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Research Article

Nonlocal Elasticity Theory for Transient Analysis of Higher-Order Shear Deformable Nanoscale Plates

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The small scale effect on the transient analysis of nanoscale plates is studied. The elastic theory of the nano-scale plate is reformulated using Eringen's nonlocal differential constitutive relations and higher-order shear deformation theory (HSDT). The equations of motion of the nonlocal theories are derived for the nano-scale plates. The Eringen's nonlocal elasticity of Eringen has ability to capture the small scale effects and the higher-order shear deformation theory has ability to capture the quadratic variation of shear strain and consequently shear stress through the plate thickness. The solutions of transient dynamic analysis of nano-scale plate are presented using these theories to illustrate the effect of nonlocal theory on dynamic response of the nano-scale plates. On the basis of those numerical results, the relations between nonlocal and local theory are investigated and discussed, as are the nonlocal parameter, aspect ratio, side-to-thickness ratio, nano-scale plate size, and time step effects on the dynamic response. In order to validate the present solutions, the reference solutions are employed and examined. The results of nano-scale plates using the nonlocal theory can be used as a benchmark test for the transient analysis.

1. Introduction

Due to their superior properties, nanoscale plates such as graphene [1], the two-dimensional (2D) counterpart of three-dimensional (3D) graphite, have attracted attention within the fields of solid-state physics, materials science, and nano-electronics. Experimentation with nano-scale size specimens is both difficult and expensive. Development of appropriate mathematical models for nanostructures, therefore, is essential. Approaches to the modeling of nanostructures are classified into three main categories: atomistic [2, 3], continuum [4–6], and hybrid atomistic-continuum mechanics [7–9]. The continuum mechanics approach is less computationally expensive and, furthermore, generates results that tend to be in good agreement with those of the atomistic and hybrid approaches [10, 11].

As small size analysis using local theory over-predicts results, correct prediction of micro-/nanostructures requires consideration of small-scale effect. Peddieson et al. [12] applied nonlocal elasticity to the formulation of a nonlocal version of the Euler-Bernoulli beam model and concluded that nonlocal continuum mechanics could potentially play a useful role in nanotechnology applications. One of the well-known continuum mechanics theories that include small scale effects with good accuracy is Eringen's nonlocal theory [13, 14]. This, compared with classical continuum mechanics theory, can predict the behavior of the large nanosized structures, without a large number of equations.

Nonlocal elasticity theory has been widely applied to the bending, vibration, and buckling behavior of one-dimensional (1D) nanostructures including nanobeams, nanorods, and carbon nanotubes. Contrastingly, there has been no such work related to the transient analysis of nano-scale plate based on higher-order shear deformation theory (HSDT). However, some researchers have applied nonlocal elasticity theory to the study of various applications of micro- and nanostructures [15, 16]. Understanding the importance of employing nonlocal elasticity theory to small scale structures, a number of researchers have reported on static, dynamic and stability analysis of 1D and 2D micro-/nanostructures [17–25]. Kiani [26] studied the free vibration of embedded single-walled nanotubes accounting for nonlocal effect using a meshless method. Nonlocal continuum-based modeling of a nanoplate subjected to a moving nanoparticle was presented by Kiani [27, 28]. Jomehzadeh and Saidi [29] investigated the decoupling of the nonlocal elasticity equations for three dimensional vibration analysis of nanoplates.

These studies [17–25] were based on classical and first-order theories of plates. The analysis of 2D nano-scale plates with accurate stress fields requires further study. The third-order shear deformation theory (TSDT) of Reddy [30] is based on a displacement field that includes the cubic term in the thickness coordinate; hence, the transverse shear strain and stress are represented as quadratic through the plate thickness and vanish on the bounding planes of the plate. Some studies, in fact, have incorporated the TSDT to obtain more accurate results [31–33]. It seems appropriate and potentially fruitful, therefore, to explore TSDT's extension to include size effects. The present study, accordingly, applies the nonlocal TSDT to the transient dynamic response of nano-scale plates.

To avoid the resonant behavior of such structures, a vibration analysis of their design is very important. Also, most of these structures, whether they are used in the civil, marine or aerospace field, are subjected to dynamic loads; therefore, there is a need to assess their transient response. However, transient analysis of nano-scale plates has not received adequate consideration. Work that exists on the vibration analysis of laminated composite and FGM plates can be found in [34, 35].

In the present work, nonlocal elasticity theory is applied to a transient analysis of nano-scale plates. Navier's method is used to solve the governing equations for simply supported boundary conditions. The effects of (i) the nonlocal parameter, (ii) size of the nano-scale plates, (iii) aspect ratios, and (iv) side-to-thickness ratios on the nondimensional dynamic responses are investigated. The present work will be helpful to the design of nano-electro-mechanical system or micro-electro-mechanical systems devices incorporating nano-scale plates.

