \langle \mu_{c\mathcal{G}}, \nu_{c\mathcal{G}}; \rho_{cG}\rangle$ are two CFFNs, then $CFFCSM(c\mathcal{F}, c\mathcal{G}) = 1$ iff $c\mathcal{F} = c\mathcal{G}$.*

Proof. If $c\mathcal{F} = c\mathcal{F}$, then equation (3) implies

$$\mathrm{CFFCSM}(c\mathcal{F}, c\mathcal{F}) = \frac{1}{2}\left(\frac{\mu_{c\mathcal{F}}^3\mu_{c\mathcal{F}}^3 + \nu_{c\mathcal{F}}^3\nu_{c\mathcal{F}}^3}{\sqrt[3]{\mu_{c\mathcal{F}}^6 + \nu_{c\mathcal{F}}^6}\sqrt[3]{\mu_{c\mathcal{F}}^6 + \nu_{c\mathcal{F}}^6}} + 1 - \frac{\left|\rho_{cF} - \rho_{cF}\right|}{\sqrt{2}}\right) = 1. \tag{6}$$

□

Proposition 9. *If $c\mathcal{F} = \langle \mu_{c\mathcal{F}}, \nu_{c\mathcal{F}}; \rho_{cF}\rangle$, $c\mathcal{G} = \langle \mu_{c\mathcal{G}}, \nu_{c\mathcal{G}}; \rho_{cG}\rangle$ are two CFFNs, then all of the subsequent traits hold.*

(i) $0 \leq CFFDSM(c\mathcal{F}, c\mathcal{G}) \leq 1$.

(ii) $CFFDSM(c\mathcal{F}, c\mathcal{G}) = CFFDSM(c\mathcal{G}, c\mathcal{F})$.

(iii) $CFFDSM(c\mathcal{F}, c\mathcal{G}) = 1$ iff $c\mathcal{F} = c\mathcal{G}$.

Proof. Proof is similar to Propositions 6–8.

Numerical Example If $c\mathcal{F} = \langle 0.94, 0.4; 0.3\rangle$ and $c\mathcal{G} = \langle 0.65, 0.83; 0.25\rangle$ are two CFFNs, then $\mathrm{CFFCSM}(c\mathcal{F}, c\mathcal{G}) = 0.69$ and $\mathrm{CFFDSM}(c\mathcal{F}, c\mathcal{G}) = 0.72$. □

4. Application of Circular Fermatean Fuzzy Number in Pattern Recognition

In this section, we provide some applications to understand the pattern recognition model. In references [3, 10, 16], the authors have taken assumed data to validate their proposed model. In the same way, to check the effectiveness of our proposed model, we also have considered assumed data provided in Sections 4.1 and 4.2 for loan allocation and laptop selection. With our new algorithm, anyone can have expertise to suggest best out of available loans, products, electing leaders, etc., according to their eligibility and requirements.

4.1. Bank Loan Allocation Using Circular Fermatean Fuzzy Similarity Measure. In the olden days, farmers used seeds and grains to borrow capital, with farm animals as repayment options. The Arthasastra [67, 68] written by Kautilya, a classical Indian book on the art of politics emanating from the 4th or 3rd century BC, indicates the existence of debtors, financiers, and interest rates. Since then, financing has evolved into a complicated financial procedure before shifting into a modern, simplified method in the modern world. A short-term loan is a loan that can be received and reimbursed very quickly, typically within a few hours or even minutes after completing the application for the loan. This type of loan is mostly offered by digital lenders, and the application procedure often happens quickly and online, with little documentation required. In this section, we entail how the pattern recognition model is used to allocate bank loans for customers.

The approaches in this procedure are described as follows:

Step 1. The expert team of the bank fixes the evaluation parameters of the specific loan in terms of FFN including the no loan option.

Step 2. FFN evaluation criteria are converted into CCFN using Proposition 3 and it is taken as a CFF loan pattern.

Step 3. The client's applications undergo scrutiny, and FFNs are given away to their parameters and expressed as CFFN, which is taken as CFF borrower patterns.

Step 4. The similarity between the borrowers' criteria and evaluation parameters is measured using the CFFCSM and the CFFDSM.

Step 5. Possible loans for a customer are ranked according to the similarity values, from higher to lower.

Step 6. The loan pattern that has the highest similarity value to the customer data pattern is allocated for the particular customer.

The loan allocation framework is illustrated in the flowchart (Figure 2).

For allocation of loans to the customers, every bank has their own criteria. In this study, the personal loan (PL), jewelry loan (JL), mortgage loan (ML), and no loan (NL) are fixed with criteria, namely, regular income (RI), CIBIL score (CS), jewelry availability (JA), and asset value (AV). The imprecise boundary makes it difficult to quantify the available loans and loan requirements by exact numbers. We consider the following loan pattern with assumed data in terms of DA and DNA using FFN for the criteria of the loans which are expressed in Table 2.

Figure 3 represents the decision makers' opinion on each criterion in the form of the circle with the mentioned colors. The blue color shaded region represents the acceptance region. The area of the circle outside the acceptance region is excluded in the further calculation procedure.

Lina, Louis, Iris, Denis, and Pierre's loan requirement application forms are scrutinized. The regular in come, CIBIL score, jewellery availability and asset value are measured in terms of the FFN according to their DA and DNA. By using Proposition 3, the FFN is converted into CFFN. The obtained CFFN is tabulated in Table 3.

For each person, their RI, CS, JA, and AV in terms of FFN are converted to CFFN. It takes the circular representation in the acceptance region and is taken as eligibility pattern corresponding to each criterion. The visualization of CFFN is given in Figure 4.

