Supplementary Material

Appendix A: Lemmas

Lemma A1: (Trenkler and Toutenburg, 1990)
Let $\hat{\beta}_1$ and $\hat{\beta}_2$ be two linear estimator of β. Suppose that $D = D(\hat{\beta}_1) - D(\hat{\beta}_2)$ is positive definite, then $\Delta = MSEM(\hat{\beta}_1) - MSEM(\hat{\beta}_2)$ is non negative if and only if $b_2'(D + b_1 b_1')^{-1} b_2 \leq 1$, where $D(\hat{\beta}_j)$, $MSE(\hat{\beta}_j)$ and b_j denote dispersion matrix, mean square error matrix and bias vector of $\hat{\beta}_j$ respectively, $j = 1, 2$.

Lemma A2: (Wang et al., 2006)
Let $n \times n$ matrices $M > 0, N \geq 0$, then $M > N$ if and only if $\lambda_* < 1$, where λ_* is the largest eigenvalue of the matrix NM^{-1}.

Lemma A3: (Baksalary and Kala, 1983)
Let B≥ 0 of type $n \times n$ matrix, b is a $n \times 1$ vector and λ is a positive real number. Then the following conditions are equivalent.

i. $\lambda B - bb' \geq 0$

ii. $B \geq 0$, $b \in \Re(B)$ and $b'B^{-1}b \leq \lambda$, where $\Re(B)$ stands for column space of B and B^{-1} is a independent choice of g-inverse of B.

Appendix B: Tables

Table B1. Bias vector, Dispersion matrix and MSEM of the estimators

Estimators ($\hat{\boldsymbol{\gamma}}_{(j)}$)	$\boldsymbol{Bias}(\hat{\boldsymbol{\gamma}}_{(j)})$, $\boldsymbol{D}(\hat{\boldsymbol{\gamma}}_{(j)})$ and $\boldsymbol{MSEM}(\hat{\boldsymbol{\gamma}}_{(j)})$
$\hat{\gamma}_{MRE}$	$Bias(\hat{\gamma}_{MRE}) = \tau A$ $D(\hat{\gamma}_{MRE}) = \sigma^2\tau$ $MSEM(\hat{\gamma}_{MRE}) = \sigma^2\tau + (\tau A)(\tau A)'$
$\hat{\gamma}_{SRRE}$	$Bias(\hat{\gamma}_{SRRE}) = (1+k)^{-1}(\tau A - k\gamma)$ $D(\hat{\gamma}_{SRRE}) = (1+k)^{-2}\sigma^2\tau$ $MSEM(\hat{\gamma}_{SRRE}) = (1+k)^{-2}(\sigma^2\tau + (\tau A - k\gamma)(\tau A - k\gamma)')$
$\hat{\gamma}_{SRAURE}$	$Bias(\hat{\gamma}_{SRAURE}) = (1+k)^{-2}\left((1+2k)\tau A - k^2\gamma\right)$ $D(\hat{\gamma}_{SRAURE}) = (1+k)^{-4}(1+2k)^2\sigma^2\tau$ $MSEM(\hat{\gamma}_{SRAURE}) = (1+k)^{-4}\Big((1+2k)^2\sigma^2\tau + \left((1+2k)\tau A - k^2\gamma\right)\big((1+2k)\tau A - k^2\gamma\big)'\Big)$
$\hat{\gamma}_{SRLE}$	$Bias(\hat{\gamma}_{SRLE}) = 2^{-1}\big((1+d)\tau A - (1-d)\gamma\big)$ $D(\hat{\gamma}_{SRLE}) = 2^{-2}(1+d)^2\sigma^2\tau$ $MSEM(\hat{\gamma}_{SRLE}) = 2^{-2}\Big((1+d)^2\sigma^2\tau + \big((1+d)\tau A - (1-d)\gamma\big)\big((1+d)\tau A - (1-d)\gamma\big)'\Big)$
$\hat{\gamma}_{SRAULE}$	$Bias(\hat{\gamma}_{SRAULE}) = 2^{-2}((1+d)(3-d)\tau A - (1-d)^2\gamma)$ $D(\hat{\gamma}_{SRAULE}) = 2^{-4}(1+d)^2(3-d)^2\sigma^2\tau$ $MSEM(\hat{\gamma}_{SRAULE}) = 2^{-4}\Big((1+d)^2(3-d)^2\sigma^2\tau + \big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)'\Big)$
$\hat{\gamma}_{SRPCR}$	$Bias(\hat{\gamma}_{SRPCR}) = (T_hT_h' - I)\gamma + T_hT_h'\tau A$ $D(\hat{\gamma}_{SRPCR}) = \sigma^2T_hT_h'\tau T_h'T_h$ $MSEM(\hat{\gamma}_{SRPCR}) = \sigma^2T_hT_h'\tau T_h'T_h + \big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big)'$
$\hat{\gamma}_{SRrk}$	$Bias(\hat{\gamma}_{SRrk}) = (1+k)^{-1}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)$ $D(\hat{\gamma}_{SRrk}) = (1+k)^{-2}\sigma^2T_hT_h'\tau T_h'T_h$ $MSEM(\hat{\gamma}_{SRrk}) = (1+k)^{-2}\Big(\sigma^2T_hT_h'\tau T_h'T_h + \big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)'\Big)$
$\hat{\gamma}_{SRrd}$	$Bias(\hat{\gamma}_{SRrd}) = 2^{-1}(1+d)\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)$ $D(\hat{\gamma}_{SRrd}) = 2^{-2}(1+d)^2\sigma^2T_hT_h'\tau T_h'T_h$ $MSEM(\hat{\gamma}_{SRrd}) = 2^{-2}(1+d)^2\Big(\sigma^2T_hT_h'\tau T_h'T_h + \big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)'\Big)$

