

Research Article

On the Leonardo Sequence via Pascal-Type Triangles

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In this study, we discussed the Leonardo number sequence, which has been studied recently and caught more attention. We used Pascal and Hosoya-like triangles to make it easier to examine the basic properties of these numbers. With the help of the properties obtained in this study, we defined a number sequence containing the new type of Leonardo numbers created by choosing the coefficients from the bicomplex numbers. Furthermore, we gave the relationship of this newly defined sequence with the Fibonacci sequence. We also provided some important identities in the literature provided by the elements of this sequence described in this paper.

1. Introduction

Recently, there have been an increasing number of studies on Leonardo numbers, which are a sequence of numbers that are closely related to Fibonacci numbers and have been the subject of some articles. The first comprehensive study on these numbers belongs to Alp and Koçer. In 2012, Kocer examined the Leonardo numbers in detail and gave an important relation between Leonardo and Fibonacci numbers [1]. Furthermore, the next detailed study belongs to Catarino and Borges [2, 3]. There are only a limited number of studies on this number sequence in the literature and one of the most recent studies is by Prasad et al. [4]. In this work, Kumari generalized these numbers and used them in Octonion algebra. On the other hand, Shattuck [5] discussed the combinatorial identities of the generalized Leonardo numbers. References [6–8] can be analyzed for more information.

Leonardo's numbers are given by, for $n \in \mathbb{N}$,

$$Le_{n+2} = Le_{n+1} + Le_n + 1, \quad (1)$$

where the initial conditions are $Le_0 = Le_1 = 1$. Writing the elements of the sequence provides computational convenience [2].

$$\{Le_n\}_{n \geq 0} = \{1, 1, 3, 5, 9, 15, 25, 41, 67, 109, 177, \dots\}. \quad (2)$$

The numbers have a relationship with the Fibonacci number, due to the nature of the repeated relation in the equation (1). In 2012, Alp and Koçer examined the Leonardo numbers in detail and mentioned Fibonacci relation [1].

$$Le_n = 2F_{n+1} - 1. \quad (3)$$

In 2022, Karatas handled the complex Leonardo numbers [9].

$$CLe_n = Le_n + iLe_{n+1} = CLe_{n-1} + CLe_{n-2} + (1 + i). \quad (4)$$

The author gave the Binet formula for these numbers in his work as follows.

$$CLe_n = \frac{2\underline{\alpha}\alpha^{n+1} - 2\underline{\beta}\beta^{n+1}}{\alpha - \beta} - (1 + i). \quad (5)$$

In this formula, the values of α, β are the roots of the characteristic equation containing the Fibonacci numbers. In addition, alpha and beta line values are as follows.

$$\begin{aligned} \underline{\alpha} &= 1 + i\alpha, \\ \underline{\beta} &= 1 + i\beta. \end{aligned} \quad (6)$$

And so, we write

$$\{CLe_n\}_{n \geq 0} = \{1 + i, 1 + 3i, 3 + 5i, 5 + 9i, 9 + 15i, \dots\}. \quad (7)$$

Definition 1. With the help of the numbers CLe_n , we can define the bicomplex Leonardo numbers as follows:

$$\begin{aligned} BLe_n &= CLe_n + jCLe_{n+2} \\ &= (Le_n + iLe_{n+1}) + j(Le_{n+2} + iLe_{n+3}), \end{aligned} \quad (8)$$

where

$$\begin{aligned} BLe_0 &= 1 + i + 3j + 5ij, \\ BLe_1 &= 1 + 3i + 5j + 9ij. \end{aligned} \quad (9)$$

In Table 1, we listed some of the n th Fibonacci, Lucas, Leonardo, complex Leonardo, bicomplex Fibonacci, and bicomplex Leonardo numbers to use later.

With the help of Table 1, the following equations can be written:

			1+i		
		1+3i		1+3i	
	3+5i	7+13i	3+7i	3+5i	
5+9i	13+23i	15+27i	7+13i	5+9i	(10)
9+15i	13+23i		13+23i	9+15i	
...	