2. Formulation

In classical local elasticity theory, stress at a point depends only on the strain at that point, whereas, in nonlocal elasticity theory, the stress at a point is a function of the strains at all points on the continuum. In other words, in nonlocal elasticity theory, stress at a point is determined by both stress at that point and spatial derivatives of it. According to Eringen [13, 14], the nonlocal constitutive behavior of a Hookean solid is represented by the differential constitutive relation

$$\left(1-\mu\nabla^2\right)\sigma_{ij}=t_{ij}, \tag{1}$$

where t_{ij} is the local stress tensor, σ_{ij} is the nonlocal stress tensor, and the nonlocal parameter μ is defined by

$$\mu=e_0^2\overline{a}^2 \tag{2}$$

in which e_0 is the material constant set by the experiment and $\overline{a}$ is the internal characteristic length.

The relations between stress resultants in local theory and in nonlocal theory are defined by integrating plate thickness into (1) as

$$\begin{aligned}\mathscr{L}\left(M_{ij}^{(0)}\right)&=M_{ij}^{(0)L},\\ \mathscr{L}\left(M_{ij}^{(1)}\right)&=M_{ij}^{(1)L},\\ \mathscr{L}\left(M_{ij}^{(2)}\right)&=M_{ij}^{(2)L},\end{aligned} \tag{3}$$

where $\mathscr{L}=1-\mu\nabla^2$ and

$$\begin{gathered}\begin{Bmatrix}M_{\alpha\beta}^{(0)},M_{\alpha\beta}^{(0)L}\\ M_{\alpha\beta}^{(1)},M_{\alpha\beta}^{(1)L}\\ M_{\alpha\beta}^{(2)},M_{\alpha\beta}^{(2)L}\end{Bmatrix}=\int_{-h/2}^{h/2}\left\{\sigma_{\alpha\beta},t_{\alpha\beta}\right\}\begin{Bmatrix}1\\ z\\ z^2\end{Bmatrix}dz,\\ \begin{Bmatrix}M_{\alpha z}^{(0)},M_{\alpha z}^{(0)L}\\ M_{\alpha z}^{(2)},M_{\alpha z}^{(2)L}\end{Bmatrix}=\int_{-h/2}^{h/2}\left\{\sigma_{\alpha z},t_{\alpha z}\right\}\begin{Bmatrix}1\\ z^2\end{Bmatrix}dz,\end{gathered} \tag{4}$$

where α and β take the symbols x, y, the superscript L denotes the quantities in local third-order shear deformation theory, and h is the thickness of the plate.

In general, differential operator ∇ in (1) is the 3D Laplace operator. For 2D problems, the operator ∇ may be reduced to the 2D one. Thus, the linear differential operator $\mathscr{L}$ becomes

$$\overline{\mathscr{L}}=1-\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right), \tag{5}$$

wherein it is clear that operator $\overline{\mathscr{L}}$ is independent of the z direction.

The third-order shear deformation theory (TSDT) extends the first-order theory by assuming that shear strain and consequently shear stress are not constant through the plate thickness [30]. The displacement field of the third-order theory of plates is given by

$$\begin{aligned}u_\alpha&=u_\alpha^0+z\phi_\alpha-c_1z^3\left(\phi_\alpha+w_{,\alpha}^0\right),\\ u_z&=w^0,\end{aligned} \tag{6}$$

where u_α^0 and w^0 are the in plane displacements and transverse displacement of point on the midplane (i.e., $z=0$), respectively, $c_1=4/3h^2$, u_z is the transverse displacement of the mid-plane of the plate, and ϕ_α denotes the slope of the transverse normal on the mid-plane.

The variational statements facilitate the direct derivation of the equations of motion in terms of the displacements. The governing equations of the third-order nonlocal plate theory can be derived, using the dynamic version of the principle of virtual displacement (Hamilton's principle), as

$$0 = \int_0^T (\delta U + \delta V - \delta K)\, dt, \tag{7}$$

where δU is the virtual strain energy, δV is the virtual work done by external forces, and δK is the virtual kinetic energy.