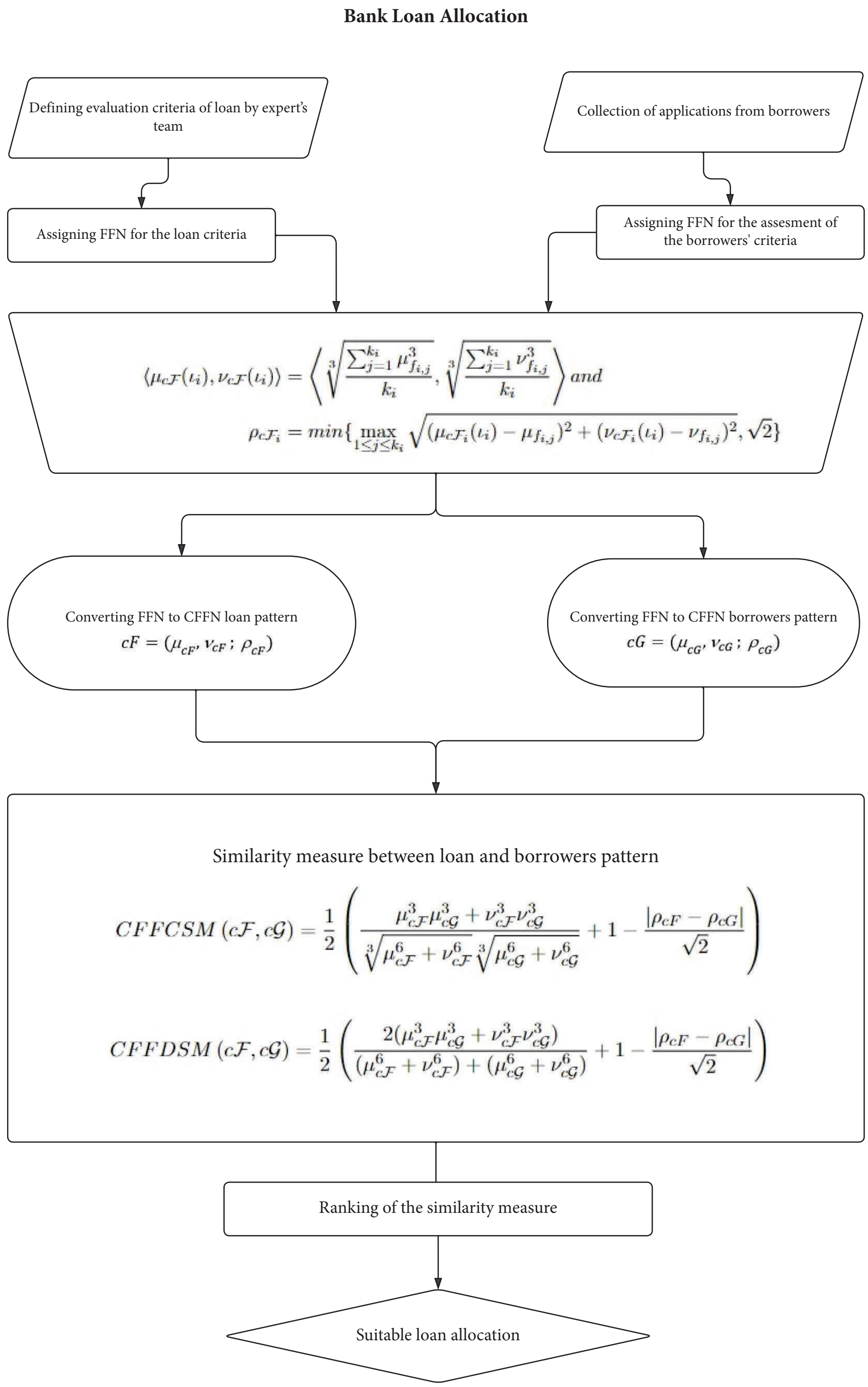


Figure 2: Loan allocation flowchart.

Bank requirement and customers' eligibility are represented as a circle instead of single numerical value. In such circumstances, measuring the similarity will give the efficient decision. The CFFCSM and CFFDSM mentioned in (3) and (4) are utilized between the customers' data and available loans. The particular loan that gets the highest similarity measure is allocated to the concerned customer. The conclusions are tabulated in Table 4.

The graph in Figure 5 exhibits the consolidated CFFCSM and CFFDSM of all five applicants to the loan criteria.

Table 2: Loan FFN and CFFN criteria.

	PL	JL	ML	NL
RI	(0.6, 0.6)	(0.1, 0.8)	(0.1, 0.96)	(0.3, 0.94)
CS	(0.9, 0.4)	(0.2, 0.5)	(0.4, 0.6)	(0.2, 0.7)
JA	(0.3, 0.9)	(0.9, 0.1)	(0.2, 0.96)	(0.1, 0.85)
AV	(0.1, 0.7)	(0.2, 0.9)	(0.85, 0.3)	(0.2, 0.75)
CFFN	(0.62, 0.70; 0.52)	(0.57, 0.70; 0.68)	(0.55, 0.80; 0.58)	(0.22, 0.82; 0.14)

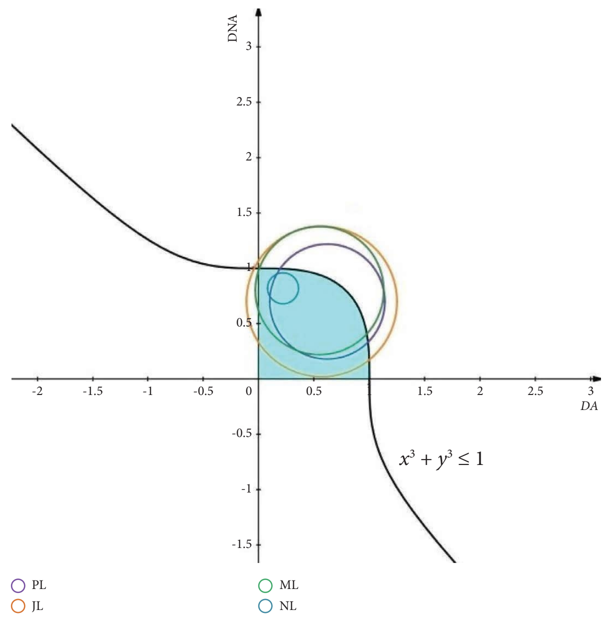


Figure 3: Graphical representation of loan requirement as CFFN.

Table 3: Customer FFN and CFFN criteria.

	Lina	Louis	Iris	Denis	Pierre
RI	(0.25, 0.86)	(0.96, 0.35)	(0.1, 0.98)	(0.63, 0.35)	(0.01, 0.9)
CS	(0.15, 0.9)	(0.5, 0.5)	(0.25, 0.9)	(0.6, 0.5)	(0.8, 0.5)
JA	(0.25, 0.8)	(0.2, 0.5)	(0.1, 0.8)	(0.1, 0.4)	(0.96, 0.3)
AV	(0.3, 0.95)	(0.1, 0.6)	(0.89, 0.1)	(0.05, 0.4)	(0.2, 0.8)
CFFN	(0.24, 0.88; 0.10)	(0.63, 0.50; 0.54)	(0.57, 0.82; 0.79)	(0.49, 0.42; 0.44)	(0.71, 0.70; 0.72)

4.1.1. Data Analysis and Simulations. The similarity values of the applicants Lina, Lois, Iris, Denis, and Pierre are displayed in Figures 6–10, respectively, that are drawn in the 3D graphic calculator which confirms that our proposed method has effectively worked between customer criteria and loan requirements.

The CFFCSM and CFFDSM of Lina criteria pattern and the loan patterns are displayed in the graph of Figure 11. Since Lina's criteria values have a high similarity to those of no loan, she is denied for allocation of loan. From Figure 6, we can confirm that the shaded circular portion which is the eligibility pattern of Lina has high similarity of the circle that represents no loan pattern.

Personal loan needs considerable regular income and good CIBIL score as per the criterion requirement. The graph in Figure 12 shows Louis has a high regular income and a sufficient CIBIL score, so he has been allocated a personal loan. Figure 7 shows that the eligibility pattern of Louis has high similarity with personal loan.

Iris does not have enough regular income and a good credit score, but she has a high value of jewelry and assets that she is allocated a jewelry loan as well as a mortgage loan. Also, from Figures 8 and 13, it is clear that eligibility criterion of Iris is similar to both jewelry loan and mortgage loan.

