Research Article

The Dynamics of H₂O Suspended by Multiple Shaped Cu Nanoadditives in Rotating System

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Heat transfer investigation in the nanofluids is significant for real world applications. The investigation of heat transfer over a stretchable magnetized surface has broad applications in various industries. Therefore, heat transfer featuring in the nanofluid synthesized by various shaped Cu and H₂O is organized over a shrinking surface. The problem is organized properly via similarity equations by inducing the influences of magnetic field. Then, OVIM is adopted and performed the solutions for the particular model. The results are furnished for the governing quantities over the feasible region and deeply discussed in the view of their physical significance. It is examined that the nanoliquids angular motion and shear stresses drops by strengthen magnetic field effects. Moreover, nanoliquid containing brick Cu-particles is better heat conductor and could be used broadly for industrial applications as for as heat transport concerned. In end, authentication of the study is provided by comparing the results with previous science literature and an excellent agreement is seems between them.

1. Introduction

Investigation of mass and thermal transport models over stretching or shrinking boundaries of different geometries has versatile range of applications in various technological processes. These comprised in polymer, roofing shingles, paper production, and many others.

The analysis of flow in a rotating frame is one of the significant motives in the light of its wide applications comprised in the field of engineering and geophysics. The engineering utilizations of such type of flows prevail in the industries of food processing and chemical, process of centrifugal filtration, and revolting machinery. The earlier work on three-dimensional rotating flow convinced by a stretch-

ing surface was reported by Wang [1]. They explored a flow model by considering a very fascinating parameter λ that manifestoes the ratio between revolutions to stretching rate of the surface. Furthermore, they investigated the series solution of the flow model by employing regular perturbation technique for smaller values of parameter λ . They inspected that velocity field above the stretching surface starts decreasing by increasing parameter λ (ratio of rotation to stretching rate of the surface). The study of entropy generation is attained much fame among the engineers due to its applications in mechanical engineering particularly and generally in aerodynamics. Therefore, Rashidi et al. [2] reported the inspection of entropy generation in steady magnetohydrodynamic flow of nanoliquid over a porous rotating disk. It

seems that the numerical study of the nanoliquids became very popular to investigate the dynamics of the nanoliquids. Therefore, Sheikholeslami et al. [3] investigated numerical results for nanofluid in a rotating system by contemplating magnetohydrodynamic effects.

In many industrial and practical purposes, thermal transport is very significant. In conventional liquids, thermal transport is poor due to their low thermal conductance ability. In order to enhance thermal transport, a new class of fluid was presented and entitled as nanofluids. These fluids are mixture of nanoparticles and base fluids. Pioneering work on such important class of liquids was done by Choi [4], and they entitled as nanofluids. In class of nanofluids, thermal conductivity plays significant role for transfer of heat. For this purpose, different theoretical models have been proposed for thermal conductivity of the nanofluid. Among that models, a model that deals the multiple shaped nanomaterials is called Hamilton and Crosser's model [5]. A parameter comprised in the model tends to three different shapes of nanoparticles against distinct values. The study of fluids between two plates subject to magnetic field is an imperative research area. Keeping in mind, Khan et al. [5] presented magnetized colloidal analysis between two plates. They considered the colloidal suspension comprising multiple shapes nanomaterials. Furthermore, they inspected the behavior of distinct flow parameters comprised in the colloidal model in the flow regimes.

The purification of various industrial products is a significant and challenging motive for industrialists. In this regard, magnetic field is an important insight in the fluids because this resists the fluid motion due to which the impurities stepdown at the bottom, and the products become purify. Therefore, Sheikholeslami et al. [6, 7] explored an MHD flow of viscous incompressible fluid and thermal transport analysis bounded by two plates in revolting the Cartesian coordinate system. They ignored the influences of dissipative phenomena on the flow field. The analysis of thermal and physical characteristics of water composed by ferrite nanomaterial reported in [8]. In the flow characteristic of non-Newtonian nanofluids, effects of mixed convection on nanofluids temperature, and free convection analysis of nanofluids [9–11], respectively, Dutta et al. [12] discussed the impacts of various flow quantities on temperature field over a stretchable surface. Andersson et al. [13] explored the time dependent two-dimensional non-Newtonian flow characteristics over stretchable geometry. Sheikholeslami et al. [14] studied the influences of thermal radiations on MHD nanofluid flow by using two phase model. They portrayed the effects of multiple physical quantities on nondimensional flow characteristics.