It can be seen from (1) that nonlocal behavior enters into the problem through the constitutive relations. The principal of virtual work, because it is independent of such relations, can be applied to derive the equilibrium equations of nonlocal nano-scale plates. On the basis of the principle of virtual displacements, then the governing equations are obtained [32]

$$\begin{aligned}
&\delta u_x^0\colon M_{xx,x}^{(0)} + M_{xy,y}^{(0)} - \overline{\mathscr{L}}\left(I_0 \ddot{u}_x^0\right) = 0,\\
&\delta u_y^0\colon M_{xy,x}^{(0)} + M_{yy,y}^{(0)} - \overline{\mathscr{L}}\left(I_0 \ddot{u}_y^0\right) = 0,\\
&\delta w^0\colon M_{zx,x}^{(0)} + M_{zy,y}^{(0)} - c_2\left(M_{zx,x}^{(2)} + M_{zy,y}^{(2)}\right)\\
&\qquad + c_1\left(M_{xx,xx}^{(2)} + M_{yy,yy}^{(2)} + 2M_{xy,xy}^{(2)}\right)\\
&\qquad - \overline{\mathscr{L}}\Big[- q_z + I_0 \ddot{w}^0 + c_1 I_4 \left(\ddot{\phi}_{x,x} + \ddot{\phi}_{y,y}\right)\\
&\qquad\qquad - c_1^2 I_6 \left(\ddot{\phi}_{x,x} + \ddot{\phi}_{y,y} + \ddot{w}_{,xx}^0 + \ddot{w}_{,yy}^0\right)\Big] = 0,\\
&\delta \phi_x\colon M_{xx,x}^{(1)} + M_{xy,y}^{(1)} - c_1\left(M_{xx,x}^{(2)} + M_{xy,y}^{(2)}\right) - M_{zx}^{(0)} + c_2 M_{zx}^{(2)}\\
&\qquad - \overline{\mathscr{L}}\Big[I_2 \ddot{\phi}_x - c_1 I_4 \left(\ddot{\phi}_x + \ddot{w}_{,x}^0\right)\\
&\qquad\qquad - c_1 \left(I_4 \ddot{\phi}_x - c_1 I_6 \left(\ddot{\phi}_x + \ddot{w}_{,x}^0\right)\right)\Big] = 0,\\
&\delta \phi_y\colon M_{xy,x}^{(1)} + M_{yy,y}^{(1)} - c_1\left(M_{xy,x}^{(2)} + M_{yy,y}^{(2)}\right) - M_{zy}^{(0)} + c_2 M_{zy}^{(2)}\\
&\qquad - \overline{\mathscr{L}}\Big[I_2 \ddot{\phi}_y - c_1 I_4 \left(\ddot{\phi}_y + \ddot{w}_{,y}^0\right)\\
&\qquad\qquad - c_1 \left(I_4 \ddot{\phi}_y - c_1 I_6 \left(\ddot{\phi}_y + \ddot{w}_{,y}^0\right)\right)\Big] = 0,
\end{aligned} \tag{8}$$

where $I_k = \int_{-h/2}^{h/2} \rho(z)^k dz$ ($k = 0, 2, 4, 6$) and $c_2 = 3c_1$, in which ρ is the mass density. From these, by setting $\overline{\mathscr{L}} = 1$, the equations of motion of the conventional third-order plate theory are obtained.

3. Navier Solutions of Nonlocal Third-Order Shear Deformation Theory

Here, analytical solutions to the free and forced vibration of simply supported plates are presented using the nonlocal third-order plate theory in order to illustrate the small scale effects on deflections and natural frequencies of the plates. For the set of simply supported boundary conditions, the Navier solution can be obtained [32]. According to Navier solution theory, the generalized displacements at middle of the plane ($z = 0$) are expanded in double Fourier series as

$$\begin{aligned}
\left\{u_x^0(x,y,t), \phi_x(x,y,t)\right\} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left\{U_{mn}, X_{mn}\right\} \Lambda_1,\\
\left\{u_y^0(x,y,t), \phi_y(x,y,t)\right\} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left\{V_{mn}, Y_{mn}\right\} \Lambda_2,\\
\left\{w^0(x,y,t), q_z(x,y,t)\right\} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left\{W_{mn}, Q_{mn}\right\} \Lambda_3,
\end{aligned} \tag{9}$$

where $\Lambda_1 = \cos\xi x \sin\eta y \cdot e^{i\varpi_{mn}t}$, $\Lambda_2 = \sin\xi x \cos\eta y \cdot e^{i\varpi_{mn}t}$, and $\Lambda_3 = \sin\xi x \sin\eta y \cdot e^{i\varpi_{mn}t}$, in which, $\xi = m\pi/a, \eta = n\pi/b$, $i = \sqrt{-1}$, t is the time and ϖ_{mn} is the natural frequency.

Substituting (9) into (8), the matrix form obtained is

$$[\mathbf{K}]\{\Delta\} + [\mathbf{M}]\left\{\ddot{\Delta}\right\} = \{Q\}, \tag{10}$$

where $\{\Delta\} = \{U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}\}$, $[\mathbf{K}]$ is the stiffness matrix, $[\mathbf{M}]$ is the mass matrix, dot-superscript convention indicates the differentiation with respect to the time variable t, and $\{Q\}$ is the force vector. For this study, the Newmark-β time integration method is used.

