Hindawi
Journal of Mathematics
Volume 2024, Article ID 6650997, 9 pages
https://doi.org/10.1155/2024/6650997

Research Article

Local Automorphisms and Local Superderivations of Model Filiform Lie Superalgebras

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Received 4 June 2023; Revised 16 February 2024; Accepted 13 March 2024; Published 27 March 2024

Academic Editor: Ljubisa Kocinac

In this paper, we give the forms of local automorphisms (resp. superderivations) of model filiform Lie superalgebra $L_{n,m}$ in the matrix version. Linear 2-local automorphisms (resp. superderivations) of $L_{n,m}$ are also characterized. We prove that each linear 2-local automorphism of $L_{n,m}$ is an automorphism.

1. Introduction and Basics

As a significant class of nilpotent Lie algebras, filiform Lie algebras were introduced by Vergne [1] and have been studied extensively, see [2–6] and references in them. Model filiform Lie algebra L_n is the simplest filiform Lie algebra. Vergne proved that each filiform Lie algebra can be obtained by deformations of model filiform Lie algebra (see [1]). Similarly, model filiform Lie superalgebra $L_{n,m}$ is the simplest filiform Lie superalgebra.

Automorphisms and superderivations are also important in the study of the structure of Lie superalgebras. In recent years, some new generalized derivations of finite-dimensional Lie algebras and Lie superalgebras were proposed and studied (see [7–9]). Local automorphisms and local derivations were introduced by Kadison in [10] and Larson and Sourour in [11]. The idea of local came from [12, 13]. The idea of 2-local was introduced by Šemrl in [14]. Later, more and more results of such problem on various algebras were obtained by many scholars (see [15–21] and references in them). In particular, local and 2-local automorphisms (resp. derivations) on some Lie algebras were proved to be automorphisms (resp. derivations) (see [22–26]). For Lie superalgebras, such problem were studied in [27–30] and some other papers.

In this paper, we will use matrices to study local automorphisms (resp. superderivations) of model filiform Lie superalgebra $L_{n,m}$. We will give concrete forms of local automorphisms (resp. superderivations) of $L_{n,m}$. For finite-dimensional nilpotent Lie algebra L with $\dim L \geqslant 2$. In [23], Ayupov and Kudaybergenov proved that there is a 2-local automorphism of L which is not an automorphism. Then, it is impossible that every 2-local automorphism of Lie superalgebra $L_{n,m}$ is an automorphism. But if a 2-local automorphism is linear, then we can prove that it must be an automorphism. So, we add an additional linear condition in the definition of 2-local automorphism, we call it linear 2-local automorphism. We will prove that all linear 2-local automorphisms of $L_{n,m}$ are automorphisms. But for 2-local superderivation of $L_{n,m}$, the situation is different. We also add an additional linear condition in the definition of 2-local superderivation, and we call it linear 2-local superderivation. In this paper, we will show that not all linear 2-local superderivations of $L_{n,m}$ are superderivations, but they is very close to a superderivations. The same situation also occurs in 2-local automorphisms (resp. derivations) of model filiform Lie algebra L_n. We find that not all linear 2-local automorphisms (resp. derivations) of Lie algebra L_n are automorphisms (resp. derivations), and the linear 2-local automorphisms (resp. derivations) which are not automorphisms (resp. derivations) are very close to automorphisms (resp. derivations).

Model filiform Lie superalgebra $L_{n,m}$ is a superalgebra with multiplication

$$\begin{aligned}[x_0, x_i] &= x_{i+1}, \quad 1 \leqslant i \leqslant n-1, \\ [x_0, y_j] &= y_{j+1}, \quad 1 \leqslant j \leqslant m-1,\end{aligned} \tag{1}$$

where $\{x_0, x_1, \cdots, x_n \mid y_1, \cdots, y_m\}$ is the homogeneous basis and the other brackets vanished. If we only consider the Lie algebra with $\{x_0, x_1, \cdots, x_n\}$ a basis, their multiplication are same to (1), then it is the model filiform Lie algebra L_n.

For a Lie superalgebra $\mathcal{G} = \mathcal{G}_{\bar{0}} \oplus \mathcal{G}_{\bar{1}}$, a linear bijective map $\varphi: \mathcal{G} \longrightarrow \mathcal{G}$ is called an automorphism of Lie superalgebra $\mathcal{G}$ if

$$\varphi([x, y]) = [\varphi(x), \varphi(y)], \quad \forall x, y \in \mathcal{G}. \tag{2}$$

Denote the group consisting of all automorphisms of $\mathcal{G}$ by $\mathrm{Aut}(\mathcal{G})$. Suppose $\mathcal{D}: \mathcal{G} \longrightarrow \mathcal{G}$ is a linear map of degree α, we call $\mathcal{D}$ a superderivation of degree α if

$$\mathcal{D}([x, y]) = [\mathcal{D}(x), y] + (-1)^{\beta\alpha}[x, \mathcal{D}(y)], \quad \forall x \in \mathcal{G}_\beta, y \in \mathcal{G}. \tag{3}$$

Denote all superderivations of degree α by $\mathrm{Der}_\alpha(\mathcal{G}), \alpha \in \mathbb{Z}_2$. The elements of $\mathrm{Der}(\mathcal{G}) = \mathrm{Der}_{\bar{0}}(\mathcal{G}) \oplus \mathrm{Der}_{\bar{1}}(\mathcal{G})$ are called superderivations of $\mathcal{G}$. Linear map ϕ: $\mathcal{G} \longrightarrow \mathcal{G}$ is called a local automorphism (resp. superderivation), if for any $x \in \mathcal{G}$, there exists $\phi_x \in \mathrm{Aut}(\mathcal{G})$ (resp. $\mathrm{Der}(\mathcal{G})$) such that $\phi(x) = \phi_x(x)$. A linear map $\sigma: \mathcal{G} \longrightarrow \mathcal{G}$ is called a linear 2-local automorphism (resp. superderivation), if for any $x, y \in \mathcal{G}$, there exists $\sigma_{xy} \in \mathrm{Aut}(\mathcal{G})$ (resp. $\mathrm{Der}(\mathcal{G})$) such that $\sigma(x) = \sigma_{xy}(x)$ and $\sigma(y) = \sigma_{xy}(y)$. Denote the group consisting of all local automorphisms of $\mathcal{G}$ by $\mathrm{LAut}(\mathcal{G})$ and the superalgebra consisting of all local superderivations of $\mathcal{G}$ by $\mathrm{LDer}(\mathcal{G})$, respectively.

