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Research Article

New Mean and Median Techniques to Solve Multiobjective Linear Fractional Programming Problem and Comparison with Other Techniques

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In the field of operation research, both linear and fractional programming problems have been more encountered in recent years because they are more realistic in expressing real-life problems. Fractional programming problem is used when several rates need to be optimized simultaneously such as resource allocation planning, financial and corporate planning, healthcare, and hospital planning. There are several techniques to solve the multiobjective linear fractional programming problem. However, because of the use of scalarization, these techniques have some limitations. This paper proposed two new mean and median techniques to solve the multiobjective linear fractional programming problem by overcoming the limitations. After utilizing mean and median techniques, the problem is converted into an equivalent linear fractional programming problem; then, the linear fractional programming problem is transformed into linear programming problem and solved by the conventional simplex method or mathematical software. Some numerical examples have been illustrated to show the efficiency of the proposed techniques and algorithm. The performance of these solutions was evaluated by comparing their results with other existing methods. The numerical results have shown that the proposed techniques are better than other techniques. Furthermore, the proposed techniques solve a pure multiobjective maximization problem, which is even impossible with some existing techniques. The present investigation can be improved further, which is left for future research.

1. Introduction

Optimization is the process of finding the maxima or minima of a single or several objective functions subject to constraints. Many real-world problems are modeled as optimization problems. A linear programming approach has been frequently used to optimize a single linear objective function subject to finite linear constraints [1]. This approach is used to solve many real-world problems such as manufacturing, marketing, finance, advertising, and agriculture. However, it can be difficult to optimize two or more objectives at the same time, and it is even more difficult if the objectives are conflicting in nature. The goal was to generate a compromise solution that achieves all the objective functions simultaneously. Multiobjective optimization techniques are helpful in improving the decision-making process in such situations. Several methods have been proposed to solve such problems by various scholars [2]. However, due to the limitations of the methods, choosing a proper technique remains a subject of active research. In the optimization problem, linear fractional programming concerns the optimization problem of a ratio of two linear functions subject to some constraints. Recently, linear fractional programming problems have attracted the interest of many researchers due to their application in different disciplines such as production planning, financial and corporative planning, economics and management science, healthcare, and hospital planning [3].

In this paper, we focus our interest on a multiobjective linear fractional programming problem where more than one linear fractional objective function is optimized simultaneously subject to finite linear constraints. Different researchers have proposed different scalarizing techniques to solve multiobjective optimization problems. Most of these techniques have been tested with nonconflicting objectives that were not appropriate. A multiobjective optimization problem with multiple conflicting objectives is the prerequisite for the application of every multiobjective optimization technique. Besides this, the existing scalarizing techniques have been evaluated when the denominator (scalarizing) quantity of the combined multiobjective function is nonzero. After analyzing the limitations of the previous scalarization techniques, improved scalarization techniques are proposed in this study. These proposed scalarizing quantities are very rare to be zero when compared to other existing techniques.

This study was motivated by the fact that different researchers solved nonconflicting multiobjective programming problems by converting them into single-objective programming problems, such as Chandra Sen, advanced transformation, new arithmetic average, new geometric average, advanced harmonic average, advanced mean deviation, and Pearson 2 skewness coefficient. However, solving the conflicting multiobjective fractional programming problem using the proposed techniques gave a better result and overcame the limitations of other techniques. This study tried to fill this gap.

The rest of the research is organized as follows. An overview of the literature is given in Section 2. Section 3 presented the methodology of the proposed methods. Section 4 described the problem formulation and the solution concept. Section 5 presented existing and proposed solving approaches. Section 6 discussed our algorithm. Section 7 provides illustrative examples, and the result is compared with the other existing approaches. Section 8 discussed the comparative study. Lastly, Section 9 presented conclusion and suggestions for future work.

2. Review of the Literature

This section presents a review of different pieces of the literature related to mean and median scalarizing techniques for multiobjective linear fractional programming problems.

Life is about making decisions, and the choice of optimal solutions is not an exclusive subject of scientists, engineers, and economists. Decision-making is present in daily life; when looking for an enjoyable vacancy, everyone will formulate an optimization problem like with a minimum amount of money, visiting a maximum number of places in a minimum amount of time and with the maximum comfort [4]. Multiobjective programming is a part of mathematical programming dealing with decision problems characterized by multiple conflicting objective functions that must be simultaneously optimized on a feasible set of decisions [5]. Fractional programming is used in several practical applications, such as cutting stock problems, shipping schedule problems, and different fields such as education, hospital administration, court systems, air force maintenance units, resource allocation, transportation, and bank branches. Different methods were suggested to solve linear fractional programming problem such as the variable transformation approach [6] and updated solution procedures [3].

