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Research Article

Stability Analysis of SIRS Model considering Pulse Vaccination and Elimination Disturbance

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It is well known that many natural phenomena and human activities do exhibit impulsive effects in the fields of epidemiology. At the same time, compared with a single control strategy, it is obvious that the multiple control strategies are more beneficial to restrain the spread of infectious diseases. In this paper, we consider pulse vaccination and pulse elimination strategies at the same time and establish an SIRS epidemic model with standard incidence. Firstly, according to the stroboscopic mapping method of the discrete dynamical system, the disease-free T periodic solution of the model under the condition of pulse vaccination and pulse elimination is obtained. Secondly, the basic reproductive number R_0 is defined, and the local asymptotic stability of the disease-free T periodic solution is proved by Floquet theory for $R_0 < 1$. Finally, based on the impulsive differential inequality theory, the global asymptotic stability of the disease-free T periodic solution is given for $R_0 < 1$, and the disease dies out eventually. The results show that in order to stop the disease epidemic, it is necessary to choose the appropriate vaccination rate and elimination rate and the appropriate impulsive period.

1. Introduction

Infectious diseases have been harmful to human health since too many years ago. Some of them cause pain and panic to the human and even lead to the destruction of country, such as the plague and leprosy. So, the impact of infectious diseases to human beings is quite obvious, and its prevalence and spread may bring us a great disaster. With the development of the society, some infectious diseases that were extinct or under control are resurgent and spreading. Some new infectious diseases are also appearing. As we all know, the new coronavirus-infected pneumonia has brought new disasters and trials to the global humanity in 2019. Therefore, in order to control the spread of infectious diseases, the research on the pathogenesis, the law of infection, and the strategy of prevention and control is becoming more and more important. The study of the spread and control measures of infectious diseases by establishing mathematical models is an important research direction in applied mathematics [1–5]. A lot of results have been achieved in infectious disease modeling, but most of the models involved are ordinary differential equations or time-lag differential equations [6–10]. Methods used in infectious disease modeling include the method of constructing Lyapunov functions, the theory of limit equations, matrix theory, branching theory, the theory of K-sequence monotone systems, the theory of centralized epidemics, and so on [11–13].

In order to effectively prevent the epidemic and development of infectious diseases, it is often necessary to develop control strategies according to the different transmission laws of infectious diseases. Vaccination strategy is an effective way to prevent the outbreak and spread of infectious diseases. There are also many studies on the model of infectious diseases under the action of vaccination [14–16]. Karand and Batabyal [14] focused on the study of a nonlinear mathematical SIR epidemic model with a vaccination program and discussed the existence and the stability of both the disease-free and endemic equilibrium. Liu et al. [16] built up a novel SEIRW model with the vaccination to the newborn children

and analyzed the stability of the model with time-varying perturbation to predict the evolution tendency of the disease. In addition, the elimination strategy is also an important measure to prevent and control infectious diseases [17, 18]. It has been used to tackle diseases caused by animals or spreading in animals such as avian influenza, tuberculosis, tetanus, and rotavirus infection.

Various continuous kinetic infectious disease models have been widely used to explain the transmission mechanisms of infectious diseases and are increasingly becoming important tools for analyzing and controlling disease transmission. However, in real life, we will encounter the growth pattern of many populations and the control of human disease is not continuous but impulsive. The dynamic equation of the infectious disease given in this case is the impulsive differential equation. Impulsive differential equations are able to describe problems with periodic motions that change instantaneously at a point, such as periodic insecticide drops, periodic medication to treat diseases, and seasonal vaccinations, which are all impulsive phenomena. The basic theory and application of impulsive differential equation have attracted the attention of many scholars, and a lot of results have been obtained [19–22]. The use of infectious disease models with impulsive vaccination to study the spread of vaccine-controlled diseases can obtain a more realistic pattern of disease development, which provides a theoretical basis for the development of disease control strategies. Therefore, the pulse vaccination models have been widely concerned [23–29]. Liu et al. [23] proved the global stability of an age-structured SIR epidemic model with impulsive vaccination strategy; Sunita and Kuldeep [24] discussed the stability of an SIRS epidemic model with nonlinear incidence rate and pulse vaccination; Nie et al. [25] proposed an SIR epidemic model with state dependent pulse vaccination; Jiang and Yang [26] studied the dynamical behavior of an SIR epidemic model with birth pulse and pulse vaccination; Nie et al. [27] considered two SIVS epidemic models, where state-dependent pulse vaccination control strategies are introduced. Yang et al. [28] formulated an SIS epidemic model in a patchy environment with pulse vaccination and quarantine at two different fixed moments by impulsive differential equations. Hao et al. [29] established an SIRS epidemic model with birth pulse, pulse vaccination, and saturation incidence and discussed the stability of the infection-free periodic solution and the endemic periodic solution.