Recently, Sheikholeslami [15] reflected the flow characteristics in magnetized porous cavity. Some important and novel analysis of nanofluid flow by taking various physical phenomena like solar thermal radiations, combined effects, dissipation fluid, resistive heating, and Lorentz forces are examined in [16–19]. Many nonlinear flow models do not have exact solutions. Thus, researchers turned to tackle the models analytically. In the light of these facts, various approximated methods were proposed. These are homotopy

perturbation method, homotopy Analysis method, differential transform method, variation of parameters method, Adomian's decomposition method, etc. The details of these methods are available in [20–23]. Recently, Gul et al. [24] and Aman et al. [25] studied different nonlinear flow models by considering various geometries. They explored the different nanoparticle flow models and detected the influences of multiple ingrained parameters in the flow field. Noor et al. [26, 27] studied the heat flux phenomena in a porous by considering porous media.

The thermal aspects of hybrid nanoliquid synthesized by coupling of ferrite and MWCNTs in water are organized by Mourad et al. [28]. They used GFEM for mathematical treatment of the model and furnished the results for the dynamics of the fluid by taking different values of governing quantities. The flow past a stretchable surface gained huge interest in the light of their broad applications in industries and engineering systems. Therefore, the evaluation of heat featuring in casson nanoliquid by considering the solar thermal radiations is presented in [29]. As for as the heat transfer is concerned, researchers organized various studies under varying flow scenario are reported in [30] and therein.

The role of viscous dissipation is significant in the nanofluids regarding the heat performance. An imperative investigation of heat and mass transport under the potential effects of viscous dissipation is examined in [31]. The authors organized the model over an elastic surface and reported the important outcomes for the dynamics of the fluids under viscous dissipation. Insertion of magnetic field and thermal radiations on the flow is the influential physical phenomenon and has multifarious industrial uses. Therefore, the studies in this regard are presented in [32] and literature cited therein. Aforementioned physical situations are very important for purification of many industrial products and in the accomplishment of the products where huge energy transport is required. Some other recent studies of potential interest are examined in [33–35].

2. Governing Colloidal Model

Thermal performance remains an influential problem from many decades and development of the nanoliquids that overcome the problem. Heat transport in the fluid over a shrinking surface is significant from industrial and engineering point of view. Therefore, the investigation of thermal performance in the nanoliquid is conducted. An incompressible flow of the nanoliquid synthesized by Cu and pure water is contemplated over a shrinking surface. In the synthetizations, four types of Cu nanoparticles, namely, cylinder, blade, brick, and platelets are considered for thermal enhancement applications. These nanoparticles are thermally compatible with base liquid, and no slip exists between them. Moreover, important physical effects of magnetic field are induced in the model in which its strength is taken B_0 . The flow scenario is organized in Cartesian coordinates with u, v, w that are the nanoliquid velocities along the coordinate axes. Under these restrictions, a mathematical model is written in the following version [36]:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} - 2\Omega v = \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial z^2} \right) - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} u, \quad (2)$$

$$u \frac{\partial v}{\partial x} + w \frac{\partial v}{\partial z} + 2\Omega u = \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 v}{\partial z^2} \right) - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} v, \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{k_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial z^2} \right). \quad (4)$$

Figure 1 depicts the flow configuration of Cu-H₂O over a shrinking surface. Eqs. (1)–(4) describe the conservation of mass, momentum, and energy, respectively. Also, μ_{nf} , ρ_{nf} , k_{nf} , $(\rho C_p)_{nf}$ denoted the dynamic viscosity, density, thermal conductivity, and specific heat, respectively, and Ω is the angular velocity, T denotes the temperature, and B_0 is the imposed magnetic field. Furthermore, for thermal improvement, the following models are adopted [36]:

$$\left. \begin{aligned} \mu_{nf} &= \mu_f (1 + a^* \phi + b\phi^2) \\ k_{nf} &= k_f \left[\frac{k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi}{k_s + (n-1)k_f - (k_s - k_f)\phi} \right] \\ \rho_{nf} &= (1 - \phi)\rho_f + \phi\rho_s \\ (\rho C_p)_{nf} &= (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \end{aligned} \right\} \quad (5)$$

Here, nanoparticle volume fraction is ϕ , and densities of the solid tiny particles and regular liquid are ρ_s and ρ_f , respectively. The parameter n is the empirical shape factor, and this parameter is given by $n = 3/\psi$, and ψ is the sphericity. The empirical shape factors are reported in [36]. Thermophysical properties of the guest nanoparticles and host liquids are given in Table 1.