Table B2. Estimated SMSE values of the estimators

k/d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(l, p) = (4, 0)$									
$SMSE(\hat{\gamma}_{MRE})$	**0.119**	**0.119**	**0.119**	**0.119**	**0.119**	**0.119**	**0.119**	**0.119**	**0.119**
$SMSE(\hat{\gamma}_{SRRE})$	0.545	1.672	3.174	4.860	6.621	8.388	10.124	11.805	13.420
$SMSE(\hat{\gamma}_{SRAURE})$	0.116	0.142	0.246	0.456	0.777	1.206	1.729	2.333	3.002
$SMSE(\hat{\gamma}_{SRLE})$	12.104	9.550	7.302	5.359	3.722	2.390	1.364	0.644	0.229
$SMSE(\hat{\gamma}_{SRAULE})$	2.450	1.545	0.930	0.536	0.304	0.182	0.131	0.117	0.117
$SMSE(\hat{\gamma}_{SRPCR})$	3.336	3.336	3.336	3.336	3.336	3.336	3.336	3.336	3.336
$SMSE(\hat{\gamma}_{rk})$	3.695	4.731	6.131	7.713	9.369	11.035	12.674	14.263	15.791
$SMSE(\hat{\gamma}_{rd})$	14.546	12.132	10.011	8.182	6.644	5.399	4.445	3.784	3.414
$(l, p) = (3, 1)$									
$SMSE(\hat{\gamma}_{MRE})$	2.974	2.974	2.974	2.974	2.974	2.974	2.974	2.974	2.974
$SMSE(\hat{\gamma}_{SRRE})$	1.171	**0.395**	**0.255**	**0.511**	1.014	1.668	2.411	3.202	4.014
$SMSE(\hat{\gamma}_{SRAURE})$	2.771	2.323	1.803	1.312	0.900	0.588	**0.379**	**0.269**	**0.248**
$SMSE(\hat{\gamma}_{SRLE})$	3.349	2.156	1.251	0.634	**0.305**	**0.263**	0.509	1.043	1.865
$SMSE(\hat{\gamma}_{SRAULE})$	**0.259**	0.437	0.767	1.185	1.633	2.064	2.440	2.730	2.912
$SMSE(\hat{\gamma}_{SRPCR})$	13.712	13.712	13.712	13.712	13.712	13.712	13.712	13.712	13.712
$SMSE(\hat{\gamma}_{rk})$	13.273	13.276	13.541	13.959	14.464	15.015	15.586	16.159	16.725
$SMSE(\hat{\gamma}_{rd})$	16.263	15.395	14.673	14.097	13.667	13.384	13.247	13.256	13.411
$(l, p) = (2, 2)$									
$SMSE(\hat{\gamma}_{MRE})$	8.499	8.499	8.499	8.499	8.499	8.499	8.499	8.499	8.499
$SMSE(\hat{\gamma}_{SRRE})$	4.873	2.617	1.251	0.477	**0.102**	**0.001**	**0.085**	**0.297**	**0.598**
$SMSE(\hat{\gamma}_{SRAURE})$	8.128	7.285	6.253	5.197	4.203	3.314	2.544	1.897	1.365
$SMSE(\hat{\gamma}_{SRLE})$	**0.347**	**0.040**	**0.036**	**0.336**	0.938	1.844	3.053	4.565	6.381
$SMSE(\hat{\gamma}_{SRAULE})$	1.792	2.787	3.847	4.904	5.898	6.780	7.508	8.051	8.386
$SMSE(\hat{\gamma}_{SRPCR})$	12.232	12.232	12.232	12.232	12.232	12.232	12.232	12.232	12.232
$SMSE(\hat{\gamma}_{rk})$	12.916	13.542	14.111	14.627	15.095	15.521	15.910	16.265	16.591
$SMSE(\hat{\gamma}_{rd})$	16.326	15.784	15.264	14.765	14.288	13.834	13.400	12.989	12.599