Many researchers and scholars have studied how to convert multiobjective optimization problems into single-objective optimization problems and solve them using several techniques. Some of them are listed in Ref. [7]. Improved scalarization techniques using mean, harmonic mean, and geometric mean have been applied for solving multioptimization linear programming problems. Reference [8] solved the multiobjective linear fractional programming problem by using mean and median. New average techniques (geometric and arithmetic) had been proposed by Refs. [9, 10], which suggested harmonic average and advanced harmonic average techniques to solve multiobjective linear fractional programming problems. Advanced transformation and new statistical averaging techniques have been applied to solve a multiobjective linear programming problem [11–13]. Also, new mean deviation and advanced mean deviation to solve a multiobjective fractional programming problem have been proposed. Moreover, regarding mean and median, Ref. [14] offered the Pearson 2 skewness coefficient technique to turn a multiobjective linear fractional programming problem into a single-objective linear fractional programming problem. The main goal of this research is to solve a multiobjective linear fractional programming problem with less computational difficulties using the new proposed techniques. Furthermore, the proposed techniques address the limitations of existing approaches.

3. Methodology

In this methodology, we have used two new mean and median techniques to solve a multiobjective linear fractional programming problem. First, the multiobjective linear fractional programming problem was converted into an equivalent linear fractional programming problem by the proposed techniques. Second, the linear fractional programming problem was transformed into a linear programming problem by variable transformation and solved by the conventional simplex method or mathematical software, which compared their results with the other existing methods. Lastly, to illustrate the proposed methods, numerical examples are given.

4. Mathematical Formulation and Solution Concept

4.1. Linear Fractional Programming Problem. The general linear fractional programming problem is formulated as follows:

$$\begin{aligned} &\operatorname{Max} z == \frac{c^t x + \alpha}{d^t x + \beta}, \\ &\text{subject to} \\ &S = \{x \in \mathbb{R}^n\colon\ Ax \le b, \quad x \ge 0\}, \end{aligned} \tag{1}$$

where $c^t x + \alpha, d^t x + \beta$ are the real valued and continuous functions on S, $d^t x + \beta \neq 0$ for all $x \in S$, and $c, d \in \mathbb{R}^n, b \in \mathbb{R}^m$ are the column vectors,$\alpha, \beta \in \mathbb{R}$ are scalars, and $A \in \mathbb{R}^{m\times n}$ represent the $m \times n$ matrix.

According to the method introduced by Charnes and Cooper [6], problem (1) is changed into the following linear problem by the use of variable transformations.

Let $\lambda = 1/d^t x + \beta, y = \lambda x$. Then,

$$\begin{aligned} &\operatorname{Max} c^t y + \alpha\lambda \\ &\text{subject to} \\ &\quad Ay - b\lambda \leq 0 \\ &\quad d^t y + \beta\lambda \leq 1, \\ &\quad y, \quad \lambda \geq 0. \end{aligned} \tag{2}$$

Theorem [15]: let (y^*, λ^*) be the optimal solution of (1), then $x^* = (y^*/\lambda^*)$ is the optimum solution for (1).

4.2. Multiobjective Linear Fractional Programming Problem (MOLFPP). In this section, the techniques for transforming a multiobjective linear fractional programming problem into a linear fractional linear programming problem will be explained.

The general problem of linear fractional programming with multiple objectives may be expressed as [16]

$$\left.\begin{aligned} \operatorname{Max} Z_1 &= \frac{c_1^t x + \alpha_1}{d_1^t x + \beta_1} \\ \operatorname{Max} Z_2 &= \frac{c_2^t x + \alpha_2}{d_2^t x + \beta_2} \\ &\vdots \\ \operatorname{Max} Z_r &= \frac{c_r^t x + \alpha_r}{d_r^t x + \beta_r} \\ \min Z_{r+1} &= \frac{c_{r+1}^t x + \alpha_{r+1}}{d_{r+1}^t x + \beta_{r+1}} \\ &\vdots \\ \operatorname{Min} Z_n &= \frac{c_n^t x + \alpha_n}{d_n^t x + \beta_n} \end{aligned}\right\}. \tag{3}$$

Subject to

$$S = \{x \in \mathbb{R}^n\colon Ax \leq b, \quad x \geq 0\}, \tag{4}$$

where r denotes the quantity of objective functions that must be maximized, $n - r$ is the number of objective functions that need to be minimized, and n is the total number of objective functions that must be maximized plus minimized, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, and $c_i, d_i \in \mathbb{R}^n$, $\alpha_i, \beta_i \in \mathbb{R}$ for all $i = 1, 2, \ldots, n$.

Definition 1. A point $x^* \in \mathbb{R}^n$ is said to be an efficient solution of a multiobjective linear fractional programming problem (3) if there is no $x \in \mathbb{R}^n$ such that $Z_i(x) \geq Z_i(x^*)$ for all $i = 1, 2, \ldots r$, and $Z_i(x) > Z_i(x^*)$ for at least one i, and $Z_{r+1}(x) \leq Z_i(x^*)$ for all $i = r+1, r+2, \ldots, n$, and $Z_i(x) < Z_i(x^*)$ for at least one $i = r+1$.

5. Applied Techniques of Converting Multiobjective Linear Fractional Programming Problem into Single Linear Fractional Programming Problem

There are many techniques, such as Chandra Sen, advanced transformation, new arithmetic average, new geometric average, advanced harmonic average, advanced mean deviation, and Pearson 2 skewness coefficient, presented in the literature to solve multiobjective programming problems. These techniques are concisely described in the following.