Supporting boundary conditions for under observation flow model are the following [36]:

$$\left. \begin{aligned} u &= ax, v = 0, w = -W, T = T_w \\ u &\longrightarrow 0, v = 0, T \longrightarrow T_\infty \end{aligned} \right\}. \quad (6)$$

In defined boundary conditions, positive a denotes the shrinking of surface, and W greater than zero is the suction velocity. The feasible nondimensional transformations defined in the following manner [36] are as follows:

$$\begin{aligned} u &= -F'(\eta)ax, v = G(\eta)ax, w = -F(\eta)\sqrt{av_f}, \beta(\eta) \\ &= \frac{T - T_\infty}{T_w - T_\infty}, \text{ and } \eta = \sqrt{\frac{a}{v_f}}z. \end{aligned} \quad (7)$$

By entreating these dimensionless variables in the colloidal governing model, the following self-similar colloidal model is attained:

FIGURE 1: Cu-H₂O colloidal flow configuration.

TABLE 1: Thermo physical properties of regular liquid and tiny particles [36].

Model	$\rho(\text{kgm}^{-3})$	$c_p(\text{kg}^{-1}\text{k}^{-1})$	$k(\text{wm}^{-1}\text{k}^{-1})$	$\beta^*(10^{-5}\text{k}^{-1})$
H ₂ O	997.1	4179	0.613	21
C ₂ H ₆ O ₂	1.115	0.58	0.1490	6.5
Cu	8933	385	401	1.67
TiO ₂	4250	686.2	8.9528	0.9
Ag	10500	235	429	1.89
Al ₂ O ₃	3970	765	40	0.85
Fe ₃ O ₄	5180	670	9.7	0.5

$$\begin{aligned} F''' + \frac{(1-\phi) + \phi[\rho_s/\rho_f]}{(1+a^*\phi + b\phi^2)} (-F'2 + FF'' + 2\lambda G) \\ - \frac{\{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi/(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi\}}{(1+a^*\phi + b\phi^2)} M^2 F' = 0, \end{aligned} \quad (8)$$

$$\begin{aligned} G'' + \frac{(1-\phi) + \phi[\rho_s/\rho_f]}{(1+a^*\phi + b\phi^2)} (-F'G + FG' - 2\lambda F') \\ - \frac{\{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi/(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi\}}{(1+a^*\phi + b\phi^2)} M^2 G = 0, \end{aligned} \quad (9)$$

$$\begin{aligned} \beta'' + \frac{(1-\phi) + \phi[(\rho C_p)_s/(\rho C_p)_f]}{[k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi/k_s + (n-1)k_f - (k_s - k_f)\phi]} \text{Pr } F\beta' \\ = 0. \end{aligned} \quad (10)$$

In Eqs. (8)–(10), F , G , and β are the velocity and temperature functions of independent variable η .

$$\begin{aligned} \text{at } \eta = 0, \\ F(\eta) = S, F'(\eta) = -1, G(\eta) = 0, \beta(\eta) = 1. \\ \text{at } \eta \longrightarrow \infty, \\ F'(\eta) \longrightarrow 0, \beta(\eta) \longrightarrow 0, G(\eta) \longrightarrow 0 \end{aligned} \quad (11)$$

FIGURE 2: Behavior of $F'(\eta)$ for λ and S . FIGURE 3: Behavior of $F'(\eta)$ for M .

The ingrained parameters are Pr (Prandtl number), M is the magnetic parameter, and λ is the rotation parameter and shows the relation between rotation rate and rate of shrinking. These parameters are given by the following expressions:

$$Pr = \frac{\mu_f (c_p)_f}{k_f}, \lambda = \frac{\Omega}{a}, M^2 = \frac{\sigma_f B_0^2 v_f}{a \mu_f}. \quad (12)$$