Table B3. Estimated SMSE values of the predictors

k/d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(l, p) = (4, 0)$									
$SMSE(\hat{y}_{MRE})$	7954.9	7954.9	7954.9	7954.9	7954.9	7954.9	7954.9	7954.9	7954.9
$SMSE(\hat{y}_{SRRE})$	6460.6	5334.0	4464.9	3781.5	**3235.0**	**2791.9**	**2428.0**	**2125.8**	**1872.5**
$SMSE(\hat{y}_{SRAURE})$	7812.7	7481.8	7060.7	6606.0	6149.6	5709.1	5293.2	4905.9	4548.1
$SMSE(\hat{y}_{SRLE})$	**2076.5**	**2541.7**	**3053.8**	**3613.0**	4219.3	4872.4	5572.6	6319.7	7113.8
$SMSE(\hat{y}_{SRAULE})$	4838.6	5428.8	5977.7	6474.8	6910.9	7278.3	7570.6	7783.0	7911.8
$SMSE(\hat{y}_{SRPCR})$	7954.8	7954.8	7954.8	7954.8	7954.8	7954.8	7954.8	7954.8	7954.8
$SMSE(\hat{y}_{SRrk})$	6460.5	5333.9	4464.8	3781.4	3235.1	2790.0	2428.1	2125.9	1872.6
$SMSE(\hat{y}_{SRrd})$	2076.6	2541.8	3053.9	3613.1	4219.2	4872.3	5572.5	6319.6	7113.7
$(l, p) = (3, 1)$									
$SMSE(\hat{y}_{MRE})$	3496.8	3496.8	3496.8	3496.8	3496.8	3496.8	3496.8	3496.8	3496.8
$SMSE(\hat{y}_{SRRE})$	2966.2	2557.4	2235.1	1976.2	**1764.7**	**1589.4**	**1442.4**	**1317.7**	**1210.9**
$SMSE(\hat{y}_{SRAURE})$	3446.7	3330.0	3180.7	3018.3	2854.2	2694.5	2542.4	2399.5	2266.3
$SMSE(\hat{y}_{SRLE})$	**1297.0**	**1488.7**	**1693.5**	**1911.5**	2142.7	2387.1	2644.7	2915.5	3199.5
$SMSE(\hat{y}_{SRAULE})$	2374.6	2592.1	2792.0	2971.3	3127.3	3257.9	3361.4	3436.3	3481.6
$SMSE(\hat{y}_{SRPCR})$	5417.8	5417.8	5417.8	5417.8	5417.8	5417.8	5417.8	5417.8	5417.8
$SMSE(\hat{y}_{SRrk})$	4572.5	3922.8	3412.0	3002.4	2668.7	2392.8	2161.8	1966.4	1799.3
$SMSE(\hat{y}_{SRrd})$	1934.1	2234.4	2556.5	2900.2	3265.6	3652.7	4061.5	4491.9	4944.0
$(l, p) = (2, 2)$									
$SMSE(\hat{y}_{MRE})$	4864.5	4864.5	4864.5	4864.5	4864.5	4864.5	4864.5	4864.5	4864.5
$SMSE(\hat{y}_{SRRE})$	4110.3	3530.3	3073.9	2707.9	2409.5	2162.6	1955.9	1780.8	1631.1
$SMSE(\hat{y}_{SRAURE})$	4793.3	4627.3	4415.0	4184.3	3951.3	3724.7	3509.1	3306.6	3118.0
$SMSE(\hat{y}_{SRLE})$	1751.9	2020.9	2309.1	2616.5	2943.2	3289.0	3654.1	4038.3	4441.8
$SMSE(\hat{y}_{SRAULE})$	3271.3	3579.5	3863.0	4117.5	4339.1	4524.8	4671.9	4778.4	4842.9
$SMSE(\hat{y}_{SRPCR})$	847.0	847.0	847.0	847.0	847.0	847.0	847.0	847.0	847.0
$SMSE(\hat{y}_{SRrk})$	737.8	652.6	584.6	529.3	**483.6**	**445.3**	**412.8**	**385.0**	**360.9**
$SMSE(\hat{y}_{SRrd})$	**380.3**	**423.1**	**468.1**	**515.4**	565.0	616.8	670.9	727.3	786.0