Throughout the paper, we assume that $3 \leqslant n \leqslant m$. All mappings mentioned in this paper are linear. The matrices of mappings of $L_{n,m}$ are all with respect to the homogeneous basis $\{x_0, x_1, \cdots, x_n \mid y_1, \cdots, y_m\}$, and the matrices of mappings of L_n are all with respect to the basis $\{x_0, x_1, \cdots, x_n\}$. $\mathbb{F}$ stands for an arbitrary field of characteristic zero, $\mathbb{F}^*$ is the set of all nonzero elements of $\mathbb{F}$, and $\mathbb{F}^n$ is the n-dimensional column vector space over $\mathbb{F}$. E_{ij} and e_i represent the matrix unit and unit vector, respectively.

Denote block matrices

$$\begin{pmatrix} A & O \\ O & B \end{pmatrix}, \quad \begin{pmatrix} O & C \\ D & O \end{pmatrix}, \tag{4}$$

by $A \oplus B$ and $C \oplus D$, respectively.

2. Local Automorphism and Linear 2-Local Automorphism of $L_{n,m}$

Suppose $a, a_1, \cdots, a_n \in \mathbb{F}$ and $\alpha \in \mathbb{F}^n$. Denote

$$\begin{aligned}\overline{B}(a, a_1, \cdots, a_n) &= \begin{pmatrix} a_1 & & & & \\ a_2 & aa_1 & & & \\ a_3 & aa_2 & a^2a_1 & & \\ \vdots & \vdots & \vdots & \ddots & \\ a_n & aa_{n-1} & a^2a_{n-2} & \cdots & a^{n-1}a_1 \end{pmatrix}, \\ \overline{A}(a, a_1, \cdots, a_n, \alpha) &= \begin{pmatrix} a & \\ \alpha & \overline{B}(a, a_1, \cdots, a_n) \end{pmatrix}.\end{aligned} \tag{5}$$

Lemma 1

(1) *Let A be an $(n+1)\times(n+1)$ invertible lower triangular matrix. Then, for any $X = (k_0, k_1, \cdots, k_n)^T \in \mathbb{F}^{n+1}$, there exists A_X such that $AX = A_X X$, where A_X is of the form $\overline{A}(a, a_1, \cdots, a_n, \alpha)$, with $aa_1 \neq 0$;*

(2) *Let B be an $m \times m$ invertible lower triangular matrix. Then, for any $a \in \mathbb{F}^*$ and $X \in \mathbb{F}^m$, there exists B_X such that $BX = B_X X$, where B_X is of the form $\overline{B}(a, b_1, \cdots, b_m)$, with $b_1 \in \mathbb{F}^*$.*

Proof

(1) Denote

$$A = \begin{pmatrix} u & \\ \beta & U \end{pmatrix}, \tag{6}$$

where $u \in \mathbb{F}^*, \beta \in \mathbb{F}^n$ and U is an $n \times n$ invertible lower triangular matrix. For any $X = (x_0, x_1, \cdots, x_n)^T \in \mathbb{F}^{n+1}$, put $X_1 = (x_1, \cdots, x_n)^T$. We will prove that there exist $\alpha \in \mathbb{F}^n$ and $\overline{B}(a, a_1, \cdots, a_n)$ with $aa_1 \in \mathbb{F}^*$ such that

$$\begin{pmatrix} u & \\ \beta & U \end{pmatrix}\begin{pmatrix} x_0 \\ X_1 \end{pmatrix} = \begin{pmatrix} a & \\ \alpha & \overline{B}(a, a_1, \cdots, a_n) \end{pmatrix}\begin{pmatrix} x_0 \\ X_1 \end{pmatrix}. \tag{7}$$

Case 1. $x_0 \neq 0$. Put $a = u, a_1 = \cdots = a_n = 1$. Then, it is easy to see that there exists $\alpha \in \mathbb{F}^n$ such that (7) holds.

Case 2. $x_0 = 0$. Assume that the first nonzero component of vector X_1 is the r–th. Put $a = 1, \alpha = 0, a_{n-r} = \cdots = a_n = 0$. Then, it is easy to prove that there exist $a_1 \in \mathbb{F}^*$ and $a_2, \cdots, a_{n-r-1} \in \mathbb{F}$ such that (7) holds.

(2) In a similarly way to the proof of (1), one can come to the conclusion. □

Theorem 2. *Let φ be a linear mapping of $L_{n,m}$. Then, $\varphi \in Aut(L_{n,m})$ if and only if the matrix of φ is of the form*

$$\overline{A}(a, a_1, \cdots, a_n, \alpha) \oplus \overline{B}(a, b_1, \cdots, b_m), \tag{8}$$

with $aa_1b_1 \neq 0$.