The mathematical relations for the surface shear stresses and heat transfer rate are

$$C_{Fx} = \frac{\tau_{wx}}{\rho_{nf} u^2}, C_{Fy} = \frac{\tau_{wy}}{\rho_{nf} u^2} \text{ and } Nu_x = \frac{q_w x}{k(T_w - T_\infty)}, \quad (13)$$

where

$$\tau_{wx} = \mu_{nf} \left(\frac{\partial u}{\partial z} \right) \Big|_{z=0}, \tau_{wy} = \mu_{nf} \left(\frac{\partial v}{\partial z} \right) \Big|_{z=0} \text{ and } q_w = -k_{nf} \left(\frac{\partial T}{\partial z} \right) \Big|_{z=0}, \quad (14)$$

and finally, the following version is obtained:

$$C_{Fx} \sqrt{Re_x} = \frac{A_2}{A_1} F''(\eta) \Big|_{\eta=0}, C_{Fy} \sqrt{Re_y} = \frac{A_2}{A_1} G'(\eta) \Big|_{\eta=0},$$

$$Nu_x (Re_x)^{-\frac{1}{2}} = -A_5 \beta'(\eta) \Big|_{\eta=0}, \quad (15)$$

where $A_1 = (1 - \phi) + \phi[\rho_s/\rho_f]$, $A_2 = 1 + a^* \phi + b \phi^2$,

$$A_5 = k_f \left[\frac{k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi}{k_s + (n-1)k_f - (k_s - k_f)\phi} \right]. \quad (16)$$

3. Mathematical Analysis of the Colloidal Model

For nonlinear flow problems, exact solutions are very rare. In the light of this fact, we turned toward the series solution. For said purpose, we adopted the optimal variational iteration method [37]. According to modified algorithm, we have the following correction functions for our flow model.

$$F_{n+1} = F_n(\eta) + h \int_0^\eta \lambda(\eta, \zeta) \left[F_n'(\zeta) + F_n'(\zeta) - \tilde{F}_n'(\zeta) + \frac{(1-\phi) + \phi[\rho_s/\rho_f]}{(1+a^*\phi + b\phi^2)} \cdot \left(-\tilde{F}_n'^2(\zeta) + \tilde{F}(\zeta) \tilde{F}_n'(\zeta) + 2\lambda \tilde{G}(\zeta) - \left\{ \frac{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi} \right\} M^2 \tilde{F}_n'(\zeta) \right) \right] d\zeta,$$

FIGURE 4: Behavior of $G(\eta)$ for M and S .

$$G_{n+1}(\eta) = G_n(\eta) + h_2 \int_0^\eta \lambda_g(\eta, \zeta) \left[\widetilde{G}'_n(\zeta) F'_n(\zeta) - \widetilde{F}'_n(\zeta) + \widetilde{F}_n(\zeta) \right. \\ \left. + \frac{(1-\phi) + \phi \left[\rho_s / \rho_f \right]}{(1 + a^* \phi + b \phi^2)} \left(-\widetilde{F}'_n(\zeta) \widetilde{G}_n(\zeta) - 2\lambda \widetilde{F}'_n(\zeta) \right. \right. \\ \left. \left. - \left\{ \frac{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi} \right\} M^2 \widetilde{G}_n(\zeta) \right) \right] d\zeta,$$

$$\beta_{n+1}(\eta) = \beta_n(\eta) + h_3 \int_0^\eta \lambda_\beta(\eta, \zeta) \\ \cdot \left[\frac{(1-\phi) + \phi \left[(\rho C_p)_s / (\rho C_p)_f \right]}{[k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi / k_s + (n-1)k_f - (k_s - k_f)\phi]} \right. \\ \left. \cdot \left(\text{Pr} \widetilde{F}_n(\zeta) \widetilde{\beta}'_n(\zeta) \right) \right] d\zeta. \quad (17)$$

Here, feasible Lagrange multipliers are the following:

$$\begin{aligned} \lambda(\eta, \zeta) &= 1 - \frac{e^{\eta-\zeta}}{2} + \frac{e^{\zeta-\eta}}{2}, \\ \lambda_g(\eta, \zeta) &= e^{\zeta-\eta} - 1, \\ \lambda_\beta(\eta, \zeta) &= e^{\zeta-\eta} - 1. \end{aligned} \quad (18)$$

Above correction functions can be reduce in the following manner:

FIGURE 5: Behavior of $G(\eta)$ for λ .