However, there are few literature studies about the infectious models considering both pulse vaccination and pulse elimination. Therefore, motivated by the above works and [17, 18, 23–29], in this paper, we will consider pulse vaccination and pulse elimination strategies at the same time and establish an SIRS epidemic model with standard incidence. Our paper is organized as follows. In Section 2, we formulate an SIRS model under pulse vaccination and impulsive elimination disturbance. In Section 3, we discuss the existence of the disease-free periodic solution. In Section 4, we prove the local stability and the global stability of the disease-free periodic solution.

2. Model Formulation

In this section, an SIRS model under pulse vaccination and elimination disturbance is proposed.

Assume that the total host population is partitioned into three classes: the susceptible, the infected, and the recovered individuals, denoted by S, I, and R, respectively. The total host population size at time t is denoted by (t), with $N(t) = S(t) + I(t) + R(t)$. According to the modeling idea of infectious disease dynamics warehouse, an SIRS epidemic model with pulse vaccination and elimination disturbance can be established:

$$\begin{cases} S' = aN - \lambda \dfrac{SI}{N} - \mathrm{d}S + \delta R, \\ I' = \lambda \dfrac{SI}{N} - (\gamma + \alpha + d)I, \qquad t \neq t_n, t_{n+1} = t_n + T, \\ R' = \gamma I - (\delta + d)R, \end{cases} \tag{1}$$

and

$$\begin{cases} S(t^+) = (1-p)S(t^-), \\ I(t^+) = (1-k)I(t^-), \qquad t = t_n, n = 0, 1, 2, \ldots, \\ R(t^+) = R(t^-) + pS(t^-), \end{cases} \tag{2}$$

where $f(t^+) = \lim_{t \longrightarrow t_n^+} f(t)$, $f(t^-) = \lim_{t \longrightarrow t_n^-} f(t)$. The parameters of the systems (1) and (2) are illustrated in Table 1.

The equation set and a set of variables above define connections between the variables that make up a mathematical model, which often explains the system under study. Following Animasaun et al. [30], mathematical modeling offers precision and a strategy for problem resolution and allows for a systematic understanding of the system being studied. Mathematical models are groups of variables, equations, and beginning values that logically describe a process or behavior. Mathematical models are also experimental tools for testing theories and theorems to assess conjectures and conclusions. In this paper, the stability of the SIRS mathematical model is studied to reveal the epidemic pattern of infectious diseases, predict the epidemic trend, and provide certain theoretical basis and strategies for the detection, prevention, and control of infectious disease epidemics.

3. The Existence of the Disease-Free Periodic Solution

In this section, the existence and the stability of the disease-free T periodic solution for the systems (1) and (2) are obtained.

Summing system (1), we have that the total population $N(t)$ satisfies the differential equation:

$$N'(t) = (a-d)N - \alpha I. \tag{3}$$

Table 1: Description of the system parameters.

