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Retraction

Retracted: The Mathematical Analysis of the New Fractional Order Ebola Model

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

1. Discrepancies in scope
2. Discrepancies in the description of the research reported
3. Discrepancies between the availability of data and the research described
4. Inappropriate citations
5. Incoherent, meaningless and/or irrelevant content included in the article
6. Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] F. M. Khan, A. Ali, E. Bonyah, and Z. U. Khan, "The Mathematical Analysis of the New Fractional Order Ebola Model," *Journal of Nanomaterials*, vol. 2022, Article ID 4912859, 12 pages, 2022.

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Research Article

The Mathematical Analysis of the New Fractional Order Ebola Model

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This research study focuses on the analytical behavior and numerical computation of the fractional order Ebola model. In this study we have calculated the conditions for the existence, uniqueness, and stability of the solution with the help of the fixed point results. In addition to this, we calculated the numerical solution of the fractional order smoke model with the help two-step fractional Adam's Bashforth method using the Caputo's fractional derivative of order μ. Furthermore, the results obtained for different orders of the fractional derivative μ have been shown graphically with the help of Matlab.

1. Introduction

The concept of fractional calculus (FC) was raised in seventeen century from famous correspondence between Leibniz and L'Hoptial. In the consequences of aforementioned correspondence, Leibniz wrote a letter to Guillaume de L'Hôpital that what will be the half order derivative of dependent variable y w.r.t x, i.e, $d^{1/2}x/dy^{1/2}$. In the response, he wrote that this will bear some useful consequence in near future. Later on, it was traced that fractional calculus was introduced by Abel in one of his papers, where the author discussed the idea of fractional-order derivatives (FOD), fractional-order integration (FOI), and the mutual inverse relationship between them [1]. In 1832, one of the greatest French mathematicians (of his era) Liouville presented the definitions for the fractional derivative and fractional integration named as Riemann-Liouville fractional derivative and integration [2]. Later, on in 1890, Heaviside practically used the fractional differential operator in electrical transmission line analysis circa [3]. Recently, the researchers of the 19th and 20th century have made their significant contributions to introduce new definitions of fractional differential and integral operators and in the study of the practical applications of FC [4].

In modern era, the uses of FC in various engineering problems have been raised [5–7] 2014). For instance, FC has various applications in different diffusion phenomenon including heat transfer, gaseous exchange, and water transfer through permeable materials [8–11]. Bagley and Torvik presented FC as an instrument for displaying tissue viscoelasticity during the 1980s (Uchaikin, 2013). Study of intricacy gives another view to a few genuine wonders which appeared to be odd, and during the most recent years, new strategies have been utilized to separate secret properties of complex frameworks [12]. Further, a variety of FC tools have been widely used in several complex phenomena [13–15]. In some circumstances, FC has been perceived for taking care of issues in viscoelasticity, electrochemistry, and dispersion [16–19]. A few analysts featured FC as a tool for examination of complex phenomenon by bringing the techniques of FC and its applications to a more extensive crowd [20, 21] .

The technique by which a real world problem is described in mathematical concepts or language is known as mathematical modeling [22]. Mathematical modeling of infectious diseases has been the main focus for the scientists and researchers over the last two decades. Mathematical modelers used to model the infectious diseases in the form of mathematical models consist of classical differential equations (CDEs). Recently, the researchers have diverted their focus to model the diseases in the form of fractional differential equations (FDEs) which has the potential to describe the real world phenomena more accurate and considered reliable as compared to the conventional derivatives. FDEs are global in nature, more realistic, and give great degree of freedom to modelers as compared to the CDEs. Modeling via FDEs has produced highly influential results in the investigation of transmission of the infectious diseases models [23, 24].

In the year 1976, a flare-up occurred in African nation of the Democratic Republic of Congo (DRC), which was then termed after the name of the lake "Ebola" flows near to the DRC. The infection has five sorts, four out of these five spread illnesses in people. The infection use to attack on the immune system which then cause internal bleeding and affect each organ of the individuals. This terrifying infection spread by contacting directly with the tainted individuals either via body fluids or direct skin contact. The infection can also be pass through connection with the creatures like monkeys, etc. Nonetheless, the infection cannot be transmit through air and food. Later on, in 2013, the infection arose in Guckduo and Guinea, where 28,616 casualties were reported, and out of these casualties, 11,310 lost their lives. Today, where the advanced world is confronting another pandemic flare-up as COVID-19, the investigation of such irresistible sicknesses is still a center of focus for the researchers [25].

2. Model Formulation

In this section of the article, we have presented the formulation of the model, which we will be studying in this paper. For this, we have considered a population and divided it into five different compartments with some assumptions. The assumptions considered for the formulation of the model are stated below

(i) $\mathbb{S}$: the first class of the model has been named as susceptible class. This class contains individuals who have no symptoms or any infection of the disease but can be attacked by the virus

(ii) $\mathbb{E}$: the second class of the population has been named as exposed class. This class contains individuals who have been attacked by the virus but not yet shown the symptoms of the infection or not yet infectious

(iii) $\mathbb{I}$: this is the third class of the population containing individuals who have been attacked by the virus and are being able to transfer the disease to not yet attacked individuals of the populations

(iv) $\mathbb{V}$: this class has been named as vaccinated class containing those individuals of the susceptible class who have been vaccinated against the virus

(v) $\mathbb{R}$: this class is the recovered class which contains those individuals who have survived the disease

The transition or transfer among the compartments has been considered in the following manner

(i) $\mathbb{S} \xrightarrow{\tau} \mathbb{E}$: an individual of the population $\mathbb{S}$ move to the population $\mathbb{E}$ through the rate τ_1

(ii) $\mathbb{S} \xrightarrow{\beta} \mathbb{I}$: an individual of the class $\mathbb{S}$ joins the class $\mathbb{I}$ with the rate β after getting infectious

(iii) $\mathbb{S} \xrightarrow{\psi} \mathbb{V}$: the given parameter is used for the rate of the vaccination which transfer an individual from $\mathbb{S}$ to $\mathbb{V}$

(iv) $\mathbb{I} \xrightarrow{\xi} \mathbb{R}$: the rate of transfer of the individuals from $\mathbb{I}$ to $\mathbb{R}$ after surviving the disease

$$\begin{aligned} \frac{d\mathbb{S}}{dt} &= \Lambda - d_0\mathbb{S}(t) - \tau_1\mathbb{S}(t)\mathbb{E}(t) - \beta\mathbb{S}(t)\mathbb{I}(t) - \psi\mathbb{S}(t) \\ \frac{d\mathbb{E}}{dt} &= \tau_1\mathbb{S}(t)\mathbb{E}(t) - (d_0 + d_1 + \kappa)\mathbb{E}(t) \\ \frac{d\mathbb{I}}{dt} &= \beta\mathbb{S}(t)\mathbb{I}(t) + \kappa\mathbb{E}(t) - \xi\mathbb{I}(t) - (d_0 + d_2)\mathbb{I}(t) \\ \frac{d\mathbb{V}}{dt} &= \psi\mathbb{S}(t) - d_0\mathbb{V}(t) \\ \frac{d\mathbb{R}}{dt} &= \xi\mathbb{I}(t) - d_0\mathbb{R}(t) \end{aligned} \tag{1}$$

And the corresponding fractional form of the system (1) is

$$\begin{aligned} {}^cD^\mu\mathbb{S}(t) &= \Lambda - d_0\mathbb{S}(t) - \tau\mathbb{S}(t)\mathbb{E}(t) - \beta\mathbb{S}(t)\mathbb{I}(t) - \psi\mathbb{S}(t) \\ {}^cD^\mu\mathbb{E}(t) &= \tau\mathbb{S}(t)\mathbb{E}(t) - (d_0 + d_1 + \kappa)\mathbb{E}(t) \\ {}^cD^\mu\mathbb{I}(t) &= \beta\mathbb{S}(t)\mathbb{I}(t) + \kappa\mathbb{E}(t) - \xi(t) - (d_0 + d_2)\mathbb{I}(t) \\ {}^cD^\mu\mathbb{V}(t) &= \psi\mathbb{S}(t) - d_0\mathbb{V}(t) \\ {}^cD^\mu\mathbb{R}(t) &= \xi(t) - d_0\mathbb{R}(t) \end{aligned} \tag{2}$$

The paper has been organized as follows: the first section of the paper contains introduction. The second section has been restricted to the formulation of the model, while the third section has been devoted to the preliminaries. The fourth section of the paper contains the existence and uniqueness of the solution of the model. The fifth section of the paper includes the stability of the solution, while the sixth section contains qualitative study where we formulated the disease free, disease endemic, and basic reproduction number R_0 and then test the stability of the R_0 locally with the help of theorems. In the seventh section, we have formulated the numerical solution of the model via Adam's

Bashforth scheme, and the eighth section contains the numerical simulation of the results obtained in the section seventh. At last, we have concluded our work in the conclusion section.