Table B4. Estimated SMSE values of the estimators when $\mathrm{n} = 50$ and $\rho = 0.9$

k/d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(l, p) = (3, 0)$									
$SMSE(\hat{\gamma}_{MRE})$	**5.73**	**5.73**	**5.73**	**5.73**	**5.73**	**5.73**	**5.73**	5.73	5.73
$SMSE(\hat{\gamma}_{SRRE})$	7.38	12.01	18.23	25.25	32.60	39.98	47.23	54.27	61.03
$SMSE(\hat{\gamma}_{SRAURE})$	5.70	5.78	6.18	7.01	8.32	10.08	12.24	14.74	17.52
$SMSE(\hat{\gamma}_{SRLE})$	55.52	44.84	35.44	27.33	20.51	14.98	10.74	7.78	6.11
$SMSE(\hat{\gamma}_{SRAULE})$	15.23	11.48	8.95	7.34	6.40	5.93	5.74	**5.70**	**5.71**
$SMSE(\hat{\gamma}_{SRPCR})$	165.01	165.01	165.01	165.01	165.01	165.01	165.01	165.01	165.01
$SMSE(\hat{\gamma}_{SRrk})$	165.18	166.59	168.68	171.13	173.74	176.40	179.04	181.60	184.08
$SMSE(\hat{\gamma}_{SRrd})$	182.06	178.16	174.76	171.87	169.47	167.58	166.18	165.29	164.90
$(l, p) = (4, 1)$									
$SMSE(\hat{\gamma}_{MRE})$	31.497	31.497	31.497	31.497	31.497	31.497	31.497	31.497	31.497
$SMSE(\hat{\gamma}_{SRRE})$	17.883	9.773	5.207	2.969	**2.280**	**2.631**	**3.679**	**5.192**	7.009
$SMSE(\hat{\gamma}_{SRAURE})$	30.085	26.888	23.008	19.078	15.433	12.229	9.522	7.312	**5.568**
$SMSE(\hat{\gamma}_{SRLE})$	**5.503**	**3.268**	**2.314**	**2.640**	4.248	7.136	11.305	16.755	23.486
$SMSE(\hat{\gamma}_{SRAULE})$	6.961	10.369	14.144	17.998	21.683	24.985	27.733	29.792	31.066
$SMSE(\hat{\gamma}_{SRPCR})$	44.136	44.136	44.136	44.136	44.136	44.136	44.136	44.136	44.136
$SMSE(\hat{\gamma}_{SRrk})$	37.801	34.469	33.032	32.809	33.368	34.431	35.814	37.393	39.085
$SMSE(\hat{\gamma}_{SRrd})$	37.694	35.326	33.729	32.903	32.848	33.563	35.050	37.308	40.336
$(l, p) = (2, 2)$									
$SMSE(\hat{\gamma}_{MRE})$	61.869	61.869	61.869	61.869	61.869	61.869	61.869	61.869	61.869
$SMSE(\hat{\gamma}_{SRRE})$	41.520	27.719	18.282	11.828	**7.455**	**4.559**	**2.724**	**1.661**	**1.161**
$SMSE(\hat{\gamma}_{SRAURE})$	59.848	55.213	49.447	43.407	37.559	32.139	27.250	22.918	19.130
$SMSE(\hat{\gamma}_{SRLE})$	**1.533**	**3.238**	**6.192**	**10.397**	15.851	22.555	30.509	39.712	50.166
$SMSE(\hat{\gamma}_{SRAULE})$	22.190	28.818	35.416	41.703	47.435	52.406	56.448	59.428	61.254
$SMSE(\hat{\gamma}_{SRPCR})$	38.090	38.090	38.090	38.090	38.090	38.090	38.090	38.090	38.090
$SMSE(\hat{\gamma}_{SRrk})$	34.346	32.175	31.013	30.508	30.438	30.657	31.066	31.601	32.216
$SMSE(\hat{\gamma}_{SRrd})$	31.708	30.913	30.494	30.451	30.784	31.493	32.579	34.040	35.877