Parameters	Description
a	The birth rate
λ	The effective contact rate between the susceptible class and the infective class
d	The natural death rate of the population
α	The disease-related death rate of the infective class
γ	The natural recovery rate of the infective class
p	The vaccination rate of the susceptible class
δ	The loss immunity rate of the recovered class
k	The elimination rate of the infective class
t_n	The time for vaccination and elimination
T	The impulsive period

Let $s = S/N$, $i = I/N$, $r = R/N$, and the systems (1) and (2) then become

$$\begin{cases} s' = a - as + \delta r - (\lambda - \alpha)si, \\ i' = \lambda si - (a + \gamma + \alpha)i + \alpha i^2, \quad t \neq t_n, t_{n+1} = t_n + T, \\ r' = ri - (a + \delta)r + \alpha ri, \end{cases} \tag{4}$$

and

$$\begin{cases} s(t^+) = (1 - p)s(t^-), \\ i(t^+) = (1 - k)i(t^-), \qquad t = t_n, n = 0, 1, 2, \cdots. \\ r(t^+) = r(t^-) + ps(t^-). \end{cases} \tag{5}$$

Since $s + i + r = 1$, only the following systems need to be considered:

$$\begin{cases} i' = \lambda(1 - i - r)i - (a + \gamma + \alpha)i + \alpha i^2, \\ r' = ri - (a + \delta)r + \alpha ir, \qquad t \neq t_n, t_{n+1} = t_n + T, \end{cases} \tag{6}$$

and

$$\begin{cases} i(t^+) = (1 - k)i(t^-), \\ r(t^+) = r(t^-) + p(1 - i(t^-) - r(t^-)), \quad t = t_n, n = 0, 1, 2, \cdots. \end{cases} \tag{7}$$

To study the existence of disease-free T periodic solution, we need to find the T periodic solution which satisfies systems (6) and (7) when $i = 0$. When $i = 0$, the systems (6) and (7) then become

$$\begin{cases} r'(t) = -(a + \delta)r, & t \neq t_n, \\ r(t^+) = r(t^-) + p(1 - r(t^-)), & t = t_n. \end{cases} \tag{8}$$

When $t_n \leq t < t_{n+1}$, the solution of system (8) is

$$r(t) = r(t_n^+)e^{-(a+\delta)(t-t_n)}. \tag{9}$$

When $t = t_{n+1}$, the solution of system (8) is

$$r(t) = r(t_{n+1}^+) = p + (1 - p)r(t_{n+1}^-). \tag{10}$$

Let $r(t_{n+1}^+) = r_{n+1}$; by solving the above equation, it can be obtained that

$$r_{n+1} = p + (1 - p)r_n e^{-(a+\delta)T}. \tag{11}$$

Set F: $r_n \longrightarrow r_{n+1}$; F is a mapping, satisfying

$$r_{n+1} = F(r_n) = p + (1 - p)r_n e^{-(a+\delta)T}. \tag{12}$$

The map F has a unique fixed point r_0; it follows that

$$r_0 = F(r_0) = \frac{pe^{(a+\delta)T}}{e^{(a+\delta)T} + p - 1}. \tag{13}$$

Since $|\mathrm{d}F(r_n)/\mathrm{d}r|_{r=r_0} = (1 - p)e^{-(a+\delta)T} < 1$, this fixed point r_0 is stable and r_n must converge to r_0. Therefore, the systems (6) and (7) have the disease-free T periodic solution $(\tilde{i}(t), \tilde{r}(t))$, where

$$\tilde{r}(t) = \begin{cases} \dfrac{pe^{(a+\delta)T}}{e^{(a+\delta)T} + p - 1} e^{-(a+\delta)(t-t_n)}, & t_n < t \leq t_{n+1}, \\ \tilde{r}(t) = r_0, & t = t_n, \end{cases} \tag{14}$$

$$\tilde{i}(t) = 0.$$

4. The Stability of the Disease-Free Periodic Solution

In this section, the stability of the disease-free T periodic solution $(\tilde{i}(t), \tilde{r}(t))$ is considered.

Firstly, the local stability of the disease-free T periodic solution is discussed.

