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Research Article

Estimation of Population Mean in Chain Ratio-Type Estimator under Systematic Sampling

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A chain ratio-type estimator is proposed for the estimation of finite population mean under systematic sampling scheme using two auxiliary variables. The mean square error of the proposed estimator is derived up to the first order of approximation and is compared with other relevant existing estimators. To illustrate the performances of the different estimators in comparison with the usual simple estimator, we have taken a real data set from the literature of survey sampling.

1. Introduction and Literature Review

Incorporating the knowledge of the auxiliary variables is very important for the construction of efficient estimators for the estimation of population parameters and increasing the efficiency of the estimators in different sampling design. Using the knowledge of the auxiliary variables, several authors have proposed different estimation technique for the finite population mean of the study variable; Cochran [1], Tripathy [2], Kadilar and Cingi [3, 4], Singh et al. [5], Khan and Arunachalam [6], Lone and Tailor [7], Khan [8], Khan and Hussain [9], and Khan et al. [10] have worked on the estimation of population parameters using auxiliary information.

In the present paper, we will work on the estimation of population mean using the knowledge of the auxiliary variables under systematic sampling. Various statisticians have worked on the estimation of population mean in systematic sampling: Cochran [11], Hansen et al. [12], Robson [13], Swain [14], Singh [15], Shukla [16], Kushwaha and Singh [17], Banarasi et al. [18], R. Singh and H. P. Singh [19], Singh et al. [20], Singh and Solanki [21], Singh and Jatwa [22], Singh et al. [23, 24], Tailor et al. [25], Verma and Singh [26], and Verma et al. [27], and so forth.

Consider a finite population $U = \{U_1, U_2, \ldots, U_N\}$ of size N units, numbered from 1 to N in some order. A sample of size n units is taken at random from the first k units and every kth subsequent unit; then, $N = nk$ where n and k are positive integers; thus, there will be k samples (clusters) each of size n and observe the study variate y and auxiliary variate x for each and every unit selected in the sample. Let (y_{ij}, x_{ij}), for $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, n$: denote the value of jth unit in the ith sample. Then, the systematic sample means are defined as follows: $\overline{y}_{sy} = t_0 = (1/n)\sum_{j=1}^{n} y_{ij}$ and $\overline{x}_{sy} = (1/n)\sum_{j=1}^{n} x_{ij}$ are the unbiased estimators of the population means $\overline{Y} = (1/N)\sum_{j=1}^{N} y_{ij}$ and $\overline{X} = (1/N)\sum_{j=1}^{N} x_{ij}$ of y and x, respectively. Let $S_y^2 = (1/(N-1))\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij} - \overline{Y})^2$, $S_x^2 = (1/(N-1))\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij} - \overline{X})^2$, and $S_z^2 = (1/(N-1))\sum_{i=1}^{k}\sum_{j=1}^{n}(z_{ij} - \overline{Z})^2$ be the population variances of the study variable and the auxiliary variables, respectively, with the corresponding population covariance's $S_{yx} = (1/(N-1))\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij} - \overline{X})(y_{ij} - \overline{Y})$, $S_{yz} = (1/(N-1))\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij} - \overline{Y})(z_{ij} - \overline{Z})$, and $S_{xz} = (1/(N-1))\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij} - \overline{X})(z_{ij} - \overline{Z})$ among the three variables y, x, and z, respectively. Also C_y^2, C_x^2, and C_z^2 are the known population coefficients of variation of the study variable and the auxiliary variables, respectively.

To obtain the properties of the estimators up to first order of approximation, we use the following errors terms:

$e_0 = (\overline{y}_{sys} - \overline{Y})/\overline{Y}$, $e_1 = (\overline{x}_{sys} - \overline{X})/\overline{X}$, and $e_2 = (\overline{z}_{sys} - \overline{Z})/\overline{Z}$, such that $E(e_i) = 0$, for $i = 0, 1$, and 2.