Proof. If $\varphi \in \mathrm{Aut}(L_{n,m})$, we can assume that the matrix of φ is $A \oplus B$, where B is an $m \times m$ matrix. Denote

$$A = \begin{pmatrix} a_0 & a_{10} & a_{20} & \cdots & a_{n0} \\ a_1 & a_{11} & a_{21} & \cdots & a_{n1} \\ a_2 & a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & \vdots & & \vdots \\ a_n & a_{1n} & a_{2n} & \cdots & a_{nn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{21} & \cdots & b_{m1} \\ b_{12} & b_{22} & \cdots & b_{m2} \\ \vdots & \vdots & & \vdots \\ b_{1m} & b_{2m} & \cdots & b_{mm} \end{pmatrix}. \tag{9}$$

For any $1 \leqslant i \leqslant n-1$, $1 \leqslant j \leqslant m-1$ and $2 \leqslant l \leqslant n$, using $\varphi(x_{i+1}) = \varphi([x_0, x_i])$, $\varphi([x_0, x_n]) = \varphi([x_1, x_l]) = 0$ and $\varphi(y_{j+1}) = \varphi([x_0, y_j])$ successively, we obtain

$$a_{i+1,0} = a_{i+1,1} = 0, a_{i+1,k+1} = \begin{vmatrix} a_0 & a_{i0} \\ a_k & a_{ik} \end{vmatrix}, \tag{10}$$

$$a_0 a_{nk} = a_{10} a_{lk} = 0, \tag{11}$$

$$b_{j+1,1} = 0, a_0 b_{js} = b_{j+1,s+1}, \tag{12}$$

where $1 \leqslant k \leqslant n-1$, $1 \leqslant s \leqslant m-1$.

By (12), we have $B = \overline{B}(a_0, b_{11}, \cdots, b_{1m})$.

If $a_{10} \neq 0$, then by (11), we have $a_{lk} = 0, 2 \leqslant l \leqslant n, 1 \leqslant k \leqslant n-1$. Note that in (10), they contradict the invertibility of A. Therefore, $a_{10} = 0$. Consequently, we have $a_0 \neq 0$ since A is invertible. For any $1 \leqslant k \leqslant n-1$, according to (10) and (11), we have $a_{i+1,k+1} = a_0 a_{ik}$ and $a_{nk} = 0$. Denote $\alpha = (a_1, \cdots, a_n)^T$. Thus, $A = \overline{A}(a, a_{11}, \cdots, a_{1n}, \alpha)$.

Conversely, if the matrix of φ is $\overline{A}(a, a_1, \cdots, a_n, \alpha) \oplus \overline{B}(a, b_1, \cdots, b_n)$ with $a a_1 b_1 \neq 0$, then φ is a Lie automorphism of $L_{n,m}$ by verification. □

Theorem 3. *Let φ be a linear mapping of $L_{n,m}$. Then, $\varphi \in LAut(L_{n,m})$ if and only if the matrix of φ is of the form $A \oplus B$, where A and B are $(n+1) \times (n+1)$ and $m \times m$ invertible lower triangular matrices, respectively.*

Proof. Assume that the matrix of φ is

$$\begin{pmatrix} A & C \\ D & B \end{pmatrix}, \tag{13}$$

where B is an $m \times m$ matrix.

If $\varphi \in \mathrm{LAut}(L_{n,m})$, by Theorem 2, for any $X = (k_0, k_1, \cdots, k_n, l_1, \cdots, l_m)^T \in \mathbb{F}^{n+m+1}$, there exist A_X and B_X such that

$$\begin{pmatrix} A & C \\ D & B \end{pmatrix} X = \begin{pmatrix} A_X & \\ & B_X \end{pmatrix} X, \tag{14}$$

where $A_X \oplus B_X$ is of the form (8).

We let $l_i = 0$ for all $i \in \{1, \cdots, m\}$. Since each k_j (where $0 \leq j \leq n$) in (14) is arbitrary, we see that $D = 0$.

Similarly, we let $k_j = 0$ for all $j \in \{1, \cdots, n\}$. Since each l_i (where $1 \leq i \leq m$) in (14) is arbitrary, we see that $C = 0$.

If A is not invertible, then there exists a nonzero vector $\alpha \in \mathbb{F}^{n+1}$ such that $A\alpha = 0$. Substituting $X = (\alpha^T, O)^T$ into (14), we have $A_X \alpha = 0$, which contradicts the invertibility of A_X. Thus, A is invertible. Similarly, B is also invertible.

Substituting $X = (0, \cdots 0, k_i, \cdots, k_{n+1}, 0, \cdots, 0)^T \in \mathbb{F}^{n+m+1}, i = 2, 3, \cdots, n+1$ into (14) in turn, by the arbitrariness of $k_i, \cdots, k_{n+1}$, we obtain that A is a lower triangular matrix. Similarly, B is also a lower triangular matrix.

Conversely, if the matrix of φ is $A \oplus B$, where A and B are $(n+1) \times (n+1)$ and $m \times m$ invertible lower triangular matrices respectively, then by Lemma 1, φ is a local automorphism of $L_{n,m}$. □

Theorem 4. *Every linear 2-local automorphism of $L_{n,m}$ is an automorphism.*

Proof. If φ is a linear 2-local Lie superalgebra automorphism of $L_{n,m}$, then $\varphi \in \mathrm{LAut}(L_{n,m})$. By Theorem 3, the matrix of φ is of the form $A \oplus B$, where A and B are $(n+1) \times (n+1)$ and $m \times m$ invertible lower triangular matrices, respectively. Denote

$$A = \begin{pmatrix} c & & & & \\ v_1 & c_{11} & & & \\ v_2 & c_{21} & c_{22} & & \\ \vdots & \vdots & \vdots & & \\ v_n & c_{n1} & c_{n2} & \cdots & c_{nn} \end{pmatrix}, \quad B = \begin{pmatrix} d_{11} & & & \\ d_{21} & d_{22} & & \\ \vdots & \vdots & & \\ d_{m1} & d_{m2} & \cdots & d_{mm} \end{pmatrix}, \tag{15}$$

where $c c_{11} \cdots c_{nn} d_{11} \cdots d_{mm} \neq 0$.

By Theorem 2, for any $X, Y \in \mathbb{F}^{n+m+1}$, there exist A_{XY} and B_{XY} such that

$$(A \oplus B)(X, Y) = (A_{XY} \oplus B_{XY})(X, Y), \tag{16}$$

where $A_{XY} = \overline{A}(a, a_1, \cdots, a_n, \alpha), B_{XY} = \overline{B}(a, b_1, \cdots, b_m)$. In fact, $a, \alpha, a_i, b_j, 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m$ are all related to X and Y. But for the sake of simplicity, we still denote them in this way without causing confusion.

