Melting Heat Transition in a Spinning Flow of Silver-Magnesium Oxide/Engine Oil Hybrid Nanofluid Using Parametric Estimation

Muhammad Bilal, Taza Gul, Abir Mouldi, Safyan Mukhtar, Wajdi Alghamdi, Souhail Mohamed Bouzgarrou, and Nosheen Feroz

1Department of Mathematics, City University of Science and Information Technology, Peshawar 25000, Pakistan
2Department of Industrial Engineering, College of Engineering, King Khalid University, Abha 61421, Saudi Arabia
3Department of Basic Sciences, Deanship of Preparatory Year, King Faisal University, Hafuf, Al Ahsa, Saudi Arabia
4Department of Information Technology, Faculty of Computing and Information Technology, King Abdulaziz University, Jeddah 80261, Saudi Arabia
5Department of Civil Engineering, Faculty of Engineering, Jazan University, Saudi Arabia
6Department of Mathematics, Bacha Khan University Charsadda, KP, Pakistan

Correspondence should be addressed to Taza Gul; tazagul@cusit.edu.pk

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This study reports the three-dimensional (3D) flow of Ag-MgO hybrid nanofluid (HNF) over a spinning disc of flexible thickness in the presence of modified Fourier law. The HNF is contained of silver and magnetic nanoparticulate in the base fluid engine oil. The energy transition has been examined in the involvement of melting heat propagation. The highly nonlinear system of partial differential equations (PDEs) is processed by adopting the proper similarity conversions to attain the coupled ODE system. The obtained system of modeled equations is numerically solved by employing the Parametric Continuation Method (PCM). The nature of various constraints, as opposed to the velocities, energy, and mass transmission, is portrayed and described. In comparison to the simple nanoliquid flow, the hybrid nanoliquid flow’s velocity and heat conduction are observed to have a significant influence. As a result, the functionality of the hybrid nanoliquid is significantly superior to that of the conventional nanofluid. The positive variation in power-law exponent n and Reynold number Re significantly enhances the fluid velocity. The effect of both melting coefficient and thermal relaxation term reduces fluid temperature.

1. Introduction

In the analysis of HNF due to its substantial participation in engineering constraints and modern machinery, the examination of HNF flow over a turning disk with energy communication has taken significant interest [1, 2]. The well-recognized uses consist of electric control techniques, cocirculating apparatus, aerodynamic systems, whirling machines, biochemical reactions, supercomputer management, and hydrothermal sectors [3]. Lv et al. [4] investigated the effects of magnetism and Hall potential on nanofluid flow across a revolving disc. Their target was to increase the level of heat dissipation for technological reasons. As per the conclusions, the modification of CNTs in water is substantially more favorable than that of other nanoparticles due to their C–C interaction. Li et al. [5] employed the bvp4c packages to perform a percentage approximation for Darcy HNF flow over a pierced rotation disc with heat slip. Khan et al. [6] investigated the chemical reaction that influences Maxwell fluid flow over a diagonally gyrating oscillating disc with the magnetic flux during unstable motion. It should be observed that the energy transference ratio raises drastically when the disc radiation and rotation factors increase. The unsteady slip flow with entropy production over a revolving disc under the action of a ferromagnetic material was studied by Shuaib et al. [7] and Bilal et al. [8]. The slip factor...
Phase develops. HNF fluid potential spectrum is also lowered as the thermal relaxation tics. The Oldroyd-B fluid is increased, the et al. [9] using a whirling disc. As the relaxation time factor aid the presented rebuttal. Tassaddiq et al. [11] created an and a full simple geometric presentation for key variables Waqas et al. [10]. A concordance with previous research and a substantial improvement in thermal properties was seen [22]. In the present analysis, we have utilized the MgO (magnesium oxide) and Ag (silver) nanomaterials in the base fluid. MgO is a chemical made up of Mg\(^{2+}\) and O\(^{2-}\) ions at 700–1500°C [23]. For metallurgical and electronic operations, MgO is more practical [24]. Similarly, Ag nanoparticles’ might be exploited to control bacterial movement in an array of products, involving dental work, injuries and wound therapy, surgery, and biomedical apparatus [25, 26]. Ahmadian et al. [27] investigate a 3D simulation of an unstable Ag-MgO HNF flow with heat conduction induced by a curvy spinning disc going up and downwards. With the dispersion of Ag-MgO nanocrystals, the HNF is created. The issue was solved using the PCM technique. The usage of Ag-MgO is thought to be more effective in overcoming poor energy transfer. Among metal and metal oxide, silver and magnesium oxide nanoparticle have been widely recorded to have broad-spectrum antibiotic assets [28]. Silver nanoparticles are the most widely utilized inorganic nanoparticles, having several applications in biomaterial detection and antibacterial activities [29]. Anuar et al. [30] used Ag and MgO nanocrystals in water to evaluate the energy distribution of a hybrid nanoliquid through an extended sheet with suction and buoyant force effects. The findings show that improving the quantity of Ag nanoparticles in HNF lowers the energy transference. Gangadhar et al. [31] arithmetically addressed the heat transport properties of a hybrid nanofluid mixture combining Au and MgO nanoparticulate. Hiba et al. [32] evaluated the thermal performance of HNF including magnesium oxide and Silver and across a highly permeable hollow microplate under magnetic impact. Recently, several researchers have been reported on the study of hybrid nanofluid flow [33–36].

