Cooling a Hot Semiannulus with Constant Heat Flux by Using Fe₃O₄-Water Nanofluid and a Magnetic Field: Natural Convection Mechanism

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1. Introduction

Nowadays, heat transfer processes have widely exerted for many applications such as cooling battery, CPU, forging, radiator in vehicles, heating the home, and powerplant [1–5]. One method that is popular between thermal engineering is injecting various nanoparticles such as metal, nonmetal, and oxide nanoparticles and nanoencapsulated phase change material (NEPCM) to host fluid [6–8], because mixing nanoparticles into host fluid can changed the thermophysical properties, specifically, thermal conductivity of host fluid [9–11].

In recent years, the heat transfer rates by computational fluid dynamics (CFD) and entropy generation are significantly investigated by researchers and companies due to having accurate with experimental studies [12–15]. Moreover, owing to its large specific area and higher solid thermal conductivity, there has been growing interest in heat transfer inside the porous media. According to previous studies, porous structure improves thermal performance of
nanofluid [16–19]. A numerical investigation of heat transfer and fluid flow in a parabolic trough solar receiver with internal annular porous structure and synthetic oil-Al2O3 nanofluid was carried out by Bozorg et al. [20]. According to their results, utilization of porous structure and nanofluids enhances heat transfer coefficient 7% and 20%, respectively. Jamal-Abad et al. [21] experimentally studied the thermal efficiency of a solar parabolic trough collector filled with porous media; they illustrated an enhancement in efficiency of the collector by increasing the mass flow rate.

It is confirmed that in nanofluid simulation, two-phase approach provides better accuracy compared to single-phase approach [22–24]. A two-phase model presented by Buongiorno [25] has received significant attention; in this model, it is suggested that among seven slip mechanisms, Brownian motion and thermophoresis diffusion play vital role in nanofluid distribution. Natural convection of nanofluid in an inclined cavity and inside porous medium considering two-phase approach is studied by studies [6] and [15]. Their results demonstrate good agreement with experimental studies. Thermophoresis and Brownian motion effect on boundary layer flow of nanofluid in presence of thermal stratification due to solar energy is analyzed by Anbuezhian et al. [26]. Their results highlighted that Brownian motion and thermophoresis distribution can affect the heat transfer properties; they reported a substantial impact on the boundary layer flow field by Brownian motion in the presence of thermal stratification. Kaloudis et al. [27] numerically investigated on parabolic trough solar collector with nanofluid using a two-phase model. They reported that two-phase simulation of nanofluids in solar studies shows better agreement with experimental studies. Also, the presence of nanoparticles improves collector’s efficiency.

To obtain the optimal configuration of solar collectors, it is essential to analyze the entropy generation. Farshad and Sheikholeslami [28] scrutinize exergy loss and heat transfer of mixture of aluminum oxide and H2O through a solar collector. Thermal performance and entropy generation analysis of a high concentration ratio parabolic trough solar collector was studied by Mwesigye et al. [29]. They reported a decrement in entropy generation with augmentation of nanofluid volume fraction for some ranges of Re number. Verma et al. [30] experimentally analyzed exergy efficiency and entropy generation in flat plate solar collectors for different types of nanofluids. They highlighted that rise of Bejan number towards unity illustrates the improvement of system performance due to efficient conversion of the available energy into useful functions. Also, Sheikholeslami et al. [31] scrutinized impact of Lorentz forces on magnetic nano fluid of Fe3O4 with entropy and exergy analyzing inside a semiannulus. Afrand et al. [32] studied free convective heat transfer and entropy generation of Al2O3-water nanofluid in a triangular enclosure. They illustrated that the Bijan number increases by decreasing the Ra and increasing the Ha. The maximum heat transfer rate takes place at the enclosure angle of 60°.

To the best knowledge of the authors, there has been no detailed investigation of impacts of nonuniform magnetic fields on PTC thermal performance using nanofluid, considering Brownian motion and thermophoresis distribution as well as entropy generation. In the present work, a coil is wrapped around semiannulus to produce a variable magnetic field. Also, it focuses on the local distribution of nanoparticles, entropy generation due to fluid friction and heat transfer, and Nusselt number variation.