Let $i(t) = \tilde{i}(t) + x(t)$, $r(t) = \tilde{r}(t) + y(t)$. When $\neq t_n$, the linearized system of the system (6) with respect to the periodic solution $(\tilde{i}(t), \tilde{r}(t))$ is

$$\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} \lambda - (\alpha + \gamma + a) & 0 \\ \gamma + \alpha\tilde{r}(t) & -(a+\delta) \end{pmatrix} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}. \quad (15)$$

Let $A(t)$ be the basis solution matrix of the linearized system, and $A(0) = E$. E is the unit matrix. $A(t)$ is the following matrix:

$$A(t) = \begin{pmatrix} a_{11}(t) & 0 \\ a_{21}(t) & a_{22}(t) \end{pmatrix}, \quad (16)$$

where

$$\begin{aligned} a_{11}(t) &= e^{\left(\lambda - (\alpha+\gamma+a)t - \lambda \int_0^u \tilde{r}(u)\mathrm{d}u\right)}, \\ a_{21}(t) &= e^{-(a+\delta)t} \int_0^u (\gamma + \alpha\tilde{r}(u)) a_{11} e^{a+\delta}\mathrm{d}u, \\ a_{22}(t) &= e^{-(a+\delta)t}. \end{aligned} \quad (17)$$

When $= t_n$, obtaining

$$\begin{aligned} \begin{pmatrix} x(t_n^+) \\ y(t_n^+) \end{pmatrix} &= \begin{pmatrix} 1-k & 0 \\ p & 1-p \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \\ M &= \begin{pmatrix} 1-k & 0 \\ p & 1-p \end{pmatrix} \begin{pmatrix} a_{11}(t) & 0 \\ a_{21}(t) & a_{22}(t) \end{pmatrix} = \begin{pmatrix} (1-k)a_{11}(T) & 0 \\ -pa_{11}(T) + (1-p)a_{21}(T) & (1-p)e^{-(a+\delta)} \end{pmatrix}. \end{aligned} \quad (18)$$

According to the Floquet theorem, the necessary and sufficient condition for the stability of the disease-free T periodic solution is that the modulus of the eigenvalues of the matrix M is all less than 1, i.e., only $a_{11}(T) < 1$. The following inequality is obtained:

$$\frac{1}{T}\int_0^T \tilde{r}(t)\mathrm{d}t > 1 - \frac{d+\gamma+a}{\lambda} + \frac{\ln(1-k)}{T}. \quad (19)$$

Using $\tilde{s}(t) = 1 - \tilde{r}(t)$, the above inequality is equivalently written as

$$\frac{1}{T}\int_0^T \tilde{s}(t)\mathrm{d}t < \frac{d+\gamma+a}{\lambda} - \frac{\ln(1-k)}{T}. \quad (20)$$

Define the basic reproductive number R_0 as follows:

$$R_0 = \frac{\lambda}{d+\gamma+a} \frac{\ln(1-k) + \int_0^T \tilde{s}(t)\mathrm{d}t}{T}, \quad (21)$$

where T is the pulse vaccination and pulse elimination cycle. Theorem 1 is obtained by the above analysis.

Theorem 1. *For systems (1) and (2), the disease-free T periodic solution is locally asymptotically stable if $R_0 < 1$.*

Now, we will prove the global stability of the disease-free T periodic solution. In order to facilitate the global stability of the disease-free T periodic solution, we need to introduce Lemma 2.

Lemma 2. *Let $f(t), g(t) \in C^1[0, +\infty)$, and $\lim_{t\longrightarrow+\infty} e^{f(t)} = 0$, $\lim_{t\longrightarrow+\infty} f'(t) = A \neq 0$, $g(t) > 0$, $\lim_{t\longrightarrow+\infty} g(t) = 0$. Then, $\lim_{t\longrightarrow+\infty} e^{f(t)} \int_0^t e^{-f(u)g(u)}\mathrm{d}u = 0$.*

Proof. Because of the product function $\exp(-f(t))g(t) \geq 0$, the generalized integral $\int_0^{+\infty} \exp(-f(u))g(u)\mathrm{d}u \geq 0$. If the generalized integral converges, let it converge to some positive number, then having

$$\begin{aligned} &\lim_{t\longrightarrow+\infty} e^{f(t)} \int_0^t \exp(-f(u))g(u)\mathrm{d}u \\ &= \lim_{t\longrightarrow+\infty} \exp(f(t))A = 0. \end{aligned} \quad (22)$$