Substituting $X = e_s, Y = e_{s+1}$ into (16), we have

$$c_{is} = k_s c_{i-1,s-1}, \quad k_s \in \mathbb{F}^*, \\ i = s, s+1, \cdots, n, \quad s = 2, 3, \cdots, n. \tag{17}$$

Then, substituting $X = e_t, Y = e_{t+1} + e_{t+2}$ into (16), we have $k_t = k_{t+1}, t = 2, 3, \cdots, n-1$.

Similarly, we can conclude that $d_{is} = l d_{i-1,s-1}, l \in \mathbb{F}^*$, $i = s, s+1, \cdots, m, s = 2, 3, \cdots, m$. Next, substituting $X = e_2 + e_{n+2}, Y = e_3 + e_{n+3}$ into (16), we have $l = k_2$. Finally, substituting $X = e_1 + e_{n+2}, Y = e_{n+3}$ into (16), we obtain $l = c$. Thus, by Theorem 2, we have $\varphi \in \mathrm{Aut}(L_{n,m})$.

From the proof of the above theorems, we get the following conclusions immediately. □

Corollary 5

$$\mathrm{Aut}\left(L_{n,m}\right)=\left\{\left(\sum_{0\leqslant i\leqslant n}k_ig^i+\sum_{0\leqslant j\leqslant m-1}l_jh^j\right)d(a)+(a-k_0)t_0+\sum_{1\leqslant s\leqslant n}p_st_s\,\middle|\,k_i,l_j,p_s\in\mathbb{F},a,k_0,l_0\in\mathbb{F}^*,1\leqslant i\leqslant n,1\leqslant j\leqslant m-1,1\leqslant s\leqslant n\right\}\tag{18}$$

where

$$\begin{aligned}d(a)(x_i)&=a^ix_i,d(a)\left(y_j\right)=a^jy_j,\quad 0\leqslant i\leqslant n,1\leqslant j\leqslant m,\\t_s(x_0)&=x_s,\quad 1\leqslant s\leqslant n,\\g(x_i)&=x_{i+1},\quad 0\leqslant i\leqslant n-1,\\h\left(y_j\right)&=y_{j+1},\quad 1\leqslant j\leqslant m-1.\end{aligned}\tag{19}$$

Corollary 6

$$\mathrm{LAut}\left(L_{n,m}\right)=\left\{\sum_{0\leqslant j\leqslant i\leqslant n}k_{ij}a_{ij}+\sum_{1\leqslant t\leqslant s\leqslant m}l_{st}b_{st}k_{ij},\ \mid l_{st}\in\mathbb{F},k_{pp},l_{qq}\in\mathbb{F}^*,0\leqslant j<i\leqslant n,1\leqslant t<s\leqslant m,0\leqslant p\leqslant n,0\leqslant q\leqslant n\right\}\tag{20}$$

where $a_{ij}(x_j)=x_i,0\leqslant j\leqslant i\leqslant n;b_{st}(y_t)=y_s,1\leqslant t\leqslant s\leqslant m.$

Corollary 7. *Let* φ *be a linear mapping of* L_n.

(1) *The automorphism group of* L_n *is*

$$Aut\left(L_n\right)=\left\{\sum_{0\leqslant i\leqslant n}k_ig^id(a)+(a-k_0)t_0+\sum_{1\leqslant s\leqslant n}p_st_s\,\middle|\,a,k_0\in\mathbb{F}^*,k_i,p_i\in\mathbb{F},1\leqslant i\leqslant n\right\}.\tag{21}$$

where $d(a)(x_i)=a^ix_i,0\leqslant i\leqslant n;g(x_i)=x_{i+1},0\leqslant i\leqslant n-1;t_s(x_0)=x_s,1\leqslant s\leqslant n;$

(2) *The local automorphism group of* L_n *is*

$$\mathrm{LAut}\left(L_n\right)=\left\{\sum_{0\leqslant j\leqslant i\leqslant n}k_{ij}a_{ij}\,\middle|\,k_{ij}\in\mathbb{F},k_{pp}\in\mathbb{F}^*,0\leqslant j<i\leqslant n,0\leqslant p\leqslant n\right\}.\tag{22}$$

where $a_{ij}(x_j)=x_i,0\leqslant j\leqslant i\leqslant n;$

(3) *The linear mapping* φ *is a linear 2-local automorphism of* L_n *if and only if there exist* $\psi\in Aut(L_n)$ *and* $k\in\mathbb{F}$ *such that*

$$\varphi=\psi+k\sigma,\tag{23}$$

where σ *is a linear mapping of* L_n *whose matrix is* E_{11}, $\alpha\in\mathbb{F}^n$, *and* $a,k,a_1,a_2,\cdots,a_n\in\mathbb{F}$ *with* $(a+k)a_1\in\mathbb{F}^*$ *such that the matrix of* φ *is* $\overline{A}(a,a_1,\cdots,a_n,\alpha)+kE_{11}$.

Proof. From the proof of the above theorems, (1), (2), and the necessity of (3) hold immediately. Next, we only need to prove the sufficiency of (3).

Assume the matrix of φ is $\overline{A}(a,a_1,\cdots,a_n,\alpha)+kE_{11}$. For any $X,Y\in\mathbb{F}^{n+1}$, we will find appropriate A_{XY} such that

$$\left(\overline{A}(a,a_1,\cdots,a_n,\alpha)+kE_{11}\right)(X,Y)=A_{XY}(X,Y),\tag{24}$$

where $A_{XY}=\overline{A}(a',a_1',\cdots,a_n',\alpha')$ with $a'a_1'\neq 0$, and $a',a_1',\cdots,a_n',\alpha'$ are all related to X and Y. □

Case 8. If X and Y are linear dependent, then by Theorem 3, the existence of A_{XY} is obvious.