PCM tackles a lot of challenging nonlinear boundary value problems that other numerical techniques cannot solve. Convergence is subject to the relaxation variables and initial strategy for many problems that are generally addressed by traditional computational approaches [37–40]. The PCM’s goal is to determine that the proposed methodology can be used to solve complex nonlinear problems related to industry [41]. Shuaib et al. [42] emphasized the 3D oscillating fluid and energy conductivity across the surface of an irregular elastic revolving disc. The fluid flow has been examined in the context of an external magnetism flux. The phenomena of an ionic fluid flow throughout a spinning disc were discovered by Shuaib et al. [43]. The Poisson’s and Planck models were used to computing the molecular interactions. Dombovari et al. [44] investigated the robustness of nonlinear hydrological systems using a parametric continuation technique. They also looked into static bifurcation, which arises while addressing complex initial value systems with distinctive roots, and devised a method for efficiently determining the points of bifurcation. Ref. [45, 46] may be used to solve the stated challenge in the future.

The assessment was aimed at reporting the 3D flow of Ag- and MgO-based HNF over a spinning disk of flexible thickness. The HNF is synthesized with the composition of silver and magnetic nanomaterials in the engine oil. The energy transition is examined with the involvement of melting heat propagation. To evaluate the behaviors of the fluid flow, Tiwari and Das’s model is employed. The nonlinear system of PDEs is processed through the proper similarity conversions to attain the coupled ODE system. The obtained system of modeled equations is numerically solved employing the Parametric Continuation Method (PCM). In the next section, the formulation, solution methodology, and results and discussion have been discussed in detail.
2. Mathematical Formulation

In this study, we considered the steady and incompressible flow of Ag-MgO hybrid nanoliquid over a gyration disk of variable thickness \( z = a(1 + r^m)^{1/n} \), moving with fixed angular velocity \( \Omega \) about the z-axis. Here, \( u, v, \) and \( w \) are the velocity component along \( r, \theta, z \) direction, respectively. \( T_{\infty} \) is the free stream temperature, and \( T_m \) is the temperature of the melting surface. Figure 1 reveals the flow mechanism over a spinning disk. The modeled equations can be rebound as [47–49]

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{1}
\]

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial u}{\partial z} = \nu_{hnf} \frac{\partial^2 u}{\partial z^2} + \frac{\nu^2}{r} - \sigma_{hnf} B_{hnf}^2 u, \tag{2}
\]

\[
\frac{\partial v}{\partial r} + \frac{w}{r} + \frac{\partial v}{\partial z} = \nu_{hnf} \frac{\partial^2 v}{\partial z^2} - \frac{\nu u}{r} - \sigma_{hnf} B_{hnf}^2 v, \tag{3}
\]

\[
u \frac{\partial T}{\partial r} + \frac{u}{r} \frac{\partial T}{\partial z} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial z^2} - \lambda \tag{4}
\]

\[
\left( u^2 \frac{\partial^2 T}{\partial r^2} - w \frac{\partial^2 T}{\partial z^2} + \left( \frac{\partial u}{\partial r} \frac{\partial T}{\partial z} + \frac{u}{r} \frac{\partial u}{\partial z} \frac{\partial T}{\partial r} \right) ight) + 2\frac{uw}{r} \frac{\partial^2 T}{\partial zdz} + \frac{\partial T}{\partial z} \left( \frac{\partial w}{\partial r} + \frac{\partial w}{\partial z} \right),
\]

\[
u \frac{\partial C}{\partial r} + \frac{v}{r} \frac{\partial C}{\partial z} = D_{hnf} \frac{\partial^2 C}{\partial z^2} - k(C - C_0). \tag{5}
\]

