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Research Article

A Novel Opportunity Losses-Based Polar Coordinate Distance (OPLO-POCOD) Approach to Multiple Criteria Decision-Making

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The ability to make decisions is crucial for achieving success in any field, particularly in areas that involve managing extensive information and knowledge. The process of decision-making in real-world scenarios often involves considering numerous factors and aspects. It can be challenging to make decisions in such complex environments. In this paper, we present a new technique that solves multicriteria decision-making (MCDM) problems by considering opportunity losses-based polar coordinate distance (OPLO-POCOD). MCDM is a subdiscipline of operations research in which some alternatives are evaluated concerning some criteria to choose the most optimal alternative(s). Opportunity loss is a fundamental concept in economics and management, which can be used as a basis for determining the value associated with information. The authors emphasize that the technique incorporates the concept of opportunity losses and uses distance vectors in polar coordinates, making it a compelling approach. By considering opportunity losses, decision-makers gain a better understanding of the trade-offs involved in selecting alternatives, enabling them to make more informed decisions. Finally, the proposed method is exhibited through the use of numerical an example to illustrate its process. Additionally, a comparative sensitivity analysis is conducted to evaluate the outcomes of OPLO-POCOD and compare them with existing MCDM methods. The OPLO-POCOD method is found to have high reliability compared to other methods, as indicated by Spearman's correlation coefficient, which is greater than 0.9. The method shows a correlation of over 98.5% with TOPSIS, COPRAS, ARAS, and MCRAT methods, demonstrating its robustness and effectiveness. These analyses show the efficiency of the proposed method and highlight the stability of the results.

1. Introduction

1.1. Multicriteria Decision Making (MCDM). The process of decision-making is a complex mental program that aims to solve problems by considering multiple aspects and determining the most desirable outcome. The process is impacted by a variety of factors, including physiological, biological, cultural, and social. This process can be either rational or irrational, and depending on the complexity of the problem, it may require implicit or explicit assumptions. The decision-making process can be classified as structured or unstructured [1]. Today, complex decision problems can be solved using mathematical equations, statistical models, mathematics, econometrics, and computing devices that facilitate the automatic calculation and estimation of solutions to decision problems. Decision analysis examines the problem of decision research, which comprises procedures and methods for decision-making [2]. Decision analysis is strongly related to the term decision support, which refers to providing aid in finding solutions to concerns raised by the decision-maker throughout the decision-making process [3].

Decision support approaches can be categorized into single-criteria and multicriteria, precisely according to the decision problems that they are used to solve [4].

In contrast to multiple-criteria approaches, single-criteria methods focus on optimizing the solution to a problem. The steps used to solve discrete problems are outlined in [5].

Real-world problems typically require complex and sophisticated analysis. Even simple daily decisions taken by an individual can lead to an incredibly complicated process [6].

One of the most accurate methods of decision-making is multicriteria decision-making (MCDM), which has been considered a revolution in this field [7, 8]. MCDM techniques are used to evaluate and select options from multiple criteria after a decision has been made [9].

Benjamin Franklin published one of the earliest research papers on multicriteria decision-making about the moral algebra concept.

Since the 1950s, numerous researchers have been working on MCDM methods to evaluate their mathematical modeling capabilities. This framework has provided a structure for decision-making problems and generates preferences from various options. Based on the research literature, philosophers and mathematicians, such as Ramon Llull (1232–1316) and Nicolaus Cusanus (1401–1464) (Spanos, 2004), are the pioneers of MCDM techniques.

Mathematicians, such as Le Chevalier Jean-Charles de Borda (1733–1799), Antoine Nicolas de Caritat (1743-1799-1794), and Francis Ysidro Edgeworth (1845–1926), also investigated MCDM (Paraskevopoulos, 2008). V. F Damaso Pareto (1896), a famous scientist and economist, introduced the "efficiency" concept, which Koopman (1951) extended [6].

In 1944, John von proposed and developed the formulation of the "utility theory". Roy [10] developed the theory of outranking relations for dealing with multidimensional decision-making problems.

After 1970, there was a significant increase in scientific research and practical applications related to multi-criteria decision-making at both theoretical and practical levels.

Table 1 outlines some of the most essential multicriteria decision-making techniques that researchers have explored.

Roy categorized models into three main groups: unique synthesis, relationships of superiority, and interactive approach [31].

According to Siskos, criteria models can be categorized into two groups: compensatory and noncompensatory models. Furthermore, criteria synthesis and models may be classified into three categories: functional methods, relational methods, and analytical methods based on calculus [32].

Pardalos et al. [31] classified the models concerning the forming process (multiobjective programming, multi-attribute decision making, utility theory, outranking relations theory, and analytical synthetic). An examination of the relevant literature indicates that different MCDM methods can be used to solve the same decision problems. Selecting the right MCDM method for a decision-making process is vital to guarantee that the final solution accurately reflects the preferences of the decision-maker [33].

The primary distinction between these approaches is regarding the complexity of the algorithms, the weighting of criteria, the means of displaying the priority assessment criteria, the extent of certainty or uncertainty in the data, and finally the type of data aggregation [34].

The purpose of this research is to introduce and present a new method called the OPLO-POCOD method for handling multicriteria decision-making (MCDM) problems. The researchers aim to address some limitations of existing MCDM methods and propose a new approach that takes into account the concept of opportunity losses and converts them into distances in the polar coordinate system. The hypothesis is that the OPLO-POCOD method can provide a more comprehensive and accurate assessment of alternatives in MCDM problems by considering the concept of opportunity losses. The researchers also aim to validate the efficiency of the new method through a numerical example and provide a conclusion based on their findings.

The article also describes the proposed methodology and demonstrates its application through a numerical example. It emphasizes the unique features of this method, particularly its use of opportunity losses converted into distances in the polar coordinate system. The lower distances indicate less lost opportunity and are considered more desirable in alternative selection.

Additionally, the article validates the efficiency of the OPLO-POCOD method and concludes with a discussion of its findings.

The rest of this paper is organized as follows. Section 2 explains opportunity cost, opportunity losses, and polar coordinate distance. Section 3 describes the new proposed methodology. In Section 4, we employ a numerical example to explain the process of the OPLO-POCOD method. In tabular and graphical forms, in Section 5, we validate the efficiency of the new method, and finally, the conclusion is discussed in Section 6.

1.2. The Concept of Opportunity Cost and Opportunity Losses.

Opportunity cost is a fundamental principle in the realms of decision-making and economics [35, 36]. Given the limited resources available, individuals endeavor to make the most suitable decisions. Consequently, individuals are worried about the efficient utilization of limited resources. The scarcity of resources necessitates optimal decision-making.

Economists use the term "opportunity cost" to describe what must be sacrificed to acquire something desired. This concept is an essential principle of economics, as it asserts that any choice has an accompanying opportunity cost. The concept of opportunity cost implies that the cost of one item is the lost opportunity to do or consume something else.

What is the consequence of decision-making? What is the result of good or bad decision-making? To answer these questions, managers use the concept of opportunity lost

Table 1: Some of the multicriteria decision-making (MCDM) techniques.