Case 9. If X and Y are linear independent, then without loss of generality, we only need to consider the case of

$$X = e_i + \sum_{k=i+1}^{n+1} x_k e_k, \quad Y = e_{i+s} + \sum_{k=i+s+1}^{n+1} y_k e_k, \tag{25}$$

where $1 \leqslant s \leqslant n+1-i, 1 \leqslant i \leqslant n$.

Subcase 10. $i > 1$. Put $A_{XY} = \overline{A}(a, a_1, \cdots, a_n, \alpha)$, and, therefore, (24) holds.

Subcase 11. $i = 1$. Denote $\alpha = (u_1, \cdots, u_n)^T$. We will find $a', a_1', \cdots, a_n', \alpha' = (u_1', \cdots, u_n')^T$ such that (24) holds, i.e.,

$$\begin{cases} a' = a + k, \\ u_1' + a_1' x_2 = u_1 + a_1 x_2, \\ u_2' + a_2' x_2 + a' a_1' x_3 = u_2 + a_2 x_2 + a a_1 x_3, \\ u_3' + a_3' x_2 + a' a_2' x_3 + (a')^2 a_1' x_4 = u_3 + a_3 x_2 + a a_2 x_3 + a^2 a_1 x_4, \\ \cdots\cdots\cdots\cdots, \\ u_n' + a_n' x_2 + a' a_{n-1}' x_3 + \cdots + (a')^{n-1} a_1' x_{n+1} = u_n + a_n x_2 + a a_{n-1} x_3 + \cdots + a^{n-1} a_1 x_{n+1}, \end{cases} \tag{26}$$

$$\begin{cases} (a')^{i+s-2} a_1' = a^{i+s-2} a_1, \\ (a')^{i+s-2} a_2' + (a')^{i+s-1} a_1' y_{i+s+1} = a^{i+s-2} a_2 + a^{i+s-1} a_1 a y_{i+s+1}, \\ \cdots\cdots\cdots\cdots, \\ (a')^{i+s-2} a_{n+2-i-s}' + (a')^{i+s-1} a_{n+1-i-s}' y_{i+s+1} + \cdots + (a')^{n-1} a_1' y_{n+1} \\ = a^{i+s-2} a_{n+2-i-s} + a^{i+s-1} a_{n+1-i-s} y_{i+s+1} + \cdots + a^{n-1} a_1 y_{n+1}. \end{cases} \tag{27}$$

First, we find $a' = a + k$. Then, it is easy to find $a_1', \cdots, a_{n+2-i-s}'$ to satisfy (27) and $a_1' \neq 0$. Put $a_{n+2-i-s}' = \cdots = a_n' = 0$. Finally, it is easy to find $u_1', \cdots, u_n'$ to satisfy (26).

3. Local Superderivations and Linear 2-Local Superderivations of $L_{n,m}$

Suppose $a, a_i, c_i, d_i, b_j \in \mathbb{F}, 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant j, \alpha, \gamma \in \mathbb{F}^n, \beta \in \mathbb{F}^{m-n}$. Denote

$$J(d_1, \cdots, d_n) = \begin{pmatrix} d_1 & & & & \\ d_2 & d_1 & & & \\ \vdots & d_2 & d_1 & & \\ d_{n-1} & \ddots & \ddots & \ddots & \\ d_n & d_{n-1} & \cdots & d_2 & d_1 \end{pmatrix},$$
$$L(a, n) = \begin{pmatrix} 0 & & & & \\ & a & & & \\ & & 2a & & \\ & & & \ddots & \\ & & & & (n-1)a \end{pmatrix}, \tag{28}$$
$$B(a, b_1, \cdots, b_m) = J(b_1, \cdots, b_m) + L(a, m),$$

$$A(a, a_1, \cdots, a_n, \alpha) = \begin{pmatrix} a & \\ \alpha & B(a, a_1, \cdots, a_n) \end{pmatrix}, \tag{29}$$

$$D(d_1, \cdots, d_n) = \begin{pmatrix} & O \\ J(d_1, \cdots, d_n) & O \end{pmatrix}_{(n+1)\times m}, \tag{30}$$

$$C(c_1, \cdots, c_n, \beta, \gamma) = \begin{pmatrix} \beta & \\ \gamma & J(c_1, \cdots, c_n) \end{pmatrix}_{m\times(n+1)}. \tag{31}$$

As early as 1996, Goze and Khakimdjanov had characterized derivations of L_n in [31]. The following lemma comes from [31].

Lemma 12

$$\mathrm{Der}(L_n) = \mathrm{span}\{adx_0, adx_1, \cdots, adx_{n-1}, h_2, h_3, \cdots, h_{n-1}, t_1, t_2, t_3\}, \tag{32}$$

where

$$\begin{aligned} &adx_i(x_j) = [x_i, x_j], \quad 0 \leqslant j \leqslant n, 0 \leqslant i \leqslant n-1, \\ &h_k(e_i) = e_{i+k}, \quad 1 \leqslant i \leqslant n-k, \\ &t_1(e_j) = e_j, t_1(e_0) = 0, \quad 1 \leqslant j \leqslant n, \\ &t_2(e_0) = e_0, \\ &t_2(e_i) = (i-1)e_i, \quad 2 \leqslant i \leqslant n, \\ &t_3(e_0) = e_1. \end{aligned} \tag{33}$$

From this lemma, we can easily get the following conclusion.

Corollary 13. *Let φ be a linear mapping of L_n. Then, $\varphi \in Der(L_n)$ if and only if there exist $a, a_1, a_2, \cdots, a_n \in \mathbb{F}$ and $\alpha \in \mathbb{F}^n$ such that the matrix of φ is $A(a, a_1, \cdots, a_n, \alpha)$.*

Next, we will characterize the matrix form of the superderivation of $L_{n,m}$.