Here, \((u,v,w)\) exhibit the velocity element, and \(\nu_{hnf}, k_{hnf}\) and \((\rho C_p)_{hnf}\) reveal the kinematic viscosity, thermal conductivity, and volumetric heat capacity, respectively. The boundary conditions are

\[
\begin{align*}
&u = 0, \quad w = 0, \quad v = r \Omega, \quad T = T_{\infty}, \quad C = C_{\infty} \text{at} \quad z = 0, \\
&k_{hnf} \frac{\partial T}{\partial z} \bigg|_{z=a(1-r^m)^{1/n}} = \rho_{hnf} (\lambda^* + C_s(T_m - T_0)) w(r, z), \\
&u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_{\infty}, \\
&C \rightarrow C_{\infty} \text{when} \quad z \rightarrow \infty.
\end{align*}
\]

Incorporating the following transformation in Equations (1)–(5) and (6)

\[
\begin{align*}
u &= \Omega r F(\eta), \quad v = \Omega r G(\eta), \quad w = -R_0 \Omega (1 + r^m)^{-m} \frac{R_0^2 \Omega r}{\mu_f} (1/n+1) J(\eta), \theta(\eta), \\
T &= T - T_m = \frac{C - C_m}{C_{\infty} - C_m}, \\
\eta &= \frac{z}{R_0} (1 + r^m)^{-m} \frac{R_0^2 \Omega r}{\mu_f} (1/n+1) J(\eta), \tag{7}
\end{align*}
\]

we get

\[
J'(\eta) + 2 F(\eta) + \eta m F(\eta)' = 0, \tag{8}
\]

\[
\left( \frac{\mu_{hnf}}{\mu_f} \right) R e^{1-n+\eta} (1 + r^m)^2 F'(\eta) + \left( \frac{\rho_{hnf}}{\rho_f} \right) \cdot \left[ \frac{2 - G(\eta) F(\eta)}{J(\eta) G'(\eta)} - \eta m F(\eta) G'(\eta) \right] = 0, \tag{9}
\]

\[
\left( \frac{\mu_{hnf}}{\mu_f} \right) R e^{1-n+\eta} (1 + r^m)^2 G'(\eta) + \left( \frac{\rho_{hnf}}{\rho_f} \right) \cdot \left[ \frac{2 - G(\eta) F(\eta)}{J(\eta) G'(\eta)} - \eta m F(\eta) G'(\eta) \right] = 0, \tag{10}
\]

\[
\left( \frac{k_{hnf}}{k_f} \right) \left( \frac{R e^{1-n+\eta} (1 + r^m)^2 m}{\lambda^* + C_s(T_m - T_0)} \right) \theta''(\eta) - \gamma \left( \frac{\rho C_p}{(\rho C_p)_{hnf}} \right) \theta(\eta) \tag{11}
\]

\[
\begin{align*}
&\left[ m(m - 1) \eta^2 F(\eta) \theta'(\eta) + \eta^2 m^2 \varepsilon^2 F(\eta) \theta'(\eta) \right] \\
&+ J^2(\eta) \theta''(\eta) + \eta m^2 \varepsilon^2 F(\eta) \theta'(\eta) \\
&+ \eta^2 m^2 \varepsilon^2 F(\eta) \theta'(\eta) + \eta m^2 \varepsilon^2 F(\eta) \theta'(\eta) \\
&+ \eta m^2 \varepsilon^2 F(\eta) \theta'(\eta) + J(\eta) F(\eta) + J(\eta) \theta(\eta) \\
&- \eta m^2 \varepsilon^2 F(\eta) \theta(\eta) - J(\eta) \theta'(\eta) = 0,
\end{align*}
\]

\[
\begin{align*}
\varphi''(\eta) &\a^2 (1 + r^m)^{2m} - \frac{1}{\nabla} a G(\eta) \varphi(\eta) \\
&+ \frac{1}{\nabla} a G(\eta) \varphi(\eta) \\
&- \frac{1}{\nabla} a G(\eta) \varphi(\eta) = 0. \tag{12}
\end{align*}
\]
Table 1: The thermophysical properties of hybrid nanofluid and model [47].