Techniques	Acronyms	References
Weighted sum model	(WSM)	[11]
Weighted product model	(WPM)	[12]
Weighted aggregated sum product assessments	(WASPAS)	[13]
Analytical hierarchy process	(AHP)	[14]
ÉLimination Et Choix Traduisant la REalité	ELECTRE	[15]
Technique for Order of Preference by Similarity to an Ideal Solution	(TOPSIS)	[16]
Preference Ranking Organization Method for Enrichment of Evaluations	(PROMETHEE)	[17]
Complex proportional assessment	(COPRAS)	[18]
Visekriterijumska optimizacija I KOmpromisno Resenje	VIKOR	[19]
Multiobjective optimization by ratio analysis plus the full multiplicative for	MULTIMOORA	[20]
Additive ratio assessment	(ARAS)	[21]
Evaluation based on Distance From The Average Solution	(EDAS)	[22]
Multiattribute utility theory	MAUT	[23]
Measuring attractiveness by a categorical based evaluation technique	MACBETH	[24]
Analytical network process	ANP	[25]
Qualitative flexible multiple criteria method	QUALIFLEX	[26]
Organization, rangement et synthese de donnees relationnelles	ORESTE	[27]
Evaluation of mixed data method	EVAMIX	[28]
Derived from the Greek word tactix	TACTIC	[29]
UTilités Additives	UTA	[30]

(losses). The best decision is chosen based on the least lost opportunity. Understanding the value of an alternative chosen requires knowledge of the value of opportunity lost based on the best alternative. Briefly, opportunity lost is the value of the next best alternative.

The opportunities losses have different forms according to the type of business. Manufacturing, service, and transportation organizations have completely different opportunities for losses. Numerous factors may lead to lost opportunities in a manufacturing organization described as follows:

(i) Products are not being made
(ii) Income is lost
(iii) Upcoming income is impacted
(iv) Inventory levels are affected
(v) Orders cannot be filled
(vi) Morale negatively impacts

Lost profit analysis is a widely utilized approach to estimating the intangible losses resulting from risks related to systems [37].

The analysis begins with an assessment of information risks and then progresses to analyzing their implications in the business domain. Business process analysis should be focused on losses resulting from information availability problems that have differences in terms of assumptions about the nature of business losses, information risks, and the content of analysis methods.

Several methods are included such as labor cost analysis, lost profit analysis [37, 38], information asset value analysis [39], business process analysis [40–42], and stock market reaction analysis [43]. The most important potential business losses include operative business losses, competitive losses, shareholder losses, company image losses, customer service processes, or losses resulting from legal processes.

2. The Concept of Distance and Its Measurement

The dimension of distance per unit of time and space has specific meanings. In mathematics, the polar coordinate system is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a reference point and an angle from a reference.

To convert a point from the polar coordinate system to the Cartesian coordinate system, the functions sine and cosine are used to convert the radius and angle of the point circle into its corresponding x and y components.

2.1. Polar Coordinates Distance. The Cartesian coordinates (rectangular coordinates) of a point are a pair or a triplet of numbers (in two or three dimensions) that state signed distances from the coordinate axis. The Cartesian coordinates of a point in the two-dimensional plane are represented by (x, y) as shown in Figure 1, while three-dimensional space requires (x, y, z).

Instead of Cartesian coordinates, polar coordinates specify the location of a point P in the plane by its distance r from the origin and an angle θ measured from the horizontal axis anticlockwise to the line r, giving coordinates (r, θ).

The polar coordinates (r, θ) of a point P and the Cartesian plane are illustrated in Figure 2.

Each angle can be represented on a grid so that its position relative to other angles can be better understood [44, 45].

As θ ranges from 0 to 2π and r ranges vary from 0 to infinity, the point $P\ (r, \theta)$ covers every point in the plane.

Therefore, to more clearly illustrate the angle between two vectors, we can utilize a grid illustrated as follows. Let us explore the process of determining the distance between two polar points on the grid in Figures 3 and 4.

If we have two points as $(4,105^{\circ})$ and $(3,225^{\circ})$ in Figure 3, we plotted a graph to show two points on the polar grid.

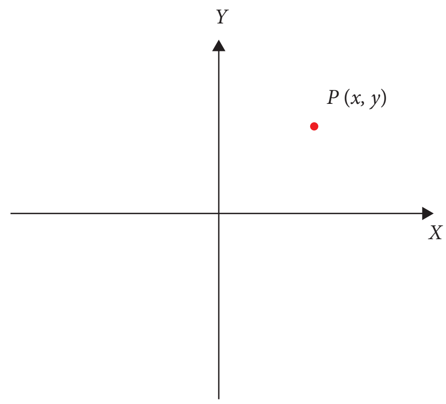

Figure 1: A point in Cartesian coordinates.

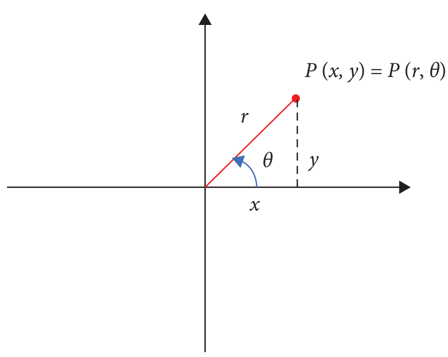

Figure 2: A point in polar coordinates.

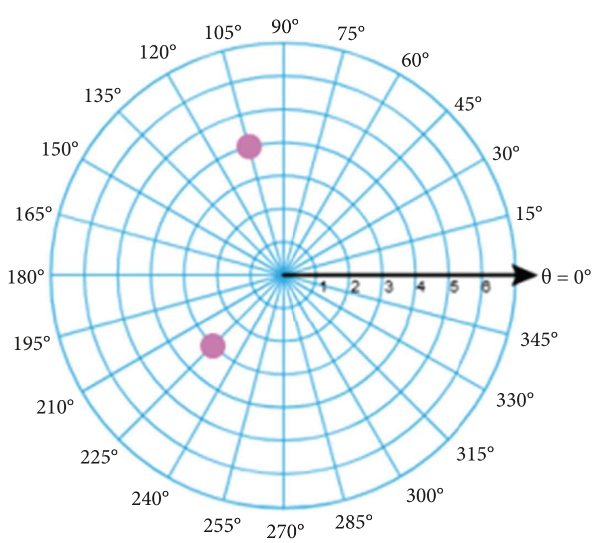

Figure 3: Two points on the grid.

The x-axis represents the length of the vector and the other axis (y-axis) represents the angle of the vector, as shown in Figure 4, and the angle between the two points is shown in Figure 5.

To calculate the angle of two points (4,105°) and (3,225°) assuming that we have the angle of each point concerning the x-axis, first we must rotate 105° and 225° counterclockwise to get from the polar axis. The angle between the two points is obtained by subtracting the larger angle minus the smaller angle 225° – 105° = 120°, as shown in Figure 6.

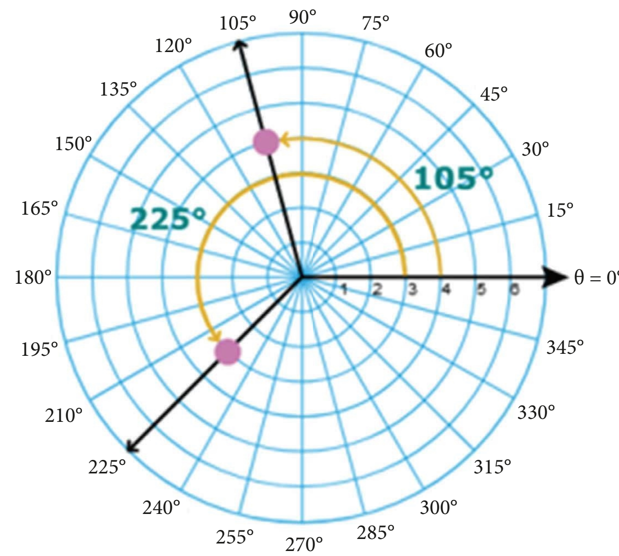

Figure 4: The angle between any point and the origin.