Denis has a regular income and a good CIBIL score, so he is allocated a personal loan. The graph in Figure 14 shows the similarity values of all the five loans. Also, it is evident from Figure 9 that Denis eligibility pattern has high similarity with personal loan.

Since Pierre has considerable jewelry compared to all other criteria, he is allocated a jewelry loan. The graphs in

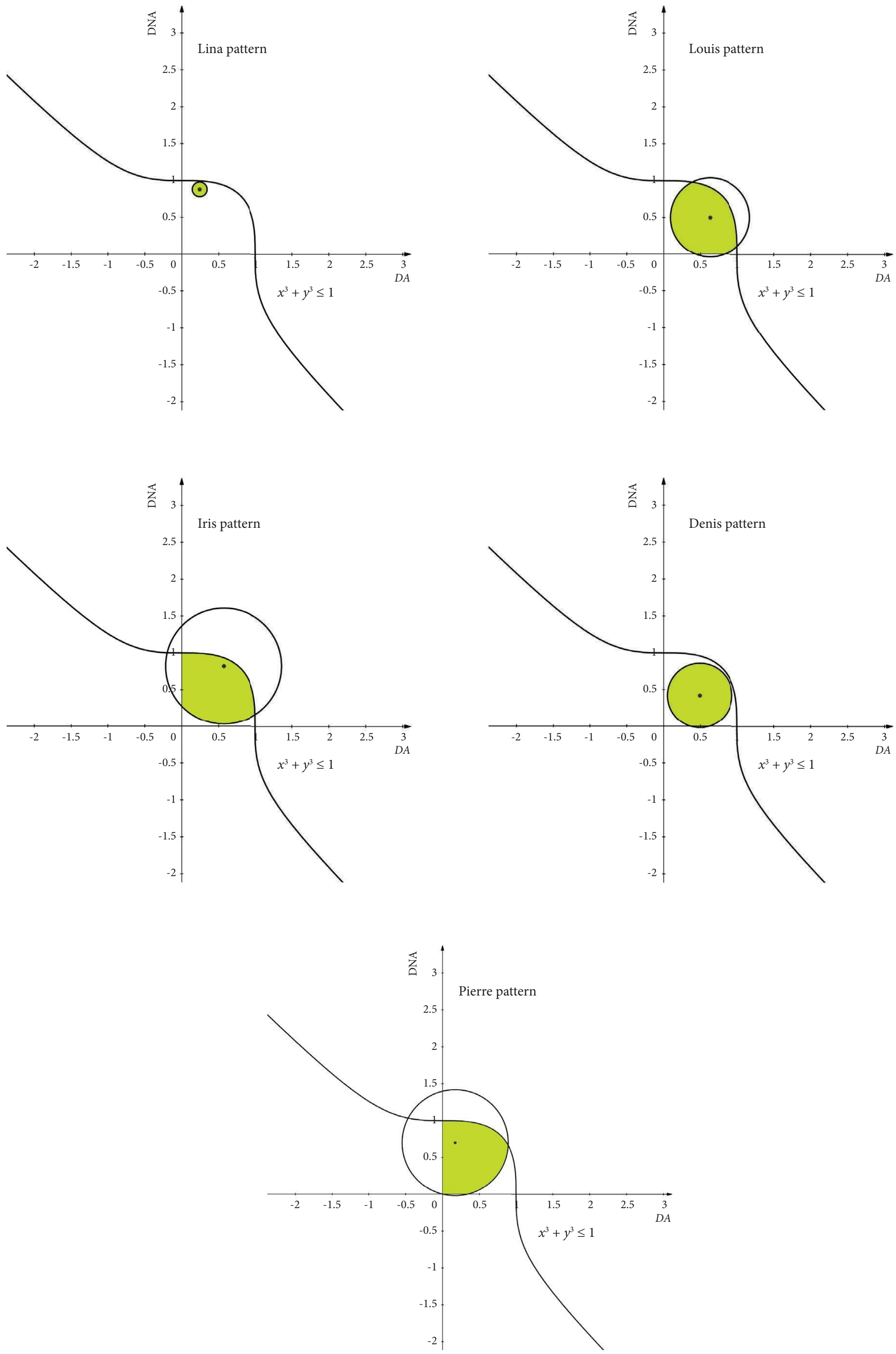


Figure 4: Customers' eligibility pattern.

Figures 10 and 15 show the similarity values and visualization of all the five loans which confirms that Pierre eligibility pattern has high similarity with jewelry loan.

The regression lines and the correlation coefficient between CFFCSM and CFFDSM were obtained using IBM SPSS software 27 for the five borrowers and displayed in

Table 4: CFFCSM and CFFDSM between loans and customer criteria.

	CFFSM	PL	JL	ML	NL	Allocated loan
Lina	CFFCSM	0.62	0.58	0.67	0.85	No loan
	CFFDSM	0.72	0.68	0.79	0.97	
Louis	CFFCSM	0.71	0.65	0.68	0.48	Personal loan
	CFFDSM	0.91	0.84	0.78	0.55	
Iris	CFFCSM	0.70	0.76	0.76	0.60	Jewelry loan/mortgage loan
	CFFDSM	0.86	0.92	0.92	0.75	
Denis	CFFCSM	0.65	0.58	0.61	0.51	Personal loan
	CFFDSM	0.75	0.69	0.64	0.53	
Pierre	CFFCSM	0.72	0.76	0.74	0.53	Jewelry loan
	CFFDSM	0.92	0.95	0.89	0.65	

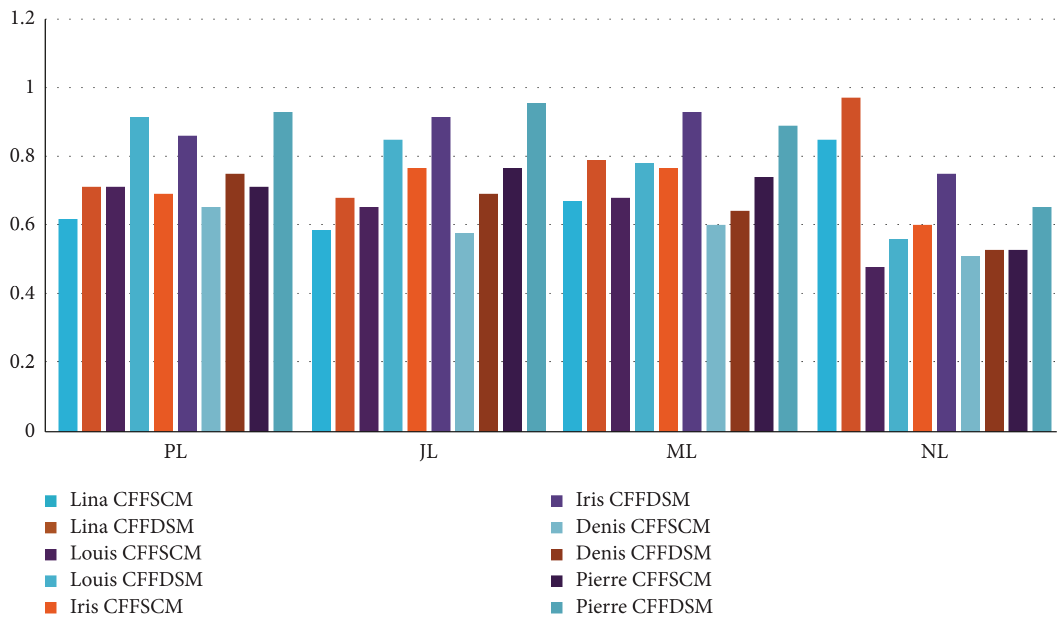


Figure 5: Consolidated similarity measures of loan and customer pattern.