It may be useful to use some notations here. These are

$$1 + i = \epsilon_i, 1 + 3i = \epsilon_i + 2i, 3 + 5i = 3\epsilon_i + 2i, \dots, \quad (11)$$

We can also examine the numbers CLe_n and the elements of the Pascal-like triangle with the help of the table below.

It is observed that the first column of Table 2 consists of CLe_n numbers. The following equation can be written to represent the notation $C(n, k)$ for the n th row and k th column element. For $n, k \in \mathbb{Z}^+$ and $n \geq k + 1$,

$$C(n, k) = C(n - 1, k - 1) + C(n - 1, k) + \epsilon_i. \quad (12)$$

For $n = 3, k = 2$, we get

$$\begin{aligned} C(3, 2) &= (3 + 7i) + (3 + 5i) + (1 + i) \\ &= 7 + 13i \\ &= 7\epsilon_i + 6i. \end{aligned} \quad (13)$$

In addition, the sum of the first column and last row elements of Table 2 gives the number BLe_n . Therefore, we can write the formula that gives these numbers as follows.

$$BLe_n = C(n, 0) + jC(n + 2, 0). \quad (14)$$

For $n = 1$,

$$\begin{aligned} BLe_1 &= C(1, 0) + jC(3, 0) \\ &= 1 + 3i + 5j + 9ij. \end{aligned} \quad (15)$$

- (i) $F_n + L_n + Le_n = 2Le_n + 1.$
- (ii) $F_n + L_n - 1 = Le_n.$
- (iii) $F_n + L_n + Le_n - 1 = 2Le_n.$
- (iv) $F_n + L_n + Le_n + CLe_n - 1 = 3Le_n + iLe_{n+1}.$
- (v) $F_n + L_n + Le_n + CLe_n + BF_n - 1 = F_n + 3Le_n + i(F_{n+4} - 1) + jCF_{n+2}.$
- (vi) $F_n + L_n + Le_n + CLe_n + BLe_n - 1 = 2(Le_n + CLe_n) + jCLe_{n+2}.$

As is known, using Pascal and Pascal-like triangles is another way of obtaining various new identities. In this section, we introduce Pascal-like and Hosoya-like triangles and give new identities. Let us give new Pascal-like triangles using the numbers Le_n and CLe_n . First, we construct a Pascal-like triangle using the numbers CLe_n . Next, we introduce Hosoya-like triangles using the numbers Le_n .

Using this information and the numbers BLe_n , we can construct the following Pascal-like triangle.

We give the elements of the Pascal-like triangle involving the numbers BLe_n in the table below.

Notice that the first column of Table 3 gives the numbers BLe_n , and the n th row and the k th column elements give the numbers $B(n, k)$. For example, for $(n, k) = (2, 1)$ and $(n, k) = (4, 3)$ we get

$$\begin{aligned} B(2, 1) &= B(1, 0) + B(1, 1) + \epsilon_i \epsilon_j \\ &= 3 + 7i + 11j + 19ij \\ &= 3\epsilon_i + 4i + j(11\epsilon_i + 8i), \\ B(4, 3) &= B(3, 2) + B(3, 3) + \epsilon_i \epsilon_j \\ &= 13 + 23i + 37j + 61ij \\ &= 13\epsilon_i + 10i + j(37\epsilon_i + 24i), \end{aligned} \quad (16)$$

respectively.

Theorem 2. For $n \geq k + 1$ with $n, k \in \mathbb{Z}^+$, the following equality is satisfied.

$$B(n, k) = B(n - 1, k - 1) + B(n - 1, k) + \epsilon_i \epsilon_j. \quad (17)$$

Now, let us recognize Hosoya-like triangles using the numbers Le_n .