3. Preliminaries

In this section of the present article, we provide some basic definitions, theorems, and results that will be used and fruitful in understanding the rest of the article.

Definition 1. (see [26]). The Caputo's fractional differential operator of any arbitrary order $\mu > 0$ is defined as

$$ {}^{c}D^{\mu}\theta(t) = \frac{1}{\Gamma(n-\mu)}\int_0^t f(s,\theta(s))^n (t-s)^{n-\mu-1} ds. \quad (3)$$

Lemma 2. (see [27]). *"The following result holds for fractional differential equations*

$$I^{\mu}[{}^{c}D^{\mu}\theta(t)] = \theta(t) + \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \cdots + \alpha_{m-1}t^{m-1}, \quad (4)$$

for arbitrary $\alpha_i \in R$, $i = 0, 1, 2, 3, \cdots, m-1$, where $m = [\mu] + 1$ and $[\mu]$ symbolizes the integer part of μ".

Lemma 3. (see [28]). *Let $\theta \in AC^n[0, T], \mu > 0,$ and $n = [\mu]$, then the following result holds*

$$\mathbb{I}^{\mu}[{}^{c}D^{\mu}\theta(\vartheta)] = \theta(\vartheta) - \sum_{j=0}^{n-1}\frac{D^j\theta(a)}{j!}(t-a)^j. \quad (5)$$

Lemma 4. (see [28]). *In view of Lemma 3, the solution of $\mathbb{D}^{\mu}\theta(t) = y(t)$, $n-1 < \mu < n$ is given by*

$$\theta(t) = \mathbb{I}^{\mu}y(t) + c_0 + c_1(t) + c_2t^2 + + c_{n-1}t^{n-1}, \quad (6)$$

where $c_j \in R$.

Definition 5. (see [26]). Suppose we have Caputo's fractional differential equation of order μ

$${}^{c}D^{\mu}\theta(t) = f(t, \theta(t)), \quad (7)$$

then the solution is given as

$$\theta(t_{n+}) = \theta(t_n) + \frac{f(t_n,\theta_n)}{h\Gamma(\mu)}\left\{\frac{2h}{\mu}t_{n+1}^{\mu} - \frac{t_{n+1}^{\mu+1}}{\mu+1} + \frac{h}{\mu}t_n^{\mu} - \frac{t^{\mu+1}}{\mu}\right\} \cdot \frac{f(t_{n-1},\theta_{n-1})}{h\Gamma(\mu}\left\{\frac{h}{\mu}t_{n+1}^{\mu} - \frac{t_{n+1}^{\mu+1}}{\mu+1} + \frac{t_n^{\mu}}{\mu+1}\right\} + R_n^{\mu}(t), \quad (8)$$

where $R_n^{\mu}(t)$ represent the remainder term. For the study of convergence and uniqueness of the solution of the scheme, we refer to ([26]).

Theorem 6. (see [29]). *"Let X be a Banach space and $\mathfrak{P} : X \longrightarrow X$ is compact and continuous, if the set,*

$$E = \{\theta \in X : \theta = m\mathfrak{P}\theta, m \in (0, 1)\}, \quad (9)$$

is bounded, then $\mathfrak{P}$ has a unique fixed point."

4. Existence of the Solution

In this section of the paper, we construct the conditions for the existence and uniqueness of the solution, and to get the desired results, we construct the following function.

$$\begin{cases} \vartheta_1(t,\mathbb{S},\mathbb{E},\mathbb{I},\mathbb{V},\mathbb{R}) = \Lambda - d_0\mathbb{S}(t) - \tau\mathbb{S}(t)\mathbb{E}(t) - \beta\mathbb{S}(t)\mathbb{I}(t) - \psi\mathbb{S}(t), \\ \vartheta_2(t,\mathbb{S},\mathbb{E},\mathbb{I},\mathbb{V},\mathbb{R}) = \tau\mathbb{S}(t)\mathbb{E}(t) - (d_0 + d_1 + \kappa)\mathbb{E}(t), \\ \vartheta_3(t,\mathbb{S},\mathbb{E},\mathbb{I},\mathbb{V},\mathbb{R}) = \beta\mathbb{S}(t)\mathbb{I}(t) + \kappa\mathbb{E}(t) - \xi\mathbb{I}(t) - (d_0 + d_2)\mathbb{I}(t), \\ \vartheta_4(t,\mathbb{S},\mathbb{E},\mathbb{I},\mathbb{V},\mathbb{R}) = \psi\mathbb{S}(t) - d_0\mathbb{V}(t), \\ \vartheta_5(t,\mathbb{S},\mathbb{E},\mathbb{I},\mathbb{V},\mathbb{R}) = \xi\mathbb{I}(t) - d_0\mathbb{R}(t). \end{cases} \quad (10)$$

Suppose that the considered space $\mathbb{C}[0, T] = \mathbb{B}$ be a Banach space with norm

$$\|\theta(t)\| = \sup_{t\in[0,T]}[|\mathbb{S}(t)| + |\mathbb{E}(t)| + |\mathbb{I}(t)| + |\mathbb{V}(t)| + |\mathbb{R}(t)|], \quad (11)$$

where

$$\theta(t) = \begin{cases} \mathbb{S}(t) \\ \mathbb{E}(t) \\ \mathbb{I}(t) \\ \mathbb{V}(t) \\ \mathbb{R}(t) \end{cases}, \theta_0(t) = \begin{cases} \mathbb{S}^0 \\ \mathbb{E}^0 \\ \mathbb{I}^0 \\ \mathbb{V}^0 \\ \mathbb{R}^0 \end{cases}, \mathfrak{T}(t,\theta(t)) = \begin{cases} \vartheta_1(t,\mathbb{S},\mathbb{E},\mathbb{I},\mathbb{V},\mathbb{R}) \\ \vartheta_2(t,\mathbb{S},\mathbb{E},\mathbb{I},\mathbb{V},\mathbb{R}) \\ \vartheta_3(t,\mathbb{S},\mathbb{E},\mathbb{I},\mathbb{V},\mathbb{R}) \\ \vartheta_4(t,\mathbb{S},\mathbb{E},\mathbb{I},\mathbb{V},\mathbb{R}) \\ \vartheta_5(t,\mathbb{S},\mathbb{E},\mathbb{I},\mathbb{V},\mathbb{R}) \end{cases}. \quad (12)$$

With the help of (12), the system (1) can be written in as

$$\begin{aligned} {}^{c}D^{\mu}\theta(t) &= \mathfrak{T}(t,\theta(t)), t \in [0, T], \\ \theta(0) &= \theta_0, \end{aligned} \quad (13)$$

By Lemma 2, equation (13) converts into the following form

$$\theta(t) = \theta_0 + \int_0^t \frac{(t-s)^{\mu-1}}{\Gamma(\mu)}\mathfrak{T}(s,\theta(s))ds, t \in J = [0, T]. \quad (14)$$