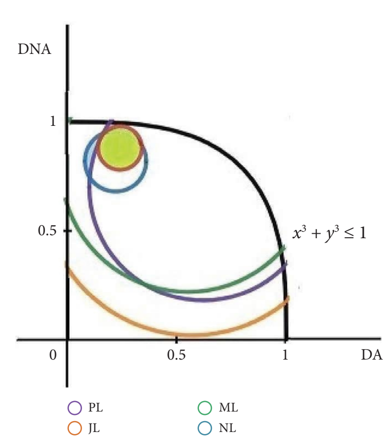


Figure 6: Circular representation of Lina similarity measures for loans.

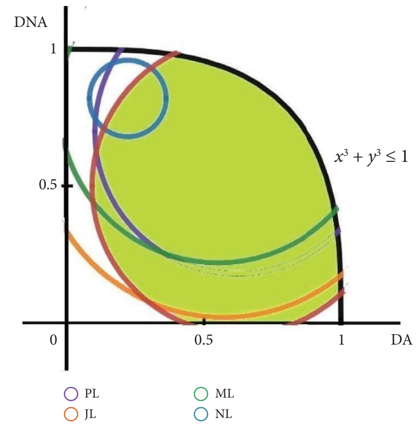


Figure 7: Circular representation of Louis similarity measures for loans.

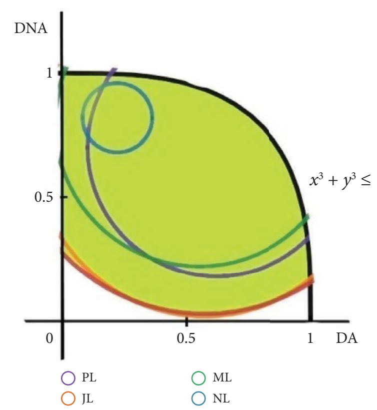


Figure 8: Circular representation of Iris similarity measures for loans.

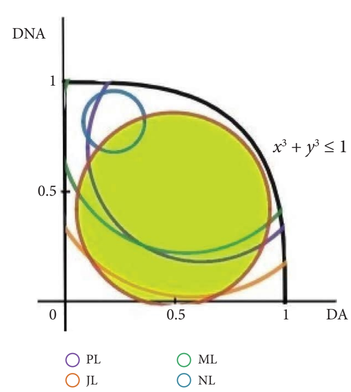


Figure 9: Circular representation of Denis similarity measures for loans.

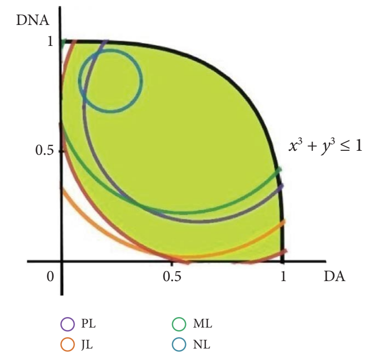


Figure 10: Circular representation of Pierre similarity measures for loans.

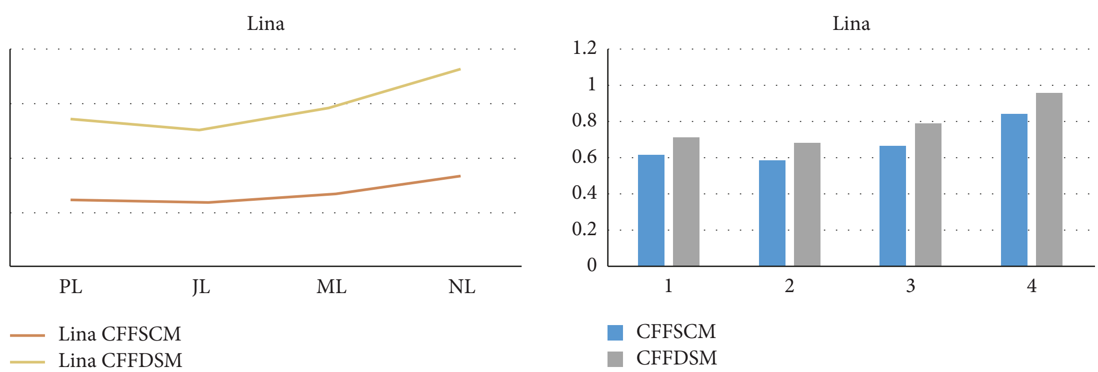


Figure 11: Lina similarity measures for loans.

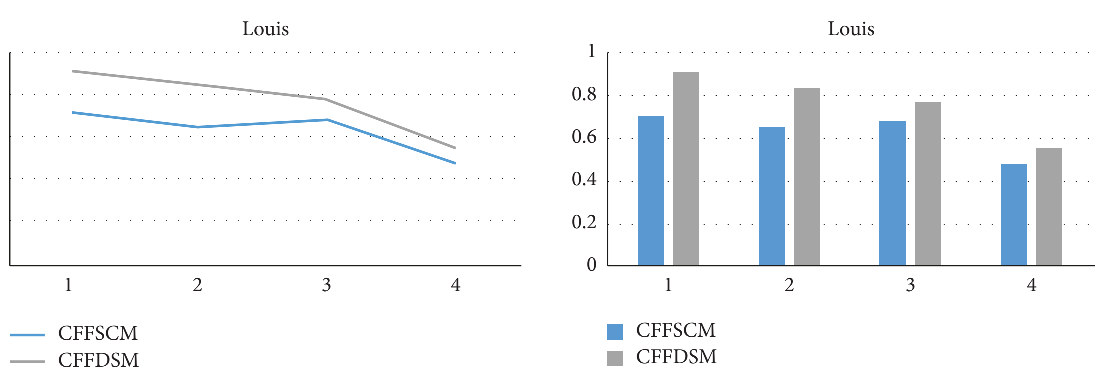


Figure 12: Louis similarity measures for loans.