Theorem 14. *Let $\mathscr{D}$ be a linear mapping of $L_{n,m}$, then $\mathscr{D}$ is a superderivation if and only if its matrix is of the form $(A \oplus B) + (D \bar{\oplus} C)$, where A, D, C, and B are in the forms of (28)–(31), respectively.*

Proof. Clearly, a direct verification can prove the sufficiency, and so we only need to prove the necessity of the theorem.

If $\mathscr{D} \in \mathrm{Der}(L_{n,m})$, we can assume that the matrix of $\mathscr{D}$ is $(A \oplus B) + (D \bar{\oplus} C)$, where

$$A = \begin{pmatrix} a_0 & a_{10} & a_{20} & \cdots & a_{n0} \\ a_1 & a_{11} & a_{21} & \cdots & a_{n1} \\ a_2 & a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & \vdots & & \vdots \\ a_n & a_{1n} & a_{2n} & \cdots & a_{nn} \end{pmatrix},$$
$$B = \begin{pmatrix} b_{11} & b_{21} & \cdots & b_{m1} \\ b_{12} & b_{22} & \cdots & b_{m2} \\ \vdots & \vdots & & \vdots \\ b_{1m} & b_{2m} & \cdots & b_{mm} \end{pmatrix},$$
$$C = \begin{pmatrix} c_{01} & c_{11} & \cdots & c_{n1} \\ c_{02} & c_{12} & \cdots & c_{n2} \\ \vdots & \vdots & & \vdots \\ c_{0m} & c_{1m} & \cdots & c_{nm} \end{pmatrix},$$
$$D = \begin{pmatrix} d_{10} & d_{20} & \cdots & d_{m0} \\ d_{11} & d_{21} & \cdots & d_{m1} \\ d_{12} & d_{22} & \cdots & d_{m2} \\ \vdots & \vdots & & \vdots \\ d_{1n} & d_{2n} & \cdots & d_{mn} \end{pmatrix}. \quad (34)$$

First, we will deduce the form of the matrix of even derivation.

By Corollary 13,

$$A = A(a_0, a_{11}, \cdots, a_{1n}, \alpha), \quad (35)$$

where $\alpha = (a_1, \cdots, a_n)^T$.

Using $[\mathscr{D}_{\bar{0}}(x_0), y_i] + [x_0, \mathscr{D}_{\bar{0}}(y_i)] = \mathscr{D}_{\bar{0}}([x_0, y_i]) = \mathscr{D}_{\bar{0}}(y_{i+1})$, we can conclude that

$$\begin{aligned} b_{i+1,1} &= 0, b_{i+1,k+1} = b_{ik}, b_{i+1,i+1} \\ &= a_0 + b_{ii}, \quad 1 \leqslant i, k \leqslant m-1 \text{ and } k \neq i. \end{aligned} \quad (36)$$

That is,

$$B = B(a_0, b_{11}, \cdots, b_{1m}). \quad (37)$$

Next, we will deduce the form of the matrix of odd derivation. Similar to the above process, substituting $(x, y) = (x_0, x_i), (x_0, x_n), (y_1, y_k), (x_0, y_m), (x_0, y_j)$ into the next equation successively,

$$[\mathscr{D}_{\bar{1}}(x), y] + (-1)^{|x|}[x, \mathscr{D}_{\bar{1}}(y)] = \mathscr{D}_{\bar{1}}([x, y]), \quad (38)$$

then we can get the following equations in turn:

$$\begin{aligned} & c_{i+1,1} = 0, c_{i+1,s+1} = c_{is}, \quad 1 \leqslant i \leqslant n-1, 1 \leqslant s \leqslant m-1, \\ & c_{n1} = \cdots = c_{n,n-1} = 0, \\ & d_{k0} = 0, \quad 1 \leqslant k \leqslant m, \\ & d_{m1} = \cdots = d_{m,n-1} = 0, \\ & d_{j+1,0} = d_{j+1,1} = 0, d_{j+1,t+1} = d_{jt}, \quad 1 \leqslant j \leqslant m, 1 \leqslant t \leqslant n-1. \end{aligned} \quad (39)$$

Then,

$$\begin{aligned} & D = D(d_{11}, \cdots, d_{1n}), \\ & C = C(c_{nm}, c_{n-1,m}, \cdots, c_{1m}, \beta, \gamma), \end{aligned} \quad (40)$$

where $|x|$ refers the degree of x, $(\beta^T, \gamma^T) = (c_{01}, c_{02}, \cdots, c_{0m})$.

By (35), (37), and (40), we complete the proof of the necessity of the theorem. □

Theorem 15. *Let φ be a linear mapping of $L_{n,m}$ whose matrix is $(A \oplus B) + (D \bar{\oplus} C)$, where B is an $m \times m$ matrix. Then, $\varphi \in LDer(L_{n,m})$ if and only if A and B are both lower triangular matrices, and D and C are of the form*

$$\begin{pmatrix} v_1 & & & & \\ \vdots & & & & \\ v_{m-n} & & & & \\ v_{m-n+1} & c_{11} & & & \\ v_{m-n+2} & c_{21} & c_{22} & & \\ \vdots & \vdots & \vdots & \ddots & \\ v_m & c_{n1} & c_{n2} & \cdots & c_{nn} \end{pmatrix}, \quad (41)$$

$$\begin{pmatrix} 0 & & & & & & \\ d_{11} & & & & & & \\ d_{21} & d_{22} & & & & & \\ \vdots & \vdots & \ddots & & & & \\ d_{n1} & d_{n2} & \cdots & d_{nn} & 0 & \cdots & 0 \end{pmatrix}, \quad (42)$$

respectively.