Definition 5. For $n \geq 0$, the numbers BLE_n can be defined as follows:

$$\begin{aligned}
 BLE_n &= \sum_{s=0}^3 Le_{n+s}e_s \\
 &= Le_n e_0 + Le_{n+1}e_1 + Le_{n+2}e_2 + Le_{n+3}e_3,
 \end{aligned}
 \tag{26}$$

where $e_0 = 1, e_1 = i, e_2 = j, e_3 = ij$ and

$$\begin{aligned}
 BLE_0 &= \sum_{s=0}^3 Le_s e_s, \\
 BLE_1 &= \sum_{s=0}^3 Le_{s+1}e_s, \\
 BLE_2 &= \sum_{s=0}^3 Le_{s+2}e_s.
 \end{aligned}
 \tag{27}$$

Corollary 6. For $n \geq 3$, we have

$$BLE_n = 2BLE_{n-1} - BLE_{n-3}.
 \tag{28}$$

Theorem 7. For the numbers BLE_n , we have

$$g(BLE_n, x) = \frac{\sum_{s=0}^3 (Le_s + (Le_{s+1} - 2Le_s)x + (Le_{s+2} - 2Le_{s+1})x^2)e_s}{x^3 - 2x + 1}.
 \tag{29}$$

Proof. First, we write the equations $x^3g(BLE_n, x)$ and $2xg(BLE_n, x)$.

$$\begin{aligned}
 g(BLE_n, x) &= BLE_0 + BLE_1x + BLE_2x^2 + \dots + BLE_nx^n + \dots \\
 -2xg(BLE_n, x) &= -2BLE_0x - 2BLE_1x^2 - 2BLE_2x^3 - \dots - 2BLE_{n-1}x^n - \dots \\
 x^3g(BLE_n, x) &= BLE_0x^3 + BLE_1x^4 + BLE_2x^5 + \dots + BLE_{n-3}x^n + \dots
 \end{aligned}
 \tag{30}$$

Later, by arranging these equations, we get

$$\begin{aligned}
 (x^3 - 2x + 1)g(BLE_n, x) &= BLE_0 + x(BLE_1 - 2BLE_0) + x^2(BLE_2 - 2BLE_1) + \dots \\
 g(BLE_n, x) &= \frac{BLE_0 + x(BLE_1 - 2BLE_0) + x^2(BLE_2 - 2BLE_1)}{(x^3 - 2x + 1)} \\
 g(BLE_n, x) &= \frac{\sum_{s=0}^3 (Le_s + (Le_{s+1} - 2Le_s)x + (Le_{s+2} - 2Le_{s+1})x^2)e_s}{x^3 - 2x + 1}.
 \end{aligned}
 \tag{31}$$

Thus, the proof is completed. □

Theorem 8. For nonnegative integers n , we have

$$BLe_n = Ar_1^n + Br_2^n + Cr_3^n, \tag{32}$$

where $r_1 = 1 + \sqrt{5}/2, r_2 = 1 - \sqrt{5}/2, r_3 = 1,$

$$\begin{aligned} A &= \frac{\sum_{s=0}^3 Le_{s+2}e_s + (-r_2 - r_3)\sum_{s=0}^3 Le_{s+1}e_s + r_2r_3\sum_{s=0}^3 Le_s e_s}{(r_1 - r_2)(r_1 - r_3)}, \\ B &= \frac{\sum_{s=0}^3 Le_{s+2}e_s + (-r_1 - r_3)\sum_{s=0}^3 Le_{s+1}e_s + r_1r_3\sum_{s=0}^3 Le_s e_s}{(r_2 - r_1)(r_2 - r_3)}, \\ C &= \frac{\sum_{s=0}^3 Le_{s+2}e_s + (-r_1 - r_2)\sum_{s=0}^3 Le_{s+1}e_s + r_1r_2\sum_{s=0}^3 Le_s e_s}{(r_3 - r_1)(r_3 - r_2)}. \end{aligned} \tag{33}$$