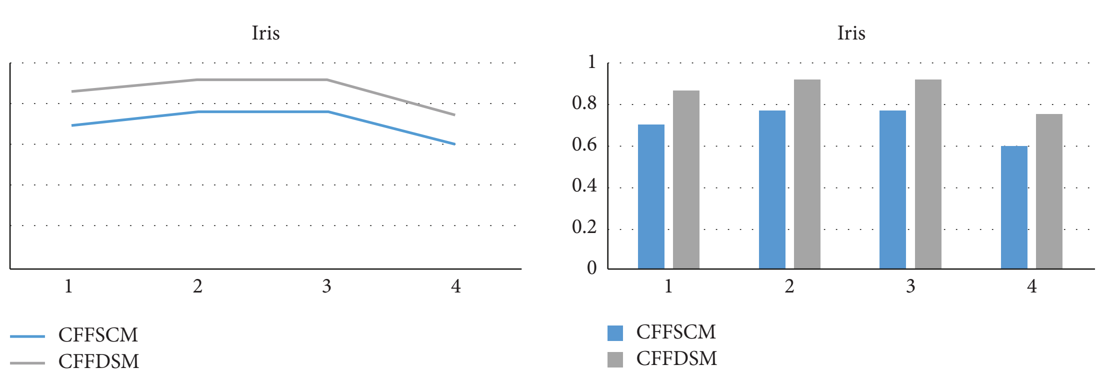


Figure 13: Iris similarity measures for loans.

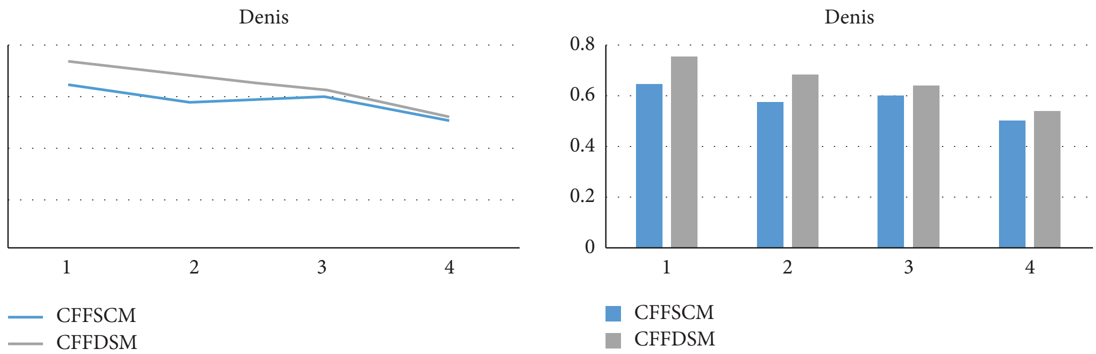


FIGURE 14: Denis similarity measures for loans.

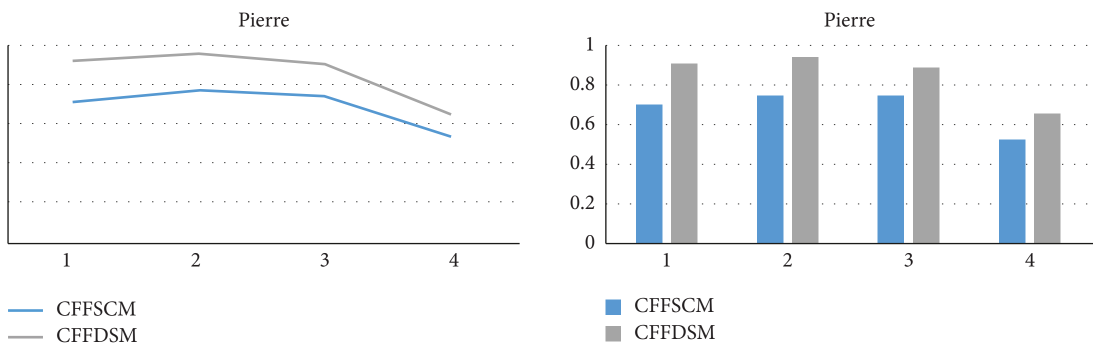


FIGURE 15: Pierre similarity measures for loans.

Figure 16. All five customers' CFFCSM and CFFDSM are positively correlated by which we can ensure that our proposed similarity measures are consistent.

4.2. Laptop Selection Using Circular Fermatean Fuzzy Similarity Measure. In this section, we suggest a suitable laptop for the customer according to their requirement by using CFFCSM and CFFDSM like the procedure and flowchart (Figure 2) in Section 4.

By taking into account the criteria of software adaptability, price, graphics quality, and security features like being free of bugs and hacking, we consider four types of laptops: gaming laptops (GL) for games; home and student laptops (HSL) for regular use; business laptops (BL) for commercial purposes; and MacBooks (MB) for high security. The specifications of all the laptops are tabulated in Table 5 in the form of FFN and CFFN.

Figure 17 illustrates the patterns of the available laptops. The region enclosed between the circles and the curve $x^3 + y^3 \leq 1$ in the yellow color shaded region represents the pattern.

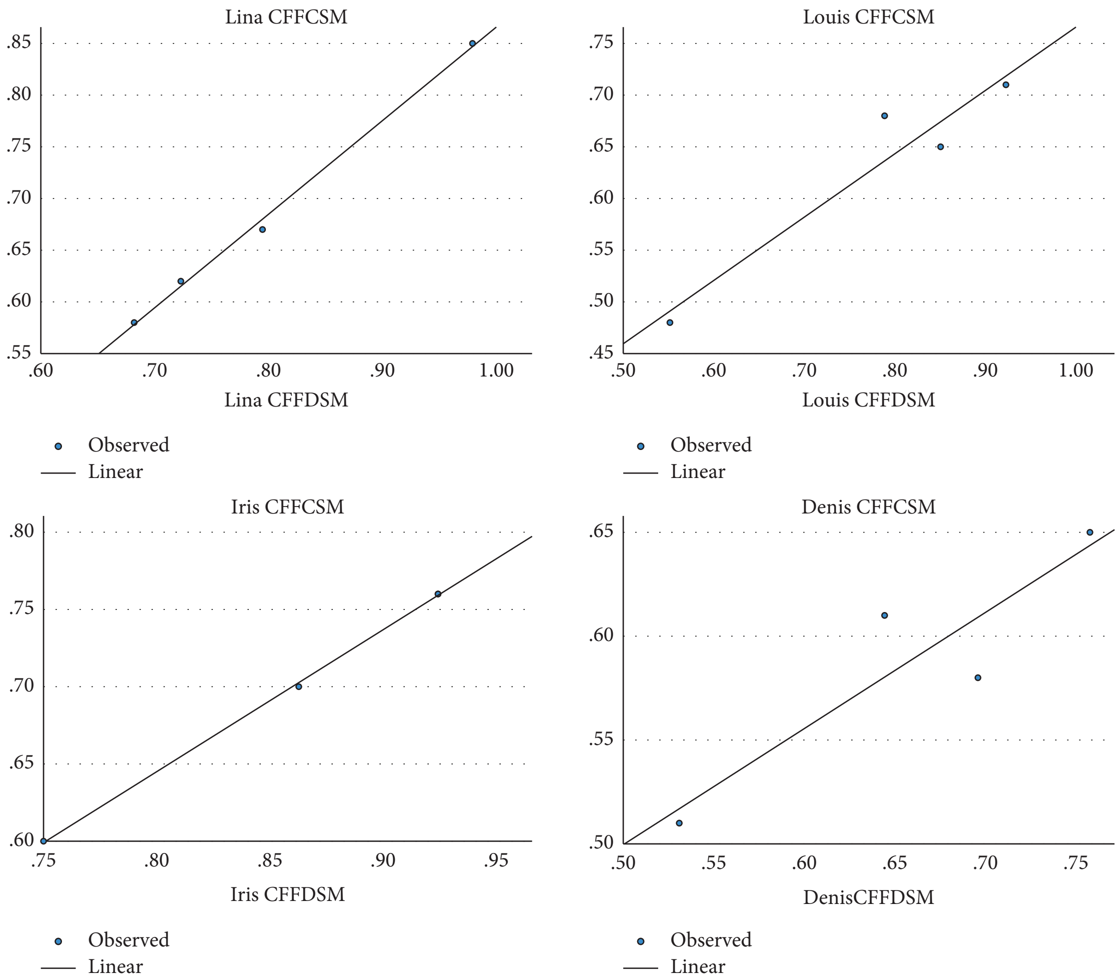
Five persons' requirements under each criterion are tabulated in Table 6 in the form of FFN and CFFN. Using equations (3) and (4) , the CFFCSM and CFFDSM between the requirement criteria and the laptop pattern are calculated. The laptop that has the highest similarity is suggested as in Table 7.