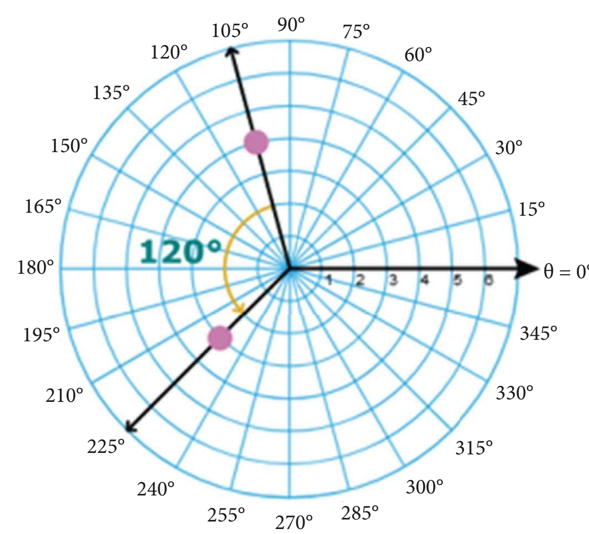

Figure 5: The angle between two points on the grid.

Figure 7 shows the distance between the first and second points which is a line segment from the pole to (4,105°) and another from the pole to (3,225°).

On the other hand, according to the law of cosines (also called the cosine rule), it can be stated as follows:

$$\mathbf{a}^2 = \mathbf{b}^2 + \mathbf{c}^2 - 2\mathbf{bc}\cos\mathbf{A}. \tag{1}$$

Equation (1) helps us solve some triangle problems. For example, if two known sides will be (b: (4,105°)) and (c: (3,225°)) and side (a), the distance between the two polar points is unknown,

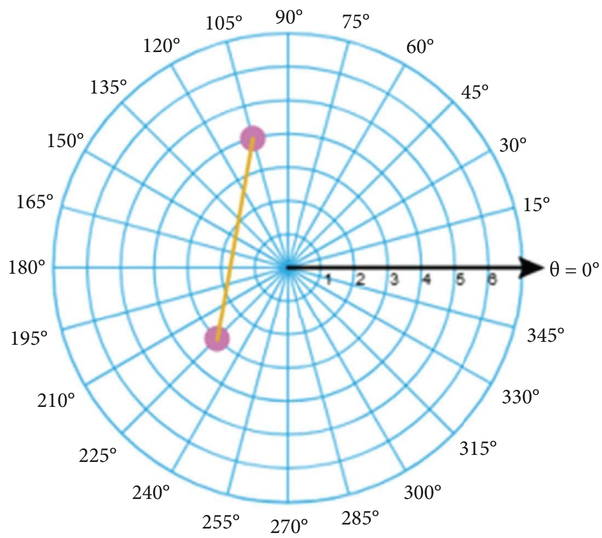

Figure 6: The distance between two points on the grid.

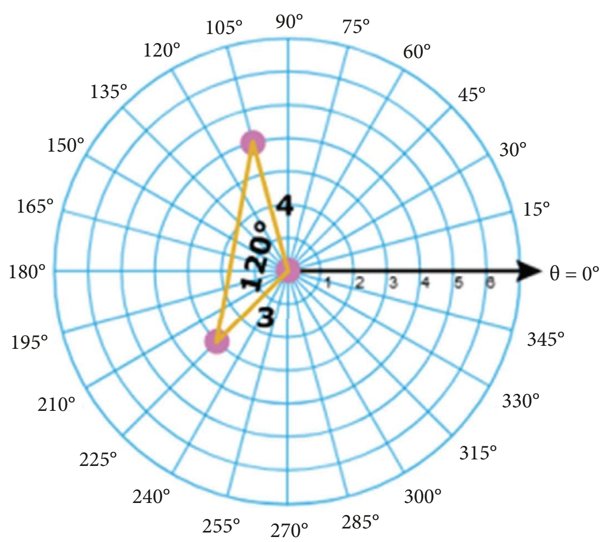

Figure 7: Geometric interpretation of the distance.

$$
\begin{aligned}
\mathbf{a^2} &= \mathbf{4^2 + 3^2 - 2\,(4)\,(3)\cos 120^{\circ}} \\
\mathbf{a^2} &= \mathbf{16 - 9 - \left(24 * -\frac{1}{2}\right)} \\
\mathbf{a^2} &= \mathbf{25 - (-12) = 37} \\
\mathbf{a} &= \sqrt{\mathbf{37}},
\end{aligned} \tag{2}
$$

(https://greenemath.com/Trigonometry/43/Polar-Equations-Graphs-IILesson.html).

In general, if we have two polar points indicated by (r_1, θ_1) and (r_2, θ_2), with r_1 and r_2 representing the two known sides, then the angle between those two sides can be determined as ($\theta 2 - \theta 1$).

We can use the law of cosines, substituting our two known sides as r_1 and r_2, along with the angle between them ($\theta_2 - \theta_1$), to solve for our unknown side.

$$\mathbf{a^2 = b^2 + c^2 - 2bc\cos A}. \tag{3}$$

Instead of the unknown side length, we will use d for the distance between two known sides, according to equations (4) and (5):

$$\mathbf{d^2 = r_1{}^2 + r_2{}^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}, \tag{4}$$

$$d = \sqrt{r_1{}^2 + r_2{}^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}. \tag{5}$$

2.2. Cosine Similarity. The cosine similarity calculation starts by computing the cosine of two nonzero vectors. Cosine similarity is an indication of the similarity between two nonzero vectors. This can be derived using the Euclidean dot product formula (equation (6)) which can be written as follows [46, 47]:

$$\mathbf{A.B = \|A\|\|B\|\cos\theta}. \tag{6}$$

The cosine similarity, cos (θ), between vectors A and B is calculated using the dot product and magnitude (equation (7)) which are defined as follows:

$$
\begin{aligned}
\textbf{cosine similarity} = \mathbf{S_C(A, B)} &\coloneqq \mathbf{\cos(\theta)} = \frac{\mathbf{A.B}}{\mathbf{\|A\|\|B\|}} \\
&= \frac{\sum_{i=1}^{n}\mathbf{A_i B_i}}{\sqrt{\sum_{i=1}^{n}\mathbf{A_i^2}}\sqrt{\sum_{i=1}^{n}\mathbf{B_i^2}}},
\end{aligned} \tag{7}
$$

where the angle θ represents the angle between the vectors A and B, and A.B represents the dot product between A and B.

$$A.B = A^T B = \sum_{i=1}^{n} A_i B_i = A_1 B_1 + A_2 B_2 + \ldots + A_n B_n, \tag{8}$$

and ‖A‖ represents the $L2$ norm or magnitude of the vector which is calculated as follows:

$$A.B = \sqrt{A_1^2 + A_2^2 + \ldots + A_n^2}. \tag{9}$$

3. The Proposed Methodology

The OPLO-POCOD method is concerned with structuring problems involving multiple criteria. This approach is based on the concept of selecting the alternative with the minimum opportunity losses and shortest distance in the polar coordinate distance. This is a method of compensatory aggregation that evaluates and ranks a set of alternatives. The fundamental notion behind compensatory methods is that they permit trade-offs between criteria, where an inadequate outcome in one criterion may be offset by superior performance in another.