Proof. First, we prove the necessity of the theorem. If $\varphi \in \mathrm{LDer}(L_{n,m})$, then for any $X = (k_0, k_1, \cdots, k_n, l_1, \cdots, l_m)^T \in \mathbb{F}^{n+m+1}$, there exist A_X, B_X, C_X, and D_X such that

$$\begin{pmatrix} A & C \\ D & B \end{pmatrix} X = \begin{pmatrix} A_X & C_X \\ D_X & B_X \end{pmatrix} X, \quad (43)$$

where A_X, B_X, C_X, and D_X are of the forms A, B, C, and D in Theorem 14, respectively.

If we let $l_1 = \cdots = l_m = 0$, then by the arbitrariness of $k_0, k_1, \cdots, k_n$ and (43) and in a similar way to the proof of Theorem 3, we obtain that A and D have the required forms.

Similarly, if we let $k_0 = k_1 = \cdots = k_n = 0$, then by the arbitrariness of $l_1, \cdots, l_m$ in (43) and in a similar way to the proof of Theorem 3, we deduce that C and B have the required forms.

Next, we will prove the sufficiency of the theorem.

For any $X_1 \in \mathbb{F}^{n+1}$, in a similar way to prove Lemma 1, we have A_{X_1} and D_{X_1} such that $AX_1 = A_{X_1}X_1$ and $DX_1 = D_{X_1}X_1$, where A_{X_1} and D_{X_1} are of the forms A and D in Theorem 14, respectively.

Similarly, for any $X_2 \in \mathbb{F}^m$ and $a \in \mathbb{F}$ which is (1, 1)-entry of A_{X_1}, there exist C_{X_2} and B_{X_2} such that $CX_2 = C_{X_2}X_2$ and $BX_2 = B_{X_2}X_2$, where C_{X_2} and B_{X_2} are of the forms C and B in Theorem 14, respectively.

Thus, for any $X = (X_1^T\ X_2^T)^T \in F^{n+1+m}$, we have

$$\begin{pmatrix} A & C \\ D & B \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} AX_1 + CX_2 \\ DX_1 + BX_2 \end{pmatrix} = \begin{pmatrix} A_{X_1}X_1 + C_{X_2}X_2 \\ D_{X_1}X_1 + B_{X_2}X_2 \end{pmatrix} = \begin{pmatrix} A_{X_1} & C_{X_2} \\ D_{X_1} & B_{X_2} \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}. \quad (44)$$

Hence, $\varphi \in \mathrm{LDer}(L_{n,m})$. □

Corollary 16

$$\mathrm{Der}\left(L_{n,m}\right) = \mathrm{span}\left\{adx_i, ady_j, a^k, b^l, c_t, d_s, g, h, t, u, v \mid 1 \leqslant i, k \leqslant n-1,\ 1 \leqslant j, l \leqslant m-1,\ 1 \leqslant s, t \leqslant n\right\}, \quad (45)$$

where

$$\begin{aligned}
&adx_i\left(x_j\right) = \left[x_i, x_j\right], adx_i\left(y_k\right) = \left[x_i, y_k\right], \quad 0 \leqslant j \leqslant n, 1 \leqslant k \leqslant m, 1 \leqslant i \leqslant n-1,\\
&ady_j\left(x_i\right) = \left[y_i, x_i\right], ady_j\left(y_k\right)\\
&a\left(x_i\right) = x_{i+1}, \quad 1 \leqslant i \leqslant n-1,\\
&b\left(y_j\right) = y_{j+1}, \quad 1 \leqslant j \leqslant m-1,\\
&d_s\left(y_1\right) = x_s, d_s\left(y_2\right) = x_{s+1}, \ldots, d_s\left(y_{n+1-s}\right) = x_n, \quad 1 \leqslant s \leqslant n,\\
&c_t\left(x_1\right) = y_{m+1-t}, c_t\left(x_2\right) = y_{m+2-t}, \ldots, c_t\left(x_t\right) = y_m, \quad 1 \leqslant t \leqslant n,\\
&g\left(x_0\right) = x_0, g\left(x_i\right) = (i-1)x_i,\\
&g\left(y_j\right) = (j-1)y_j, \quad 2 \leqslant i \leqslant n, 2 \leqslant j \leqslant m,\\
&h\left(x_i\right) = x_i, \quad 1 \leqslant i \leqslant n,\\
&t\left(y_j\right) = y_j, \quad 1 \leqslant j \leqslant m,\\
&u\left(x_0\right) = x_1; v\left(x_0\right) = y_1.
\end{aligned} \quad (46)$$

Corollary 17

$$\mathrm{LDer}\left(L_{n,m}\right) = \mathrm{span}\left\{a_{ij}, b_{kl}, c_{p,m-n+q}, d_{s,n+t}, h_u \mid 0 \leqslant j \leqslant i \leqslant n, n+1 \leqslant l \leqslant k \leqslant n+m, 1 \leqslant t \leqslant s \leqslant n, 1 \leqslant q \leqslant p \leqslant n, 1 \leqslant u \leqslant m\right\}, \quad (47)$$

where

$$\begin{aligned}
a_{ij}(x_i) &= x_j, \quad 0 \leqslant j \leqslant i \leqslant n,\\
b_{kl}(y_k) &= x_l, \quad 1 \leqslant l \leqslant k \leqslant m,\\
h_u(x_0) &= y_{n+u}, \quad 1 \leqslant u \leqslant m,\\
c_{p,m-n+q}(x_p) &= y_{m-n+q}, \quad 1 \leqslant q \leqslant p \leqslant n,\\
d_{s,n+t}(y_s) &= y_{n+t}, \quad 1 \leqslant t \leqslant s \leqslant n.
\end{aligned} \tag{48}$$

Theorem 18. *Let φ be a linear mapping of $L_{n,m}$. Then, φ is a linear 2-local superderivation of $L_{n,m}$ if and only if there exist $\psi \in Der(L_{n,m})$ and $k \in \mathbb{F}$ such that*

$$\varphi = \psi + k\sigma, \tag{49}$$

where σ is a linear mapping of $L_{n,m}$ whose matrix is E_{11}.