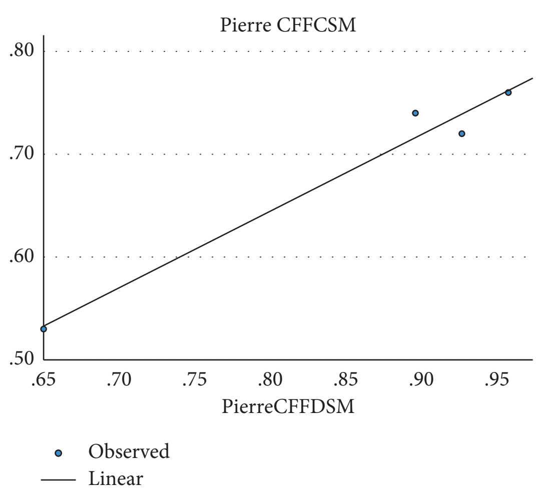
If the difference between the similarity measures of two laptops is very small, then both the laptops are suggested for the customer. Since the requirement of person 3 and person 5 has nearly equal similarity with business and gaming laptop, those two are suggested to them.

Figure 18 exhibits the consolidated CFFCSM and CFFDSM of all five customers to the requirement criteria.

4.2.1. Data Analysis and Simulations. Figures 19–23 illustrate the suggestions of the laptop to the customers according to their requirement. Figures 19(a)–23(a) indicate the requirement pattern of the customer, and the shaded portion in Figures 19(b)–23(b) portrays the high similarity to the suggested laptop. All the five persons are allocated with the laptop that has high similarity with their requirement.

4.2.2. Comparison Analysis. Circular Fermatean fuzzy sets, which capture data in circular form more effectively than existing fuzzy sets, provide a more robust framework for managing uncertainty in some fields depending on the particular requirements of the application and the type of





Borrowers	R square value between CFFCSM and CFFDSM
Lina	**0.997**
Louis	**0.915**
Iris	**0.999**
Denis	**0.827**
Pierre	**0.972**

Figure 16: The regression lines of CFFCSM and CFFDSM.

Table 5: Specification of FFN and CFFN criteria.

	GL	HSL	BL	MB
Software	(0.5, 0.6)	(0.6, 0.4)	(0.7, 0.8)	(0.3, 0.8)
Graphics	(0.9, 0.3)	(0.5, 0.6)	(0.2, 0.9)	(0.2, 0.9)
Price	(0.4, 0.9)	(0.4, 0.8)	(0.8, 0.3)	(1, 0)
Security	(0.5, 0.7)	(0.3, 0.8)	(0.8, 0.5)	(0.99, 0.1)
CFFN	(0.64, 0.69; 0.47)	(0.48, 0.69; 0.31)	(0.70, 0.70; 0.54)	(0.79, 0.68; 0.71)

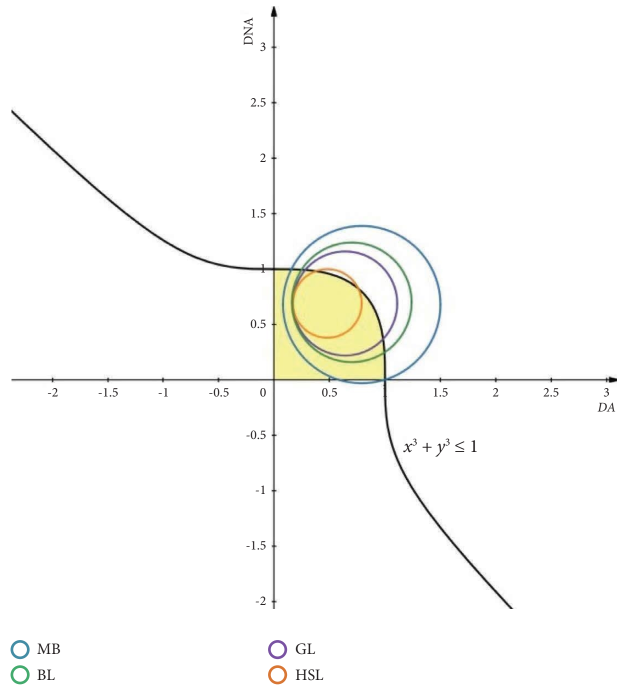


Figure 17: CFFN pattern of the laptops.

Table 6: Requirement of FFN and CFFN criteria.

	Person 1	Person 2	Person 3	Person 4	Person 5
Software	(0.7, 0.65)	(0.3, 0.6)	(0.4, 0.65)	(0.3, 0.9)	(0.7, 0.55)
Graphics	(0.3, 0.6)	(0.5, 0.9)	(0.95, 0.2)	(0.2, 0.85)	(0.85, 0.35)
Price	(0.5, 0.9)	(0.85, 0.1)	(0.3, 0.45)	(1, 0)	(0.6, 0.2)
Security	(0.3, 0.9)	(0.8, 0.1)	(0.2, 0.55)	(1, 0)	(0.4, 0.8)
CFFN	(0.51, 0.79; 0.28)	(0.68, 0.62; 0.54)	(0.62, 0.51; 0.45)	(0.80, 0.70; 0.72)	(0.68, 0.57; 0.37)

Table 7: CFFCSM and CFFDSM of laptops and customer criteria.