$$F(\eta) = C_1 + C_2 \eta + C_3 e^{-\eta} + h_1 \int_0^\eta \left(1 - \frac{e^{\eta-\zeta}}{2} + \frac{e^{\zeta-\eta}}{2} \right) \\ \cdot \left[F'_n(\zeta) + \frac{(1-\phi) + \phi \left[\rho_s / \rho_f \right]}{(1 + a^* \phi + b \phi^2)} \right. \\ \cdot \left(-\widetilde{F}_n'^2(\zeta) + \widetilde{F}(\zeta) \widetilde{F}'_n(\zeta) + 2\lambda \widetilde{G}(\zeta) \right. \\ \left. \left. - \left\{ \frac{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi} \right\} M^2 \widetilde{F}'_n(\zeta) \right) \right] d\zeta,$$

$$G(\eta) = C_4 + C_5\eta + h_2 \int_0^\eta \left(e^{\zeta-\eta} - 1 \right) \left[\widetilde{G}_n'(\zeta) + \frac{(1-\phi) + \phi [\rho_s/\rho_f]}{(1+a^*\phi + b\phi^2)} \right. \\ \left. \cdot \left(-\widetilde{F}_n'(\zeta) \widetilde{G}_n'(\zeta) - 2\lambda \widetilde{F}_n'(\zeta) - \left\{ \frac{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi} \right\} M^2 \widetilde{G}_n'(\zeta) \right) \right] d\zeta,$$

$$\beta(\eta) = C_6 + C_7\eta + h_3 \int_0^\eta \lambda_\beta(\eta, \zeta) \\ \cdot \left[\beta_n'(\eta) + \frac{(1-\phi) + \phi [(\rho C_p)_s/(\rho C_p)_f]}{[k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi/k_s + (n-1)k_f - (k_s - k_f)\phi]} \right. \\ \left. \cdot \left(\Pr \widetilde{F}_n(\zeta) \widetilde{\beta}_n'(\zeta) \right) \right] d\zeta. \quad (19)$$

By entreating given boundary conditions, we can find the values of constant C_i , for $i = 1, 2, 3, \dots, 7$. Further, initial approximations for F , G , and β are as under.

$$\begin{aligned} F_0(\eta) &= S - 1 + e^{-\eta}, \\ G_0(\eta) &= 0, \\ \beta_0(\eta) &= e^\eta. \end{aligned} \quad (20)$$

Finally, we have the following recursive relations for the flow model:

$$\begin{aligned} F_{n+1}(\eta) &= h_1 \int_0^\eta \left(1 - \frac{e^{\eta-\zeta}}{2} + \frac{e^{\zeta-\eta}}{2} \right) \left[\sum_{i=0}^n F_i''(\zeta) + \frac{(1-\phi) + \phi [\rho_s/\rho_f]}{(1+a^*\phi + b\phi^2)} \right. \\ &\quad \cdot \left(-\sum_{i=0}^n A_i + \sum_{i=0}^n y_i + 2\lambda \sum_{i=0}^n G_i(\zeta) - \left\{ \frac{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi} \right\} M^2 \sum_{i=0}^n F_i(\zeta) \right) \Big] d\zeta \\ &\quad + h_1 \int_0^\infty \left(-e^\zeta + \frac{e^{\zeta-\eta}}{2} + \frac{e^{-\zeta-\eta}}{2} \right) \left[\sum_{i=0}^n F_i''(\zeta) + \frac{(1-\phi) + \phi [\rho_s/\rho_f]}{(1+a^*\phi + b\phi^2)} \right. \\ &\quad \cdot \left(-\sum_{i=0}^n A_i + \sum_{i=0}^n y_i + 2\lambda \sum_{i=0}^n G_i(\zeta) - \left\{ \frac{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi} \right\} M^2 \sum_{i=0}^n F_i(\zeta) \right) \Big] d\zeta, \\ G_{n+1}(\eta) &= h_2 \int_0^\eta \left(e^{\zeta-\eta} - 1 \right) \left[\sum_{i=0}^n G_i'(\zeta) + \frac{(1-\phi) + \phi [\rho_s/\rho_f]}{(1+a^*\phi + b\phi^2)} \right. \\ &\quad \cdot \left(\sum_{i=0}^n Q_i + \sum_{i=0}^n g_i - 2\lambda \sum_{i=0}^n F_i'(\zeta) - M^2 \left\{ \frac{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi} \right\} \sum_{i=0}^n G_i(\zeta) \right) \Big] d\zeta \\ &\quad + h_2 \int_0^\infty (1 - e^{-\eta}) \left[\sum_{i=0}^n G_i'(\zeta) + \frac{(1-\phi) + \phi [\rho_s/\rho_f]}{(1+a^*\phi + b\phi^2)} \right. \\ &\quad \cdot \left(\sum_{i=0}^n Q_i + \sum_{i=0}^n g_i - 2\lambda \sum_{i=0}^n F_i'(\zeta) - M^2 \left\{ \frac{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi} \right\} \sum_{i=0}^n G_i(\zeta) \right) \Big] d\zeta, \\ \beta_{n+1}(\eta) &= h_3 \int_0^\eta \left(e^{\zeta-\eta} - 1 \right) \left[\sum_{i=0}^n \beta_i'(\eta) + \frac{(1-\phi) + \phi [(\rho C_p)_s/(\rho C_p)_f] \Pr}{[k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi/k_s + (n-1)k_f - (k_s - k_f)\phi]} \left(\sum_{i=0}^n k_i \right) \right] d\zeta \\ &\quad + h_3 \int_0^\infty (1 - e^{-\eta}) \left[\sum_{i=0}^n \beta_i'(\eta) + \frac{(1-\phi) + \phi [(\rho C_p)_s/(\rho C_p)_f] \Pr}{[k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi/k_s + (n-1)k_f - (k_s - k_f)\phi]} \left(\sum_{i=0}^n k_i \right) \right] d\zeta. \end{aligned} \quad (21)$$