After individually optimizing all fractional objective functions subject to the given constraints using the above variable transformation (2), we get

$$\left.\begin{aligned} \operatorname{Max} Z_1 &= \gamma_1 \\ \operatorname{Max} Z_2 &= \gamma_1 \\ &\vdots \\ \operatorname{Max} Z_r &= \gamma_r \\ \min Z_{r+1} &= \gamma_{r+1} \\ &\vdots \\ \operatorname{Min} Z_n &= \gamma_n \end{aligned}\right\}, \tag{5}$$

where $\gamma_i\,(i = 1, 2, \ldots, n)$ are the aspiration values of the objective functions.

5.1. Chandra Sen Technique [17]. According to Chandra Sen's technique, the multiobjective linear fractional programming problem given in (3) can be converted into a single-objective function as

$$\operatorname{Max} Z = \frac{\sum_{i=1}^r Z_i}{|\gamma_i|} - \frac{\sum_{i=r+1}^n Z_i}{|\gamma_i|}. \tag{6}$$

This can be solved using the simplex method with the same constraints (3).

5.2. Advanced Transformation Technique [11]. According to advanced transformation technique, the multiobjective linear fractional programming problem given in (3) is converted into a single-objective function as

$$\operatorname{Max} Z = \frac{\sum_{i=1}^r Z_i - \sum_{i=r+1}^n Z_i}{O_{At}}, \tag{7}$$

where $O_{At} = 1/1/m, m = \min\{m_1, m_2\}$, where $m_1 = \min\{|\gamma_i|\}, \forall i = 1, 2, \ldots, r$. $m_2 = \min\{|\gamma_i|\}, \forall i = r+1, r+2, \ldots, n$,

subject to the same constraints as in (3).

5.3. New Arithmetic Average Technique [9]. According to a new arithmetic average technique, the combined objective function under the same constraints (3) can be formulated as

$$\text{Max}\, Z = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{n} Z_i}{m}, \tag{8}$$

where $m = m_1 + m_2/2$, where $m_1 = \min\{|\gamma_i|\}$, $\forall i = 1, 2, \ldots, r$ and $m_2 = \min\ \{|\gamma_i|\} \forall i = r+1, r+2, \ldots, n.$

5.4. New Geometric Average Technique [9]. The combined objective function by a new geometric average technique under the same constraints (3) is expressed as follows:

$$\text{Max}\, Z = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{n} Z_i}{NGA}, \tag{9}$$

where $NGA = \sqrt{m_1 m_2}$, where $m_1 = \min\ \{|\gamma_i|\}$, $\forall i = 1, 2, \ldots, r.$

$$m_2 = \min\left\{\left|\gamma_i\right|\right\}, \forall i = r+1, r+2, \ldots, n. \tag{10}$$

5.5. Advanced Harmonic Average Technique [10]. The combined objective function by the advanced harmonic average technique under the same constraints (3) is expressed as follows:

$$\text{Max}\, Z = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{n} Z_i}{AH_{av}}, \tag{11}$$

where $AH_{av} = ((2|m_1||m_2|)/(|m_1| + |m_2))$, where $m_1 = \min\{|\gamma_i|\}, \forall i = 1, 2, \ldots, r.$

$$m_2 = \max\left\{\left|\gamma_i\right|\right\}, \forall i = r+1, r+2, \ldots, n. \tag{12}$$

5.6. Advanced Mean Deviation Technique [13]. According to the advanced mean deviation technique, the combined objective function under the same constraints (3) can be formulated as

$$\text{Max}\, Z = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{n} Z_i}{AMD}, \tag{13}$$

where $AMD = MD_1 + MD_2/S$, where $MD_1 = \sum_{i=1}^{r} |\gamma_i - \overline{\gamma_1}|/r, \overline{\gamma_1} = \sum_{i=1}^{r} \gamma_i/r, \forall i = 1, 2, \ldots, r,$ $MD_2 = ((\sum_{i=r+1}^{r} |\gamma_i - \overline{\gamma_2}|)/(n-r)), \overline{\gamma_2} = ((\sum_{i=r+1}^{r} \gamma_i)/(n-r)), \forall i = r+1, r+2, \ldots. n.$ S is the total number of objective functions.

5.7. Pearson 2 Skewness Coefficient Technique [14]. According to Pearson 2 skewness coefficient technique, the combined objective function under the same constraints (3) can be formulated as

$$\text{Max}\, Z = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{n} Z_i}{S_{k2}}, \tag{14}$$

where $S_{k2} = 3|mean(\gamma_i) - median(\gamma_i)/s|\ \forall i = 1, 2, \ldots, n,$ where s is a standard deviation for the value of all objective functions.

5.8. Proposed Techniques. The mean, median, and standard deviation have been used for scalarizing the multiobjective linear fractional programming problem. The proposed techniques to solve the multiobjective fraction programming problem based on the mean and median idea are briefly described in the following.