<table>
<thead>
<tr>
<th>Property</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity ( \mu )</td>
<td>( \mu_j / (1 - \phi_{Au}) ) ( \mu_j / (1 - \phi_{Ag}) )</td>
</tr>
<tr>
<td>Density ( \rho )</td>
<td>( (1 - \phi_{Au}) \rho_f + \phi_{Au} \rho_{Au} ) ( 1 - \phi_{hnf} \rho_f + \phi_{MgO} \rho_{MgO} + \phi_{Ag} \rho_{Ag} )</td>
</tr>
<tr>
<td>Heat capacity ( \rho C_p )</td>
<td>( (1 - \phi_{Au}) (\rho C_p)<em>j + \phi</em>{Au} (\rho C_p)<em>{Au} ) ( 1 - \phi</em>{hnf} (\rho C_p)<em>j + \phi</em>{MgO} (\rho C_p)<em>{MgO} + \phi</em>{Ag} (\rho C_p)_{Ag} )</td>
</tr>
<tr>
<td>Thermal conductivity ( k )</td>
<td>( (k_{Au} + 2k_j - 2\phi_{Au}(k_j - k_{Au}))k_{Au} + 2k_j + 2\phi_{Au}(k_j - k_{Au})k_j ) ( k_{Ag} + 2k_{hnf} - 2\phi_{Ag}(k_{hnf} - k_{Ag})k_{Ag} + 2k_{hnf} + 2\phi_{Ag}(k_{hnf} - k_{Ag}) \times (k_{MgO} + 2k_j - 2\phi_{MgO}(k_j - k_{MgO}))k_{MgO} + 2k_j + 2\phi_{MgO}(k_j - k_{MgO}) )</td>
</tr>
<tr>
<td>Diffusivity ( \alpha )</td>
<td>( k_{hnf} / (\rho C_p)<em>{hnf} ) ( k</em>{hnf} / (\rho C_p)_{hnf} )</td>
</tr>
</tbody>
</table>
The transform conditions are

\[ F(0) = 0, G(0) = 1, \frac{k_{hnf}}{k_f} \text{Me} Re^{1-1/n+1(1 + r^*)^{2m}} \theta'(\alpha) + \frac{\rho_{hnf}}{\rho_f} \text{Pr} J(\alpha) = 0, \theta(\alpha) = 1, \phi(\alpha) = 1, \]

\[ F(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0, G(\infty) = 0. \]

(13)

Here, \( \alpha \) is the disk thickness coefficient, \( \text{Re} \) is the Reynolds number, \( \text{Me} \) is the melting constant, \( \varepsilon \) is the constant coefficient, \( r^* \) is the dimensionless radius parameter, \( \gamma \) is the thermal relaxation parameter, and \( \text{Pr} \) is the Prandtl number defined as [47]

\[ \alpha = \frac{a}{R_0} \left( \frac{R^2 \Omega \rho_f}{\mu_f} \right)^{1/n+1}, \text{Re} = \frac{R^2 \Omega \rho_f}{\mu_f}, \]

\[ \text{Me} = \frac{(T_{co} - T_m) C_p}{C_v (T_m - T_0) + \lambda^2}, \]

\[ r^* = \frac{r}{R_0}, \gamma = \Omega \lambda, \text{Pr} = \frac{\mu_f (C_p)_{nf}}{k_{hnf}}. \]

where \( F, G, \) and \( J \) denote the radial, tangential, and axial velocities, and \( \phi, \theta \) show the dimensionless concentration and temperature. The deformations are expressed as

\[ f(\xi) = f(\eta - \alpha) = F(\eta), \]

\[ j(\xi) = j(\eta - \alpha) = J(\eta), \]

\[ g(\xi) = g(\eta - \alpha) = G(\eta), \]

\[ \theta(\xi) = \theta(\eta - \alpha) = \theta(\eta), \]

\[ \phi(\xi) = \phi(\eta - \alpha) = \phi(\eta). \]

(15)

Using Equation (15), Equations (7)–(12) take the form

\[ f'(\xi) + \xi \epsilon \phi'(\xi) + \phi f'(\xi) + 2f(\xi) = 0, \]