2. Physical Model

In the present study, 2-dimensional and steady-state natural convection is simulated in a semiannulus enclosure. Schematic presentation of the problem is presented in Figure 1. In Figure 1, two horizontal walls are thermally insulated, the inner semicircle wall at constant heat flux (q0), and the outer semicircle wall is fixed at constant temperature (Tw). The working fluid is Fe3O4-water nanofluid. Natural convection fluid flow is simulated based on Boussinesq’s approximation. Also, heat transfer, nanoparticles distribution, and entropy generation in the presence of a nonuniform magnetic field are investigated. Therefore, it is assumed that nanoparticle distribution is based on Bouguirno two-phase model. According to the aforementioned assumption, the governing equations are presented in the next section.

3. Relations and Hypothesis

The nondimensional parameters are as follows: \( X = x/L, Y = y/L, T∗ = k_f/(T − T_w)/q_0^l, m = d/L, m∗ = P_l^2/ρ_f/β_f^2, H∗ = H/H_0, M∗ = M/M_0, q∗ = q/β_f H_0 \), \( D_f = D_f/D_{b0}, D_{b0} = γ μ_f/ρ_f/β_f H_0 \), \( H_0 = l/2π l, D_{b0} = K_B T_w/3π μ_f d_n f, \) and \( M_0 = \chi(β_f H_0, T_w) H_0 \) for \( β_f H_0 = 0.02, \) and \( T_m = T_w + q_0^l L/2k_f. \)

The dimensionless forms of continuity equations, momentum, energy, and volume fraction are as follows:

Nondimensional continuity equation:

\[
\nabla^∗ V^∗ = 0.
\]

Nondimensional momentum equation:

\[
\frac{1}{\rho_f} \frac{D}{D_0} (V^∗ V^∗) V^∗ = −\nabla^∗ P^∗ + \left( \frac{μ_{nf}}{μ_f} \right) \frac{1}{Da} V^∗ + \nabla^∗ \cdot \left( \frac{μ_{nf}}{μ_f} V^∗ \right) + \frac{(ρβ_f)_{nf}}{ρ_f/β_f} \frac{R_A}{Pr} T^∗ \nabla^∗ + Mn (M^∗ V^∗) H^∗.
\]
Nondimensional heat transfer equation:

\[
\frac{(\rho C_p)_{sf}}{[\rho C_p]_f} \nabla^* \nabla^* T^* = \frac{1}{Pr} \nabla T^* + \frac{k_f}{k_f} \nabla^* T^*
\]

\[
+ \frac{1}{Pr Le} \left( D_f^* \nabla^* \nabla^* T^* + \frac{D_f^*}{N_{BT}^*} \frac{\nabla T^*}{1 + \left( \frac{T^*}{\delta} \right)} - D_f^* \xi^* \frac{\nabla^* H^*}{H^*} \right).
\]

(3)

Nondimensional mass transfer equation:

\[
\frac{1}{\xi} \nabla^* \nabla^* \phi^* = \frac{1}{Sc} \nabla \phi^*
\]

\[
+ \left( \frac{D_f^* \nabla^* \phi^*}{N_{BT}^*} + \frac{D_f^*}{N_{BT}^*} \frac{\nabla^* T^*}{1 + \left( \frac{T^*}{\delta} \right)} - D_f^* \xi^* \frac{\nabla^* H^*}{H^*} \right).
\]

(4)

The dimensionless numbers in the above relationships are defined as:

\[
Da = \frac{K}{L^2}, Pr = \frac{v_f}{\alpha_f}, \alpha_f = \frac{k_f}{(\rho C_p)_f}, Ra_f = \frac{g B_1 q^4 L_4}{k_f \alpha_f v_f}, \]

\[
Ra_p = \frac{Ra_f \, Da, \tilde{a} = \frac{g}{\rho} \frac{B_1}{\alpha_f}, Mn = \frac{H_0 H_3 M_0 L^2}{\rho_j q^2}, Lc = \frac{v_f}{\rho C_p}_f \frac{D_B^*}{D_0}, Sc = \frac{v_f}{D_B^*}.
\]

(5)

\[Mn\] is the magnetic number, which is defined as the ratio of the Kelvin force to the kinematic viscosity. Moreover, \(Da, Ra, Pr, Le,\) and \(Sc\) denote the Darcy, Rayleigh, Prandtl, Lewis, and Schmitt numbers, respectively.