Table B5. Estimated SMSE values of the estimators when $\mathrm{n} = 50$ and $\rho = 0.99$

k/d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(l, p) = (5, 0)$									
$SMSE(\hat{\gamma}_{MRE})$	5.89	5.89	**5.89**	**5.89**	**5.89**	**5.89**	5.89	5.89	5.89
$SMSE(\hat{\gamma}_{SRRE})$	7.38	11.93	18.13	25.16	32.54	39.97	47.29	54.38	61.21
$SMSE(\hat{\gamma}_{SRAURE})$	**5.84**	**5.88**	6.23	7.02	8.29	10.02	12.16	14.65	17.42
$SMSE(\hat{\gamma}_{SRLE})$	55.65	44.87	35.41	27.25	20.41	14.88	10.67	7.76	6.17
$SMSE(\hat{\gamma}_{SRAULE})$	15.13	11.41	8.91	7.34	6.44	6.01	**5.85**	**5.84**	**5.87**
$SMSE(\hat{\gamma}_{SRPCR})$	195.79	195.79	195.79	195.79	195.79	195.79	195.79	195.79	195.79
$SMSE(\hat{\gamma}_{SRrk})$	195.66	196.47	197.82	199.45	201.22	203.04	204.85	206.63	208.36
$SMSE(\hat{\gamma}_{SRrd})$	206.95	204.25	201.91	199.94	198.34	197.10	196.22	195.71	195.57
$(l, p) = (4, 1)$									
$SMSE(\hat{\gamma}_{MRE})$	19.939	19.939	19.939	19.939	19.939	19.939	19.939	19.939	19.939
$SMSE(\hat{\gamma}_{SRRE})$	10.064	**5.145**	**3.333**	**3.494**	4.914	7.131	9.844	12.850	16.014
$SMSE(\hat{\gamma}_{SRAURE})$	18.862	16.462	13.629	10.873	8.458	6.497	**5.017**	**4.004**	**3.417**
$SMSE(\hat{\gamma}_{SRLE})$	13.416	8.899	5.691	3.795	**3.209**	**3.934**	5.969	9.315	13.972
$SMSE(\hat{\gamma}_{SRAULE})$	**3.866**	5.457	7.647	10.142	12.685	15.060	17.091	18.640	19.609
$SMSE(\hat{\gamma}_{SRPCR})$	47.194	47.194	47.194	47.194	47.194	47.194	47.194	47.194	47.194
$SMSE(\hat{\gamma}_{SRrk})$	42.791	41.218	41.373	42.592	44.459	46.710	49.175	51.742	54.339
$SMSE(\hat{\gamma}_{SRrd})$	52.213	48.337	45.290	43.073	41.686	41.128	41.400	42.502	44.433
$(l, p) = (3, 2)$									
$SMSE(\hat{\gamma}_{MRE})$	49.955	49.955	49.955	49.955	49.955	49.955	49.955	49.955	49.955
$SMSE(\hat{\gamma}_{SRRE})$	31.645	19.652	11.823	6.804	**3.718**	**1.979**	**1.190**	**1.076**	**1.443**
$SMSE(\hat{\gamma}_{SRAURE})$	48.113	43.906	38.710	33.317	28.157	23.440	19.254	15.617	12.509
$SMSE(\hat{\gamma}_{SRLE})$	**1.112**	**1.367**	**2.914**	**5.755**	9.889	15.316	22.036	30.049	39.355
$SMSE(\hat{\gamma}_{SRAULE})$	15.014	20.589	26.283	31.807	36.908	41.372	45.025	47.731	49.394
$SMSE(\hat{\gamma}_{SRPCR})$	44.297	44.297	44.297	44.297	44.297	44.297	44.297	44.297	44.297
$SMSE(\hat{\gamma}_{SRrk})$	40.730	38.850	38.034	37.900	38.206	38.796	39.565	40.445	41.389
$SMSE(\hat{\gamma}_{SRrd})$	40.613	39.293	38.406	37.951	37.928	38.337	39.179	40.452	42.158