The process of translating a multiobjective fraction programming problem into a single linear fractional programming problem is as follows:

$$\text{MaxZ} = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{n} Z_i}{\text{NMMT1}} \text{ and } \text{MaxZ} = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{n} Z_i}{\text{NMMT2}}, \tag{15}$$

where NMMT denotes new mean and median techniques and is calculated as

$$\text{NMMT1} = \left(\frac{\min\{M, M_d, S\}}{M + M_d + S}\right)\left(\frac{1}{N}\right), \tag{16}$$

$$\text{NMMT2} = \left(\frac{1}{M + M_d + S}\right)\left(\frac{1}{N}\right), \tag{17}$$

where M = Mean of $|\gamma_i| \forall i = 1, 2, \ldots, n.$ M_d = Median of $|\gamma_i|, \forall i = 1, 2, \ldots, n.$ S = Standard of deviation of $|\gamma_i|$, $\forall i = 1, 2, \ldots, n.$ N = Total number of objective functions.

6. The New Mean and Median Algorithm for Solving MOLFPP

We propose an algorithm for solving multiobjective linear fractional programming problem (3) as follows:

Step 1: find the individual optimal value of each of the fractional objective functions subject to the constraints using variable transformation.

Step 2: calculate NMMT1 and NMMT2 using formula (17).

Step 3: construct the combined single fractional objective functions using formula (15).

Step 4: optimize the combined fractional objective function under the same constraints.

$$\text{MaxZ} = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{n} Z_i}{\text{NMMT1}} \text{ and } \text{MaxZ} = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{n} Z_i}{\text{NMMT2}}. \tag{18}$$

7. Numerical Examples

In this section, illustrative examples are presented to demonstrate how the techniques and the algorithm work. Some examples have been taken from previous studies to show the differences with our proposed techniques.

Example 1. (see Ref. [18]):

$$\begin{aligned}
&\operatorname{Max} Z_1 = \frac{12x_1 + 13x_2}{40x_1 + 55x_2 + 500},\\
&\operatorname{Max} Z_2 = \frac{12x_1 + 13x_2}{1.5x_3 + 1.6x_4},\\
&\quad \text{Subject to}\\
&\quad 2x_1 + x_2 \le 250\\
&\quad 5x_1 + 4x_2 \le 500\\
&\quad 45x_1 + 30x_2 \le 1500\\
&\quad 0.1x_1 + 0.1x_2 - x_3 - x_4 \le 0\\
&\quad 0.1x_1 - x_3 \le 0\\
&\quad 0.05x_2 - x_4 \le 0\\
&\quad -x_1 + x_3 \le 0\\
&\quad -x_2 + x_4 \le 0\\
&\quad x_1, x_2, x_3, x_4 \ge 0
\end{aligned} \tag{19}$$

It is clear from the results of Table 1 that both objectives are individually achieved by different solutions (values of $x_1, x_2, x_3,$ *and* x_4).

7.1. New Mean and Median Techniques 1. $\operatorname{Max}Z = (\sum_{i=1}^{2} Z_i/NMMT1), NMMT1 = ((\min\{41.946, 41.946, 41.728\})/(41.949 + 41.946 + 41.728)) \quad (1/2) = (41.728/251.24) = 0.166.$

So,

$$\operatorname{Max} Z = 6.024\left[\frac{12x_1 + 13x_2}{40x_1 + 55x_2 + 500} + \frac{12x_1 + 13x_2}{1.5x_3 + 1.6x_4}\right],$$

$$\text{Subject to}\quad \begin{aligned}
&2x_1 + x_2 \le 250\\
&5x_1 + 4x_2 \le 500\\
&45x_1 + 30x_2 \le 1500\\
&0.1x_1 + 0.1x_2 - x_3 - x_4 \le 0\\
&0.1x_1 - x_3 \le 0\\
&0.05x_2 - x_4 \le 0\\
&-x_1 + x_3 \le 0\\
&-x_2 + x_4 \le 0\\
&x_1, x_2, x_3, x_4 \ge 0
\end{aligned} \tag{20}$$

Table 1: After all objective functions have been optimized individually, the results of Example 1 are given below.

Objective functions	(x_1, x_2, x_3, x_4)	$Z_i^* = \|\gamma_i\|$	Mean = (Z_i^*)	Median = (Z_i^*)	Standard deviation
1	(33.333, 0, 33.333, 0)	0.218	41.946	41.946	41.728
2	(2.564, 46.154, 2.564, 2.3077)	83.674			

After solving problem (20), we get Max $Z = 505.05$ at (2.564, 46.153, 2.564, 2.308).

7.2. New Mean and Median Technique 2. MaxZ = $(\sum_{i=1}^{2} Z_i)$, NMMT2 = (1/41.946 + 41.946 + 41.728) (1/2) = (1/251.24) = 0.004.

So,

$$\text{Max Z} = 250\left[\frac{12x_1 + 13x_2}{40x_1 + 55x_2 + 500} + \frac{12x_1 + 13x_2}{1.5x_3 + 1.6x_4}\right],$$

$$\text{Subject to} \quad \begin{aligned} &2x_1 + x_2 \le 250 \\ &5x_1 + 4x_2 \le 500 \\ &45x_1 + 30x_2 \le 1500 \\ &0.1x_1 + 0.1x_2 - x_3 - x_4 \le 0 \\ &0.1x_1 - x_3 \le 0 \\ &0.05x_2 - x_4 \le 0 \\ &-x_1 + x_3 \le 0 \\ &-x_2 + x_4 \le 0 \\ &x_1, x_2, x_3, x_4 \ge 0 \end{aligned} \quad . \tag{21}$$

After solving problem (21), we get Max $Z = 20968$ at (2.564, 46.153, 2.564, 2.308).