(16)

\[ \left( \frac{\mu_{hnf}}{\mu_f} \right) \text{Re}^{1-1/n+1(1 + r^*)^{2m}} f''(\xi) + \left( \frac{\rho_{hnf}}{\rho_f} \right) \cdot \left[ -f^2(\xi) - j(\xi) f'(\xi) - \xi \epsilon \phi f(\xi) f'(\xi) + g^2(\xi) - \epsilon \phi f(\xi) f'(\xi) \right] = 0, \]

(17)

\[ \left( \frac{\mu_{hnf}}{\mu_f} \right) \text{Re}^{1-1/n+1(1 + r^*)^{2m}} \left[ -j(\xi) g'(\xi) + \xi \epsilon \phi f(\xi) g'(\xi) + 2g(\xi) f(\xi) - \epsilon \phi f(\xi) g'(\xi) \right] = 0, \]

(18)

The skin friction is stated as

\[ C_{fr} = \frac{\tau_{wr}}{\rho_f (R_0 \Omega)^2}, C_{f0} = \frac{\tau_{w0}}{\rho_f (R_0 \Omega)^2}. \]

(22)

Shear forces are

\[ \tau_{wr} = \frac{\partial u}{\partial z} \bigg|_{z = (1 + r^*)^{-m}}, \tau_{w0} = \frac{\partial v}{\partial z} \bigg|_{z = (1 + r^*)^{-m}}. \]

(23)

The nondimensional form is

\[ \text{Re}^{m + 1} C_{fr} = \left[ \frac{\mu_{hnf}}{\mu_f} (1 + r^*)^m \right] r^* f'(0), \text{Re}^{m + 1} C_{f0} = \left[ \frac{\mu_{hnf}}{\mu_f} (1 + r^*)^m \right] r^* g'(0). \]

(24)

3. Numerical Solution

The basic steps of PCM are as follows: Step 1: simplifying Equations (16)–(20) to 1st order with the boundary conditions
\( X_1 = f(\xi), X_2 = f'(\xi), X_2' = \frac{B^* \left[ 2(x_1)^2 + x_2(x_5 + m(\xi + \alpha)\epsilon x_1) - (x_5)^2 \right]}{A^* (1 + r^*)^{2m} [Re]^{1-n+1}}, \)

\( X_3 = g(\xi), X_4 = g'(\xi), X_4' = \frac{B^* \left[ 2x_1 x_3 + x_4(x_5 + m(\xi + \alpha)\epsilon x_1) - x_5 x_2 \right]}{A^* (1 + r^*)^{2m} [Re]^{1-n+1}}, \)

\( X_5 = j(\xi), X_5' = \left[ -2x_1 + m(\xi + \alpha)\epsilon(x_2)^2 \right], X_6 = \theta(\xi), X_7 = \theta'(\xi), \)

\[
D^* (\gamma Pr)^* \left[ X_5 X_6 X_5' + X_7 \left( \frac{m(\xi + \alpha)(\epsilon x_1)^2 + m\epsilon x_5 x_1 + m}{(\xi + \alpha)\epsilon x_1 x_2 + m(\xi + \alpha)\epsilon x_1 x_5' - m\epsilon x_1 x_5} \right) \right] - m(\xi + \alpha)\epsilon x_1 x_7 + x_5 x_7
\]

\[
X_7' = \frac{C^* C_1^* (1 + r^*)^{2m} (Re)^{1-n+1} \left[ x_5^2 + 2m(\xi + \alpha)^2 \epsilon^2(x_1)^2 \right]}{1}, \]

\[
X_9' = \frac{1}{Scd x_8} + \frac{x_5 - 1}{Scma^2(\xi) x_1 - 1} \frac{1}{Scma^2(\xi) x_1 + 1} \frac{1}{Sc(1 + r^*)^{1-n+1}}, \]

\[
X_1(0) = 0, X_5(0) = 1, C^* C_1^* \left[ (1 + r^*)^{2m} Re^{1-n+1} \right] X_7 + D^* Pr^* x_5 x_6 = 1, \]

\[
X_1(\infty), X_5(\infty), X_6(\infty), X_8(\infty). \]