The criteria used in this method include qualitative and quantitative criteria that should be quantified in the process of implementing the technique.

3.1. The Executive Steps of the OPLO-POCOD Method

Step 1. Formation of the initial matrix

Construct an initial matrix composed of m alternatives and n criteria, denoted as $x_{ij}(m\times n)$ or $x_{m\times n}$. This matrix is generated from the information received from the decision maker, as outlined in the following equation:

$$\mathbf{X}=\begin{bmatrix} \mathbf{x_{11}} & \cdots & \mathbf{x_{1j}} & \cdots & \mathbf{x_{1n}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{x_{i1}} & \cdots & \mathbf{x_{ij}} & \cdots & \mathbf{x_{in}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{x_{m1}} & \cdots & \mathbf{x_{mj}} & \cdots & \mathbf{x_{mn}} \end{bmatrix}_{\mathbf{m\times n}}, \quad \mathbf{i=1\ to\ m; j=1\ to\ n}, \tag{10}$$

where x_{ij} represents the element of the decision matrix for the ith alternative in jth criteria.

Numerous methods of rating scales have been developed to measure attitudes directly. The Likert scale (1932) is perhaps the most widely used scientific instrument. According to the Likert scale, an individual can express a particular statement in the form of a five (or seven) point scale. Each response has a numerical value equal to which would be used to measure the attitude under the survey (Table 2) [48].

Step 2. Constitution of the opportunity loss matrix.

The opportunity loss or regret values for each state of criteria can be determined by subtracting the payoff values associated with each state of criteria from their respective maximum payoffs in the case of a profit or gain (or, conversely, from the minimum payout in cases of cost or loss).

$$\textbf{Opportunity loss} = \textbf{best value for each action} - \textbf{value course of action}, \tag{11}$$

$$\begin{array}{rc} & \begin{matrix} & & c_1 & & c_j & & c_n \end{matrix} \\ X = & \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix}_{m\times n} \\ \text{Best value} & \begin{matrix} x_{i1}^{*} & & x_{ij}^{*} & & x_{in}^{*} \end{matrix} \end{array}, \quad i=1\ to\ m;\ j=1\ to\ n. \tag{12}$$

Here, for each state, x_{best} equals the minimum number for negative criteria and the maximum number for positive criteria.

$$\begin{array}{rc} & \begin{matrix} & \boldsymbol{c_1} & & \boldsymbol{c_j} & & \boldsymbol{c_n} \end{matrix} \\ X = & \begin{bmatrix} x_{11}-x_1^{*} & \cdots & x_{1j}-x_j^{*} & \cdots & x_{1n}-x_n^{*} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1}-x_1^{*} & \cdots & x_{ij}-x_j^{*} & \cdots & x_{in}-x_n^{*} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{m1}-x_1^{*} & \cdots & x_{mj}-x_j^{*} & \cdots & x_{mn}-x_n^{*} \end{bmatrix}_{m\times n} \end{array}, \quad i=1\ to\ m; \quad j=1\ to\ n. \tag{13}$$

$$\textbf{opportunity losses} = \left|\mathbf{x_{ij}} - \mathbf{x_{best}}\right| \quad \textbf{for}\ \forall\, \mathbf{x_{ij}}\ \mathbf{i=1\ to\ m; j=1\ to\ n}. \tag{14}$$

According to equations (13) and (14), we create the opportunity losses (OPL) matrix, based on the following equation:

Table 2: Measurement of attitudes using a seven-point scale.

Poor	Fairly weak	Medium	Fairly good	Good	Very good	Excellent
1	2	3	4	5	6	7

$$\mathrm{OPL} = \begin{array}{c} \begin{array}{ccccc} & & c_1 & c_j & c_n \end{array} \\ \begin{bmatrix} opl_{11} & \cdots & opl_{1j} & \cdots & opl_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ opl_{i1} & \cdots & opl_{ij} & \cdots & opl_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ opl_{m1} & \cdots & opl_{mj} & \cdots & opl_{mn} \end{bmatrix}_{m\times n} \end{array}, \quad i = 1\ to\ m\ ;\ j = 1\ to\ n. \tag{15}$$

Step 3. At this stage, the X_{pair} (ordered pair matrix) should be formed. The components of the ordered pair are x_{ij} (equation (12)) and opportunity losses opl_{ij} (equation (14)).

$$\mathbf{X}_{\mathbf{pair}_{\mathbf{ij}}} = \left(\mathbf{x}_{\mathbf{ij}}, \mathbf{OPL}_{\mathrm{ij}}\right), \quad \mathbf{i = 1\ to\ m\,;j = 1\ to\ n}, \tag{16}$$

$$\mathbf{X}_{\mathbf{pair}} = \begin{bmatrix} (\mathbf{x}_{11}, \mathbf{opl}_{11}) & \cdots & (\mathbf{x}_{1j}, \mathbf{opl}_{1j}) & \cdots & (\mathbf{x}_{1n}, \mathbf{opl}_{1n}) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ (\mathbf{x}_{i1}, \mathbf{opl}_{i1}) & \cdots & (\mathbf{x}_{ij}, \mathbf{opl}_{ij}) & \cdots & (\mathbf{x}_{in}, \mathbf{opl}_{in}) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ (\mathbf{x}_{m1}, \mathbf{opl}_{m1}) & \cdots & (\mathbf{x}_{mj}, \mathbf{opl}_{mj}) & \cdots & (\mathbf{x}_{mn}, \mathbf{opl}_{mn}) \end{bmatrix}_{\mathbf{m\times n}}, \quad \mathbf{i = 1\ to\ m; j = 1\ to\ n}. \tag{17}$$

Step 4. Calculating the distance matrix in polar coordinates.

In this step, the distance of each point from the best point for that criterion should be calculated. Here, a point is (x_{ij}, opl_{ij}) and a point is the best column value corresponding to the point (x_{best},$\text{opl}_{x\text{best}}$) where $\text{opl}_{x\text{best}}$ is equal 0.

The distance between these two points is obtained here using the following equation:

$$\mathbf{d}_{\mathbf{ij}} = \sqrt{\mathbf{A}_{\mathbf{ij}}^2 + \mathbf{B}_{\mathbf{ij}}^2 - 2\mathbf{A}_{\mathbf{ij}}\mathbf{B}_{\mathbf{ij}}\cos\left(\boldsymbol{\theta}_2 - \boldsymbol{\theta}_1\right)}, \tag{18}$$

where $A = (x_{\mathrm{ij}}, \mathrm{pl}_{\mathrm{ij}})$ and $B = (x_{\text{best}}, \text{opl}_{x_{\text{best}}})$ and (A, θ_1) and (B, θ_2).

$$\mathbf{A.B} = \|\mathbf{A}\|\|\mathbf{B}\|\cos\boldsymbol{\theta}$$

$$\cos\left(\theta_2 - \theta_1\right) = \frac{A.B}{\|A\|\|B\|} = \frac{\sum_{i=1}^{n} A_i B_i}{\sqrt{\sum_{i=1}^{n} A_i^2}\sqrt{\sum_{i=1}^{n} B_i^2}}. \tag{19}$$

As mentioned, in this equation, the cosine similarity according to equation (7) can be used to calculate the cosine of the angle difference between two vectors. Based on equation (1), matrix D should be formed.