FIGURE 6: Behavior of $\beta(\eta)$ for λ and M . FIGURE 7: Behavior of surface shear stresses for λ and M .

Now, define the residual functions in the following way:

$$R_n(\eta, h_1) = F'_n + \frac{(1-\phi) + \phi \left[\frac{\rho_s}{\rho_f} \right]}{(1+a^*\phi + b\phi^2)} \left(-F_n'^2 + F_n F_n' + 2\lambda G_n \right) - \frac{\{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi / (\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi\}}{(1+a^*\phi + b\phi^2)} M^2 F_n',$$

$$p_n(\eta, h_2) = G'_n + \frac{(1-\phi) + \phi \left[\frac{\rho_s}{\rho_f} \right]}{(1+a^*\phi + b\phi^2)} \left(-F_n' G_n + F_n G_n' - 2\lambda F_n' \right) - \frac{\{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi / (\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi\}}{(1+a^*\phi + b\phi^2)} M^2 G_n,$$

$$b_n(\eta, h_3) = \beta'_n + \frac{(1-\phi) + \phi \left[\frac{(\rho C_p)_s}{(\rho C_p)_f} \right]}{[k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi / k_s + (n-1)k_f - (k_s - k_f)\phi]} \text{Pr } F_n \beta'_n, \quad (22)$$

and error squared residual functions define as

$$r_n = \frac{1}{100} \sum_{i=0}^{100} \left(R_n \left(\frac{i}{20} \right) + p_n \left(\frac{i}{20} \right) + b_n \left(\frac{i}{20} \right) \right). \quad (23)$$

4. Geometrical Interpretation of the Results

The influences of governing quantities in the dynamics of the nanoliquids are decorated in this section. These

FIGURE 8: Behavior of shear stresses for λ and M along the y direction. FIGURE 9: Behavior of heat transport rate for λ and M .

parameters potentially alter the fluid motion behavior, thermal storage, and trends in the shear stresses on the surface. For this, Figures 2–9 are portrayed over the feasible region.

Figures 2 and 3 portray the behavior of Cu (platelets)/ H_2O_{nf} , Cu (blades)/ H_2O_{nf} , Cu (cylinders)/ H_2O_{nf} , and Cu (bricks)/ H_2O_{nf} nanoliquids over the surface. Figure 2 highlights that the velocity rises by taking upturns in λ and S , respectively. The peak variations are examined for Cu (bricks)/ H_2O_{nf} and Cu (blades)/ H_2O_{nf} than Cu (platelets)/ H_2O_{nf} and Cu (cylinders)/ H_2O_{nf} . Physical aspects of this behavior are the thermophysical attributes of the various Cu-nanomaterials, sphericity and density. Density of the nanomaterials is one of the potential ingredients in the study of nanoliquids. The nanoliquid becomes more dense due to higher density which ultimately affect the fluid motion. At the surface, the motion of the fluid obeys imposed BCs,

and then the fluid tends to its ambient position as described in ambient BCs. In Figure 3, the momentum BL region minimizes due to stronger magnetic field variations.