The average Nusselt on the constant heat flux wall (inner cylinder wall) is calculated as follows:

\[
Nu_{ave} = \frac{K_{eff}}{k_{eff}} \frac{1}{T^*}, Nu_{ave} = \frac{1}{\pi} \int_0^\pi Nu_{loc}(\zeta) d\zeta.
\]

(6)

In this study, the entropy generation is considered due to the irreversibility of the velocity gradients and temperature gradients. According to Shavik et al. [33], the entropy gener-

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
 & \(Ra = 10^5\) & \(Be_{ave}\) & \(Ra = 10^5\) \\
\hline
\hline
\text{Present work} & 1.15 & 0.97 & 23.2 & 0.193 \\
Shavik et al. [33] & 1.15 & 0.97 & 23.27 & 0.194 \\
\hline
\end{tabular}
\caption{Comparison of total entropy generation (\(S_{ave}\)) and Bejan number (\(Be_{ave}\)), for \(Pr = 0.71\) and irreversibility (\(X = 10^{-4}\)).}
\end{table}

\[
s_s = \frac{\mu_{nf}}{T_m^4} \left[ \frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 v^*}{\partial y^2} \right] + \frac{k_{nf}}{T_m^4} \left[ \frac{\partial^2 T^*}{\partial x^2} + \frac{\partial^2 T^*}{\partial y^2} \right].
\]

(7)

In the mentioned equation, the first and second terms are local entropy generation due to the fluid friction (\(S_{L,FF}\)) and the heat transfer (\(S_{L,HT}\)) irreversibility, respectively. Also, the dimensionless equation of entropy generation is as follows:

\[
S_{L,HT} = \frac{k_{nf}}{k_f} \left( \frac{\partial T^*}{\partial x} + \frac{\partial T^*}{\partial y} \right)^2,
\]

\[
S_{L,FF} = X \frac{\mu_{nf}}{\mu_f} \left[ \left( \frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} \right)^2 + \left( \frac{\partial u^*}{\partial y} + \frac{\partial v^*}{\partial x} \right)^2 \right],
\]

\[
= \mu_j k_f T_m \left( \frac{\alpha_f}{L^4 q^2} \right)^2 X,
\]

\[
S_{L_s} = s_s \times \frac{k_f T_m^2}{q} = S_{L,FF} + S_{L,HT}.
\]

(8)

Bejan number is the ratio of the entropy generation due to the heat transfer to the total entropy generation. This is as follows:

\[
Be_L = \frac{S_{L,HT}}{S_{L,s}}.
\]

(9)
Total entropy generation and the total Bejan number are obtained by integrating the relations of local entropy generation and Bejan number.

\[
S_{T,HT} = \int_A S_{L,HT} dA, \quad S_{T,FF} = \int_A S_{L,FF} dA, \quad S_{T,s} = \int_A S_{L,s} dA, \quad \text{Be}_{ave} = \frac{\int_A \text{Be}_{L} dA}{\int_A dA}.
\]  

(10)

4. CFD Setting

The presented nonlinear governing PDE equations are solved based on finite volume method (FVM). The continuity and momentum equations are coupled and solved in an algorithm termed SIMPLE. The energy and concentration equations simultaneously solved. The advection terms in the governing equations are discretized based on first-order upwind schemes, and diffusion terms are solved based on second-order central schemes. The numerical convergence criterion was residual values. The

Figure 2: Nondimensional distribution of nanoparticles on the outer and inner cylinder walls for $\varphi_{Ave} = 0.03$, $Ra_p = 10$ and 1000, and different magnetic numbers at (a) $\varepsilon = 0.4$ and (b) $\varepsilon = 0.7$. 
residual values at convergence for velocity and pressure fields were $10^{-5}$, and for temperature and $\phi$ were $10^{-6}$. Under relaxation factors for velocity, pressure, temperature, and $\phi$ were 0.4, 0.6, 0.2, and 0.01, respectively. Square uniform mesh is selected for presented study, based on nanoparticle distribution on the hot wall 40,000 ($200 \times 200$) numbers of mesh selected. Moreover, for validation of the entropy generation, the results of the present study is compared with the work of Shavik et al. [33] (Table 1). In all cases, the results of this study are in good agreement.