If the generalized integral diverges, then by L'Hospital law, it follows that

$$\begin{aligned} \lim_{t\longrightarrow+\infty} e^{f(t)} \int_0^t \exp(-f(u))g(u)\mathrm{d}u &= \lim_{t\longrightarrow+\infty} \frac{\int_0^t \exp(-f(u))g(u)\mathrm{d}u}{\exp(-f(t))} \\ &= \lim_{t\longrightarrow+\infty} \frac{g(t)}{-f'(t)} = 0. \end{aligned} \quad (23)$$

□

Theorem 3. *For systems (1) and (2), the disease-free T periodic solution is globally asymptotically stable if $R_0 < 1$.*

Proof. Firstly, when $R_0 < 1$, we will prove $\lim_{t \longrightarrow +\infty} i(t) = 0$.

From the first equations of the systems (4) and (5), it follows that

$$\begin{cases} s'(t) \le (a+\delta)(1-s), & t \ne t_n, \\ s(t^+) = (1-p)s(t^-), & t = t_n. \end{cases} \tag{24}$$

For system (24), by the impulsive differential inequality, it is obtained that

$$\begin{aligned}
s(t) &\le s(0^+) \prod_{0<nT<t} (1-p)e^{-(a+\delta)t} + \int_0^t \prod_{u<nT<t} (1-p)(a+\delta)e^{-d(t-u)}\mathrm{d}u \\
&\le s(0^+) \prod_{0<nT<t} (1-p)e^{-(a+\delta)t} + (a+\delta)e^{-(a+\delta)t} \\
&\quad \cdot \left[\int_0^T (1-p)^n e^{(a+\delta)u}\mathrm{d}u + \int_T^{2T} (1-p)^{n-1} e^{(a+\delta)u}\mathrm{d}u + \cdots + \int_{(n-1)T}^{nT} (1-p)e^{(a+\delta)u}\mathrm{d}u + \int_{nT}^t e^{(a+\delta)u}\mathrm{d}u \right] \\
&\le s(0^+) \prod_{0<nT<t} (1-p)e^{-(a+\delta)t} + (a+\delta)e^{-(a+\delta)t} \\
&\quad \cdot \left[(1-p)^n \int_0^T e^{(a+\delta)u}\mathrm{d}u + (1-p)^{n-1} \int_T^{2T} e^{(a+\delta)u}\mathrm{d}u + \cdots + (1-p) \int_{(n-1)T}^{nT} e^{(a+\delta)u}\mathrm{d}u + \int_{nT}^t e^{(a+\delta)u}\mathrm{d}u \right] \\
&\le s(0^+) \prod_{0<nT<t} (1-p)e^{-(a+\delta)t} + (a+\delta)e^{-(a+\delta)t} \\
&\quad \cdot \left[(1-p)^n \frac{e^{(a+\delta)T}-1}{a+\delta} + (1-p)^{n-1} \frac{e^{(a+\delta)2T} - e^{(a+\delta)T}}{a+\delta} + \cdots + (1-p) \frac{e^{(a+\delta)nT} - e^{(a+\delta)(n-1)T}}{a+\delta} + \frac{e^{(a+\delta)t} - e^{(a+\delta)nT}}{a+\delta} \right] \\
&\le s(0^+) \prod_{0<nT<t} (1-p)e^{-(a+\delta)t} + (1-p)^n e^{-(a+\delta)t} \\
&\quad \cdot \left[\left(e^{(a+\delta)T} - 1\right) + \frac{e^{(a+\delta)T}}{1-p} \left(e^{(a+\delta)T} - 1\right) + \cdots + \left(\frac{e^{(a+\delta)T}}{1-p}\right)^{n-1} \left(e^{(a+\delta)T} - 1\right) + e^{(a+\delta)t} - e^{(a+\delta)[t/T]t} \right] \\
&\le s(0^+) \prod_{0<nT<t} (1-p)e^{(a+\delta)T} + (1-p)^n e^{-(a+\delta)t} \frac{\left(e^{(a+\delta)T} - 1\right)\left(1 - \left(e^{(a+\delta)T}/1-p\right)^n\right)}{1 - e^{(a+\delta)T}/1-p} + e^{(a+\delta)t} - e^{(a+\delta)[t/T]t} \\
&\le s(0^+) e^{-(a+\delta)T} (1-p)^{[t/T]} - \frac{\left(e^{(a+\delta)T} - 1\right)(1-p)^{[t/T]+1}}{e^{(a+\delta)T} + p - 1} + \left(1 - \frac{pe^{(a+\delta)([t/T]T+1-t)}}{e^{(a+\delta)T} + p - 1}\right) \\
&\le r(t) + \left(1 - \frac{pe^{(a+\delta)([t/T]T+1-t)}}{e^{(a+\delta)T} + p - 1}\right).
\end{aligned} \tag{25}$$