Proof. If φ is a linear 2-local superderivation of $L_{n,m}$, then by Theorem 18, we can assume that the matrix of $L_{n,m}$ is $(A \oplus B) + (D \bar{\oplus} C)$, where A and B are both lower triangular matrices, and D and C are of the forms (41) and (42), respectively. Thus, for any $X, Y \in \mathbb{F}^{n+m+1}$, there exist A_{XY}, D_{XY}, C_{XY}, and B_{XY} such that

$$\begin{pmatrix} A & C \\ D & B \end{pmatrix}(X, Y) = \begin{pmatrix} A_{XY} & C_{XY} \\ D_{XY} & B_{XY} \end{pmatrix}(X, Y), \tag{50}$$

where A_{XY}, D_{XY}, C_{XY}, and B_{XY} are of the forms of (28)–(31), respectively.

Denote $A = \begin{pmatrix} a & O \\ \alpha & A_1 \end{pmatrix}$ and $A_1 = (a_{ij})_{n\times n}$ with $a_{ij} = 0\,(i < j)$. For any $i \in \{2, 3, \cdots, n-1\}$, substituting $X = e_i, Y = e_{i+1}$ into (50), we have $a_{i,i-1} = a_{i+1,i}$. Then, for any $j \in \{2, 3, \cdots, n-1\}$, substituting $X = e_j + e_{j+1}, Y = e_{j+1} + e_{j+2}$ into (50), we have $a_{j+1,j+1} - a_{jj} = a_{jj} - a_{j-1,j-1}$. Denote $\alpha = (u_1, \cdots, u_n)^T, a_{22} - a_{11} = k, a_{s1} = a_s, 1 \leqslant s \leqslant n$. Thus,

$$A = A(k, a_1, \cdots, a_n, \alpha) + (a-k)E_{11}. \tag{51}$$

Similarly, we conclude that

$$\begin{aligned}
C &= C(c_1, \cdots, c_n, \beta, \gamma),\\
D &= D(d_1, \cdots, d_n),\\
B &= B(l, b_1, \cdots, b_n).
\end{aligned} \tag{52}$$

If $m - n = 1$, substituting $X = e_2 + e_{n+3}, Y = e_3 + e_{n+4}$ into (50), we have $k = l$. Else if $m - n \neq 1$, substituting $X = e_2 + e_{n+2}, Y = e_3 + e_{n+3}$ into (50), we have $k = l$. Thus, φ is desired.

Next, we will prove the sufficiency of the theorem. Assume the matrix of φ is $(A \oplus B) + (D \bar{\oplus} C)$, where C, D, and A are the same as in (51) and (52), respectively, and $B = B(k, b_1, \cdots, b_n)$. For any $X, Y \in \mathbb{F}^{n+m+1}$, we want to find A_{XY}, B_{XY}, C_{XY} and D_{XY} such that (50) holds, where A_{XY}, D_{XY}, C_{XY}, and B_{XY} are of the forms (29)–(31), respectively. □

Case 19. If X and Y are linear dependent, then by Theorem 18, the existence of A_{XY}, B_{XY}, C_{XY}, and D_{XY} is obvious.

Case 20. If X and Y are linear independent, then without loss of generality, we only need to consider the case of

$$\begin{aligned}
X &= e_i + \sum_{k=i+1}^{n+m+1} x_k e_k,\\
Y &= e_{i+s} + \sum_{k=i+s+1}^{n+m+1} y_k e_k,
\end{aligned} \tag{53}$$

where $1 \leqslant s \leqslant n + m + 1 - i, 1 \leqslant i \leqslant n + m$.

Subcase 21. $i > 1$. Put $A_{XY} = A - (a-k)E_{11} B_{XY} = B, C_{XY} = C$ and $D_{XY} = D$, and, therefore, (50) holds.

Subcase 22. $i = 1$ and $s \leqslant n$. Let

$$\begin{aligned}
A_{XY} &= A(a', a_1', \cdots, a_n', \alpha',), D_{XY} = D(d_1', \cdots, d_n')\\
C_{XY} &= C(c_1', \cdots, c_n', \beta', \gamma'), B_{XY} = B(a', b_1', \cdots, b_n').
\end{aligned} \tag{54}$$

We will choose appropriate $a', \alpha, \beta, \gamma, a_i', c_i', d_i', b_j', 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m$ such that (50) holds.

Put $a' = a$. For any $a_t, c_t, n - s + 2 \leqslant t \leqslant n$, it is easy to choose appropriate $a_k', c_k', d_i', b_j', 1 \leqslant k \leqslant n - s + 1, 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m$ such that

$$\begin{pmatrix} A & C \\ D & B \end{pmatrix} Y = \begin{pmatrix} A_{XY} & C_{XY} \\ D_{XY} & B_{XY} \end{pmatrix} Y, \tag{55}$$

then we can choose α', β', γ' such that (50) holds.

Subcase 23. $i = 1$ and $s > n$. Similar to the proof in Subcase 22, we can achieve the goal.

From the proof of the above theorems, we get the following conclusion immediately.

Corollary 24. *Let φ be a linear mapping of L_n. Then,*

(1) $\varphi \in Der(L_n)$ if and only if there exist $a, a_1, a_2, \cdots, a_n \in \mathbb{F}$ and $\alpha \in \mathbb{F}^n$ such that the matrix of φ is $A(a, a_1, \cdots, a_n, \alpha)$;

(2) $\varphi \in LDer(L_n)$ if and only if the matrix of φ is a lower triangular matrix;

(3) φ is a linear 2-local automorphism of L_n if and only if there exist $a, a_1, a_2, \cdots, a_n, k \in \mathbb{F}$ and $\alpha \in \mathbb{F}^n$ such that the matrix of φ is $A(a, a_1, \cdots, a_n, \alpha) + kE_{11}$.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This study was supported by the NSF of Hainan Province of China (nos. 121MS0784 and 120RC587), the NSF of Heilongjiang Province of China (no. LH2020A020)}, and the NSF of China (no. 12061029).

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