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}_{11} & \cdots & \mathbf{d}_{1j} & \cdots & \mathbf{d}_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{d}_{i1} & \cdots & \mathbf{d}_{ij} & \cdots & \mathbf{d}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{d}_{m1} & \cdots & \mathbf{d}_{mj} & \cdots & \mathbf{d}_{mn} \end{bmatrix}_{\mathbf{m\times n}}, \quad \mathbf{i = 1\ to\ m; j = 1\ to\ n}. \tag{20}$$

Step 5. Creating a weighted distance matrix

Since the criteria may have different values, appropriate weight should be assigned to each criterion. Therefore, the weighted matrix D_w (equation (22)) should be obtained using equation (21):

$$\bar{\mathbf{d}}_{\mathbf{ij}} = \mathbf{w}_{\mathbf{j}} * \mathbf{d}_{\mathbf{ij}}, \tag{21}$$

$$\mathbf{D}_{\mathbf{w}} = \begin{bmatrix} \bar{\mathbf{d}}_{11} & \cdots & \bar{\mathbf{d}}_{1j} & \cdots & \bar{\mathbf{d}}_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{\mathbf{d}}_{i1} & \cdots & \bar{\mathbf{d}}_{ij} & \cdots & \bar{\mathbf{d}}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{\mathbf{d}}_{m1} & \cdots & \bar{\mathbf{d}}_{mj} & \cdots & \bar{\mathbf{d}}_{mn} \end{bmatrix}_{\mathbf{m\times n}}, \quad \mathbf{i = 1\ to\ m; j = 1\ to\ n}. \tag{22}$$

Step 6. Calculating the total distance

In this step, the sum distance obtained for each alternative (each row) is calculated according to the following equation:

$$\mathbf{S_i} = \sum_{\mathbf{j=1}}^{\mathbf{n}} \overline{\mathbf{d}}_{\mathbf{ij}}, \quad \mathbf{i = 1\ to\ m}, \tag{23}$$

$$S_T = \sum_{i=1}^{m} S_i. \tag{24}$$

Here, S_T is the total distance.

Step 7. Calculating the degree of opportunity loss and achievement opportunity

The degree of opportunity loss for each option is determined by the following equation:

$$\textbf{degree opportunity loss}\left(\mathbf{DOL_i}\right) = \frac{\mathbf{S_i}}{\mathbf{S_T}}. \tag{25}$$

Here, $\sum_{i=1}^{m} \mathrm{DOL}_i = 1$.

$$\textbf{Percentage of opportunity achievement}\left(\mathbf{POA_i}\right) = \mathbf{1 - DOL_i}. \tag{26}$$

The value of DOL_i lies in the range between 0 and 1, with a closer value of 0 indicating lower opportunities for the best-ranked alternative and a closer value of 1 implying more opportunities for the lowest-ranked alternative. In the explanation of the value of the POA_i, it should be noted that a value close to zero indicates a lower level of opportunity achievement for that alternative (the lowest rank), while any value close to one implies greater opportunity achievement (the highest rank).

This method offers several benefits. First, it considers the concept of opportunity losses, providing a more comprehensive evaluation of alternatives by taking into account potential benefits or opportunities that are foregone when choosing one alternative over another. Second, the method utilizes the polar coordinate system to represent the distances between alternatives, allowing for a more intuitive understanding of the relative positions and distances between alternatives. Third, it provides a comprehensive assessment of alternatives by considering multiple criteria simultaneously, capturing the complexity of real-world decision problems. Fourth, the OPLO-POCOD method introduces unique features, such as the conversion of opportunity losses into distances in the polar coordinate system, offering a novel approach to decision-making. Finally, the method aims to select the alternative that minimizes opportunity losses, resulting in a more accurate representation of the decision-makers' preferences and leading to more informed and effective decision-making. Overall, the OPLO-POCOD method offers valuable benefits and is a valuable technique in the field of multicriteria decision-making.

3.2. Schematic Diagram. A schematic diagram of the developed OPLO-POCOD method for determining the priorities of alternatives is provided in Figure 8.

4. Illustrative Example

To illustrate the process of the OPLO-POCOD method, we use an example in this section. The example "material selection of break booster valve body in a vehicle" is a suitable example for testing our proposed method that was investigated in Moradian's study. This example has also been solved using several MCDM methods by Abdulaal and Bafail [9]. Now, to solve the material selection problem with the proposed technique, we need to do the following steps:

Step 1. We created the initial table, which consists of 4 criteria and 16 alternatives, according to Table 3.

The weight of the criteria was determined based on a pairwise comparison based on the expert's judgment. It is given in the last row of Table 3.

Step 2. In this step, we can calculate the opportunity loss for each column and then the table DOL, by specifying the best criteria in each column of Table 4.

For c_3 and c_4, the best value is the smallest element in the corresponding column, and for other criteria, the largest element is the best value. Based on equations (11) and (14), the value of the opportunity losses table is obtained (Table 5).

Step 3. Based on equations (16) and (17), we create the ordered pair table (Table 6).

Step 4. We calculate the distance table for the ordered pairs obtained in the x_{pair} table and create the D table using equations (18) and (20) (Table 7).

Step 5. Calculating the weighted distance table.

Based on the weights expressed by the experts, we should obtain the weighted distance table (Table 8).

Step 6. Using equations (23)–(26), we calculate the sum of distances, degree opportunity loss (DOL), and percentage of opportunity achievement (POA) for each alternative. Based on this, an alternative that has fewer opportunity losses or more percentage of opportunity achievement (POA) can certainly be a better option to choose. Therefore, based on the DOL and POA indexes, the ranking of the rooms is given in Table 9.

According to the DOL index results, we conclude that A_2 is in the first rank with 0.0001% and is the best option. After that, A_8 with 0.0051% is in the second rank and then A_6 and A_4 with 0.0387% and 0.0458% are in the third and fourth ranks, respectively.

Based on the POA results, we can say A_2 with 0.9999 achievement opportunities as the first rank, and after that, the alternatives A_8, A_6, and A_4, with 0.9949, 0.9613, and 0.9542 achievement opportunities are ranked second, third, and fourth.

According to the philosophy of the technique, which is based on the concept of lost opportunity, it can be said that the alternative that has the least lost opportunity is the best. Since the lost opportunity has been converted into the distance dimension according to the proposed method, Figure 9 shows the total amount of opportunity losses based on the weighted distance of the alternatives.