And we have

$$s(t) \le r_1(t) + \left(1 - \frac{pe^{(a+\delta)T}([t/T]T + 1 - t)}{e^{(a+\delta)T} + p - 1}\right), \tag{26}$$

where

$$r_1(t) = e^{-(a+\delta)t} \exp\left[s(0^+)(1-p)^{[t/T]} - \frac{(1-p)^{[t/T]+1}\left(e^{(a+\delta)T} - 1\right)}{e^{(a+\delta)T} + p - 1} \right]. \tag{27}$$

From the second equations of the systems (4) and (5), it follows that

$$\begin{cases} i'(t) \le i\left(\lambda - (\alpha+\gamma+a) + \lambda r_1(t) + \alpha i - \dfrac{\lambda p e^{(a+\delta)T(1+[t/T]-(a+\delta)t)}}{e^{(a+\delta)T}+p-1}\right), \\ i(t^+) = (1-k)i(t^-). \end{cases} \tag{28}$$

Obviously, system (28) does not satisfy the form of the differential equation inequality. Therefore, let $(t) = -1/i(t)$; system (28) becomes the following impulsive differential equation:

$$\begin{cases} x'(t) \le -\left(\lambda - (\alpha+\gamma+a) + \lambda r_1(t) + \alpha - \dfrac{\lambda p e^{(a+\delta)T(1+[t/T]-(a+\delta)t)}}{e^{(a+\delta)T}+p-1}\right), \\ x(t^+) = \dfrac{1}{1-k}x(t^-). \end{cases} \tag{29}$$

For system (29), by the impulsive differential inequality, we obtain

$$\begin{aligned} x(t) \le x(0^+) \prod_{0<nT<t} \frac{1}{1-p} \exp\left[\int_0^t -\left(\lambda - (\alpha+\gamma+a) + \lambda r_1(u) - \frac{\lambda p e^{(a+\delta)T(1+[u/T]-(a+\delta)u)}}{e^{(a+\delta)T}+p-1}\right)\mathrm{d}u\right] \\ + \varepsilon \int_0^t \prod_{s<nT<t} \frac{1}{1-p} \exp\left[\int_s^t -\left(\lambda - (\alpha+\gamma+a) + \lambda r_1(u) - \frac{\lambda p e^{(a+\delta)T(1+[u/T]-(a+\delta)u)}}{e^{(a+\delta)T}+p-1}\right)\mathrm{d}u\right]\mathrm{d}s. \end{aligned} \tag{30}$$

Making the inverse substitution $(t) = -1/x(t)$, with the help of the comparison principle, it follows that

$$i(t) \le \frac{i(0)e^{\ln(1-k)[t/T]+\int_0^t f(s)\mathrm{d}s}}{1-\alpha i(0)e^{\ln(1-k)[s/T]+\int_0^s f(u)\mathrm{d}u}}, \tag{31}$$

where

$$f(t) = \left(\lambda - (\alpha+\gamma+a) + \lambda r_1(t) - \frac{\lambda p e^{(a+\delta)T(1+[t/T]T-t)}}{e^{(a+\delta)T}+p-1}\right)t. \tag{32}$$