The first order of approximation of the above errors terms is given by

$$\begin{aligned} E\left(e_0^2\right) &= \theta\rho_y^* C_y^2, \\ E\left(e_1^2\right) &= \theta\rho_x^* C_x^2, \\ E\left(e_2^2\right) &= \theta\rho_z^* C_z^2, \\ E\left(e_0 e_1\right) &= \theta k C_x^2 \sqrt{\rho_y^* \rho_x^*}, \\ E\left(e_0 e_2\right) &= \theta k^* C_z^2 \sqrt{\rho_y^* \rho_z^*}, \\ E\left(e_1 e_2\right) &= \theta k^{**} C_z^2 \sqrt{\rho_x^* \rho_z^*}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \lambda &= \left(\frac{N-1}{nN}\right), \\ \rho_{yx} &= \frac{S_{yx}}{S_y S_x}, \\ \rho_{yz} &= \frac{S_{yz}}{S_y S_z}, \\ \rho_{xz} &= \frac{S_{xz}}{S_x S_z}, \\ k &= \frac{\rho_{yx} C_y}{C_x}, \\ k^* &= \frac{\rho_{yz} C_y}{C_z}, \\ \rho_y^* &= \left\{1 + (n-1)\rho_y\right\}, \\ \rho_x^* &= \left\{1 + (n-1)\rho_x\right\}, \\ \rho_z^* &= \left\{1 + (n-1)\rho_z\right\}, \\ \rho^{**} &= \frac{\rho_y^*}{\rho_x^*}, \\ \rho_2^{**} &= \frac{\rho_y^*}{\rho_z^*}, \\ \rho_1^{**} &= \frac{\rho_x^*}{\rho_z^*}, \end{aligned} \tag{2}$$

where ρ_y, ρ_x, and ρ_z are the intraclass correlation among the pair of units for the variables y, x, and z, respectively.

The variance of the usual unbiased estimator for population mean is

$$\operatorname{Var}(t_0) = \lambda \overline{Y}^2 \rho_y^* C_y^2. \tag{3}$$

The classical ratio and product estimators for finite population mean suggested by Swain [14] and Shukla [16] are given by

$$\begin{aligned} t_1 &= \overline{y}_{sy}\left(\frac{\overline{X}}{\overline{x}_{sy}}\right), \\ t_2 &= \overline{y}_{sy}\exp\left(\frac{\overline{z}_{sy}}{\overline{Z}}\right). \end{aligned} \tag{4}$$

The mean square errors of the estimators, to the first order of approximation, are given as follows:

$$\operatorname{MSE}(t_1) = \lambda \overline{Y}^2 \left[\rho_y^* C_y^2 + \rho_x^* C_x^2 \left(1 - 2k\sqrt{\rho^{**}}\right)\right], \tag{5}$$

$$\operatorname{MSE}(t_2) = \lambda \overline{Y}^2 \left[\rho_y^* C_y^2 + \rho_z^* C_z^2 \left(1 + 2k^* \sqrt{\rho_2^{**}}\right)\right]. \tag{6}$$

The usual regression estimator, using single auxiliary variable and its variance, is given as follows:

$$t_3 = \overline{y}_{sy} + b\left(\overline{X} - \overline{x}_{sy}\right), \tag{7}$$

$$\operatorname{MSE}(t_3) = \lambda \rho_y^* S_y^2 \left[1 - \rho_{yx}^2\right]. \tag{8}$$

Utilizing the known knowledge of the auxiliary variable, Singh et al. [20] suggested the following ratio and product type exponential estimators:

$$\begin{aligned} t_4 &= \overline{y}_{sy}\exp\left(\frac{\overline{X} - \overline{x}_{sy}}{\overline{X} + \overline{x}_{sy}}\right), \\ t_5 &= \overline{y}_{sy}\exp\left(\frac{\overline{x}_{sy} - \overline{X}}{\overline{x}_{sy} + \overline{X}}\right). \end{aligned} \tag{9}$$

The mean square errors of the estimators up to first order of approximation are given by

$$\operatorname{MSE}(t_4) = \lambda \overline{Y}^2 \left[\rho_y^* C_y^2 + \frac{\rho_x^* C_x^2}{4}\left(1 - 4k\sqrt{\rho^{**}}\right)\right], \tag{10}$$

$$\operatorname{MSE}(t_5) = \lambda \overline{Y}^2 \left[\rho_y^* C_y^2 + \frac{\rho_x^* C_x^2}{4}\left(1 + 4k\sqrt{\rho^{**}}\right)\right]. \tag{11}$$

After that, Tailor et al. [25] define the following ratio-cum-product estimator for the population mean $\overline{Y}$:

$$t_6 = \overline{y}_{sy}\left(\frac{\overline{X}}{\overline{x}_{sy}}\right)\left(\frac{\overline{z}_{sy}}{\overline{Z}}\right). \tag{12}$$