To prove the existence of the solution, we make the following assumptions:

$(P1)\exists$ constants $K_1^*, M_1^* \ni$

$$|\mathfrak{T}(t,\theta(t))| \le K_1^*|\theta|^q + M_1^*. \quad (15)$$

$(P2) \exists L_* > 0, \ni$ for each $\theta, \bar{\theta}$

$$\left|\mathfrak{T}(t,\theta) - \mathfrak{T}\left(t,\bar{\theta}\right)\right| \leq L_* \left\|\theta - \bar{\theta}\right\|. \tag{16}$$

And let $\mathfrak{P} : \mathbb{B} \longrightarrow \mathbb{B}$ be an operator as

$$\mathfrak{P}\theta(t) = \theta_0 + \int_0^t \frac{(t-s)^{\mu-1}}{\Gamma(\mu)} \mathfrak{T}(s, \theta(s))ds. \tag{17}$$

Theorem 7. *When the assumptions (P1) and (P2) are true, it verifies that the problem (13) has at least of one fixed point which also implies that the problem of our study has also at least one solution.*

Proof. Furthermore we proceed as. □

Step 1. First, we have to show that $\mathfrak{P}$ is continuous. To acquire the results, we suppose that $\mathfrak{T}_j$ is continuous for $j = 1, 2, 3, 4, 5, 6$. Which implies that $\mathfrak{T}(s, \theta(s))$ is also continuous. Assume $\theta_n . \theta \in X \ni \theta_n \longrightarrow \theta$, we must have $\mathfrak{P}\theta_n \longrightarrow \mathfrak{P}\theta$.

For this, we consider

$$\begin{aligned}\|\mathfrak{P}\theta_n - \mathfrak{P}\theta\| &= \max_{t \in J=[0,T]} \left| \int_0^t \frac{(t-s)^{\mu-1}}{\Gamma(\mu)} \mathfrak{T}_n(s, \theta_n(s))ds \right. \\ &\quad \left. - \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} \mathfrak{T}(s, \theta(s))ds \right| \\ &\leq \max_{t \in J=[0,T]} \int_0^t \left| \frac{(t-s)^{\mu-1}}{\Gamma(\mu)} \right| |\mathfrak{T}_n(s, \theta_n(s)) - \mathfrak{T}(s, \theta(s))| \\ &\quad \cdot ds, \leq \frac{T^t}{\Gamma(\mu+1)} \|\mathfrak{T}_n - \mathfrak{T}\| \longrightarrow 0 \; as \; n \longrightarrow \infty.\end{aligned} \tag{18}$$

As $\mathfrak{T}$ is continuous, therefore $\mathfrak{P}\theta_n \longrightarrow \mathfrak{P}\theta$, yields that $\mathfrak{P}$ is continuous.

Step 2. Now, to prove that $\mathfrak{P}$ is bounded for any $\theta \in \mathbb{X}$, we make of the supposition that $\mathfrak{P}$ satisfies the growth condition:

$$\begin{aligned}\|\mathfrak{P}\theta\| &= \max_{t \in [0,T]} \left| \theta_0 + \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} \mathfrak{T}(s, \theta(s))ds \right|, \\ &\leq |\theta_0| + \max_{t \in [0,T]} \frac{1}{\Gamma(\mu)} \int_0^t |(t-s)^{\mu-1}| |\mathfrak{T}(s, \theta(s))| ds, \\ &\leq |\theta_0| + \frac{T^\mu}{\Gamma(\mu+1)} [K_1^* \|\theta\|^q + M_1^*].\end{aligned} \tag{19}$$

Here, we assume a $\mathcal{S}$, the subset of $\mathbb{X}$ with the property of boundedness, and we need to prove that $\mathfrak{P}(\mathcal{S})$ is also bounded. To reach our destination, we assume that for any $\theta \in \mathcal{S}$, now as $\mathcal{S}$ is bounded, so $\exists K_q \geq 0 \ni$

$$\|\theta\| \leq K_q, \forall \theta \in \mathbb{S}. \tag{20}$$

Further, for any $\theta \in \mathcal{S}$ by using the growth condition, we have

$$\begin{aligned}\|\mathfrak{P}\theta\| &\leq |\theta_0| + \frac{T^\mu}{\Gamma(\mu+1)} [K_1^* \|\theta^q\| + M_1^*] \\ &\leq |\theta_0| + \frac{T^\mu}{\Gamma(\mu+1)} \left[K_1^* K_q + M_1^*\right].\end{aligned} \tag{21}$$

Therefore, $\mathfrak{P}(\mathcal{S})$ is bounded.

Step 3. Here, we attempt to prove that the operator we defined is equicontinuous, for this we assume that $t_2 \leq t_1 \in J = [0, T]$, then

$$\begin{aligned}|\mathfrak{P}\theta(t_1) - \mathfrak{P}\theta(t_2)| &= \left| \frac{1}{\Gamma(\mu)} \int_0^t (t_1 - s)^{\mu-1} \mathfrak{T}(s, \theta(s))ds \right. \\ &\quad \left. - \frac{1}{\Gamma(\mu)} \int_0^t (t_2 - s)^{\mu-1} \mathfrak{T}(s, \theta(s))ds \right|, \\ &\leq \left| \frac{1}{\Gamma(\mu)} \int_0^t (t_1 - s)^{\mu-1} - \frac{1}{\Gamma(\mu)} \int_0^t (t_2 - s)^{\mu-1} \right| \\ &\quad \cdot |\mathfrak{T}(s, \theta(s))| ds, \\ &\leq \frac{T^\mu}{\Gamma(\mu+1)} [K_1^* \|\theta\|^q + M_1^*][t_1 - t_2].\end{aligned} \tag{22}$$

By taking advantage of Arzelà-Ascoli theorem, we can say that $\mathfrak{P}(\mathcal{S})$ is relative compact.

Step 4. In this step, we need to prove that the set defined below is bounded

$$E = \{\theta \in X : \theta = m\mathfrak{P}\theta, \quad m \in (0, 1)\}. \tag{23}$$

To prove this, we suppose that $\theta \in E, \ni$ for each $t \in J$, where $J = [0, T]$ we have

$$\|\theta\| = m\|\mathfrak{P}\theta\| \leq m\left[|\theta_0| + \frac{T^\mu}{\Gamma(\mu+1)} []K_1^* \|\theta\|^q + M_1^*\right]. \tag{24}$$

From here, we can claim that the set defined above is bounded. By using Schaefer's FPT, the operator we defined, i.e., $\mathfrak{P}$ has atleast one fixed point, and hence, the model we studied in this paper has at least one solution.

Theorem 8. *The problem (13) is unique solution, if* $T^\mu K_1^* / \Gamma(\mu + 1) < 1$.

Proof. Let $\theta, \overline{\theta} \in X$, then

$$\left\|\mathfrak{P}\theta - \mathfrak{P}\overline{\theta}\right\| \leq \max_{t\in J=[0,T]} \int_0^t \left|\frac{(t-s)^{\mu-1}}{\Gamma(\mu)}\right| \left|\mathfrak{T}(s,\theta(s)) - \mathfrak{T}\left(s,\overline{\theta}(s)\right)\right| ds, \leq \frac{T^\mu L_{\mathfrak{T}}}{\Gamma(\mu+1)} \left\|\theta - \overline{\theta}\right\|. \tag{25}$$

□

Hence, we can say that the fixed point is unique, and therefore, our solution is unique.