Table B6. Estimated SMSE values of the estimators when $\mathrm{n} = 50$ and $\rho = 0.999$

k/d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(l, p) = (5,\ 0)$									
$SMSE(\hat{\gamma}_{MRE})$	**2.39**	**2.39**	**2.39**	**2.39**	**2.39**	**2.39**	**2.39**	**2.39**	**2.39**
$SMSE(\hat{\gamma}_{SRRE})$	4.52	9.46	15.90	23.06	30.49	37.93	45.21	52.26	59.02
$SMSE(\hat{\gamma}_{SRAURE})$	2.41	2.60	3.13	4.11	5.55	7.44	9.71	12.30	15.16
$SMSE(\hat{\gamma}_{SRLE})$	53.51	42.81	33.36	25.17	18.23	12.55	8.13	4.96	3.05
$SMSE(\hat{\gamma}_{SRAULE})$	12.80	8.91	6.23	4.47	3.41	2.81	2.53	2.42	2.40
$SMSE(\hat{\gamma}_{SRPCR})$	196.31	196.31	196.31	196.31	196.31	196.31	196.31	196.31	196.31
$SMSE(\hat{\gamma}_{SRrk})$	196.66	197.70	199.12	200.72	202.39	204.08	205.74	207.35	208.90
$SMSE(\hat{\gamma}_{SRrd})$	207.64	205.19	203.05	201.19	199.64	198.38	197.41	196.75	196.38
$(l, p) = (4,\ 1)$									
$SMSE(\hat{\gamma}_{MRE})$	13.160	13.160	13.160	13.160	13.160	13.160	13.160	13.160	13.160
$SMSE(\hat{\gamma}_{SRRE})$	5.343	**2.031**	**1.501**	**2.706**	4.988	7.929	11.255	14.786	18.405
$SMSE(\hat{\gamma}_{SRAURE})$	12.276	10.327	8.073	5.949	4.176	2.841	**1.962**	**1.516**	**1.461**
$SMSE(\hat{\gamma}_{SRLE})$	15.440	10.116	6.059	3.270	**1.749**	**1.496**	2.510	4.792	8.342
$SMSE(\hat{\gamma}_{SRAULE})$	**1.478**	2.206	3.608	5.401	7.336	9.204	10.835	12.095	12.889
$SMSE(\hat{\gamma}_{SRPCR})$	45.790	45.790	45.790	45.790	45.790	45.790	45.790	45.790	45.790
$SMSE(\hat{\gamma}_{SRrk})$	42.368	41.593	42.412	44.190	46.535	49.198	52.022	54.906	57.784
$SMSE(\hat{\gamma}_{SRrd})$	55.431	51.070	47.532	44.815	42.922	41.850	41.602	42.175	43.571
$(l, p) = (3,\ 2)$									
$SMSE(\hat{\gamma}_{MRE})$	42.748	42.748	42.748	42.748	42.748	42.748	42.748	42.748	42.748
$SMSE(\hat{\gamma}_{SRRE})$	26.110	15.416	8.618	4.435	**2.035**	**0.870**	**0.566**	**0.866**	**1.589**
$SMSE(\hat{\gamma}_{SRAURE})$	41.064	37.224	32.498	27.618	22.976	18.767	15.066	11.885	9.204
$SMSE(\hat{\gamma}_{SRLE})$	**0.971**	**0.590**	**1.465**	**3.595**	6.982	11.624	17.521	24.675	33.084
$SMSE(\hat{\gamma}_{SRAULE})$	11.362	16.241	21.300	26.256	30.864	34.916	38.244	40.715	42.235
$SMSE(\hat{\gamma}_{SRPCR})$	44.641	44.641	44.641	44.641	44.641	44.641	44.641	44.641	44.641
$SMSE(\hat{\gamma}_{SRrk})$	41.468	39.910	39.365	39.461	39.965	40.726	41.646	42.659	43.722
$SMSE(\hat{\gamma}_{SRrd})$	42.850	41.326	40.233	39.571	39.339	39.538	40.168	41.229	42.720