The mean square error of the estimator t_6, up to first order of approximation, is given by

$$\begin{aligned} \operatorname{MSE}(t_6) = \lambda \overline{Y}^2 \Big[&\rho_y^* C_y^2 + \rho_x^* C_x^2 \left(1 - 2k\sqrt{\rho^{**}}\right) \\ &+ \rho_z^* C_z^2 \left(1 - 2k^{**}\sqrt{\rho_1^{**}}\right) + 2k^* C_z^2 \sqrt{\rho_y^* \rho_z^*}\Big]. \end{aligned} \tag{13}$$

2. Proposed Estimator

In this section, we have proposed the following regression in ratio-cum-product type estimator for the unknown population mean under systematic sampling:

$$t_m = \overline{y}_{sy}\left(\frac{\overline{X}}{\overline{X}+b_{yx}\left(\overline{x}_{sy}-\overline{X}\right)}\right)^{\delta_1} \cdot \left(\frac{\overline{Z}+b_{yz}\left(\overline{z}_{sy}-\overline{Z}\right)}{\overline{Z}}\right)^{\delta_2}, \tag{14}$$

where δ_1 and δ_2 are the unknown constants, whose values are to be found for the minimum mean square error.

The mean square error (MSE) of the estimator up to first order of approximation is

$$\begin{aligned}\mathrm{MSE}\left(t_m\right) = \lambda\overline{Y}^2 \Big[&\rho_y^* C_y^2 + \delta_1^2\beta_{yx}^2\rho_x^* C_x^2 + \delta_2^2\beta_{yz}^2\rho_z^* C_z^2 \\ &- 2\delta_1\beta_{yx}kC_x^2\sqrt{\rho_y^*\rho_x^*} + 2\delta_2\beta_{yz}k^* C_z^2\sqrt{\rho_y^*\rho_z^*} \\ &- 2\delta_1\delta_2\beta_{yx}\beta_{yz}k^{**}C_z^2\sqrt{\rho_x^*\rho_z^*}\Big].\end{aligned} \tag{15}$$

On differentiating (15), with respect to δ_1 and δ_2, we obtain the minimum mean squared error of the estimator t_m, which is given by

$$\begin{aligned}&\mathrm{MSE}\left(t_m\right) \\ &\quad = \lambda\overline{Y}^2\rho_y^*\left[C_y^2 - k_x^{2*}C_z^2 - \frac{\left(k^*k^{**}C_z^2 - kC_x^2\right)^2}{\left(C_x^2 - k^{2**}C_z^2\right)}\right],\end{aligned} \tag{16}$$

where the optimum values are $\delta_1 = \sqrt{\rho_y^*}(kC_x^2 - k^*k^{**}C_z^2)/\beta_{yx}\sqrt{\rho_x^*}(C_x^2 - k^{2**}C_z^2)$ and $\delta_2 = (\sqrt{\rho_y^*}/\beta_{yz}\sqrt{\rho_z^*})\{k^{**}(kC_x^2 - k^*k^{**}C_z^2)/(C_x^2 - k^{2**}C_z^2) - k^*\}$.

3. Comparison

In this section, we have compared the MSE of the proposed estimator with the MSEs of simple estimator, Swain [14] estimator, Shukla [16] estimator, Singh et al. [20] estimators, and Tailor et al. [25] estimator and found some theoretical conditions under which the proposed estimator will always perform better:

(i) By (16) and (3), $\mathrm{MSE}(t_m) \le \mathrm{MSE}(t_0)$ if

$$\left[k_x^{2*}C_z^2 + \frac{\left(k^*k^{**}C_z^2 - kC_x^2\right)^2}{\left(C_x^2 - k^{2**}C_z^2\right)}\right] \ge 0. \tag{17}$$

(ii) By (16) and (5), $\mathrm{MSE}(t_m) \le \mathrm{MSE}(t_1)$ if

$$\begin{aligned}&\left[\rho_y^*\left\{k_x^{2*}C_z^2 + \frac{\left(k^*k^{**}C_z^2 - kC_x^2\right)^2}{\left(C_x^2 - k^{2**}C_z^2\right)}\right\}\right. \\ &\quad \left. + \rho_x^* C_x^2\left(1 - 2k\sqrt{\rho^{**}}\right)\right] \ge 0.\end{aligned} \tag{18}$$

(iii) By (16) and (6), $\mathrm{MSE}(t_m) \le \mathrm{MSE}(t_2)$ if

$$\begin{aligned}&\left[\rho_y^*\left\{k_x^{2*}C_z^2 + \frac{\left(k^*k^{**}C_z^2 - kC_x^2\right)^2}{\left(C_x^2 - k^{2**}C_z^2\right)}\right\}\right. \\ &\quad \left. + \rho_z^* C_z^2\left(1 + 2k^*\sqrt{\rho_2^{**}}\right)\right] \ge 0.\end{aligned} \tag{19}$$