Step 2: introducing parameter \( p \)
Figure 2: The nature of radial velocity $f(\eta)$ and tangential velocity $g(\eta)$ profiles versus (a) volume friction $\phi_1$, (b) volume friction $\phi_2$, (c) Reynold number $Re$, (d) power-law exponent $n$, and (e) Reynold number $Re$. 
Figure 3: The energy outlines $\theta(\eta)$ versus (a) volume friction $\phi_1$, (b) volume friction $\phi_2$, (c) Reynold number Re, (d) melting coefficient Me, and (e) thermal relaxation parameter $\gamma$, respectively.
Figure 4: The nature of mass transition $\varphi(\eta)$ versus (a) volume friction $\phi_1$, (b) volume friction $\phi_2$, (c) Schmidt number $Sc$, (d) chemical reaction $d_1$, respectively.

Figure 5: The comparison between PCM and Matlab built-in package bvp4c.
Step 3: apply the Cauchy principle and discretized Equation (26)

$$\frac{U^{i+1} - U^i}{\Delta \eta} = AU^{i+1}, \quad \frac{W^{i+1} - W^i}{\Delta \eta} = AW^{i+1}. \quad (27)$$

Finally, we get the iterative form as

$$U^{i+1} = (I - \Delta \eta A)^{-1} U^i, \quad W^{i+1} = (I - \Delta \eta A)^{-1} (W^i + \Delta \eta R). \quad (28)$$

4. Results and Discussion

The discussion segment analyzed the comportment of velocity, energy, and mass-circulation as compared to the deviation of numerous physical parameters for hybrid nanoliquid consisting of Ag and magnetic nanoparticles.

| Table 2: The experimental values of Ag, MO, and engine oil [47]. |

<table>
<thead>
<tr>
<th></th>
<th>(\rho) (kg/m(^3))</th>
<th>(C_p) (J/kg · K)</th>
<th>(k) (W/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine oil</td>
<td>884</td>
<td>1910</td>
<td>0.114</td>
</tr>
<tr>
<td>Magnesium oxide</td>
<td>3560</td>
<td>955</td>
<td>45</td>
</tr>
<tr>
<td>Silver</td>
<td>10,500</td>
<td>235</td>
<td>429</td>
</tr>
</tbody>
</table>

| Table 3: Comparative analysis with the existing literature, when \(n = 1\) and \(\phi_{huf} = 0\). |

<table>
<thead>
<tr>
<th>Order of approximations</th>
<th>(f' (0))</th>
<th>(-g' (0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present work</td>
<td>0.499321</td>
<td>0.500761</td>
</tr>
<tr>
<td>Zhang et al. [47]</td>
<td>0.497201</td>
<td>0.509623</td>
</tr>
<tr>
<td>Xun et al. [50]</td>
<td>0.410221</td>
<td>0.515911</td>
</tr>
<tr>
<td>Ming et al. [51]</td>
<td>0.410200</td>
<td>0.515901</td>
</tr>
</tbody>
</table>

| Table 4: Statistical outcomes for skin friction \((f' (0), -g' (0))\). |

<table>
<thead>
<tr>
<th>(m)</th>
<th>(\tau)</th>
<th>(Re)</th>
<th>(\phi_2)</th>
<th>(f' (0))</th>
<th>(-g' (0))</th>
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<tbody>
<tr>
<td>0.1</td>
<td>0.4</td>
<td>1.0</td>
<td>0.03</td>
<td>0.0474535</td>
<td>0.0702328</td>
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<tr>
<td>0.5</td>
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<td>0.4</td>
<td>1.0</td>
<td>0.03</td>
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<td>0.0815843</td>
</tr>
<tr>
<td>1.5</td>
<td>0.4</td>
<td>1.0</td>
<td>0.03</td>
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<td>0.1130261</td>
</tr>
<tr>
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<td>0.1</td>
<td>1.0</td>
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<td>0.0817175</td>
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<tr>
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<td>0.3</td>
<td>1.0</td>
<td>0.03</td>
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<td>0.0716306</td>
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<td>0.03</td>
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</tr>
<tr>
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<td>1.0</td>
<td>0.02</td>
<td>0.0705745</td>
<td>0.0893014</td>
</tr>
<tr>
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<td>1.0</td>
<td>0.03</td>
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<td>0.0852958</td>
</tr>
<tr>
<td>1.0</td>
<td>0.4</td>
<td>1.0</td>
<td>0.04</td>
<td>0.0667325</td>
<td>0.0815843</td>
</tr>
</tbody>
</table>

| Table 5: Arithmetic results for Nusselt number \((k_{huf}/k_f \theta'(0), k_{huf}/k_f \theta'(0))\). |