Now, solve Example 1 using other possible techniques such as

(1) By Chandra Sen [17], the result is ??? $Z = 1.96$ at (33.333, 0, 33.333, 0).

(2) By the new arithmetic average [9], the result is Max, $Z = 11.048$ at (2.564, 46.153, 2.564, 2.308).

(3) By advanced mean deviation [13], the result is Max $Z = 3499$ at (2.564, 46.153, 2.564, 2.308).

Here, most of recently proposed methods such as advanced transformation, new arithmetic average, advanced harmonic average, and Pearson 2 skewness coefficient did not solve the problem.

Example 2 (see Ref. [14]):

$$\begin{aligned} &\text{Max } Z_1 = \frac{x_1 - x_2 + 7.5}{2x_1 + 2x_2 + 2}, \\ &\text{Max } Z_2 = \frac{x_1 + x_2 + 4}{x_1 + x_2 + 1}, \\ &\text{Max } Z_3 = \frac{x_1 - x_2 + 4.5}{3x_1 + 3x_2 + 3} \\ &\text{Max } Z_4 = \frac{x_1 + x_2}{x_1 + x_2 + 1} \\ &\text{Min } Z_5 = \frac{-2x_1 - 2x_2}{3x_1 + 3x_2 + 3} \\ &\text{Min } Z_6 = \frac{-5x_1 - x_2}{x_1 + x_2 + 1} \\ &\text{Min } Z_7 = \frac{-4x_1 + x_2}{2x_1 + 2x_2 + 2} \end{aligned} \quad . \tag{22}$$

Subject to

$$\begin{aligned} &-x_1 + x_2 \le 2, \\ &2x_1 + 2x_2 \ge 1, \\ &9x_1 + 3x_2 \le 3 \\ &x_2 \le 1, \\ &x_1, x_2 \ge 0. \end{aligned} ,$$

It is clear from the results of Table 2; all seven objectives are not individually achieved by the same solution.

Table 2: After all objective functions have been optimized individually, the results of Example 2 are given below.

Objective functions	(x_1, x_2)	$Z_i^* = \|\gamma_i\|$	Mean (Z_i^*)	Median (Z_i^*)	Standard deviation
1	(0.25, 0.25)	2.5			
2	(0, 0.5)	3			
3	(0.25, 0.25)	1			
4	(0, 1)	0.5	1.226	1	1.01
5	(0, 1)	$\frac{1}{3}$			
6	(0.25, 0.25)	1			
7	(0.25, 0.25)	0.25			

7.3. New Mean and Median Technique 1. $\mathrm{Max}Z = (\sum_{i=1}^{4} Z_i - \sum_{i=5}^{7} Z_i / NMMT1)$, $\mathrm{NMMT1} = ((\min\{1.226, 1, 1.01\})/(1.226 + 1 + 1.01))(1/7) = (1/3.236)1/7 = 0.044$.

So,

$$\mathrm{Max} = \frac{63x_1 + 14x_2 + 55.5}{0.264(x1 + x2 + 1)},$$

$$\text{Subject to} \quad \begin{aligned} &-x_1 + x_2 \le 2, \\ &2x_1 + 2x_2 \ge 1 \\ &9x_1 + 3x_2 \le 3 \quad , \\ &x_2 \le 1, \\ &x_1, x_2 \ge 0, \end{aligned} \tag{23}$$

After solving problem (23), we get Max $Z = 188.769$ at (1/4, 1/4).

7.4. New Mean and Median Technique 2. $MaxZ = (\sum_{i=1}^{4} Z_i - \sum_{i=5}^{7} Z_i / NMMT2)$, $\mathrm{NMMT2} = (1/1.226 + 1 + 1.01)(1/7) = (1/3.236)1/7 = 0.044$.

So,

$$\mathrm{Max} = \frac{63x_1 + 14x_2 + 55.5}{0.264(x1 + x2 + 1)},$$

$$Subject\ to \quad \begin{aligned} &-x_1 + x_2 \le 2, \\ &2x_1 + 2x_2 \ge 1, \\ &9x_1 + 3x_2 \le 3 \quad . \\ &x_2 \le 1, \\ &x_1, x_2 \ge 0. \end{aligned} \tag{24}$$

After solving problem (24), we get Max $Z = 188.769$ at ((1/4), (1/4)).

Now, solve Example 2 by other techniques such as

(1) By Chandra Sen [17], the result is Max $Z = 17.853$ at ((1/4), (1/4)).

(2) By advanced transformation [11], the result is Max $Z = 33.189$ at ((1/4), (1/4)).

(3) By the new Arithmetic average [9], the result is Max $Z = 22.126$ at ((1/4), (1/4))

(4) By the new geometric average [9], the result is Max $z = 23.42$ at((1/4), (1/4)).