**Figure 3:** Nondimensional distribution of nanoparticles on inner and outer walls (with angle of $\zeta$) for (a) different magnetic numbers and volume fraction at $Ra_p = 1000$ and $\varepsilon = 0.7$, (b) different porous Rayleigh number and porosity at $\phi_{Ave} = 0.03$ and $Mn = 8 \times 10^6$.

**Figure 4:** The dimensionless local entropy generation due to heat transfer ($S_{LHT}$), fluid friction ($S_{LFF}$) and summation ($S_{LS}$) for $\phi_{Ave} = 0.03$, $\varepsilon = 0.7$, and $Mn = 0$ and $8 \times 10^6$ at (a) $Ra_p = 10$ and (b) $Ra_p = 1000$. 
5. Results and Discussion

Extended Buongiorno’s two-phase model is considered for the distribution of nanoparticles. The numerical simulation has investigated for porous Rayleigh number \( R_{ap} = 10 \) and 1000, the volume fraction of nanoparticles \( \varphi_{ave} = 0.01 \) and 0.03, porosity number \( \varepsilon = 0.4 \) and 0.7, and magnetic number \( 0 \leq Mn \leq 8 \times 10^7 \). Constant values included \( Pr = 4.623, Sc = 3.55 \times 10^4, T_c = 310 \, K, q'' = 48.01 \, (w/m^2), 1.71 \times 10^5 < Le < 6.84 \times 10^5, Da = 10^{-3} (K = 0.625 \times 10^{-6}), \delta = 161, N_{BT} = 0.245, X = 3 \times 10^{-11}, \) and \( T_m = T_c + Lq''/2k_f = 310.96 \, K \).

The effects of the mentioned parameters have studied the distribution of nanoparticles and the entropy generation contours due to fluid friction and heat transfer, and Bejan.

Figure 2 shows the nondimensional distribution of nanoparticles on the lower and upper cylinder walls in \( \varphi_{ave} = 0.03 \) and \( R_{ap} = 10 \) and 1000 for the different magnetic numbers. In Figure 2(a) and porous Rayleigh number 10, the density of nanoparticles is higher on the top wall (cold) than the down wall (hot); that is due to the thermophoresis term in the volume fractional equation. With the rising of porous Rayleigh number, the dimensional distribution of nanoparticles is almost identical due to the increasing flow velocity on the inner and outer walls. By increasing the magnetic number to \( Mn = 2 \times 10^7 \) for all of the cases, the density of nanoparticles for both walls is almost equal to the unit value, and no change occurs. But with the increasing magnetic number to \( Mn = 8 \times 10^7 \), the peak of nanoparticle density appears near the wires, which is due to the absorption of nanoparticles by the magnetic field. The above arguments are also true for Figure 2(b), which the porosity has increased to 0.7. Besides, in all graphs, the peak of nanoparticle density in the inner wall is wider than the outer wall.

Figure 3 shows a better comparison of the effect of different parameters on the dimensional distribution of nanoparticles. Figure 3(a) is shown for magnetic numbers and different volume fractions in \( R_{ap} = 1000 \) and \( \varepsilon = 0.7 \). The dimensionless nanoparticle density is the same for the volume fraction 0.01 and 0.03 in a constant magnetic number. Therefore, the volume fraction does not affect the dimensionless distribution of nanoparticles. Also, in a constant volume fraction, the density of nanoparticles increases near

Figure 5: Dimensionless total entropy generation due to heat transfer \( (S_{T,HT}) \) and fluid friction \( (S_{T,FF}) \) for \( \varepsilon = 0.7 \) and different volume fraction of nanoparticle.
the wires due to increasing the magnetic number. Figure 3 (b) is plotted for different porosity and porous Rayleigh number in $\varphi_{\text{Ave}} = 0.03$ and $Mn = 8 \times 10^7$. Porosity and porous Rayleigh numbers do not affect on the dimensionless distribution of nanoparticles at the high magnetic field.