(iv) By (16) and (8), $\mathrm{MSE}(t_m) \le \mathrm{MSE}(t_3)$ if

$$\left[k_x^{2*}C_z^2 - C_y^2\rho_{yx}^2 + \frac{\left(k^*k^{**}C_z^2 - kC_x^2\right)^2}{\left(C_x^2 - k^{2**}C_z^2\right)}\right] \ge 0. \tag{20}$$

(v) By (16) and (10), $\mathrm{MSE}(t_m) \le \mathrm{MSE}(t_4)$ if

$$\begin{aligned}&\left[\rho_y^*\left\{k_x^{2*}C_z^2 + \frac{\left(k^*k^{**}C_z^2 - kC_x^2\right)^2}{\left(C_x^2 - k^{2**}C_z^2\right)}\right\}\right. \\ &\quad \left. + \frac{\rho_x^* C_x^2}{4}\left(1 - 4k\sqrt{\rho^{**}}\right)\right] \ge 0.\end{aligned} \tag{21}$$

(vi) By (16) and (11), $\mathrm{MSE}(t_m) \le \mathrm{MSE}(t_5)$ if

$$\begin{aligned}&\left[\frac{\rho_x^* C_x^2}{4}\left(1 + 4k\sqrt{\rho^{**}}\right)\right. \\ &\quad \left. + \rho_y^*\left\{k_x^{2*}C_z^2 + \frac{\left(k^*k^{**}C_z^2 - kC_x^2\right)^2}{\left(C_x^2 - k^{2**}C_z^2\right)}\right\}\right] \ge 0.\end{aligned} \tag{22}$$

(vii) By (16) and (13), $\mathrm{MSE}(t_m) \le \mathrm{MSE}(t_6)$ if

$$\begin{aligned}&\left[\rho_x^* C_x^2\left(1 - 2k\sqrt{\rho^{**}}\right) + \rho_z^* C_z^2\left(1 - 2k^{**}\sqrt{\rho_1^{**}}\right)\right. \\ &\quad + 2k^* C_z^2\sqrt{\rho_y^*\rho_z^*} \\ &\quad \left. + \rho_y^*\left\{k_x^{2*}C_z^2 + \frac{\left(k^*k^{**}C_z^2 - kC_x^2\right)^2}{\left(C_x^2 - k^{2**}C_z^2\right)}\right\}\right] \ge 0.\end{aligned} \tag{23}$$

Table 1: The percent relative efficiency of different estimators with respect to t_0.

	Population	
Estimator	MSE(t_α)	PRE(t_α, t_0)
t_0	1455.08	100.00
t_1	373.32	389.62
t_2	768.06	189.45
t_3	45.52	3196.57
t_4	820.09	177.43
t_5	1044.42	139.32
t_6	187.08	777.79
t_m	22.74	6400.00

4. Numerical Comparison

For comparing the theoretical efficiency conditions of the different estimators numerically, we have used the following real data set.

Population 1 (source: Tailor et al. [25]). Consider

$$\begin{aligned} N &= 15, \\ n &= 3, \\ \overline{X} &= 44.47, \\ \overline{Y} &= 80, \\ \overline{Z} &= 48.40, \\ C_y &= 0.56, \\ C_x &= 0.28, \\ C_z &= 0.43, \\ S_y^2 &= 2000, \\ S_x^2 &= 149.55, \\ S_z^2 &= 427.83, \\ S_{yx} &= 538.57, \\ S_{yz} &= -902.86, \\ S_{xz} &= -241.06, \\ \rho_{yx} &= 0.9848, \\ \rho_{yz} &= -0.9760, \\ \rho_{xz} &= -0.9530, \\ \rho_y &= 0.6652, \\ \rho_x &= 0.707, \\ \rho_z &= 0.5487. \end{aligned} \tag{24}$$

For the percent relative efficiencies (PREs) of the estimator, we use the following formula and the results are shown in Table 1:

PRE(t_α, t_0) = MSE(t_0)/MSE(t_α) × 100, for $\alpha = 0, 1, 2, 3, 4, 5, 6$, and m.

5. Conclusion

A chain ratio-type estimator is proposed under double sampling scheme using two auxiliary variables, and the properties of the proposed estimator are derived up to first order of approximations. Both theoretically and empirically, it has been shown that the recommended estimator performed better than the other competing estimators in terms of higher percent relative efficiency. Hence, looking on the dominance nature of the proposed estimator may be suggested for its practical applications.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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