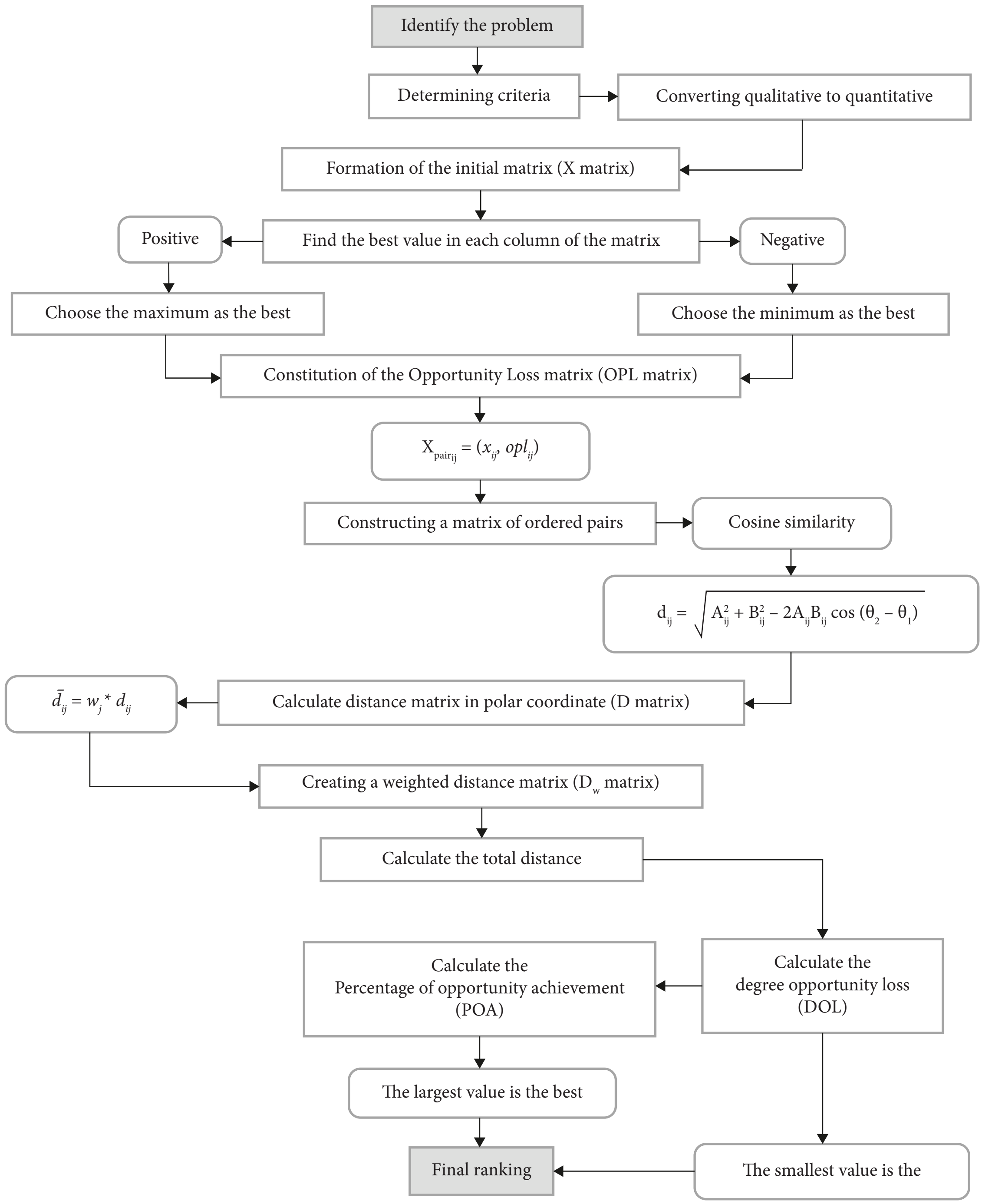

Figure 8: Schematic diagram of the steps of the proposed methodology.

Table 3: The initial table with 4 criteria and 16 alternatives.

	C1 Max	*C2* Max	*C3* Min	*C4* Min
A1	80	80	1.37	1
A2	185	222	1.66	1.5
A3	36	53	0.9	1.6
A4	110	150	1.1	2.3
A5	62	132	1.2	2
A6	128	143	1.43	2.8

Table 3: Continued.

	$C1$ Max	$C2$ Max	C3 Min	$C4$ Min
$A7$	84	63	1.13	1.5
$A8$	180	205	1.37	2.1
$A9$	46	85	1.06	1.3
$A10$	70	96	1.29	1.8
$A11$	28	44	0.96	1.1
$A12$	52	121	1.17	1.6
$A13$	54	71	1.41	1.1
$A14$	103	78	1.62	1.6
$A15$	59	86	1.05	1.3
$A16$	110	135	1.35	1.8
Weight	0.5405	0.1802	0.0688	0.2015

$C1$—tensile strength (maximized positive); $C2$—deflection temperature of the material (maximized positive); C3—materials density (minimized negative); $C4$—cost of the product (minimized negative); here, criteria $\mathbf{c}_3$ and $\mathbf{c}_4$ are negative and the other criteria are positive.

Table 4: Best values for each criterion.

	$C1$ Max	$C2$ Max	$C3$ Min	$C4$ Min
Best value	185	222	0.9	1

Table 5: The value opportunity losses for each alternative.

	$C1$	$C2$	$C3$	$C4$
$A1$	105	142	0.47	0
$A2$	0	0	0.76	0.5
$A3$	149	169	0	0.6
$A4$	75	72	0.2	1.3
$A5$	123	90	0.3	1
$A6$	57	79	0.53	1.8
$A7$	101	159	0.23	0.5
$A8$	5	17	0.47	1.1
$A9$	139	137	0.16	0.3
$A10$	115	126	0.39	0.8
$A11$	157	178	0.06	0.1
$A12$	133	101	0.27	0.6
$A13$	131	151	0.51	0.1
$A14$	82	144	0.72	0.6
$A15$	126	136	0.15	0.3
$A16$	75	87	0.45	0.8

Table 6: The ordered pair matrix.

	$C1$ Max	$C2$ Max	$C3$ Min	$C4$ Min
$A1$	(80, 105)	(80, 142)	(1.37, 0.47)	(1, 0)
$A2$	(185, 0)	(222, 0)	(1.66, 0.76)	(1.5, 0.5)
$A3$	(36, 149)	(53, 169)	(0.9, 0)	(1.6, 0.6)
$A4$	(110, 75)	(150, 72)	(1.1, 0.2)	(2.3, 1.3)
$A5$	(62, 123)	(132, 90)	(1.2, 0.3)	(2, 1)
$A6$	(128, 57)	(143, 79)	(1.43, 0.53)	(2.8, 1.8)
$A7$	(84, 101)	(63, 159)	(1.13, 0.23)	(1.5, 0.5)
$A8$	(180, 5)	(205, 17)	(1.37, 0.47)	(2.1, 1.1)
$A9$	(46, 139)	(85, 137)	(1.06, 0.16)	(1.3, 0.3)
$A10$	(70, 115)	(96, 126)	(1.29, 0.39)	(1.8, 0.8)
$A11$	(28, 157)	(44, 178)	(0.96, 0.06)	(1.1, 0.1)
$A12$	(52, 133)	(121, 101)	(1.17, 0.27)	(1.6, 0.6)

Table 6: Continued.

	C1 Max	C2 Max	C3 Min	C4 Min
A13	(54, 131)	(71, 151)	(1.41, 0.51)	(1.1, 0.1)
A14	(103, 82)	(78, 144)	(1.62, 0.72)	(1.6, 0.6)
A15	(59, 126)	(86, 136)	(1.05, 0.15)	(1.3, 0.3)
A16	(110, 75)	(135, 87)	(1.35, 0.45)	(1.8, 0.8)

Table 7: The distance calculation for each criterion.

	C1 Max	C2 Max	C3 Min	C4 Min
A1	148.4924	200.8183	0.66468	0
A2	0	0	1.074802	0.707107
A3	210.7178	239.0021	0	0.848528
A4	106.066	101.8234	0.282843	1.838478
A5	173.9483	127.2792	0.424264	1.414214
A6	80.61017	111.7229	0.749533	2.545584
A7	142.8356	224.86	0.325269	0.707107
A8	7.071068	24.04163	0.66468	1.555635
A9	196.5757	193.7473	0.226274	0.424264
A10	162.6346	178.1909	0.551543	1.131371
A11	222.0315	251.73	0.084853	0.141421
A12	188.0904	142.8356	0.381838	0.848528
A13	185.262	213.5462	0.721249	0.141421
A14	115.9655	203.6468	1.018234	0.848528
A15	178.1909	192.333	0.212132	0.424264
A16	106.066	123.0366	0.636396	1.131371

Table 8: Weighted distance matrix.