Proof. To prove, if we use the initial values BLe_0, BLe_1, BLe_2 and recurrence relation $BLe_n = 2BLe_{n-1} - BLe_{n-3}$, then we write

$$\begin{aligned} A + B + C &= \sum_{s=0}^3 Le_s e_s, \\ Ar_1 + Br_2 + Cr_3 &= \sum_{s=0}^3 Le_{s+1}e_s, \\ Ar_1^2 + Br_2^2 + Cr_3^2 &= \sum_{s=0}^3 Le_{s+2}e_s. \end{aligned} \tag{34}$$

$$BLe_{n+r}BLe_{n-r} - BLe_n^2 = (-1)^{n-r} \{ABr_3^{r-n}(r_1^{2r} + r_2^{2r} + 2(-1)^{r+1}r_3^{-r}) + ACr_2^{r-n}(r_1^{2r} + r_3^{2r} + 2(-1)^{r+1}r_2^{-r}) + BCr_1^{r-n}(r_2^{2r} + r_3^{2r} + 2(-1)^{r+1}r_1^{-r})\}. \tag{35}$$

Proof. We can write the following equation for the left-hand side of equation (36).

$$\begin{aligned} \text{LHS} &= (Ar_1^{n+r} + Br_2^{n+r} + Cr_3^{n+r})(Ar_1^{n-r} + Br_2^{n-r} + Cr_3^{n-r}) - (Ar_1^n + Br_2^n + Cr_3^n)^2, \\ \text{LHS} &= AB(r_1r_2)^n (r_1^r r_2^{-r} + r_1^{-r} r_2^r - 2) + AC(r_1r_3)^n (r_1^r r_3^{-r} + r_1^{-r} r_3^r - 2) + BC(r_2r_3)^n (r_2^r r_3^{-r} + r_2^{-r} r_3^r - 2). \end{aligned} \tag{37}$$

For the last equation, if we do some editing and necessary algebraic operations, then

$$(-1)^{n-r} \{ABr_3^{r-n}(r_1^{2r} + r_2^{2r} + 2(-1)^{r+1}r_3^{-r}) + ACr_2^{r-n}(r_1^{2r} + r_3^{2r} + 2(-1)^{r+1}r_2^{-r}) + BCr_1^{r-n}(r_2^{2r} + r_3^{2r} + 2(-1)^{r+1}r_1^{-r})\}. \tag{39}$$

Thus, the proof is completed. \square

Here are some operations and abbreviations needed. If we also do them, then

$$\begin{aligned} A &= \frac{\sum_{s=0}^3 Le_{s+2}e_s + (-r_2 - r_3)\sum_{s=0}^3 Le_{s+1}e_s + r_2r_3\sum_{s=0}^3 Le_s e_s}{(r_1 - r_2)(r_1 - r_3)}, \\ B &= \frac{\sum_{s=0}^3 Le_{s+2}e_s + (-r_1 - r_3)\sum_{s=0}^3 Le_{s+1}e_s + r_1r_3\sum_{s=0}^3 Le_s e_s}{(r_2 - r_1)(r_2 - r_3)}, \\ C &= \frac{\sum_{s=0}^3 Le_{s+2}e_s + (-r_1 - r_2)\sum_{s=0}^3 Le_{s+1}e_s + r_1r_2\sum_{s=0}^3 Le_s e_s}{(r_3 - r_1)(r_3 - r_2)}. \end{aligned} \tag{35}$$

Thus, we provided the most general formula for the number BLe_n .

In the next theorem, we give the equation, which is significantly important for integer sequences and is known as Catalan's identity. \square

Theorem 9. For $n, r \in \mathbb{Z}^+$ and $n \geq r$ we have

$$BLe_{n+r}BLe_{n-r} - BLe_n^2, \tag{38}$$

is equal to this

Corollary 10. For the numbers BLe_n , we have

$$BLe_{n+1}BLe_{n-1} - BLe_n^2 = (-1)^{n-1} \{ ABr_3^{1-n}(r_1^2 + r_2^2 + 2r_3^{-1}) + ACr_2^{1-n}(r_1^2 + r_3^2 + 2r_2^{-1}) + BCr_1^{1-n}(r_2^2 + r_3^2 + 2r_1^{-1}) \} \quad (40)$$