In order to investigate the transient response of a laminated composite and nano-scale plate, a parametric study is carried out according to different dynamic loading conditions. The load acting on the plate is distributed over its entire surface and varied with time according to

$$q(x,y,t) = q_z F(t), \tag{11}$$

where $q_z = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn} \sin\alpha x \sin\beta y$, the Q_{mn} is the coefficient for some typical load [32], $F(t) = \{1,\ 0 \le t\}$: rectangular step loading, and

$$F(t) = \begin{cases} 1, & 0 \le t \le t_1 \\ 0, & t \ge t_1 \end{cases} \text{: rectangular pulse loading,} \tag{12}$$

in which $t_1 = 1000\,\mu$s.

4. Numerical Results and Discussion

To validate this present study, several numerical examples are solved to test the performance in free and forced vibration analyses. Examples include composite materials for confirmation of crucial features or for comparison with previous published analysis results. Specially, a mistake in a reference is corrected for the purpose of an accurate comparison.

4.1. Validation. In order to validate the results of the forced vibration analysis, a transient analysis of the simply supported cross-ply (0°/90°) plate shown in Figure 1 is carried out under sinusoidal step loading intensity $q_0 = 10$ with $\Delta t = 5\,\mu$s. The rectangular plate, shown in Figure 1, is analyzed for the natural vibration of laminated composite plate, and results are presented in Figure 2.

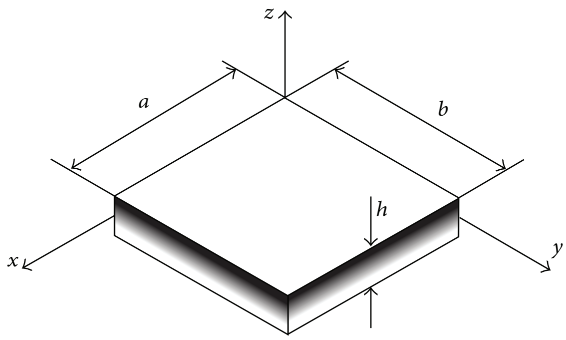

Figure 1: Geometry of laminated composite and nanoscale plates.

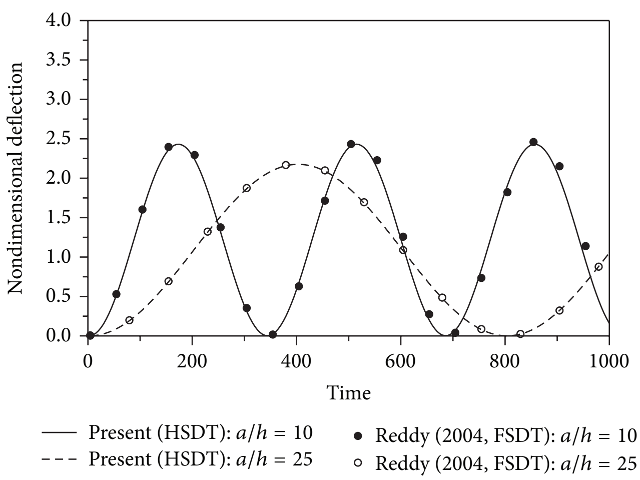

Figure 2: Dynamic response of laminated composite plate under suddenly applied step load.

The geometric and material properties applied are as follows:

$$a = b = 25\,\text{cm}, \qquad \Delta t = 5\,\mu\text{s}, \qquad \rho = 8 \times 10^{-8}\,\text{N} \cdot \text{s}^2/\text{cm}^4,$$
$$E_1 = 25E_2, \qquad E_2 = 2.1 \times 10^6\,\text{N/cm}^2,$$
$$G_{12} = G_{13} = 0.5E_2, \qquad G_{23} = 0.2E_2,$$
$$\nu_{12} = \nu_{13} = \nu_{23} = 0.25, \qquad q_0 = 10\,\text{N/cm}^2. \tag{13}$$

The center deflection versus time of a cross-ply plate (0°/90°) is compared with the results by Reddy [32]. Figure 2 shows the present results and the results by Reddy [32]. It can be seen that the difference between the two results is almost negligible.

The results are presented in the nondimensional form using the equation:

$$\overline{w} = w \times \frac{E_2 h^3}{q_0 b^4} \times 10^2. \tag{14}$$

4.2. Free Vibration of Nanoscale Plate. The analytical free vibration responses are numerically evaluated here for an isotropic plate to discuss the effects of nonlocal parameter μ on the plate vibration response. Table 1 shows the non-dimensional frequencies of simply supported nano-scale plates with various values of nonlocal parameter μ. The nano-scale plate is made of the following material properties:

$$E = 2.6, \qquad \nu = 0.3, \qquad \rho = 1.0. \tag{15}$$

The results are presented in the non-dimensional form using the equation:

$$\overline{\varpi}_{11} = \varpi_{11} \times h\sqrt{\frac{\rho}{G}}. \tag{16}$$

The results based on HSDT plate theory with various values of nonlocal parameter μ are compared with those reported by Aghababaei and Reddy [36]. It can be seen that when the aspect ratios are 1 and 2, the two sets of results are identical. Table 1 also shows that the material property ($E = 30\times10^6$) presented by Aghababaei and Reddy [36] is not accurate, and that, for accurate comparison by (15), it should be modified.