(5) Advanced harmonic average [10], the result is Max $Z = 12.458$ at((1/4), (1/4))

(6) By advanced mean deviation [13], the result is Max $Z = 44.20$ at ((1/4), (1/4)).

(7) By the Pearson 2 skewness coefficient [14], the result is M*ax*. $Z = 12.458$ at ((1/4), (1/4)).

Example 3. Consider the following multiobjective linear fractional programming problem:

$$\begin{aligned} &\mathrm{Max}\, Z_1 = \frac{x_1 + 2x_2 - x_3}{9x_1 + 6x_2 + 3x_3} \\ &\mathrm{Max}\, Z_2 = \frac{6x_1 + x_2}{3x_1 + 2x_2 + x_3} \\ &\mathrm{Max}\, Z_3 = \frac{6x_1 + x_2 + x_3 + 6}{9x_1 + 6x_2 + 3x_3} \quad , \\ &\mathrm{Min}\, Z_4 = \frac{x_1 - 3x_2 + 2x_3}{3x_1 + 2x_2 + x_3} \\ &\text{Subject to} \\ &2x_1 + 3x_2 - 5x_3 \ge 10, \\ &x_1 + x_2 + x_3 \le 8, \\ &x_1 + x_3 \ge 4, \\ &x_1, x_2, x_3 \ge 0. \end{aligned} \tag{25}$$

Using the value of mean, median, and standard deviation from Table 3, we find the following.

Table 3: After all objective functions have been optimized individually, the results of Example 3 are given below.

Objective functions	(x_1, x_2, x_3)	$Z_i^* = \lvert\gamma_i\rvert$	Mean (Z_i^*)	Median (Z_i^*)	Standard deviation
1	(4, 4, 0)	0.2			
2	(8, 0, 0)	2	0.85	0.6	0.698
3	(5, 0, 0)	0.8			
4	(4, 4, 0)	0.4			

7.5. New Mean and Median Technique 1. $\text{Max}Z = (\sum_{i=1}^{3} Z_i - \sum_{i=4}^{4} Z_i / NMMT1)$, $\text{NMMT1} = (\min\{0.85, 0.6, 0.698\}/2.148)(1/4) = 0.6/8.592 = 0.07$.

So,

$$\text{Max Z} = \frac{1}{0.07}\left[\frac{22x_1 + 15x_2 - 6x_3 + 6}{3(3x_1 + 2x_2 + x_3)}\right],$$

$$\text{subject to} \quad \begin{aligned} &2x_1 + 3x_2 - 5x_3 \geq 10,\\ &x_1 + x_2 + x_3 \leq 8,\\ &x_1 + x_3 \geq 4,\\ &x_1, x_2, x_3 \geq 0. \end{aligned} \tag{26}$$

After solving problem (26), we get Max $Z = 37.144$ at $(4, 0.667, 0)$.

7.6. New Mean and Median Technique 2. $\mathit{MaxZ} = (\sum_{i=1}^{3} Z_i - \sum_{i=4}^{4} Z_i / NMMT2)$, $\text{NMMT2} = (1/2.148)(1/4) = 1/8.592 = 0.116$

So,

$$\text{Max Z} = \frac{1}{0.116}\left[\frac{22x_1 + 15x_2 - 6x_3 + 6}{3(3x_1 + 2x_2 + x_3)}\right],$$

$$\text{subject to} \quad \begin{aligned} &2x_1 + 3x_2 - 5x_3 \geq 10,\\ &x_1 + x_2 + x_3 \leq 8,\\ &x_1 + x_3 \geq 4,\\ &x_1, x_2, x_3 \geq 0. \end{aligned} \tag{27}$$

After solving problem (27), we get Max $Z = 22.386$ at (4, 0.667, 0).

Now, solve Example 3 by other techniques such as

(1) By Chandra Sen [17], the result is Max $Z = 1.11$ at (5, 0, 0).

(2) By advanced transformation [11], the result is Max $Z = 1\ 0.56$ at $(4, 0.667, 0)$.

(3) By the new arithmetic average [9], the result is Max $Z = 8.658$ at $(4, 0.667, 0)$.

(4) By the new geometric average [9], the result is Max $Z = 9.188$ at $(4, 0.667, 0)$.

(5) By advanced harmonic average [10], the result is Max $Z = 9.75$ at $(4, 0.667, 0)$.

(6) By advanced mean deviation [13], the result is Max $Z = 15.6$ at $(4, 0.667, 0)$.

(7) By the Pearson 2 skewness coefficient [14], the result is Max $Z = 2.418$ at $(4, 0.667, 0)$.