5. Stability Results

To prove that the solution of the considered model is stable, we use the concept of Ulam and Ulam Hyer stability. To get the desired results we proceed as

Assume $\mathscr{L} : \mathbb{X} \longrightarrow X$ be an operator which satisfy

$$\theta = \mathscr{L}(\theta), \text{ where } \theta \in X. \tag{26}$$

Definition 9. (see [27]). Equation (26) has UH-stability, if for $\varsigma_1 > 0$ and assume any solution $\theta \in \mathscr{L}$ for the inequality given by

$$\|\theta - \mathscr{L}\theta\| \leq \varsigma_1, \forall t \in J = [0, T], \tag{27}$$

and the unique solution $\overline{\theta}$ for (25) with $C_q^* > 0$, such that

$$\left\|\overline{\theta} - \theta\right\| \leq C_q^* \varsigma_1, \forall t \in [0, T]. \tag{28}$$

Definition 5.2. "If $\exists \vartheta \in C(R, R)$ with $\vartheta(0) = 0$," for unique result $\overline{\theta}$ and any solution of equation (26) $\ni$

$$\left\|\overline{\theta} - \theta\right\| \leq \vartheta(\varsigma_1), \tag{29}$$

then equation (26) has GUH-stability.

Remark 10. (see [27]). "If $\ni \zeta_1^{**}(t) \in C([0, T], R)$, then $\overline{\theta} \in X$ satisfies (27) if

(i) $|\zeta_1^{**}(t)| \leq \varsigma_1, \forall t \in [0, T]$

(ii) $\mathscr{L}\overline{\theta}(t) = \overline{\theta} + \zeta_1^{**}(t), \forall t \in [0, T]$"

For further analysis, we suppose that the following is the solution of the perturbed problem of (13)

$$\begin{cases} {}^C D_{+0}^\mu \theta(t) = \mathfrak{T}(t, \theta(t)) + \zeta_1^{**}(t), \\ \theta(0) = \theta_0. \end{cases} \tag{30}$$

Lemma 11. *The result stated below holds true for equation (30),*

$$|\theta(t) - \mathfrak{P}\theta(t)| \leq a\varsigma_1, \quad \text{where} \quad a = \frac{T^\mu}{\Gamma(\mu+1)} \tag{31}$$

Theorem 12. *By making use of Lemma 11, the solution of the problem (13) is UH-stable as well as GUH-stable, if $T^\mu L_\omega/\Gamma(\mu+1) < 1$.*

Proof. Assume $\theta, \overline{\theta} \in X$ be any and unique solutions, respectively, problem (13), then

$$\begin{aligned} \left|\theta(t) - \overline{\theta}(t)\right| = \left|\theta(t) - \mathfrak{P}\overline{\theta}(t)\right|, &\leq |\theta(t) - \mathfrak{P}\theta(t)| \\ &+ \left|\mathfrak{P}\theta(t) + \mathfrak{P}\overline{\theta}(t)\right|, \leq a\varsigma_1 \\ &+ \frac{T^\mu L_\theta}{\Gamma(\mu+1)} \left|\theta(t) - \overline{\theta}(t)\right|, \\ &\leq \frac{a\varsigma_1}{1 - T^\mu L_\vartheta/\Gamma(\mu+1)}. \end{aligned} \tag{32}$$

From here, we claim that the solution of (13) is UH and GUH stability if

$$Y(\varsigma_1) = \frac{a\varsigma_1}{1 - T^\mu L_\vartheta/\Gamma(\mu+1)}. \tag{33}$$

Such that $Y(0) = 0$. □

Definition 13. The UHR-Stability of (26) is ensured for $g^* \in C([0, T], R)$, if for $\varsigma_1 > 0$ and assume $\theta \in X$ be any solution of the inequality expressed by

$$\|\theta - H\theta\| \leq g(t)\varsigma_1. \tag{34}$$

$\exists$ a unique solution $\overline{\theta}$ of (26) with $\mathscr{K}'_q > 0 \ni$

$$\left\|\overline{\theta} - \theta\right\| \leq \mathscr{K}'_q g^*(t)\varsigma_1, \quad \forall \quad t \in [0, T]. \tag{35}$$

Definition 14. (see [27]). "For $g^* \in C[0, T], R]$, if $\exists \mathscr{K}'_{q,g}$ and for $\varsigma_1 > 0$, consider that θ be any solution of (34) and $\overline{\theta}$ be any solution of (26) $\ni$

$$\left\|\overline{\theta} - \theta\right\| \leq \mathscr{K}'_{q,g} g^*(t), \forall \quad t \in J = [0, T], \tag{36}$$

then equation (26) is generalized UHR stable."

Remark 15. If $\exists \zeta_1^{**}(t) \in C(J, R)$, then for $\overline{\theta} \in X$ (27) holds, if

(i) $|\zeta_1^{**}(t)| \leq \varsigma_1 \omega(t), \forall t \in J$

(ii) $\mathscr{L}\overline{\theta}(t) = \overline{\theta} + \zeta_1^{**}(t), \forall t \in J$

Lemma 16. *The stated result below holds true for (30)*

$$|\theta(t) - \mathfrak{P}\theta(t)| \leq a\omega(t)\varsigma_1, \quad a = \frac{T^\mu}{\Gamma(\mu+1)} \tag{37}$$

Proof. The proof has been left for the readers. □

Theorem 17. *With the help of Lemma 16, our solution is Ulam and Generalized Ulam stable if $T^{\mu}L_{\theta}/\Gamma\mu + 1 < 1$.*

Proof. . Assume $\theta, \overline{\theta} \in X$ be two solutions such that θ is any and any and $\overline{\theta}$ is the unique solution of our problem, then

$$\begin{aligned} \left|\theta(t) - \overline{\theta}(t)\right| &= \left|\theta(t) - \mathfrak{P}\overline{\theta}(t)\right|, \leq |\theta(t) - \mathfrak{P}\theta(t)| \\ &+ \left|\mathfrak{P}\theta(t) + \mathfrak{P}\overline{\theta}(t)\right|, \leq a\omega(t)\varsigma_1 + \frac{T^{\mu}L_{\theta}}{\Gamma(\mu+1)} \\ &\cdot \left|\theta(t) - \overline{\theta}(t)\right|, \leq \frac{a\omega(t)\varsigma_1}{1 - T^{\mu}L_{\vartheta}/\Gamma(\mu+1)}. \end{aligned} \tag{38}$$

□

Hence, the solution possesses both type of the stabilities.

6. Qualitative Study

In this section of the article, we present disease free equilibrium, disease endemic equilibrium, the basic reproduction number R_0, and the local assymptotical stability of the R_0. To proceed, we first find the disease free equilibrium and disease endemic equilibrium of the model. The disease free equilibrium is given as $S^0 = (\Lambda/d_0 + \psi, 0, 0, 0, 0)$, while the endemic equilibrium is given below.

6.1. Endemic Equilibrium. The endemic equilibrium of the model is given as

$$\begin{aligned} S^* &= \frac{d_0 + d_1 + \kappa}{\tau}, \\ E^* &= \frac{1}{\tau\kappa}(\tau(\xi + d_0 + d_2) - \beta(d_0 + d_1 + \kappa))I^*, \\ I^* &= \left(\frac{\Lambda\tau}{d_0 + d_1 - \kappa} - (d_0 + \psi)\right)\left(\frac{\kappa}{\tau(\xi + d_0 + d_2) - \beta(d_0 + d_1 + \kappa) - \kappa\beta}\right), \\ V^* &= \frac{\psi}{d_0}\left(\frac{d_0 + d_1 - \kappa}{\tau}\right), \\ R^* &= \frac{\xi}{d_0}R^*. \end{aligned} \tag{39}$$

6.2. The Basic Reproduction Number. To find R_0, we construct two vectors such as

$$\mathscr{F} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \tau\mathbb{S}(t)\mathbb{E}(t) \\ \beta\mathbb{S}(t)\mathbb{I}(t) \end{bmatrix}, \tag{40}$$

$$\mathscr{V} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} (d_0 + d_1 + \kappa)\mathbb{E}(t) \\ (\xi + d_0 + d_2)\mathbb{I}(t) - \kappa\mathbb{E}(t) \end{bmatrix}. \tag{41}$$