Table 1: Nondimensional frequency of simply supported nanoscale plate with various nonlocal parameter (μ).

μ	$a/h = 10,\ b/a = 1$		$a/h = 10,\ b/a = 2$	
	Aghababaei and Reddy [36]	Present	Aghababaei and Reddy [36]	Present
0	0.0930	0.0930	0.0589	0.0589
1	0.0850	0.0850	0.0556	0.0556
2	0.0788	0.0788	0.0527	0.0527
3	0.0737	0.0737	0.0503	0.0503
4	0.0696	0.0695	0.0482	0.0482
5	0.0660	0.0660	0.0463	0.0463

Table 2: Nondimensional material and geometric properties.

E	ν	$a = b$	ρ	q_z
30×10^6	0.3	10	5×10^{-6}	1.0

4.3. Forced Vibration of Nanoscale Plate under Step Load. The non-dimensional geometric and material data on the nano-scale plate are provided in Table 2.

The results of a forced vibration analysis of laminated composite plate under a distributed rectangular step load with $\Delta t = 5\,\mu$s and a variable nonlocal parameter are plotted in Figure 3. The nonlocal parameters are $\mu = 0,\ 0.25,\ 1.0,\ 2.25,$ and 4, because $e_0\overline{a}$ in (2) should be smaller than 2.0 nm for carbon nanotubes (Q. Wang and C. M. Wang [37]). The results show the profound scale effect for the higher values of nonlocal parameter. In other words, the local elasticity theory under-predicts the dynamic response. Thus also, if local elasticity results are used for experimental determination of Young's modulus, the value will be under-predicted.

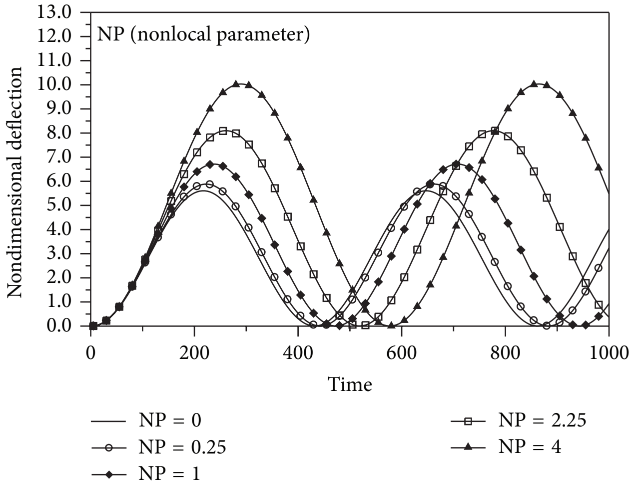

Figure 3: Dynamic response of nano-scale plate under suddenly applied step load with variable nonlocal parameter.

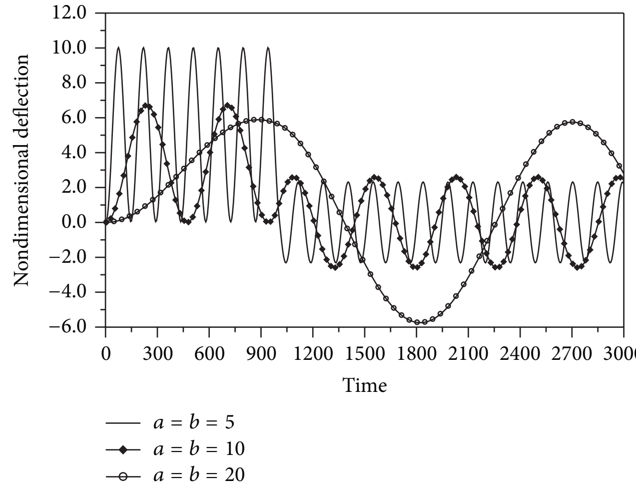

Figure 5: Dynamic response of nano-scale plate under suddenly applied pulse load of variable size ($\mu = 1$, $t_1 = 1000\,\mu$s).