<table>
<thead>
<tr>
<th>Parameters</th>
<th>PCM</th>
<th>PCM</th>
<th>bvp4c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Me</td>
<td>Re</td>
<td>(\phi_1, \phi_2)</td>
<td>(k_{huf}/k_f \theta'(0))</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0474535</td>
<td>0.0484531</td>
<td>0.0484340</td>
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<tr>
<td>0.5</td>
<td>0.0354123</td>
<td>0.0366122</td>
<td>0.0366031</td>
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<tr>
<td>1.0</td>
<td>0.0364852</td>
<td>0.0369853</td>
<td>0.0369542</td>
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<tr>
<td>1.5</td>
<td>0.0291107</td>
<td>0.0271407</td>
<td>0.0271268</td>
</tr>
<tr>
<td>0.2</td>
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<td>0.0554555</td>
<td>0.0554463</td>
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<tr>
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<td>0.0575961</td>
<td>0.0575870</td>
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<tr>
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<td>0.0578962</td>
<td>0.0589965</td>
<td>0.0589834</td>
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<tr>
<td>0.01</td>
<td>0.0673420</td>
<td>0.0683460</td>
<td>0.0683352</td>
</tr>
<tr>
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<td>0.0683241</td>
<td>0.0693271</td>
<td>0.0693160</td>
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<tr>
<td>0.03</td>
<td>0.0690324</td>
<td>0.0713142</td>
<td>0.0713021</td>
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<tr>
<td>0.04</td>
<td>0.0723419</td>
<td>0.0743319</td>
<td>0.0743237</td>
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</table>

| Table 6: Numerical outcomes for Sherwood number. |

<table>
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<tr>
<th>Sc</th>
<th>(d_1)</th>
<th>(\phi_1, \phi_2)</th>
<th>((D_{nf}/D_f) \psi'(0))</th>
<th>((D_{huf}/D_f) \psi'(0))</th>
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</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.0632328</td>
<td>0.0642322</td>
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<tr>
<td>0.4</td>
<td>0.062932</td>
<td>0.0639336</td>
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<td>0.6</td>
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<tr>
<td>0.8</td>
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<td>0.5910240</td>
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</tr>
<tr>
<td>0.1</td>
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<tr>
<td>0.2</td>
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<td>0.0771426</td>
<td></td>
<td></td>
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<tr>
<td>0.3</td>
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<tr>
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<td></td>
<td></td>
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<tr>
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<td>0.04</td>
<td>0.0726718</td>
<td>0.7906714</td>
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<td></td>
</tr>
</tbody>
</table>

The default values used while solving the set of 1st order ODEs through PCM code are \(\phi_1 = \phi_2 = 0.01, Re = 0.5, n = 1.0, Me = 0.2, \gamma = 0.1, Sc = 0.2, d_1 = 0.01\) and \(\tau = 0.4\).

4.1. Velocity Profile. Figures 2(a)–2(e) expose the nature of radial velocity \(f(\eta)\) and tangential velocity \(g(\eta)\) profiles versus volume friction \(\phi_1\), volume friction \(\phi_2\), Reynolds number \(Re\), power-law exponent \(n\), and Reynolds number \(Re\). Figures 2(a) and 2(b) particularize that the velocity field rises with the growing values of volume friction of both silver and magnetic nanoparticulates. Physically, the specific heat capacity of engine oil is much higher than silver and magnesium compounds that is why the increasing quantity of such nanomaterials reduces the average heat capacity of HNF and causes the elevation of fluid velocity. Figures 2(c) and 2(e) display the dominance of both radial \(f(\eta)\) and tangential velocity \(g(\eta)\) profiles against Reynolds number \(Re\). The upshot of Reynold's number increases the rotation of the disk, which accelerates the fluid particles and exercises their kinetic energy, which causes the improvement in the velocity.
field. Similar behavior of radial velocity has been observed versus the increment of power-law exponent \( n \) in Figure 2 (d). The positive variation in the power-law exponent significantly enhances the fluid velocity in the radial direction.