Now, we present the Jacobian of both the matrix, i.e.,

$$J_{\mathscr{F}} = \begin{bmatrix} \tau\mathbb{S}(t) & 0 \\ 0 & \beta\mathbb{S}(t) \end{bmatrix}, \tag{42}$$

$$J_{\mathscr{V}} = \begin{bmatrix} d_0 + d_1 + \kappa & 0 \\ -\kappa & \xi + d_0 + d_2 \end{bmatrix}. \tag{43}$$

with

$$(J_{\mathscr{V}})^{-1} = \begin{bmatrix} \dfrac{1}{d_0 + d_1 + \kappa} & 0 \\ \dfrac{\kappa}{(\xi + d_0 + d_2)(d_0 + d_1 + \kappa)} & \dfrac{1}{\xi + d_0 + d_2} \end{bmatrix}. \tag{44}$$

Now, to find the next generation matrix (NGM), we find the product of $J_{\mathscr{F}}$ and $(J_{\mathscr{V}})^{-1}$, i.e.,

$$\text{NGM} = \begin{bmatrix} \dfrac{\tau\mathbb{S}(t)}{d_0 + d_1 + \kappa} & 0 \\ \dfrac{\beta\mathbb{S}(t)\kappa}{(\xi + d_0 + d_2)(d_0 + d_1\kappa)} & \dfrac{\beta\mathbb{S}(t)}{\xi + d_0 + d_2} \end{bmatrix}. \tag{45}$$

Clearly, the eigen values are (say) λ_1 and λ_2 which are given as $\lambda_1 = \tau\mathbb{S}(t)/d_0 + d_1 + \kappa$ and $\lambda_2 = \beta\mathbb{S}(t)/\xi + d_0 + d_2$. Therefore, the basic reproduction number $R_0 = \max(\lambda_1, \lambda_2)$.

Theorem 18. *The basic reproduction number R_0 is locally assymptotically stable at the disease free equilibrium point that is stable if $R_0 < 1$.*

Proof. For this purpose, we construct the following Jacobian.

$$A_j = \begin{bmatrix} -(d_0 + \psi) & -\tau\mathbb{S}^0 & -\beta\mathbb{S}^0 & 0 & 0 \\ 0 & \tau\mathbb{S}^0 - (d_0 + d_1 + \kappa) & 0 & 0 & 0 \\ \beta\mathbb{S}^0 & \kappa & -\xi - (d_0 + d_2) & 0 & 0 \\ \psi & 0 & 0 & -d_0 & 0 \\ 0 & 0 & \xi & 0 & -d_0 \end{bmatrix}. \tag{46}$$

Now, let the eigen values are (say) $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$. Clearly $\lambda_1 = \lambda_2 = -d_0$, $\lambda_3 = \tau S^0 - (d_0 + d_1 + \kappa)$, $\lambda_4 = -(d_0 + \psi)$, and $\lambda_5 = -\beta^2\mathbb{S}^2 - (d_0 + \psi)(\xi + d_0 + d_2)$. From λ_3, we have $\tau\mathbb{S}^0/d_0 + d_1 + \kappa < 1$, and from λ_5, we have $d_0 + \psi/\beta\mathbb{S}^0 < \beta\mathbb{S}^0/\xi + d_0 + d - 2 < 1$. Therefore, $R_0 = \max(\lambda_3, \lambda_5) < 1$ and hence is locally assymptotically stable at the disease free equilibrium point. □

Theorem 6.2. *The basic reproduction number R_0 is locally assymptotically stable at the endemic equilibrium point if $R_0 > 1$.*

Proof. The proof of this result can be obtained on the same manner as the proof in [30]. □

7. Numerical Solution

This section of the article is devoted to the numerical solution of the considered model. For this, we will use the well-known two-step fractional order Adam's Bashforth method. The considered model is given as

$$\begin{aligned}
{}^{c}D^{\mu}\mathbb{S}(t) &= \Lambda - d_0\mathbb{S}(t) - \tau\mathbb{S}(t)\mathbb{E}(t) - \beta\mathbb{S}(t)\mathbb{I}(t) - \psi\mathbb{S}(t),\\
{}^{c}D^{\mu}\mathbb{E}(t) &= \tau\mathbb{S}(t)\mathbb{E}(t) - (d_0 + d_1)\mathbb{E}(t),\\
{}^{c}D^{\mu}\mathbb{I}(t) &= \beta\mathbb{S}(t)\mathbb{I}(t) + \mathbb{K}appa\mathbb{E}(t) - \mathbb{X}i\mathbb{I}(t) - (d_0 + d_2)\mathbb{I}(t),\\
{}^{c}D^{\mu}\mathbb{I}(t) &= \psi\mathbb{S}(t) - d_0\mathbb{V}(t),\\
{}^{c}D^{\mu}\mathbb{R}(t) &= \mathbb{X}i\mathbb{I}(t) - d_0\mathbb{R}(t).
\end{aligned} \tag{47}$$

To obtain the desired results, we apply the fundamental theorem of fractional calculus to system (3) gives

$$\begin{aligned}
\mathbb{S}(t) &= \mathbb{S}(0) + \frac{1}{\Gamma(\mu)}\int_0^t \mathscr{A}_1(\beta, \mathbb{S}(\beta))(t-\beta)^{\mu-1}d\beta,\\
\mathbb{E}(t) &= \mathbb{E}(0) + \frac{1}{\Gamma(\mu)}\int_0^t \mathscr{A}_2(\beta, \mathbb{E}(\beta))(t-\beta)^{\mu-1}d\beta,\\
\mathbb{I}(t) &= \mathbb{I}(0) + \frac{1}{\Gamma(\mu)}\int_0^t \mathscr{A}_3(\beta, \mathbb{I}(\beta))(t-\beta)^{\mu-1}d\beta,\\
\mathbb{V}(t) &= \mathbb{V}(0) + \frac{1}{\Gamma(\mu)}\int_0^t \mathscr{A}_4(\beta, \mathbb{V}(\beta))(t-\beta)^{\mu-1}d\beta,\\
\mathbb{R}(t) &= \mathbb{R}(0) + \frac{1}{\Gamma(\mu)}\int_0^t \mathscr{A}_5(\beta, \mathbb{R}(\beta))(t-\beta)^{\mu-1}d\beta.
\end{aligned} \tag{48}$$

The unknown terms $\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_4, \mathscr{A}_5$ are given below. Now, for $t = t_{n+1}$, we get

$$\begin{aligned}
\mathbb{S}(t_{n+1}) &= \mathbb{S}(0) + \frac{1}{\Gamma(\mu)}\int_0^{t_{n+1}} \mathscr{A}_1(t, \mathbb{S}(t))(t_{n+1}-t)^{\mu-1}dt,\\
\mathbb{E}(t_{n+1}) &= \mathbb{E}(0) + \frac{1}{\Gamma(\mu)}\int_0^{t_{n+1}} \mathscr{A}_2(t, \mathbb{E}(t))(t_{n+1}-t)^{\mu-1}dt,\\
\mathbb{I}(t_{n+1}) &= \mathbb{I}(0) + \frac{1}{\Gamma(\mu)}\int_0^{t_{n+1}} \mathscr{A}_3(t, \mathbb{I}(t))(t_{n+1}-t)^{\mu-1}dt,\\
\mathbb{V}(t_{n+1}) &= \mathbb{V}(0) + \frac{1}{\Gamma(\mu)}\int_0^{t_{n+1}} \mathscr{A}_4(t, \mathbb{V}(t))(t_{n+1}-t)^{\mu-1}dt,\\
\mathbb{R}(t_{n+1}) &= \mathbb{R}(0) + \frac{1}{\Gamma(\mu)}\int_0^{t_{n+1}} \mathscr{A}_5(t, \mathbb{R}(t))(t_{n+1}-t)^{\mu-1}dt.
\end{aligned} \tag{49}$$