Example 4. Consider the following multiobjective linear fractional programming problem:

$$\begin{aligned} \text{Max } Z_1 &= \frac{x_1 + 2x_2}{x_1 + x_2 + x_3}\\ \text{Max } Z_2 &= \frac{3x_1 + 2x_2}{2x_1 + 2x_2 + 2x_3}\\ \text{Max } Z_3 &= \frac{3x_1 + x_2 + 4x_3}{x_1 + x_2 + x_3}\\ \text{Min } Z_4 &= \frac{3x_1 + x_2 + x_3}{5x_1 + 5x_2 + x_3}\\ \text{Min } Z_5 &= \frac{-x_1 + x_3}{10x_1 + 10x_2 + 2x_3},\\ \text{Min } Z_6 &= \frac{4x_1 + x_2 - x_3}{10x_1 + 10x_2 + 2x_3}\\ \text{Min } Z_7 &= \frac{-6x_1 - x_2 + x_3}{5x_1 + 5x_2 + x_3}\\ &\text{Subject to}\\ &x_1 + x_2 + x_3 \leq 40,\\ &2x_1 + x_2 + x_3 \leq 100,\\ &x_1, x_2, x_3 \geq 0. \end{aligned} \tag{28}$$

Using the value of mean, median, and standard deviation from Table 4, we find the following.

7.7. New Mean and Median Technique 1. Max $.Z$ $(\sum_{i=1}^{3} Z_i - \sum_{i=4}^{4} Z_i / NMMT2)$, NMMT1 = ((min {1.357, 1.2, 1.26})/(1.357 + 1.2 + 1.26)) (1/7) = 1.2/26.72 = 0.045.

So,

$$\text{Max Z} = \frac{1}{0.045}\left[\frac{11x_1 + 8x_2 + 8x_3}{2(x_1 + x_2 + x_3)} - \frac{-x_1 + x_2 + 4x_3}{2(5x_1 + 5x_2 + x_3)}\right],$$
$$\begin{aligned} \text{subject to}\quad & x_1 + x_2 + x_3 \le 40, \\ & 2x_1 + x_2 + x_3 \le 100, \\ & x_1, x_2, x_3 \ge 0. \end{aligned} \tag{29}$$

After solving problem (29), we get Max $Z = 124.43$ at (40, 0, 0).

7.8. New Mean and Median Technique 2. $MaxZ = (\sum_{i=1}^{3} Z_i - \sum_{i=4}^{4} Z_i / NMMT2)$, NMMT2 (1/3.82) (1/7) = 1/26.74 = 0.037.

So,

$$\text{Max Z} = \frac{1}{0.037}\left[\frac{11x_1 + 8x_2 + 8x_3}{2(x_1 + x_2 + x_3)} - \frac{-x_1 + x_2 + 4x_3}{2(5x_1 + 5x_2 + x_3)}\right],$$
$$\begin{aligned} \text{subject to}\quad & x_1 + x_2 + x_3 \le 40, \\ & 2x_1 + x_2 + x_3 \le 100, \\ & x_1, x_2, x_3 \ge 0. \end{aligned} \tag{30}$$

After solving (30), we get Max $Z = 151.35$ at (40, 0, 0). Now, solve Example 4 using other techniques such as

(1) By Chandra Sen [17], the result is Max $Z = 1.217$ at (40, 0, 0).

(2) By advanced transformation [11], the result is Max $Z = 0.56$ at (40, 0, 0).

(3) By new arithmetic average [9], the result is Max $Z = 7$ at (40, 0, 0).

(4) By the new geometric average [9], the result is Max $z = 14.47$ at (40, 0, 0).

(5) By advanced harmonic average [10], the result is Max $Z = 29.014$ at (40, 0, 0).

(6) By advanced mean deviation [13], the result is Max $Z = 29.014$ at (40, 0, 0).

(7) By Pearson 2 skewness coefficient [14], the result is Max $Z = 14.98$ at (40, 0, 0).

Table 4: After all objective functions have been optimized individually, the results of Example 4 are given below.

Objective functions	(x_1, x_2, x_3)	$Z_i^* = \vert\gamma_i\vert$	Mean (Z_i^*)	Median (Z_i^*)	Standard deviation
1	$(0, 40, 0)$	2			
2	$(40, 0, 0)$	1.5			
3	$(0, 0, 40)$	4			
4	$(0, 40, 0)$	0.2	1.357	1.2	1.26
5	$(0, 40, 0)$	0.1			
6	$(0, 0, 40)$	0.5			
7	$(0, 0, 40)$	1.2			

Example 5 (see Ref. [19]):

$$
\begin{aligned}
&\operatorname{Max} Z_1 = \frac{-3x_1 + 2x_2}{x_1 + x_2 + 3}\\
&\operatorname{Max} Z_2 = \frac{7x_1 + x_2}{5x_1 + 2x_2 + 1}\\
&\operatorname{Max} Z_3 = \frac{x_1 + 4x_2}{2x_1 + 3x_2 + 2},\\
&\text{Subject to}\\
&x_1 - x_2 \geq 1,\\
&2x_1 + 3x_2 \leq 15,\\
&x_1 + 9x_2 \geq 9,\\
&x_1 \geq 3,\\
&x_1, x_2 \geq 0.
\end{aligned}
\tag{31}
$$

Using the value of mean, median, and standard deviation from Table 5, we find the following.

7.9. New Mean and Median Technique 1. $MaxZ = (\sum_{i=1}^{3} Z_i/NMMT1)$, $\text{NMMT1} = ((\min\{0.92, 0.82, 0.31\})/(0.92 + 0.82 + 0.31))(1/3) = 0.31/6.15 = 0.05$.