Next, when $R_0 < 1$, we will prove $\lim_{t\longrightarrow+\infty} e^{\int_0^t f(s)\mathrm{d}s} = 0$. Since

$$\begin{aligned} \int_0^t f(s)\mathrm{d}s = -(\lambda - (\alpha+\gamma+a))t + \int_0^t \lambda r_1(u)\mathrm{d}u - \frac{\lambda p e^{(a+\delta)T}}{e^{(a+\delta)T}+p-1} \\ \cdot \int_0^t e^{(a+\delta)([u/T]T-u)}\mathrm{d}u, \end{aligned} \tag{33}$$

where

$$\int_0^t \lambda r_1(u)\mathrm{d}u \le \int_0^t \lambda s(0)e^{-(a+\delta)u}\mathrm{d}u = \lambda s(0)\frac{1-e^{-(a+\delta)t}}{a+\delta} \le \frac{\lambda s(0)}{a+\delta},$$
$$\int_0^t e^{(a+\delta)([u/T]T-u)}\mathrm{d}u = \frac{1-e^{(a+\delta)T}}{a+\delta}\left[\frac{t}{T}\right] + \frac{1-e^{-(a+\delta)(t-[t/T]T)}}{a+\delta}. \tag{34}$$

Substituting (7) and (24) into (6), we have

$$\begin{aligned}\int_0^t f(s)\mathrm{d}s &= (\lambda-(\alpha+\gamma+a))t + \frac{\lambda s(0)}{a+\delta} - \frac{\lambda p e^{(a+\delta)T}}{e^{(a+\delta)T}+p-1}\left(\frac{1-e^{(a+\delta)T}}{a+\delta}\left[\frac{t}{T}\right] + \frac{1-e^{-(a+\delta)(t-[t/T]T)}}{a+\delta}\right)\\ &= (\lambda-(\alpha+\gamma+a))t - \frac{\lambda p e^{(a+\delta)T}}{e^{(a+\delta)T}+p-1}\frac{1-e^{(a+\delta)T}}{a+\delta}\left[\frac{t}{T}\right]\\ &\quad - \frac{\lambda p e^{(a+\delta)T}}{e^{(a+\delta)T}+p-1}\frac{1-e^{-(a+\delta)(t-[t/T]T)}}{a+\delta} + \frac{\lambda s(0)}{a+\delta}\\ &\le \left[\lambda\left(1-\frac{\lambda p e^{(a+\delta)T}}{e^{(a+\delta)T}+p-1}\frac{1-e^{(a+\delta)T}}{a+\delta}\right)-(\alpha+\gamma+a)\right]\left[\frac{t}{T}\right]\\ &\quad + \frac{\lambda s(0)}{a+\delta} - (\lambda-(\alpha+\gamma+a))\left(t-\left[\frac{t}{T}\right]T\right) - \frac{\lambda p e^{(a+\delta)T}}{e^{(a+\delta)T}+p-1}\frac{1-e^{-(a+\delta)(t-[t/T]T)}}{a+\delta},\end{aligned} \tag{35}$$

$$e^{\ln(1-k)[t/T]T+\int_0^t f(s)\mathrm{d}s} \le D_1(t)e^{(\alpha+\gamma+a)[t/T]T(R_0-1)},$$

where

$$\exp\left[-(\lambda-(\alpha+\gamma+a))\left(t-\left[\frac{t}{T}\right]T\right) + \frac{\lambda s(0)}{a+\delta} - \frac{\lambda p e^{(a+\delta)T}}{e^{(a+\delta)T}+p-1}\frac{1-e^{-(a+\delta)(t-[t/T]T)}}{a+\delta}\right], \tag{36}$$

and $D_1(t)$ is positive and has an upper bound. Thus, $i(t) \longrightarrow 0\,(t \longrightarrow +\infty)$ for $R_0 < 1$.

Finally, when $R_0 < 1$, for the disease-free periodic solution $(s(t), i(t), r(t))$ of the systems (4) and (5), we will prove $s(t) \longrightarrow \tilde{s}(t), r(t) \longrightarrow \tilde{r}(t)\,(t \longrightarrow +\infty)$.