For $t = t_n$, we get the following

$$\begin{aligned}
\mathbb{S}(t_n) &= \mathbb{S}(0) + \frac{1}{\Gamma(\mu)}\int_0^{t_n} \mathscr{A}_1(t, \mathbb{S}(t))(t_n-t)^{\mu-1}dt,\\
\mathbb{E}(t_n) &= \mathbb{E}(0) + \frac{1}{\Gamma(\mu)}\int_0^{t_n} \mathscr{A}_2(t, \mathbb{E}(t))(t_n-t)^{\mu-1}dt,\\
\mathbb{I}(t_n) &= \mathbb{I}(0) + \frac{1}{\Gamma(\mu)}\int_0^{t_n} \mathscr{A}_3(t, \mathbb{I}(t))(t_n-t)^{\mu-1}dt,\\
\mathbb{V}(t_n) &= \mathbb{V}(0) + \frac{1}{\Gamma(\mu)}\int_0^{t_n} \mathscr{A}_4(t, \mathbb{V}(t))(t_n-t)^{\mu-1}dt,\\
\mathbb{R}(t_n) &= \mathbb{R}(0) + \frac{1}{\Gamma(\mu)}\int_0^{t_n} \mathscr{A}_5(t, \mathbb{R}(t))(t_n-t)^{\mu-1}dt.
\end{aligned} \tag{50}$$

By $\mathbb{S}(t_{n+1}) - \mathbb{S}(t_n)$, $\mathbb{E}(t_{n+1}) - \mathbb{E}(t_n)$, $\mathbb{I}(t_{n+1}) - \mathbb{I}(t_n)$, $\mathbb{V}(t_{n+1}) - \mathbb{V}(t_n)$, and $\mathbb{R}(t_{n+1}) - \mathbb{R}(t_n)$ in (49) and (50), we obtain

$$\begin{aligned}
\mathbb{S}(t_{n+1}) &= \mathbb{S}(t_n) + \mathfrak{A}^1_{\mu,1} + \mathfrak{A}^1_{eta,2},\\
\mathbb{E}(t_{n+1}) &= \mathbb{E}(t_n) + \mathfrak{A}^2_{\mu,1} + \mathfrak{A}^2_{eta,2},\\
\mathbb{I}(t_{n+1}) &= \mathbb{I}(t_n) + \mathfrak{A}^3_{\mu,1} + \mathfrak{A}^3_{eta,2},\\
\mathbb{V}(t_{n+1}) &= \mathbb{V}(t_n) + \mathfrak{A}^4_{\mu,1} + \mathfrak{A}^4_{eta,2},\\
\mathbb{R}(t_{n+1}) &= \mathbb{R}(t_n) + \mathfrak{A}^5_{\mu,1} + \mathfrak{A}^5_{eta,2},
\end{aligned} \tag{51}$$

where

$$\begin{aligned}
\mathfrak{A}^1_{\mu,1} &= \frac{1}{\Gamma(\mu)}\int_0^{t_{n+1}} \mathscr{A}_1(t, \mathbb{S}(t))(t_{n+1}-t)^{\mu-1}dt,\\
A^2_{\mu,1} &= \frac{1}{\Gamma(\mu)}\int_0^{t_{n+1}} \mathscr{A}_2(t, \mathbb{E}(t))(t_{n+1}-t)^{\mu-1}dt,\\
\mathfrak{A}^3_{\mu,1} &= \frac{1}{\Gamma(\mu)}\int_0^{t_{n+1}} \mathscr{A}_3(t, \mathbb{I}(t))(t_{n+1}-t)^{\mu-1}dt,\\
\mathfrak{A}^4_{\mu,1} &= \frac{1}{\Gamma(\mu)}\int_0^{t_{n+1}} \mathscr{A}_4(t, \mathbb{V}(t))(t_{n+1}-t)^{\mu-1}dt,\\
\mathfrak{A}^5_{\mu,1} &= \frac{1}{\Gamma(\mu)}\int_0^{t_{n+1}} \mathscr{A}_5(t, \mathbb{R}(t))(t_{n+1}-t)^{\mu-1}dt.
\end{aligned} \tag{52}$$

$$\begin{aligned}
\mathfrak{A}^1_{\mu,2} &= \frac{1}{\Gamma(\mu)}\int_0^{t_n} \mathscr{A}_1(t, \mathbb{S}(t))(t_n-t)^{\mu-1}dt,\\
\mathfrak{A}^2_{\mu,2} &= \frac{1}{\Gamma(\mu)}\int_0^{t_n} \mathscr{A}_2(t, \mathbb{E}(t))(t_n-t)^{\mu-1}dt,\\
\mathfrak{A}^3_{\mu,2} &= \frac{1}{\Gamma(\mu)}\int_0^{t_n} \mathscr{A}_3(t, \mathbb{I}(t))(t_n-t)^{\mu-1}dt,\\
\mathfrak{A}^4_{\mu,2} &= \frac{1}{\Gamma(\mu)}\int_0^{t_n} \mathscr{A}_4(t, \mathbb{V}(t))(t_n-t)^{\mu-1}dt,\\
\mathfrak{A}^5_{\mu,2} &= \frac{1}{\Gamma(\mu)}\int_0^{t_n} \mathscr{A}_5(t, \mathbb{R}(t))(t_n-t)^{\mu-1}dt.
\end{aligned} \tag{53}$$

Table 1: The physical interpretation and numerical values of the parameters.

Parameters	Physical description	Numerical value	Source
Λ	The birth rate	0.4	Assumed
d_0	Natural death rate	0.7	Assumed
d_1	Disease death rate in $\mathbb{E}(t)$	0.075	Assumed
d_2	Disease death rate in $\mathbb{I}(t)$	0.35	Assumed
τ	The contact rate of $\mathbb{S}(t)$ and $\mathbb{E}(t)$	0.14280	Assumed
κ	The transference rate from $\mathbb{E}(t)$ to $\mathbb{I}(t)$	0.048	Assumed
β	The contact rate of $\mathbb{S}(t)$ and $\mathbb{I}(t)$	0.35	Assumed
ξ	Recovery rate	0.53	Assumed
ψ	Vaccination rate	0.00493	Assumed

By approximating $A^1_{\mu,1}$, $A^1_{\mu,2}$, $A^2_{\mu,1}$, $A^2_{\mu,2}$, $A^3_{\mu,1}$, $A^3_{\mu,2}$, $A^4_{\mu,1}$, $A^4_{\mu,2}$, $A^5_{\mu,1}$, and $A^5_{\mu,2}$ with the help of Lagrange's polynomials and the plugging back in (51), we get the following solution

$$\mathbb{S}(t_{n+1}) = \mathbb{S}(t_n) + \frac{\mathscr{A}_1(t_n, \mathbb{S}(t_n))}{\hbar\Gamma(\mu)}\left\{\frac{2\hbar}{\mu}t^{\mu}_{n+1} - \frac{t^{\mu+1}_{n+1}}{\mu+1} + \frac{\hbar}{\mu}t^{\mu}_{n} - \frac{t^{\mu+1}}{\mu}\right\} + \frac{\mathscr{A}_1(t_{n-1}, \mathbb{S}_{n-1})}{\hbar\Gamma(\mu)}\left\{\frac{\hbar}{\mu}t^{\mu}_{n+1} - \frac{t^{\mu+1}_{n+1}}{\mu+1} + \frac{t^{\mu}_{n}}{\mu+1}\right\} + \mathbb{R}^{\mu}_{1,n}(t),$$

$$\mathbb{E}(t_{n+1}) = \mathbb{E}(t_n) + \frac{\mathscr{A}_2(t_n, \mathbb{E}(t_n))}{\hbar\Gamma(\mu)}\left\{\frac{2\hbar}{\mu}t^{\mu}_{n+1} - \frac{t^{\mu+1}_{n+1}}{\mu+1} + \frac{\hbar}{\mu}t^{\mu}_{n} - \frac{t^{\mu+1}}{\mu}\right\} + \frac{\mathscr{A}_2(t_{n-1}, \mathbb{E}_{n-1})}{\hbar\Gamma(\mu)}\left\{\frac{\hbar}{\mu}t^{\mu}_{n+1} - \frac{t^{\mu+1}_{n+1}}{\mu+1} + \frac{t^{\mu}_{n}}{\mu+1}\right\} + \mathbb{R}^{\mu}_{2,n}(t),$$