The trends in the nanoliquid velocity $G(\eta)$ for stronger magnetic field, S and λ , are plotted in Figures 4 and 5, respectively. It is examined that by intensifying the strength of magnetic field, the fluid motion reduces abruptly. Physically, the magnetic field resists the fluid motion consequently; the fluid lost its momentum, and the motion reduces. Near the surface locality, these effects are maximizes because the strength of magnetic field is stronger in this region. For brick geometry-based nanomaterial, the velocity declines rapidly than rest of the nanoliquids. Similarly, quite rapid reduction in the velocity is noted for varying S over the surface. The fascinating trends in the velocity are examined due to rotation of the surface. The rotational

TABLE 2: Comparison of $F''(0)$ for $\phi = 0.0, M = 0, Pr = 7, S = 0$ by OVIM.

λ	Present	[1]	[38]	[36]
0.0	-1.00048	-1.0000	-1.0000	-1.0005
0.5	-1.13848	-1.1384	-1.1384	-1.1385
1.0	-1.32503	-1.3250	-1.3250	-1.3250
2.0	-1.65235	-1.6523	-1.6523	-1.6524

TABLE 3: Comparison of $G'(0)$ for $\phi = 0.0, M = 0, Pr = 7, S = 0$ by OVIM.

λ	Present	[1]	[38]	[36]
0.0	0.000000	0.00000	0.00000	0.00000
0.5	-0.512684	-0.5128	-0.5128	-0.5127
1.0	-0.837109	-0.8371	-0.8371	-0.8371
2.0	-1.28726	-1.2873	-1.2873	-1.2873

TABLE 4: Comparison of $\beta'(0)$ for $\phi = 0.0, M = 0, Pr = 7, S = 0$ BY OVIM.

λ	Present	[1]	[38]	[36]
0.0	1.89524	1.894	1.894	1.895
0.5	1.85114	1.850	1.850	1.851
1.0	1.78763	1.788	1.788	1.788
2.0	1.66436	1.664	1.664	1.669

parameter provides momentum to the fluid particles which ultimately intensifies the fluid momentum. Therefore, the nanoliquids move abruptly.

The nanoliquids (Cu (platelets)/H₂Onf, Cu (blades)/H₂Onf, Cu (cylinders)/H₂Onf, and Cu (bricks)/H₂Onf) temperature against λ and M are described in Figure 6. It is noticed that the temperature drops for higher λ . On the other side, the thermal field drops while imposing higher strength magnetic field. Physically, the nanoliquid (Cu (platelets)/H₂Onf, Cu (blades)/H₂Onf, Cu (cylinders)/H₂Onf, and Cu (bricks)/H₂Onf) particles move slowly over the surface due to higher M consequently; the particle colloids reduce which leads to decrement in the temperature. The temperature in Cu (platelets)/H₂Onf reduces slowly than the other nanoliquids. Therefore, Cu (platelets)/H₂Onf has high thermal ability, and it would be better for industrial and real world applications.

The trends in the surface shear stresses and surface heat transfer rate for multiple parameters are demonstrated in Figures 7–9. It is noted that the alterations in the surface shear stresses in the x direction increase for higher values of M and λ , respectively. Figure 8 shows the prominent changes in skin friction along the y direction. The behavior of surface shear stresses along the y direction is opposite for different values of M and λ , respectively. It is examined that the rate of heat transport intensifies for Cu (bricks)/H₂Onf.

Tables 2–4 provided the reliability of the analysis. These computations are performed for $F''(0)$, $G'(0)$ and $-\beta'(0)$, respectively. Also, BC set as $F'(0) = 1$ (stretching sheet case).

5. Conclusions

A novel heat transfer model for Cu-H₂O over a magnetized shrinking surface oriented counterclockwise is organized successfully. The model is upgraded in dimensionless version via appropriate set of similarity equations. Then, for solution purpose, OVIM is chosen and implemented effectively. The physical results are furnished for different governing quantities and explained in the view of physics lying behind them. From the careful investigation, it is examined that the nanoliquid motion reduces by increasing λ and S , and the nanoliquid composed by bricks shaped nanomaterial moves abruptly over the surface. The imposed magnetic field opposes the motion $G(\eta)$ of the colloidal suspensions. Further, it is noted that stronger Lorentz forces favor the local thermal performance of the particular nanoliquids, and maximum trends in the shear stresses are observed against λ and M , respectively. Among the nanoliquids used, brick-shaped nanomaterial has high thermal storage ability, and it would be excellent for further industrial applications as for as heat transfer concerned.

Data Availability

The data used in this work will be available on behalf of corresponding author.

Conflicts of Interest

The authors declare no financial or competing interest regarding to this work.

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