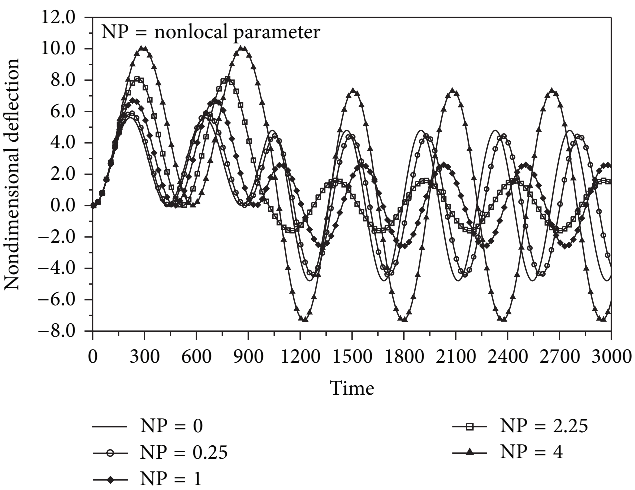

Figure 4: Dynamic response of nano-scale plate under suddenly applied pulse load ($t_1 = 1000\,\mu$s).

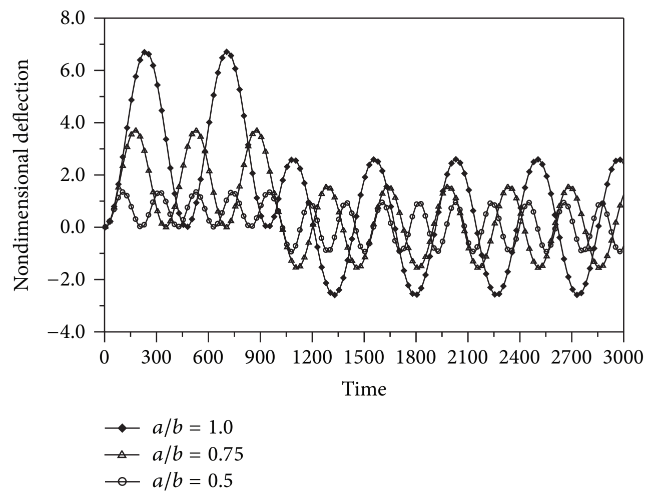

Figure 6: Dynamic response of nano-scale plate of variable aspect ratio under suddenly applied pulse load ($\mu = 1$, $t_1 = 1000\,\mu$s).

4.4. Forced Vibration of Nanoscale Plate under Pulse Load. A forced vibration analysis of center deflection under the rectangular pulse loading is carried out using the same material. The plot of the center deflection versus time is shown in Figure 4. As can be seen, the maximum deflection increases until the applied loading is removed after $t_1 = 1000\,\mu$s, due to the nonlocal parameters increase due to stiffness degradation. After removal of the load, however, a change of the maximum deflection occurs. This phenomenon is expected to vary with the point at which the load is removed. Accordingly, the amplitude also changes after the load removal.

Figure 5 shows the variation of dynamic response with respect to the length of the nano-scale plate, which varies from 5 nm to 20 nm. The nonlocal variable is assumed to be 1. As the size of the nano-scale plate increases, the amplitude and frequency of the dynamic response decrease.

The dynamic response due to the change of the aspect ratio ($a/b = 0.5, 0.75, 1.0$) of the nano-scale plate is shown in Figure 6. The nonlocal variable is again assumed to be 1. The amplitude and the period of the dynamic response decrease, as the aspect ratio decreases. When the aspect ratio changes from 1 to 0.75, the maximum deflection is reduced by 45%: when the aspect ratio changes from 0.75 to 0.5, the reduction is 64%.

Figure 7 shows the dynamic response of a nano-scale plate of variable side-to-thickness ratio under a suddenly applied pulse load. Typically in the case of isotropic plates or laminated composite plates, the side-to-thickness ratio is more than 20, and the effect of shear deformation is very small. Similarly, in the case of the nano-scale plate shown in Figure 7 with a side-to-thickness ratio of 20 or greater, there is no effect on the maximum deflection. But when the side-to-thickness ratio decreases, the frequency of the

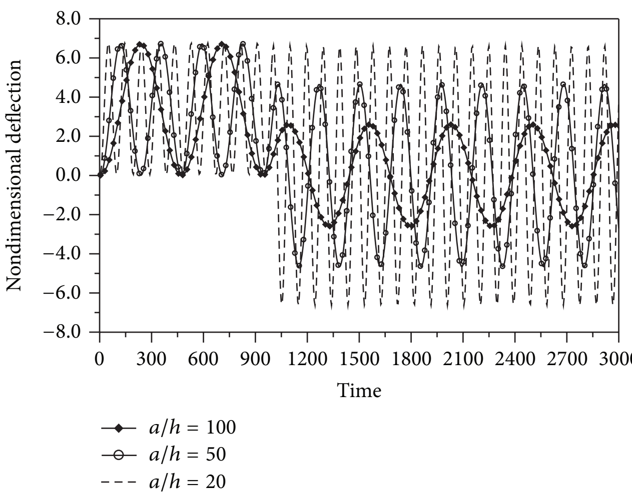

Figure 7: Dynamic response of nano-scale plate of variable side-to-thickness ratio under suddenly applied pulse load ($\mu = 1$, $t_1 = 1000\,\mu$s).