Proof. If we use the Binet formula, we write the left side of equation (40) as follows.

$$\begin{aligned} BLe_{n+1}BLe_{n-1} - BLe_n^2 &= (Ar_1^{n+1} + Br_2^{n+1} + Cr_3^{n+1})(Ar_1^{n-1} + Br_2^{n-1} + Cr_3^{n-1}) - (Ar_1^n + Br_2^n + Cr_3^n)^2, \\ BLe_{n+1}BLe_{n-1} - BLe_n^2 &= AB(r_1^{n+1}r_2^{n-1} + r_2^{n+1}r_1^{n-1} - 2r_1^n r_2^n) \\ &\quad + AC(r_1^{n+1}r_3^{n-1} + r_3^{n+1}r_1^{n-1} - 2r_1^n r_3^n) + BC(r_2^{n+1}r_3^{n-1} + r_3^{n+1}r_2^{n-1} - 2r_2^n r_3^n), \end{aligned} \quad (41)$$

If we complete the necessary arrangements, then we obtain

$$BLe_{n+1}BLe_{n-1} - BLe_n^2 = (-1)^{n-1} \{ ABr_3^{1-n}(r_1^2 + r_2^2 + 2r_3^{-1}) + ACr_2^{1-n}(r_1^2 + r_3^2 + 2r_2^{-1}) + BCr_1^{1-n}(r_2^2 + r_3^2 + 2r_1^{-1}) \} \quad (42)$$

Thus, the proof is completed. \square

Theorem 11. For positive numbers m, n , $BLe_m BLe_{n+1} - BLe_n BLe_{m+1}$ is

$$\begin{aligned} &(-1)^{-(m+n)} \{ ABr_3^{-(m+n)}(r_1^{-n}r_2^{-m+1} + r_1^{-m+1}r_2^{-n} - r_1^{-m}r_2^{-n+1} - r_1^{-n+1}r_2^{-m}) + ACr_2^{-(m+n)}(r_1^{-n}r_3^{-m+1} + r_1^{-m+1}r_3^{-n} - r_1^{-m}r_3^{-n+1} - r_1^{-n+1}r_3^{-m}) \\ &\quad + BCr_1^{-(m+n)}(r_2^{-n}r_3^{-m+1} + r_2^{-m+1}r_3^{-n} - r_2^{-m}r_3^{-n+1} - r_2^{-n+1}r_3^{-m}) \} \end{aligned} \quad (43)$$

Proof. If we want to calculate the left-hand side of equation (43), then we get

$$\text{LHS} = (Ar_1^m + Br_2^m + Cr_3^m)(Ar_1^{n+1} + Br_2^{n+1} + Cr_3^{n+1}) - (Ar_1^n + Br_2^n + Cr_3^n)(Ar_1^{m+1} + Br_2^{m+1} + Cr_3^{m+1}). \quad (44)$$

From this equality, we get

$$\begin{aligned} \text{LHS} &= AB(r_1^m r_2^{n+1} + r_1^{n+1} r_2^m - r_1^n r_2^{m+1} - r_1^{m+1} r_2^n) + AC(r_1^m r_3^{n+1} + r_1^{n+1} r_3^m - r_1^n r_3^{m+1} - r_1^{m+1} r_3^n) \\ &\quad + BC(r_2^m r_3^{n+1} + r_2^{n+1} r_3^m - r_2^n r_3^{m+1} - r_2^{m+1} r_3^n). \end{aligned} \quad (45)$$