	CFFSM	GL	HSL	BL	MB	Suggested laptops
Person 1	CFFCSM	0.706	0.766	0.681	0.592	Home and student laptop
	CFFDSM	0.884	0.952	0.843	0.708	
Person 2	CFFCSM	0.739	0.629	0.784	0.750	Business
	CFFDSM	0.956	0.823	0.981	0.904	
Person 3	CFFCSM	0.720	0.619	0.717	0.683	Gaming/business laptop
	CFFDSM	0.919	0.812	0.881	0.793	
Person 4	CFFCSM	0.712	0.588	0.761	0.850	MacBook
	CFFDSM	0.854	0.691	0.912	0.994	
Person 5	CFFCSM	0.715	0.666	0.713	0.680	Gaming laptop
	CFFDSM	0.928	0.854	0.903	0.826	

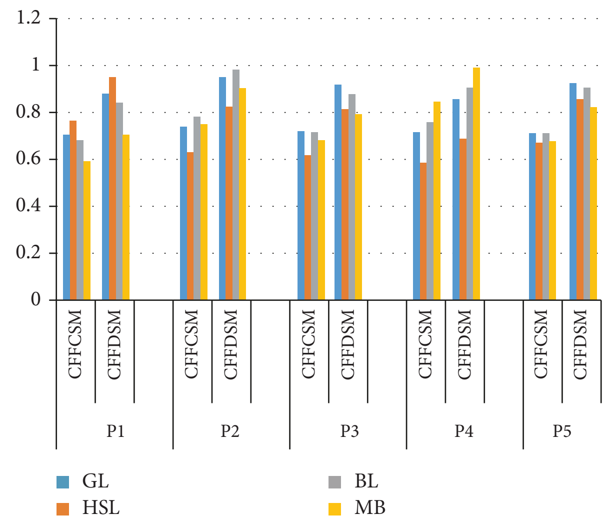


Figure 18: Consolidated similarity measures.

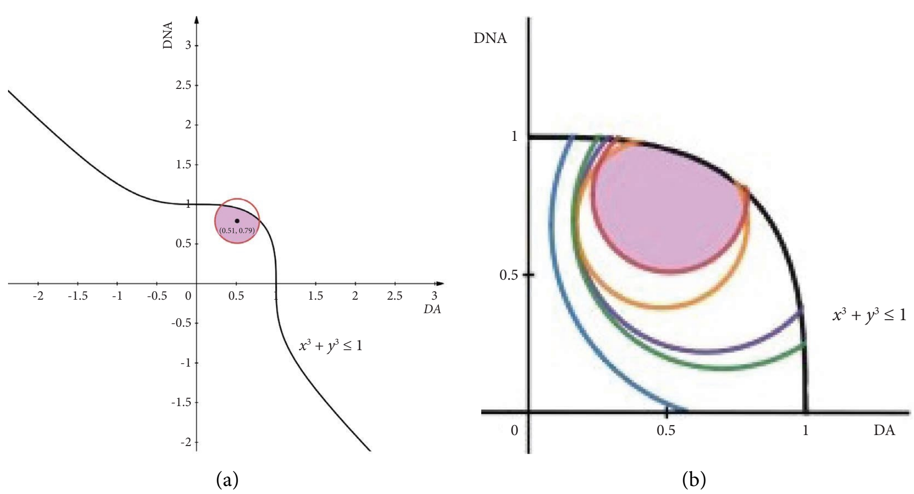


Figure 19: Person 1 laptop suggestion. (a) Requirement pattern. (b) Pattern similarity.

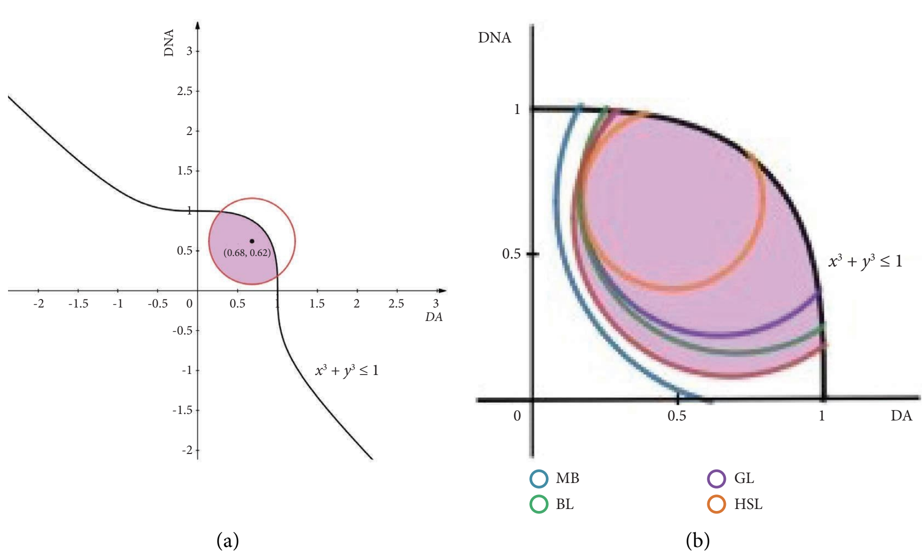
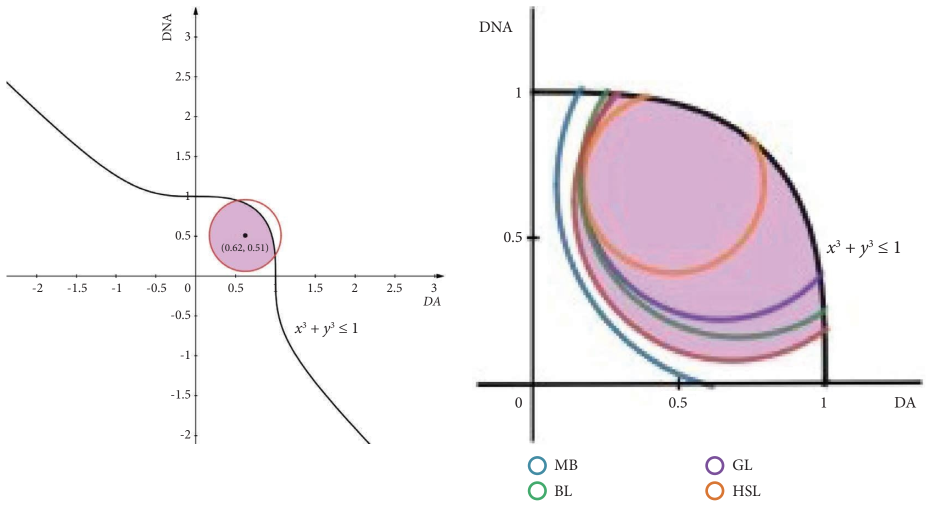
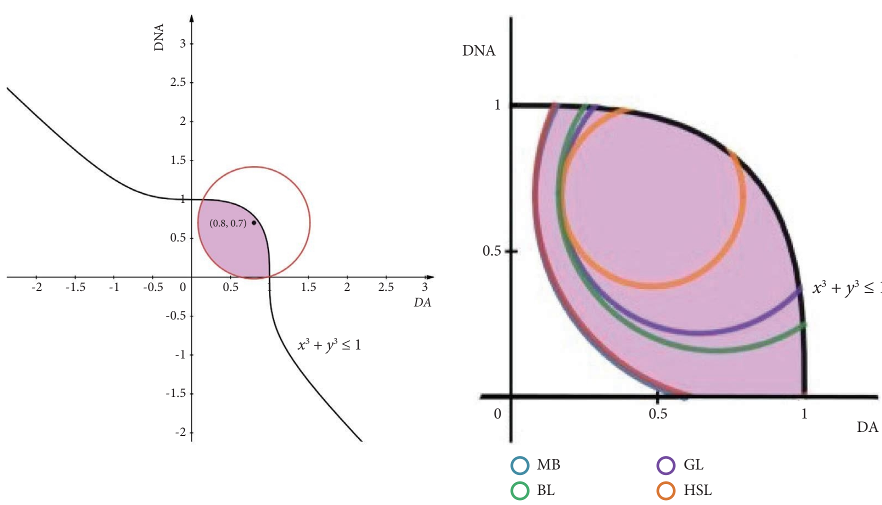