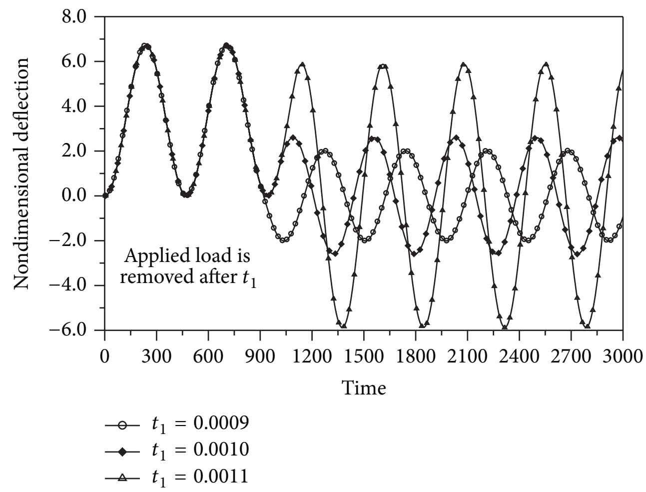

Figure 9: Dynamic response of nano-scale plate under suddenly applied pulse load ($\mu = 1$, $t_1 = 900\,\mu$s, $1000\,\mu$s, and $1100\,\mu$s).

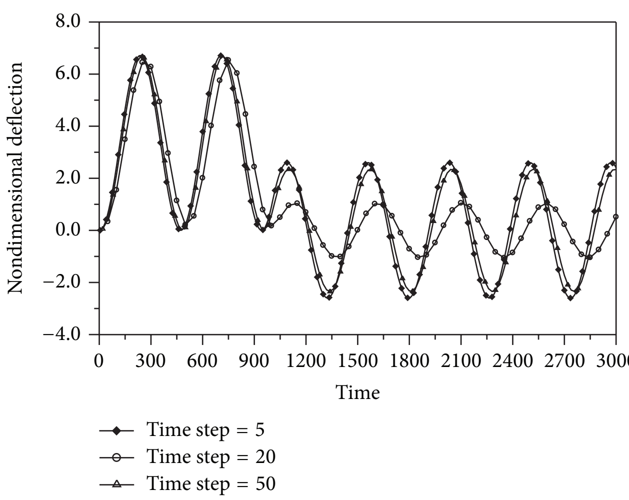

Figure 8: Dynamic response of nano-scale plate under suddenly applied pulse load with variable time step ($\mu = 1$, $t_1 = 1000\,\mu$s).

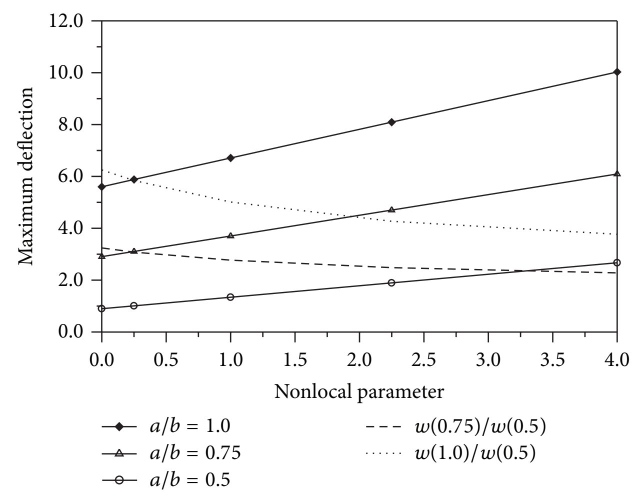

Figure 10: Maximum deflection of nano-scale plate of various aspect ratio with nonlocal parameter.

dynamic response is increased; though there is no difference in its amplitude. Notably, the frequency with a the side-to-thickness ratio of 50 is twice that with a the side-to-thickness ratio of 100, due to the doubling of the thickness of the nano-scale plate.

Next, the effect of the time step on the accuracy of the solutions is investigated using a nano-scale plate under distributed pulse loading. Figure 8 shows the nondimensionalized center transverse deflection for three different time steps. A larger time step reduces the amplitude and increases the period. For all of the time steps below $10\,\mu$s, the effect is not noticeable on the graph. In all the subsequent, cases then the time step = $5\,\mu$s is used. The difference between the maximum deflection of time step $5\,\mu$s and that of $50\,\mu$s is about 3%. It is expected therefore that, in order to obtain an accurate dynamic response, an appropriate choice of the loading time interval is required.