Thus, $BLe_m BLe_{n+1} - BLe_n BLe_{m+1}$ is as follows:

$$\begin{aligned}
 &(-1)^{-(m+n)}\{A\text{Br}_3^{-(m+n)}(r_1^{-n}r_2^{-m+1} + r_1^{-m+1}r_2^{-n} - r_1^{-m}r_2^{-n+1} - r_1^{-n+1}r_2^{-m}) + A\text{Cr}_2^{-(m+n)}(r_1^{-n}r_3^{-m+1} + r_1^{-m+1}r_3^{-n} - r_1^{-m}r_3^{-n+1} - r_1^{-n+1}r_3^{-m}) \\
 &+ B\text{Ce}_1^{-(m+n)}(r_2^{-n}r_3^{-m+1} + r_2^{-m+1}r_3^{-n} - r_2^{-m}r_3^{-n+1} - r_2^{-n+1}r_3^{-m})\}
 \end{aligned} \tag{46}$$

which is the desired result. \square

Theorem 12. For positive numbers m and n , $B\text{Le}_{n-1}B\text{Le}_m + B\text{Le}_nB\text{Le}_{m+1}$ is

$$\begin{aligned}
 &A^2r_1^{m+n-1}(1+r_1^2) + B^2r_2^{m+n-1}(1+r_2^2) + C^2r_3^{m+n-1}(1+r_3^2) + AB(1+r_1r_2)(r_1^m r_2^{n-1} + r_2^m r_1^{n-1}) \\
 &+ AC(1+r_1r_3)(r_3^m r_1^{n-1} + r_1^m r_3^{n-1}) + BC(1+r_2r_3)(r_3^m r_2^{n-1} + r_2^m r_3^{n-1}).
 \end{aligned} \tag{47}$$

Proof. In formula $B\text{Le}_{n-1}B\text{Le}_m + B\text{Le}_nB\text{Le}_{m+1}$, if we use the Binet formula, then this value will be equal to the expression below.

$$\begin{aligned}
 &(A r_1^{n-1} + B r_2^{n-1} + C r_3^{n-1})(A r_1^m + B r_2^m + C r_3^m) + (A r_1^n + B r_2^n + C r_3^n)(A r_1^{m+1} + B r_2^{m+1} + C r_3^{m+1}) \\
 &= A^2 r_1^{m+n-1}(1+r_1^2) + B^2 r_2^{m+n-1}(1+r_2^2) + C^2 r_3^{m+n-1}(1+r_3^2) \\
 &+ (-1)^{m+n-1}\{A\text{Br}_3^{m+n-1}(r_1^{-m}r_2^{-n+1} + r_2^{-m}r_1^{-n+1} + r_1^{-m+1}r_2^{-n+2} + r_2^{-m+1}r_1^{-n+2}) \\
 &+ A\text{Cr}_2^{m+n-1}(r_1^{-m}r_3^{-n+1} + r_3^{-m}r_1^{-n+1} + r_1^{-m+1}r_3^{-n+2} + r_3^{-m+1}r_1^{-n+2}) \\
 &+ B\text{Cr}_1^{m+n-1}(r_2^{-m}r_3^{-n+1} + r_3^{-m}r_2^{-n+1} + r_2^{-m+1}r_3^{-n+2} + r_3^{-m+1}r_2^{-n+2})\}
 \end{aligned} \tag{48}$$

Then, $B\text{Le}_{n-1}B\text{Le}_m + B\text{Le}_nB\text{Le}_{m+1}$ is

$$\begin{aligned}
 &A^2r_1^{m+n-1}(1+r_1^2) + B^2r_2^{m+n-1}(1+r_2^2) + C^2r_3^{m+n-1}(1+r_3^2) + AB(1+r_1r_2)(r_1^m r_2^{n-1} + r_2^m r_1^{n-1}) \\
 &+ AC(1+r_1r_3)(r_3^m r_1^{n-1} + r_1^m r_3^{n-1}) + BC(1+r_2r_3)(r_3^m r_2^{n-1} + r_2^m r_3^{n-1}).
 \end{aligned} \tag{49}$$

Thus, we prove the claim of the theorem. \square

Conflicts of Interest

The authors declare that they have no conflicts of interest.

3. Conclusion

In this study, we discussed the Leonardo number sequence and its basic properties in detail. With the help of the properties and Pascal-like triangles, we defined a new sequence of numbers containing Leonardo numbers. We have also considered the basic identities provided by the elements of the newly defined sequence, which are often used in examining integer sequences.

Data Availability

No data were used to support this study.

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