Let

$$V(t) = |s(t) - \tilde{s}(t)|. \tag{37}$$

When $\neq t_n$, we obtain

$$\begin{aligned}D^+(V(t)) &= \mathrm{sign}(s(t)-\tilde{s}(t))(s'(t)-s(t))\\ &= -(a+\delta)|s(t)-\tilde{s}(t)| + |\lambda-\alpha|si\\ &\le -(a+\delta)|s(t)-\tilde{s}(t)| + r_3(t),\end{aligned} \tag{38}$$

where $r_3(t) = |\lambda - \alpha|a/\, di(t)$. According to $i(t) \longrightarrow 0$ $(t \longrightarrow +\infty)$, we obtain

$$r_3(t) \longrightarrow 0\,(t \longrightarrow +\infty). \tag{39}$$

When $t = t_n$, we have

$$V\left(t_n^+\right) = (1-p)V\left(t_n^-\right). \tag{40}$$

Since $V(t) \in \mathrm{PC}'[R_+, R]$, and $V(t)$ is left continuous at $t = t_n$, by using the differential equation inequality for (38) and (40), it follows that

$$V(t) \le V\left(0^+\right) \prod_{0<nT<t} (1-p)e^{-(a+\delta)t} + \int_0^t \prod_{s<nT<t} (1-p)e^{\int_s^t (a+\delta)u\mathrm{d}u}\mathrm{d}s. \tag{41}$$

Obviously, the first term of (41) has the following result:

$$V\left(0^+\right) \prod_{0<nT<t} (1-p)e^{-(a+\delta)t} \longrightarrow 0\,(t \longrightarrow +\infty). \tag{42}$$

For the second item of (41), we have

$$\int_0^t \prod_{s<nT<t} (1-p)e^{\int_s^t (a+\delta)u\mathrm{d}u}\mathrm{d}s = e^{-(a+\delta)t} \int_0^t \prod_{s<nT<t} (1-p)e^{\int_0^s (a+\delta)u\mathrm{d}u}\mathrm{d}s. \tag{43}$$

Define $f(t) = -(a+\delta)t$, $g(t) = r_3(t)$. According to Lemma 2, we obtain $V(t) \longrightarrow 0\,(t \longrightarrow +\infty)$, namely, $s(t) \longrightarrow \tilde{s}(t)\,(t \longrightarrow +\infty)$. And by $(t) = 1 - s(t) - i(t)$, hence, $r(t) \longrightarrow \tilde{r}(t)\,(t \longrightarrow +\infty)$.

In summary, we complete the proof of Theorem 3. □

5. Conclusions

Nowadays, the vaccination strategy has become one of the most effective ways to control infectious diseases. The development of vaccination has saved countless lives. The elimination strategy is the most direct control measure when the disease is found. Therefore, it is of practical significance to study the effect of vaccination and elimination on the spread of infectious diseases and the prevention and control of infectious diseases. Based on this, we study the dynamical behavior of an SIRS epidemic model with pulse vaccination, pulse elimination, and standard incidence. We define the basic reproductive number $R_0 = \lambda/d + \gamma + a\ln(1-k) + \int_0^T \tilde{s}(t)\mathrm{d}t/T$ which determines whether a disease is extinct or not. If $R_0 < 1$, the disease-free T periodic solution is locally asymptotically stable by Floquet theory and the disease-free T periodic solution is globally asymptotically stable based on the impulse differential inequality. It is shown that the disease will be extinct when the average number of patients in each period is less than 1 after subtracting the number of people who are eliminated. From the expression of R_0, it can be seen that R_0 is a monotone decreasing function of the vaccination rate p and the elimination rate k. Therefore, increasing the pulse vaccination rate p and the elimination rate k is the most effective measure to stop the disease epidemic. Of course, in order to prevent the epidemic of infectious diseases, it is necessary to choose the appropriate impulsive period T. If the impulsive period T is large or the pulse vaccination rate p and the elimination rate k are too small so that $R_0 > 1$, then the disease will persist.

Interestingly, the stability of the model is under the influence of impulsive vaccination and impulsive elimination strategies. We believe that our study findings help the public health department mitigate disease by providing some theoretical guidance. In addition, there is a lot of work waiting for us to study in this field, such as the disease incidence of other forms or the numerical simulations of the stability, and we leave these for future work.

Data Availability

No underlying data were collected or produced in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Yanli Ma participated in the design of this study, carried out the study, collected important background information, and drafted the manuscript. Xuewu Zuo contributed for some resources in the revision of the manuscript. They have read and approved the final manuscript.

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