So,

$$
\begin{aligned}
&\operatorname{Max} Z = \frac{1}{0.05}\left[\frac{-3x_1 + 2x_2}{x_1 + x_2 + 2} + \frac{7x_1 + x_2}{5x_1 + 2x_2 + 1} + \frac{x_1 + 4x_2}{2x_1 + 3x_2 + 2}\right],\\
&\text{subject to}\quad
\begin{aligned}
&x_1 - x_2 \geq 1,\\
&2x_1 + 3x_2 \leq 15,\\
&x_1 + 9x_2 \geq 9,\\
&x_1 \geq 3,\\
&x_1, x_2 \geq 0.
\end{aligned}
\end{aligned}
\tag{32}
$$

Table 5: After all objective functions have been optimized individually, the results of Example 5 are given below.

Objective functions	(x_1, x_2)	$Z_i^* = \|\gamma_i\|$	Mean (Z_i^*)	Median (Z_i^*)	Standard deviation
1	$(3.6, 2.6)$	0.6			
2	$(7.2, 0.2)$	1.35	0.92	0.82	0.31
3	$(3.6, 2.6)$	0.82			

Table 6: The comparison of the five numerical results that are obtained from previous examples is presented as follows.

Techniques	Example 1	Example 2	Example 3	Example 4	Example 5
Chandra Sen	1.96	17.853	1.11	1.217	0.716
Advanced transformation	—	33.189	1.56	0.56	—
New arithmetic average	11.048	22.126	8.658	7	4.29
New geometric average	—	23.42	9.188	14.47	—
Advanced harmonic average	—	12.458	9.75	29.014	—
Advanced mean deviation	3499	44.2	15.6	29.014	4.53
Pearson 2 skewness coefficient	—	12.458	2.418	14.98	1.29
New Mean and Median technique 1	505.05	188.769	37.144	124.43	25.79
New Mean and Median Technique 2	20968	188.769	22.386	151.35	8.058

After solving problem (32), we get Max $Z = 25.79$ at (3.6, 2.6).

7.10. New Mean and Median Technique 2. $MaxZ = (\sum_{i=1}^{3} Z_i/NMMT1)$, NMMT2 = (1/2.05) (1/3) = 1/6.15 = 0.16.

So,

$$\text{Max Z} = \frac{1}{0.16}\left[\frac{-3x_1 + 2x_2}{x_1 + x_2 + 2} + \frac{7x_1 + x_2}{5x_1 + 2x_2 + 1} + \frac{x_1 + 4x_2}{2x_1 + 3x_2 + 2}\right],$$

$$\text{subject to} \quad \begin{aligned} & x_1 - x_2 \geq 1, \\ & 2x_1 + 3x_2 \leq 15, \\ & x_1 + 9x_2 \geq 9, \\ & x_1 \geq 3, \\ & x_1, x_2 \geq 0. \end{aligned} \tag{33}$$

After solving (33), we get Max $Z = 8.058$ at $(3.6, 2.6)$.

Now, solve Example 5 by other possible techniques such as

(1) By Chandra Sen [17], the result is Max $Z = 0.716$ at $(3.6, 2.6)$.

(2) By the new arithmetic average [9], the result is Max $Z = 4.29$ at $(3.6, 2.6)$.

(3) By advanced mean deviation [13], the result is Max $Z = 4.53$ at $(3.6, 2.6)$.

(4) By the Pearson 2 skewness coefficient [14], the result is Max $Z = 1.29$ at $(3.6, 2.6)$.

In Example 5, advanced transformation, advanced harmonic average, and Pearson 2 skewness coefficient did not also solve the problem. This is one of the limitations.

8. Comparative Study

The following table summarizes the results of the MOLFPP using different techniques. It shows the comparison between the techniques studied in this paper and other techniques. The solution to the problem obtained by the proposed techniques gave a better result than the other techniques that were previously studied, as shown in Table 6. In addition to this, the proposed techniques solve problems that could not be solved by some existing techniques.

From Table 6, "—" represents that the techniques did not solve the given problems. It is evident from this table that the results of Examples 1–5 show that when using the two proposed techniques called New Mean and Median Technique 1 and New Mean and Median Technique 2, the results are better than other techniques.

9. Conclusion and Future Work

In this paper, we have proposed two new techniques to convert a multiobjective linear fractional programming problem into a single linear fractional programming problem, and then the single linear fractional programming problem is solved by variable transformation in simple and easy ways. These techniques provided a more compressive and effective solution to the conflicting multiobjective linear fractional programming problem. To illustrate the solution process and motivation, some numerical examples have been solved and the techniques were compared with the other existing techniques that were previously studied, as shown in Table 6. The numerical results confirm that our proposed techniques optimize the problem better than those other techniques. Furthermore, the limitations of the existing methods have been pointed out.

Future studies would compare these new mean and median techniques with other techniques other than Chandra Sen, advanced transformation, new arithmetic average, new geometric average, advanced harmonic average, advanced mean deviation, and Pearson 2 skewness coefficient to reinforce the results and come up with better results. In addition to this, the proposed algorithm can be improved to solve multilevel multiobjective fractional programming problems.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors have no relevant financial or nonfinancial interests to disclose. The authors declare that there are no conflicts of interest with respect to the publication of this manuscript.

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