	C1	C2	C3	C4
A1	80.260	36.187	0.046	0.000
A2	0.000	0.000	0.074	0.142
A3	113.893	43.068	0.000	0.171
A4	57.329	18.349	0.019	0.370
A5	94.019	22.936	0.029	0.285
A6	43.570	20.132	0.052	0.513
A7	77.203	40.520	0.022	0.142
A8	3.822	4.332	0.046	0.313
A9	106.249	34.913	0.016	0.085
A10	87.904	32.110	0.038	0.228
A11	120.008	45.362	0.006	0.028
A12	101.663	25.739	0.026	0.171
A13	100.134	38.481	0.050	0.028
A14	62.679	36.697	0.070	0.171
A15	96.312	34.658	0.015	0.085
A16	57.329	22.171	0.044	0.228

The chart consists of 4 criteria and 16 alternatives. We can see immediately there were substantial differences between A_2 and A_{11}. As can be seen from the chart, A_2 has the least lost opportunity and A_{11} has the most.

In more detail, as shown in Table 10, it can be stated that $A2$ has the best performance so that opportunity loss in the criteria $c1$ and $c2$ is zero; it is also slightly far from the best in $c3$ and $c4$ with 0.074 and 0.142. In contrast, $A11$ has the highest opportunity loss along with the amount of 120.008 for $c1$ and $c2$ with 45.362.

5. Validity Test of the Novel Method

To validate the new method, it is necessary to compare its performance with other MCDM methods. The example solved in this article is taken from Moradian et al. [49] who used

Table 9: Ranking alternatives based on DOL and POA indexes.

No.	Distance	DOL	POA	Rank
$A1$	116.493	0.0701	0.9299	7
$A2$	0.216	0.0001	0.9999	1
$A3$	157.132	0.0946	0.9054	15
$A4$	76.067	0.0458	0.9542	4
$A5$	117.269	0.0706	0.9294	8
$A6$	64.267	0.0387	0.9613	3
$A7$	117.887	0.0710	0.9290	9
$A8$	8.513	0.0051	0.9949	2
$A9$	141.263	0.0850	0.9150	14
$A10$	120.280	0.0724	0.9276	10
$A11$	165.404	0.0995	0.9005	16
$A12$	127.599	0.0768	0.9232	11
$A13$	138.693	0.0835	0.9165	13
$A14$	99.618	0.0600	0.9400	6
$A15$	131.071	0.0789	0.9211	12
$A16$	79.772	0.0480	0.9520	5

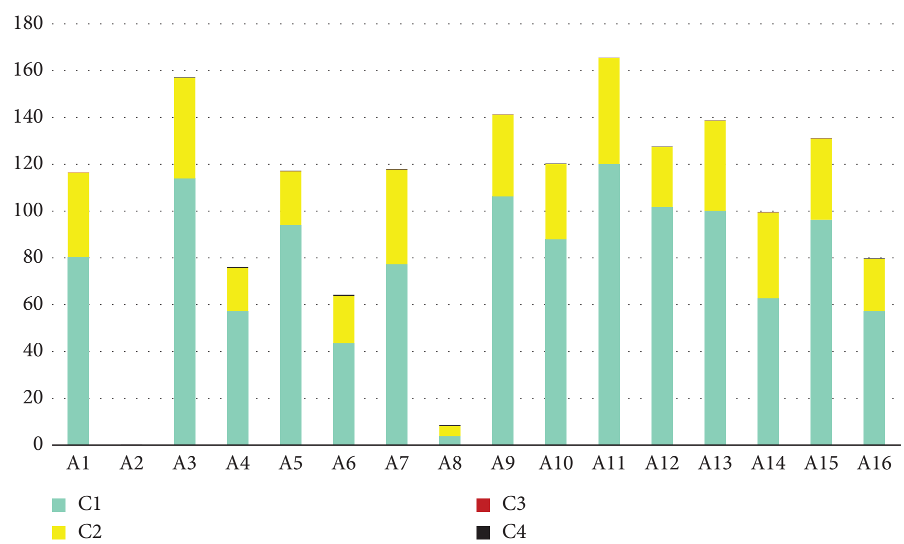

Figure 9: The total amount of opportunity losses (based on weighted distance) for each alternative.

Table 10: Degree opportunity loss for best and worst alternatives.

	$C1$ Max	$C2$ Max	$C3$ Min	$C4$ Min	Distance
Best value	185	222	0.9	1	
$A2$	185	222	1.66	1.5	185
Dol-$A2$	0.000	0.000	0.074	0.142	0.216
$A11$	28	44	0.96	1.1	28
Dol-$A11$	120.008	45.362	0.006	0.028	165.404

MCDM methods to evaluate the material selection of break booster valve body in a vehicle. This example has also been solved by Abdulaal and Bafail [9] using several MCDM methods.

For the same numerical example, the OPLO-POCOD method was applied and the results were compared to those obtained from a variety of other MCDM methods. Table 11 provides a summary of the rankings yielded by these methods.

The Spearman's rank correlation method was utilized to determine the correlation coefficients between methods. As illustrated in Table 12, there is a correlation degree between the novel methods and other methods for this numerical example.

To compare the ranking results obtained from the different methods, Spearman's rank correlation coefficient (r) is used. This is a suitable coefficient when we have ordinal variables or ranked variables. Table 12 represents the

Table 11: Comparing rankings from various MCDM methods.

No.	RAMS	RATMI	MCRAT	RAPS	ARAS	SAW	TOPSIS	COPRAS	VIKOR	WASPAS	MOORA	OPLO-POCOD
$A1$	7	7	7	7	7	6	7	7	7	6	7	7
$A2$	1	1	1	1	1	1	1	1	1	1	1	1
$A3$	16	16	15	16	16	16	16	16	15	15	15	15
$A4$	5	5	5	5	4	5	5	5	5	5	5	4
$A5$	12	12	10	12	10	12	10	11	11	11	11	8
$A6$	3	3	3	3	3	3	3	3	3	3	4	3
$A7$	8	8	8	8	8	8	8	8	8	8	8	9
$A8$	2	2	2	2	2	2	2	2	2	2	2	2
$A9$	14	14	14	14	14	14	14	14	14	14	14	14
$A10$	10	9	9	11	9	11	9	9	9	9	9	10
$A11$	15	15	16	15	15	15	15	15	16	16	16	16
$A12$	13	13	13	13	13	13	13	12	12	13	12	11
$A13$	9	11	12	9	12	10	12	13	13	12	13	13
$A14$	6	6	6	6	6	7	6	6	6	7	6	6
$A15$	11	10	11	10	11	9	11	10	10	10	10	12
$A16$	4	4	4	4	5	4	4	4	4	4	3	5

Table 12: Correlation coefficients between the ranking results of the OPLO-POCOD and the other methods.