$$\mathbb{I}(t_{n+1}) = \mathbb{I}(t_n) + \frac{\mathscr{A}_3(t_n, \mathbb{I}_{\hbar}(t_n))}{\hbar\Gamma(\mu)}\left\{\frac{2\hbar}{\mu}t^{\mu}_{n+1} - \frac{t^{\mu+1}_{n+1}}{\mu+1} + \frac{\hbar}{\mu}t^{\mu}_{n} - \frac{t^{\mu+1}}{\mu}\right\} + \frac{\mathscr{A}_3(t_{n-1}, \mathbb{I}_{n-1})}{\hbar\Gamma(\mu)}\left\{\frac{\hbar}{\mu}t^{\mu}_{n+1} - \frac{t^{\mu+1}_{n+1}}{\mu+1} + \frac{t^{\mu}_{n}}{\mu+1}\right\} + \mathbb{R}^{\mu}_{3,n}(t),$$

$$\mathbb{V}(t_{n+1}) = \mathbb{V}(t_n) + \frac{\mathscr{A}_4(t_n, \mathbb{V}(t_n))}{\hbar\Gamma(\mu)}\left\{\frac{2\hbar}{\mu}t^{\mu}_{n+1} - \frac{t^{\mu+1}_{n+1}}{\mu+1} + \frac{\hbar}{\mu}t^{\mu}_{n} - \frac{t^{\mu+1}}{\mu}\right\} + \frac{\mathscr{A}_4(t_{n-1}, \mathbb{V}_{n-1})}{\hbar\Gamma(\mu)}\left\{\frac{\hbar}{\mu}t^{\mu}_{n+1} - \frac{t^{\mu+1}_{n+1}}{\mu+1} + \frac{t^{\mu}_{n}}{\mu+1}\right\} + \mathbb{R}^{\mu}_{4,n}(t),$$

$$\mathbb{R}(t_{n+1}) = \mathbb{R}(t_n) + \frac{\mathscr{A}_5(t_n, \mathbb{R}(t_n))}{\hbar\Gamma(\mu)}\left\{\frac{2\hbar}{\mu}t^{\mu}_{n+1} - \frac{t^{\mu+1}_{n+1}}{\mu+1} + \frac{\hbar}{\mu}t^{\mu}_{n} - \frac{t^{\mu+1}}{\mu}\right\} + \frac{\mathscr{A}_5(t_{n-1}, \mathbb{S}_{n-1})}{\hbar\Gamma(\mu)}\left\{\frac{\hbar}{\mu}t^{\mu}_{n+1} - \frac{t^{\mu+1}_{n+1}}{\mu+1} + \frac{t^{\mu}_{n}}{\mu+1}\right\} + \mathbb{R}^{\mu}_{5,n}(t), \tag{54}$$

where

$$\begin{aligned}
\mathscr{A}_1 &= \Lambda - d_0\mathbb{S}(t) - \tau\mathbb{S}(t)\mathbb{E}(t) - \beta\mathbb{S}(t)\mathbb{I}(t) - \psi\mathbb{S}(t),\\
\mathscr{A}_2 &= \tau\mathbb{S}(t)\mathbb{E}(t) - (d_0 + d_1 + \kappa)\mathbb{E}(t),\\
\mathscr{A}_3 &= \beta\mathbb{S}(t)\mathbb{I}(t) + \kappa\mathbb{E}(t) - \xi\mathbb{I}(t) - (d_0 + d_2)\mathbb{I}(t),\\
\mathscr{A}_4 &= \psi\mathbb{S}(t) - d_0\mathbb{V}(t),\\
\mathscr{A}_5 &= \xi\mathbb{I}(t) - d_0\mathbb{R}(t).
\end{aligned} \tag{55}$$

And $\mathbb{R}^{\mu}_{1,n}(t)$, $\mathbb{R}^{\mu}_{2,n}(t)$, $\mathbb{R}^{\mu}_{3,n}(t)$, $\mathbb{R}^{\mu}_{4,n}(t)$, and $\mathbb{R}^{\mu}_{5,n}(t)$ are the remainder's terms.

8. Numerical Simulation

In this section of the article, we present the graphical results of the solution obtained in (54). For this purpose we have simulated the results via Matlab by assigning the values given in (Table 1) to the parameters and classes of the model. The graphical results are shown in the following.

9. Discussion

Figure 1 describes the dynamics of susceptible population for different values of the order of fractional derivatives. Each curve tends to the equilibrium solution irrespective of the value of μ. As we increase the value of μ, the rate of convergence to the stated equilibrium increases. Figure 2 represents the behavior of E(t) along the time direction, and the figure shows that for describing the slow evolution of disease, one might assume small values of μ. Infection from the community could be rapidly eliminated by increasing the order of the derivative as shown in Figure 3. A similar conclusion could be drawn from Figures 4 and 5, i.e., to capture the realistic scenario of slowly spreading diseases, one must consider the tools of fractional order derivative while modeling such epidemics.

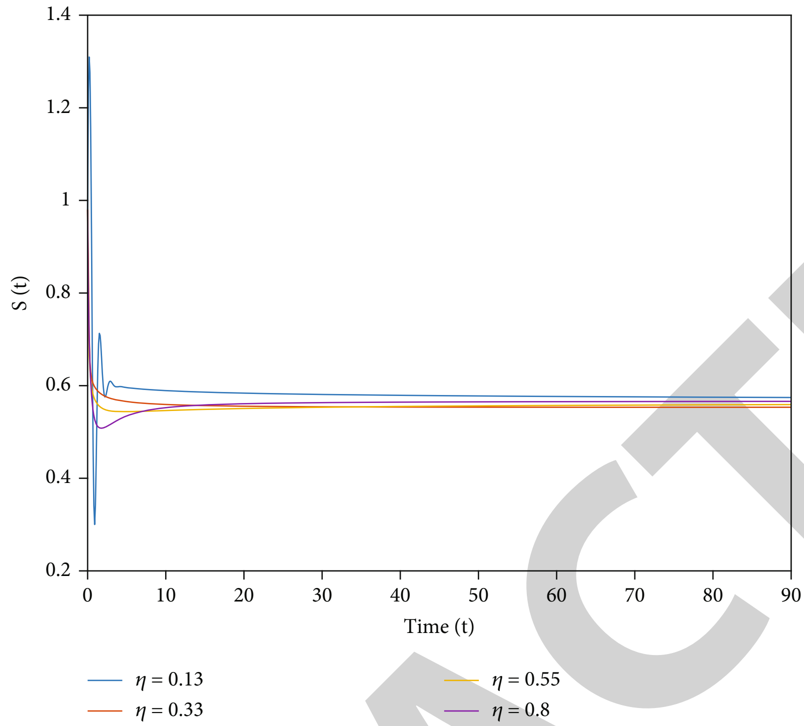

FIGURE 1: The behavior of susceptible population.