Figure 4 illustrates the dimensionless local entropy generation due to heat transfer ($S_{L, HT}$) and fluid friction ($S_{L, FF}$) and summation ($S_L$) inside a semiannulus for $\varphi_{\text{Ave}} = 0.03$, $\varepsilon = 0.7$, and $Mn = 0$ and $8 \times 10^7$ for different porous Rayleigh number. In Figure 4(a) in the absence of a magnetic field and $Ra_p = 10$, the $S_{L, HT}$ contours are similar to temperature contours. The highest value of $S_{L, HT}$ is on the inner cylinder wall (hot) due to extreme temperature gradients. In Figure 4(b), as increasing porous Rayleigh number to 1000, the shape of the contours $S_{L, HT}$ changes completely, in such a way that the densities of the contours increase near the inner cylinder wall and a core is created near the outer cylinder wall. By adding a magnetic field, the density of $S_{L, HT}$ contours increase near the wires, and, with the rise of porous Rayleigh number, there is no change in the shape of the contours. In all cases, due to gradients of high velocity on the walls and near the wires, the maximum value of $S_{L, FF}$ are in these regions. By comparing the values of $S_{L, HT}$ and $S_{L, FF}$, it can be seen that the effects of entropy generation due to heat transfer are much greater than the entropy generation due to fluid friction. As a result, the contours $S_{L, s}$ are very similar to $S_{L, HT}$ contours.

Figure 5 presents total entropy generation due to the heat transfer ($S_{T, HT}$) and fluid friction ($S_{T, FF}$) for the $\varepsilon = 0.7$ and different volume fractions of nanoparticle in porous Rayleigh numbers 10 and 1000. According to Figure 5(a), by intensifying the magnetic field for both porous Rayleigh numbers, $S_{T, HT}$ decreases linearly for the volume fraction of nanoparticles 0.01 and 0.03. In Figure 5(b), by intensifying the magnetic field at both porous Rayleigh numbers 10 and 1000, $S_{T, FF}$ decreases linearly and nonlinearly for volume fractions 0.01 and 0.03, respectively. Also, in all cases and the absence of a magnetic field, the value of $S_{T, FF}$ is almost zero. Figure 6(a) shows the dimensionless summation total entropy generation ($S_{T, s}$) at $\varepsilon = 0.7$ and different volume fraction of nanoparticles. According to the charts, the charts of $S_{T, s}$ are similar to $S_{T, HT}$ charts, because of the
values of $S_{T,HT}$ dominates the values of $S_{T,FF}$. Also, according to Figure 6(b), with the increasing magnetic number for both porous Rayleigh numbers, Bejan number decreases linearly and nonlinearly for volume fractions 0.01 and 0.03, respectively.

6. Conclusion

In present work, effects of porous Rayleigh number, the volume fraction of nanoparticles, porosity, and magnetic number are investigated on nondimensional distribution of nanoparticles and entropy generation. The main findings can be condensed following point:

(i) Adding a magnetic field increases the distribution of nanoparticles near the wires and causes the formation of vortices and increasing the flow velocity

(ii) In the presence of the nonuniform magnetic field, with the increasing porosity and porous Rayleigh number, the distribution of nanoparticles becomes uniform. But, in the absence of a magnetic field, porosity and porous Rayleigh number do not affect the dimensionless distribution of nanoparticles

(iii) By increasing magnetic number and volume fraction of nanoparticles, Bejan number, entropy generation due to heat transfer, and summation decrease but entropy generation due to fluid friction increases

(iv) Entropy generation due to heat transfer is much greater than the entropy generation due to fluid friction

(v) With increasing the magnetic number, entropy generation due to fluid friction increases near the wires

Data Availability

No data were used to support this study.

Conflicts of Interest

There is no conflict of interest regarding the publication of this article.

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