Figure 20: Person 2 laptop suggestion. (a) Requirement pattern. (b) Pattern similarity.



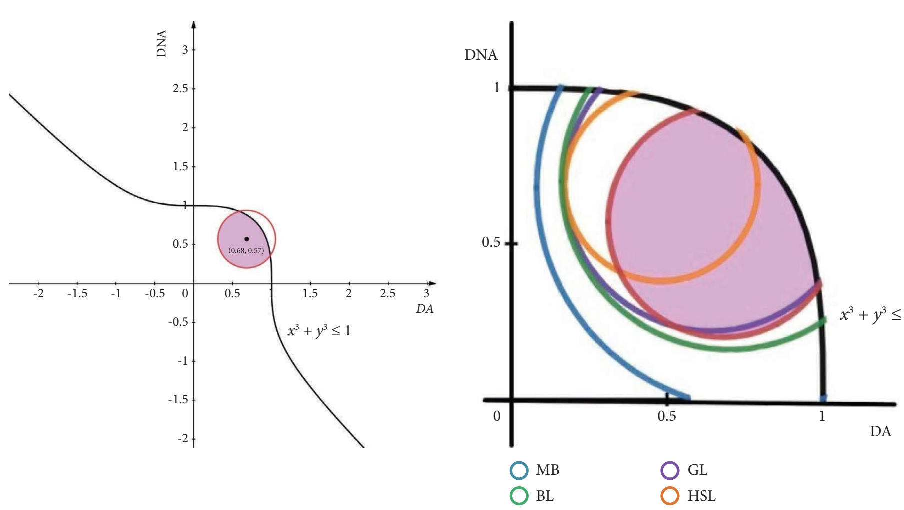
(a) (b)

FIGURE 21: Person 3 laptop suggestion. (a) Requirement pattern. (b) Pattern similarity.



(a) (b)

FIGURE 22: Person 4 laptop suggestion. (a) Requirement pattern. (b) Pattern similarity.



(a) (b)

FIGURE 23: Person 5 laptop suggestion. (a) Requirement pattern. (b) Pattern similarity.

Table 8: Comparison of proposed SM with existing SM.

Persons	Similarity measure	Suggestion
Lina	CFFCSM	No loan
	CFFDSM	No loan
	FFCSM	No loan
	S_{FF}	No loan
Louis	CFFCSM	Personal loan
	CFFDSM	Personal loan
	FFCSM	Personal loan
	S_{FF}	Personal loan
Iris	CFFCSM	Mortgage loan
	CFFDSM	Mortgage loan
	FFCSM	Mortgage loan
	S_{FF}	Mortgage loan
Denis	CFFCSM	Personal loan
	CFFDSM	Personal loan
	FFCSM	Personal loan
	S_{FF}	Personal loan
Pierre	CFFCSM	Jewelry loan
	CFFDSM	Jewelry loan
	FFCSM	Jewelry loan
	S_{FF}	Jewelry loan
Person 1	CFFCSM	Home laptop
	CFFDSM	Home laptop
	FFCSM	Home laptop
	S_{FF}	Home laptop
Person 2	CFFCSM	Business laptop
	CFFDSM	Business laptop
	FFCSM	Business laptop
	S_{FF}	Business laptop
Person 3	CFFCSM	Business laptop
	CFFDSM	Business laptop
	FFCSM	Business laptop
	S_{FF}	Business laptop
Person 4	CFFCSM	MacBook
	CFFDSM	MacBook
	FFCSM	MacBook
	S_{FF}	MacBook
Person 5	CFFCSM	Gaming laptop
	CFFDSM	Gaming laptop
	FFCSM	Gaming laptop
	S_{FF}	Gaming laptop

data. We have compared our proposed CFFSM with existing FFSM defined in [52, 57] in Table 8, which ensures the stability of the proposed CFFSM.

Compared to FFSM, the CFFSM is typically adaptable to randomness in circular data. When data points are scattered unevenly inside the circle or when the circular patterns vary over time, it can handle such scenarios with efficient execution.

5. Limitations, Managerial, and Theoretical Implications

The circle will be quite big and the center will deviate if one of the decision makers has a diverse opinion. In this case, the diverse opinion can be ignored to achieve the ideal result. The accuracy and score functions of FF sets, which are obtained from their DNA and DA, can be utilized in comparing them. However, since the radius is a part of the construction of the CFF set, comparisons are difficult. Comparability is limited to the CFF sets with same centers. This limitation prevents the alternatives from being rated and this may be overcome by defining effective CFF score function.

6. Conclusion and Future Work

Pattern recognition is an emerging field of development and research encompassing the processing of figurative, arithmetic, and other types of information generated by reality. In this study, CFF cosine and Dice similarity measures are introduced and their properties are verified. A bank loan is sanctioned to the applicants using pattern recognition by means of CFFSM between the loan criteria and the applicants' eligibility criteria. The graphical representation by 3D graph calculator for the similarity measurement is a new type of approach through our proposed circular Fermatean fuzzy set. It enhances the reliability of the application of circular Fermatean fuzzy set. Simulations of data analysis validate the potency of the suggested method. CFFCSM and CFFDSM fit is done through regression line using SPSS software 27 (see Figure 16). The same procedure is applied to suggest the suitable laptop for the customers based on their required features. Our proposed formulas can be used to construct efficient algorithms which help to implement artificial intelligence in banking sector as well as in purchase. In future work, an effective CFF score function and CFF accuracy function will be set up for ranking addressed in limitation section. Also, their topological structure will be investigated in order to create the CFF topological space. Various CFF aggregation operators, CFF distance measures, and CFF similarity measures will be defined and applied in machine learning and clustering to enhance their application in the fields of medical and image processing. Thus, with the help of programming languages, an algorithm for machine learning in artificial intelligence will be developed by our proposed similarity measure which can be extended to the problem of electing the best leader through preelection survey from the public.

Data Availability

All the used data are available within the article.

Disclosure

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All the authors contributed equally to the manuscript.

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