	$\mathbf{r_s}$											
	RAMS	RATMI	MCRAT	RAPS	ARAS	SAW	TOPSIS	COPRAS	VIKOR	WASPAS	MOORA	OPLO-POCOD
RAMS	1											
RATMI	0.991	1										
MCRAT	0.976	0.988	1									
RAPS	0.997	0.988	0.971	1								
ARAS	0.976	0.988	0.994	0.971	1							
SAW	0.988	0.988	0.971	0.994	0.971	1						
TOPSIS	0.979	0.991	0.997	0.974	0.997	0.974	1					
COPRAS	0.971	0.991	0.991	0.968	0.991	0.974	0.994	1				
VIKOR	0.968	0.988	0.994	0.965	0.988	0.971	0.991	0.997	1			
WASPAS	0.976	0.991	0.994	0.974	0.988	0.982	0.991	0.991	0.994	1		
MOORA	0.965	0.985	0.991	0.962	0.982	0.968	0.988	0.994	0.997	0.991	1	
OPLO-POCOD	0.938	0.950	0.979	0.932	0.979	0.932	0.976	0.971	0.974	0.965	0.968	1

correlation coefficients that show the association between the results of the proposed method and the selected MCDM methods. If this correlation coefficient is greater than 0.8, the relationship between variables is very strong. As can be seen in Table 12, all values of r are greater than 0.8. Therefore, we can confirm the validity and stability of the results of the OPLO-POCOD method.

The OPLO-POCOD method has the least correlation of 93.2% than the RAPS and SAW methods, as indicated in Table 12. The OPLO-POCOD method has more than 95% correlation with other methods. The proposed method has demonstrated its ability to effectively rank alternatives.

6. Result and Conclusion

In recent times, there has been a growing application of multicriteria decision-making in tackling a wide range of real-world problems. Additionally, researchers have made significant progress in suggesting and refining numerous methods and techniques for this purpose.

In this paper, we proposed a new MCDM method, namely, the OPLO-POCOD method. To assess the alternatives on multiple criteria, it evaluates the alternatives based on opportunity losses as a fundamental concept, and polar coordinate distance, a seven-step procedure, was used for the OPLO-POCOD method. A numerical example has been used to illustrate the OPLO-POCOD method. Moreover, we have performed a comparative sensitivity analysis to demonstrate the validity and stability of the proposed method. In this analysis, ten sets of criteria weights are simulated and the results of the OPLO-POCOD method have been compared with the results of some existing MCDM methods. According to the results of this analysis, we can say that the proposed method is efficient in dealing with MCDM problems.

The example of material selection of a brake booster valve body in a vehicle was investigated by several researchers with different techniques. Abdulaal and Bafail [9] used several MCDM methods for selecting the best alternative out of 16 available alternatives with four selection criteria.

The effectiveness of the proposed technique OPLO-POCOD was compared to the other MCDM methods, including ARAS, SAW, TOPSIS, VIKOR, WASPAS, MOORA, RAMS, RAPS, and MCRAT.

The results of the OPLO-POCOD method show that A_2 is the best alternative and has a minimum opportunity or distance and the final ranking is $A_2 > A_8 > A_6 > A_4 > A_{16} > A_{14} > A_1 > A_5 > A_7 > A_{10} > A_{12} > A_{15} > A_{13} > A_9 > A_3 > A_{11}$.

We also utilize Spearman's correlation coefficient to analyze the correlation between the results of the OPLO-POCOD method and other methods.

The results of this analysis are shown in Table 12. According to Table 12, all correlation coefficients are greater than 0.9; thus, this indicates that our technique is highly reliable compared to other methods. The OPLO-POCOD method demonstrated a correlation of over 98.5% with each of the TOPSIS, COPRAS, ARAS, and MCRAT methods. Considering the ranking of the evaluation criteria, we demonstrated that the OPLO-POCOD method yields the same result and has salient features based on the opportunity losses concept and the distance vector in polar coordinates, which make it a robust and compelling method.

The final ranking from the OPLO-POCOD method is highly reliable as it provides more detailed information with DOL and POA indexes being more understandable results to managers and decision-makers compared to other MCDM methods. Also, the proposed new technique provides a more detailed analysis with high accuracy of each alternative based on different criteria.

6.1. The Advantages of the OPLO-POCOD Method. The benefit of the OPLO-POCOD method is that it introduces a new approach to handling multiple criteria decision making (MCDM) problems. It incorporates the concept of opportunity losses and converts them into distances in the polar coordinate system. This allows for a more comprehensive assessment of alternatives.

One of the advantages of this method is that it considers opportunity losses, which are often overlooked in other MCDM methods. By taking into account the potential losses associated with each alternative, decision-makers can make more informed choices.

Additionally, the use of polar coordinates allows for a more intuitive representation of distances. Lower distances in the polar coordinate system indicate less lost opportunity, making those alternatives more desirable.

Overall, the OPLO-POCOD method provides a unique perspective on MCDM problems by incorporating opportunity losses and utilizing a polar coordinate distance approach. This can lead to more accurate and effective decision-making in various domains.

The advantages of the OPLO-POCOD method are as follows:

(1) Incorporation of opportunity losses: this method introduces the concept of opportunity losses into the MCDM framework. Opportunity losses refer to the potential benefits that are forgone when selecting a particular alternative. By considering these losses, the method provides a more comprehensive assessment of alternatives and helps decision-makers make informed choices.

(2) Unique features: the OPLO-POCOD method offers features that are not found in other MCDM methods. These unique features provide a fresh perspective on decision-making and can potentially lead to more accurate and effective outcomes.

(3) Conversion into polar coordinates: the method converts opportunity losses into distances in the polar coordinate system. This conversion allows for a more intuitive representation of distances, where lower distances indicate less lost opportunity. This makes it easier for decision-makers to understand and compare alternatives.

(4) Enhanced alternative selection: by considering opportunity losses and utilizing polar coordinate distances, the OPLO-POCOD method helps in identifying alternatives that have lower lost opportunities. These alternatives are considered more desirable and can result in improved decision-making.

Overall, the advantages of the OPLO-POCOD method lie in its incorporation of opportunity losses, unique features, intuitive representation of distances, and the potential for enhanced alternative selection. These advantages make it a valuable technique for handling MCDM problems.

6.2. Future Research Directions for This Article

(1) Validation and application in real-world scenarios: the authors have provided a numerical example to demonstrate the effectiveness of the OPLO-POCOD method. However, further research could involve applying the technique to real-world decision-making problems in various industries and evaluating its performance and reliability in different contexts.

(2) Comparison with additional MCDM methods: the authors have compared the OPLO-POCOD method with eleven different MCDM methods. However, there are numerous other MCDM methods available. Future research could involve comparing the OPLO-POCOD method with additional methods to further validate its superiority and robustness.

(3) Sensitivity analysis: conduct sensitivity analysis to examine the impact of changes in criteria weights and values on the final rankings obtained by the OPLO-POCOD method. This would help in understanding the stability and sensitivity of the technique and provide insights into its applicability in different decision-making scenarios.

(4) Extension to dynamic decision-making: the OPLO-POCOD method is currently applied to static decision-making problems. Future research could explore the extension of this technique to dynamic decision-making scenarios, where criteria and alternatives may change over time. This would involve developing a framework that incorporates

time-dependent information and updating the rankings accordingly.

(5) Incorporation of uncertainty and risk: decision-making often involves dealing with uncertainty and risk. Future research could focus on developing extensions of the OPLO-POCOD method that incorporate uncertainty and risk analysis techniques, such as fuzzy logic, Monte Carlo simulation, or Bayesian inference. This would provide decision-makers with a more comprehensive understanding of the potential outcomes and associated risks when selecting alternatives.

(6) User-friendly software implementation: develop user-friendly software or tools that implement the OPLO-POCOD method and provide decision-makers with an intuitive interface for inputting criteria and alternatives, calculating opportunity losses, and obtaining the final rankings. This would enhance the practical applicability of the technique and make it more accessible to a wider range of users.

Data Availability

The data used to support the findings of the study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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