4.2. Energy Distribution Profile. Figures 3(a)–3(e) illuminate the nature of energy transition \( \theta(\eta) \) versus volume friction \( \phi_1 \), volume friction \( \phi_2 \), Reynold number \( Re \), melting coefficient, and thermal relaxation parameter \( \gamma \) respectively. As discussed in Figure 2, the specific heat capacity of engine oil is much higher than silver and magnesium compounds that is why the increasing quantity of such nanomaterials reduces the average heat capacity of hybrid nanofluid and causes a rise in internal heat, which encourage both velocity and energy transmission rate. Figure 3(c) reports the Rey

4.3. Mass Transfer Profile. Figures 4(a)–4(d) spot the nature of mass transition \( \phi(\eta) \) versus volume friction \( \phi_1 \), volume friction \( \phi_2 \), Schmidt number \( Sc \), and chemical reaction \( d_1 \), respectively. Mass transmission enhances with the rising quantity of nanoparticles, because, as we have discussed earlier, the rising values of volume friction parameters \( \phi_1 \) and \( \phi_2 \) significantly elevated the heat and fluid velocity that is why the mass transition also enhances their effects as shown in Figures 4(a) and 4(b). The upshot of the Schmidt number boosts the fluid kinetic viscosity, which results in the reduction of concentration profile \( \phi(\eta) \) as illustrated in Figure 4(c). The energy transport rate is reduced by the chemical reaction variable, while the mass transport rate is increased. An increase in the intensity of \( \phi_1 \), indicates that the species concentration interaction is less with the thermal boundary layer and more with the momentum Figure 4(d).

Figure 5 reports the comparative assessment of PCM technique with Matlab code bvp4c. From Figures 5(a) and 5(b), it can be clearly observed that both techniques show best settlement and PCM procedure is a reliable method. Table 2 describes the experimental values of Ag, MgO, and engine oil. Table 3 displays the comparative analysis of the current work with the existing literature, in which the present work revealed the best settlement with them. Table 4 presents the numerical results for skin friction. It has been observed that the drag force enhances along both radial and tangential direction with the variation of parameter \( m \) and \( \tau \) while reduces the effect of Reynold number. Table 5 communicates the numerical outcomes for Nusselt number versus melting coefficient, Reynold number, and both nanofluid and hybrid nanofluid. As comparative to the simple nanofluid, hybrid nanofluid energy transition rate is faster against volume friction coefficient \( \phi_1, \phi_2 \), respectively. Table 6 displays the Sherwood number versus volume friction parameter \( \phi_1, \phi_2 \), Schmidt number, and chemical reaction \( d_1 \) constant, respectively. The mass transference rate diminishes with the rising effect of Schmidt number while enhances against the increasing quantity of chemical reaction parameter and volume friction constants.

5. Conclusion

The 3D flow of Ag and MgO HNF past over a gyrating disk of varying thickness has been reported in the present estimation. The hybrid nanoliquid is synthesized by using silver, magnetic nanoparticle, and engine oil. The energy transition consequences are examined in the involvement of melting heat propagation. The highly nonlinear system of PDEs is processed through the proper similarity conversions to attain the coupled ODE system. The obtained system of modeled equations is numerically solved through the PCM technique. The key points are rebound as follows:

(i) The radial \( f(\eta) \) and tangential \( g(\eta) \) velocities and energy propagation enhance with the rising values of volume friction of both silver \( \phi_{1=A g} \) and magnetic nanoparticulates \( \phi_{2=M g O} \)

(ii) The upshot of Reynold number \( Re \) improves the velocity and energy transition of fluid flow, due to an increase in the number of disk’s rotation

(iii) The positive variation in power-law exponent \( n \) significantly enhances the fluid velocity in the radial direction

(iv) The increasing quantity of nanomaterials reduces the average heat capacity of hybrid nanofluid and causes a rise in internal heat, which encourages both velocity and heat transition rate

(v) The effect of both melting coefficient and thermal relaxation term \( \gamma \) reduces fluid temperature

Data Availability

The relevant data exist in the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Muhammad Bilal and Taza Gul contributed to the modeling and writing manuscript and conception or design of the work. Abir Mouldi, Safyan Mukhtar, Wajdi Alghamdi, Soumail Mohamed Bouzgarrou, and Nosheen Feroz contributed to the validation and critical revision of the article.

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References