Figure 9 shows the dynamic response of the nano-scale plate under a suddenly applied pulse load and a variable time step. The change of the amplitude occurred according to the time in which the load is removed.

In order to study the effect of nonlocal parameter on the maximum deflection, the predicted results of the present nonlocal plate model have been illustrated in Figure 10. As it is obvious from Figure 10, the predicted results of nonlocal plate model generally increase with the nonlocal parameter, irrespective of the aspect ratio (a/b). As the nonlocal parameter increases, the ratios ($w_{(a/b=1.0)}/w_{(a/b=0.5)}$ and $w_{(a/b=0.75)}/w_{(a/b=0.5)}$) of the proposed nonlocal plate decrease. In other words, the effect of nonlocal parameters on maximum deflection is more significant when the aspect ratio is 0.5 ($a/b = 0.5$).

Figure 11 plots the absolute value of maximum deflection of the nano-scale plate under suddenly applied step and

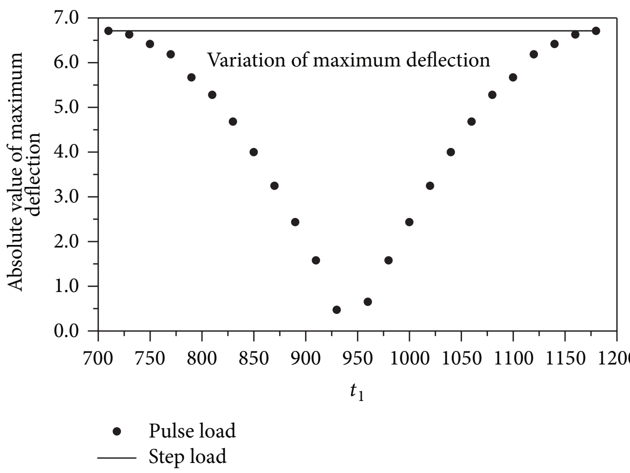

Figure 11: Absolute value of maximum deflection of nano-scale plate under suddenly applied step and pulse load with variable t_1 ($\mu = 1$).

pulse load with variable t_1. The maximum deflection when changing from 710 to 940 shows a negative value, and when changing from 950 to 1180, it shows a positive value. It would be expected that this absolute value of maximum deflection curve maintains a constant shape and that the maximum value of each deflection curves does not exceed the maximum deflection when subjected to a step load.

5. Conclusions

In this paper, equations of motion under nonlocal third-order shear deformation theory (TSDT) as well as analytical solutions for bending and free vibration are derived to elucidate the effect of nonlocal parameters. In order to illustrate the effects of nonlocal theories on plate response, numerical results for simply supported rectangular plates are presented. It is found that nonlocal theory can be applied to the analysis of nano-scale plates where the small size effects are significant.

Using the Eringen's nonlocal elasticity theory, the dynamic responses of nano-scale plates are studied. The higher-order shear deformation theory (HSDT) is applied to capture the quadratic variation of shear strain and consequently shear stress as function of plate thickness. It is determined that the effect of nonlocal constitutive relations is to increase the magnitude of deflections and decrease the amplitude of frequencies. Additionally, it is found that the difference between nonlocal theory and local theory is significant for high values of nonlocal parameter.

From the present work, the following conclusions are drawn.

(1) The dynamic response computed by nonlocal elasticity is always larger compared with the local elasticity value. The stiffness of a nano-scale plate by nonlocal elasticity is small compared with that by local elasticity. Correspondingly, the amplitude and period are large.

(2) As the size of a nano-scale plate decreases, the effect of nonlocal elasticity becomes more significant and dynamic response increased. With aspect ratio decreases of $a/b = 1.0$ to 0.75 and 0.75 to 0.5, the amplitude and frequency of the dynamic response decrease from 1.0 to 0.75, the maximum deflection is reduced by 45%, and from 0.75 to 0.5, there is a 64% reduction.

(3) In the case of a nano-scale plate of side-to-thickness ratio larger than 20, the effect of shear deformation is very small. When the side-to-thickness ratio decreases, there is no difference in the amplitude of the dynamic response, but its frequency increases.

(4) The effect of a larger time step is to reduce the amplitude and increase the period. It is expected therefore that, in order to obtain accurate dynamic response, an appropriate choice of loading time interval is required.

The dynamic response of the nano-scale plate can serve as a benchmark for future guidelines to nano-scale structural design under nonlocal elasticity theory. Especially, the maximum deflection of a nano-scale plate under a suddenly applied step and pulse load with variable t_1 will be contributed for the purpose of a future benchmark test applicable to the transient analysis of nano-scale structures. Further, it will be necessary to include, in transient analysis, the nonlocal elasticity theory for nano-scale shell and other boundary conditions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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