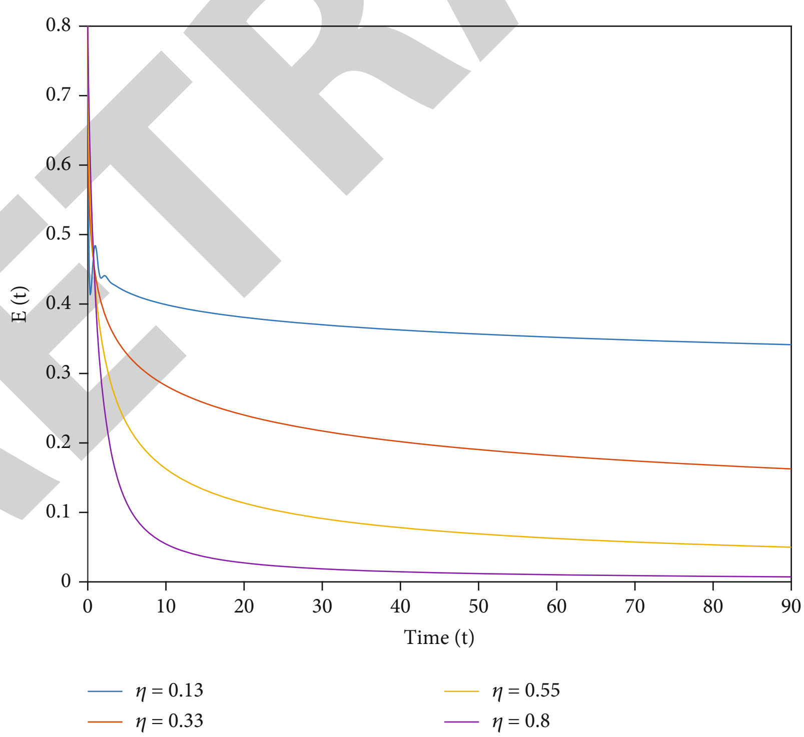

FIGURE 2: The behavior of exposed population.

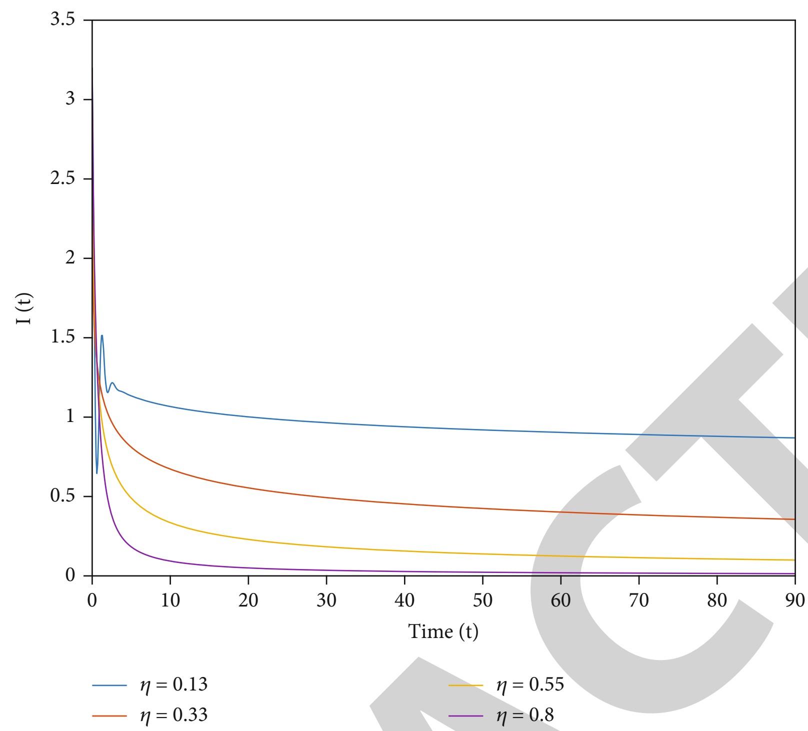

FIGURE 3: The behavior of infected population.

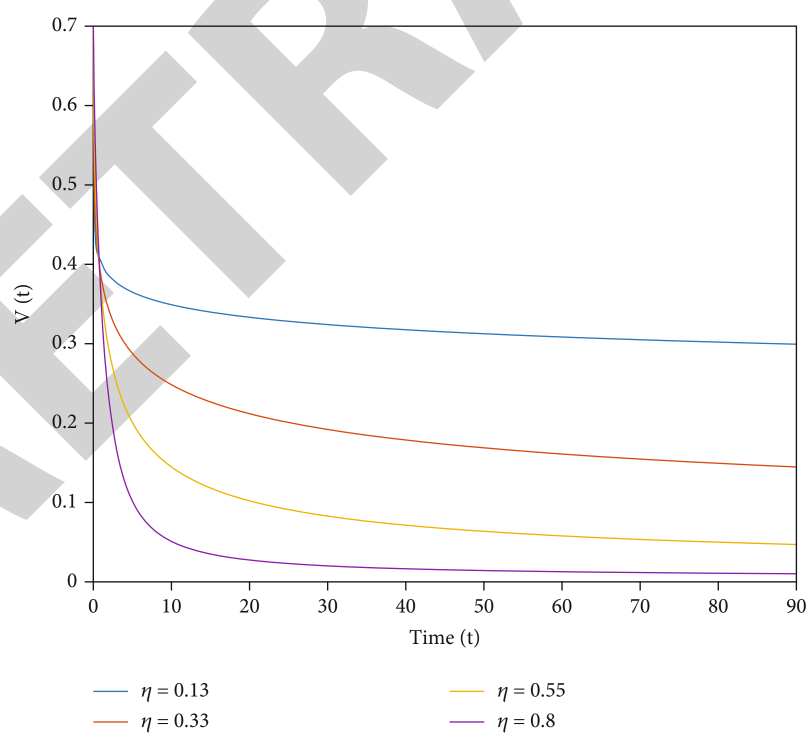

FIGURE 4: The behavior of vaccinated population.

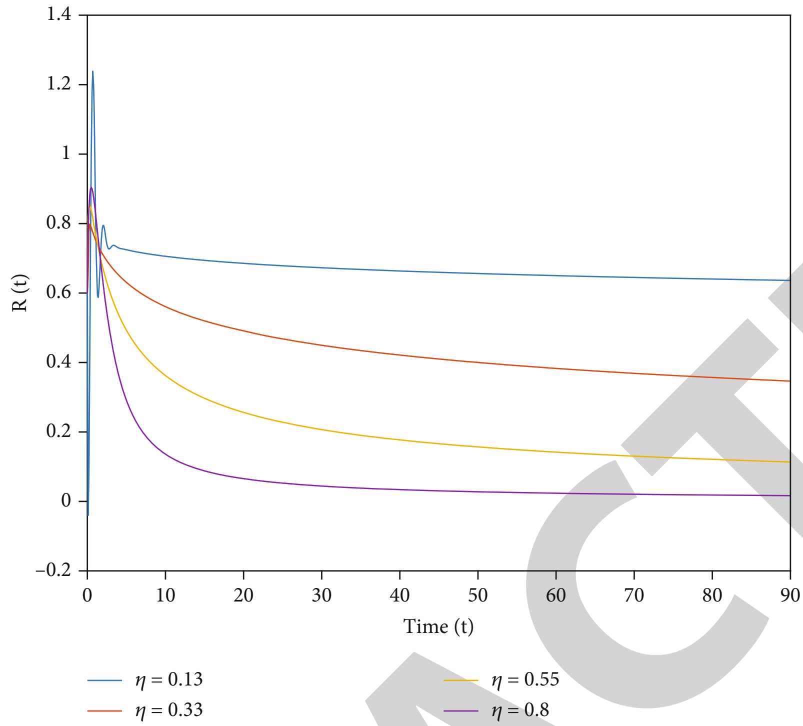

FIGURE 5: The behavior of recovered population.

10. Conclusion

In this paper, we have studied the fractional order Ebola model containing Caputo's fractional derivative of order μ. The paper contains the study related to the existence of the solution performed by using theorems of fixed point theory for the existence of fixed point. In addition, we have proved that the solution of the system is unique as well as Ulam stable. Apart from this, we have find the numerical solution of the studied model with the help of two-point fractional order Adam's Bashforth method presented for the approximation of the fractional differential equations containing the Caputo's fractional derivative. In addition, we have visualized the results graphically with the help of Matlab. At last, we have discussed the dynamical behavior of the obtained solution for all classes of the said model.

Data Availability

The data will be for public after the publication.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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