Table B7. Estimated SMSE values of the predictors when $n = 50$ and $\rho = 0.9$

k/d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(l, p) = (5, 0)$									
$SMSE(\hat{y}_{MRE})$	26713	26713	26713	26713	26713	26713	26713	26713	26713
$SMSE(\hat{y}_{SRRE})$	22354	19021	16413	14333	12646	11258	10101	9127	8298
$SMSE(\hat{y}_{SRAURE})$	26300	25340	24112	22781	21439	20136	18900	17741	16665
$SMSE(\hat{y}_{SRLE})$	8966	10464	12081	13816	15669	17641	19731	21940	24267
$SMSE(\hat{y}_{SRAULE})$	17540	19304	20931	22396	23674	24747	25598	26214	26588
$SMSE(\hat{y}_{SRPCR})$	26257	26257	26257	26257	26257	26257	26257	26257	26257
$SMSE(\hat{y}_{SRrk})$	21978	18705	16144	14101	**12444**	**11080**	**9944**	**8987**	**8172**
$SMSE(\hat{y}_{SRrd})$	**8829**	**10301**	**11889**	**13593**	15413	17350	19402	21571	23856
$(l, p) = (4, 1)$									
$SMSE(\hat{y}_{MRE})$	16251	16251	16251	16251	16251	16251	16251	16251	16251
$SMSE(\hat{y}_{SRRE})$	13669	11690	10139	8898	7891	7060	6366	5781	5282
$SMSE(\hat{y}_{SRAURE})$	16007	15438	14711	13922	13126	12352	11618	10929	10288
$SMSE(\hat{y}_{SRLE})$	5684	6584	7553	8590	9695	10869	12112	13423	14803
$SMSE(\hat{y}_{SRAULE})$	10809	11858	12824	13693	14451	15087	15591	15956	16177
$SMSE(\hat{y}_{SRPCR})$	8636	8636	8636	8636	8636	8636	8636	8636	8636
$SMSE(\hat{y}_{SRrk})$	7314	6299	5502	4863	**4342**	**3912**	**3552**	**3248**	**2988**
$SMSE(\hat{y}_{SRrd})$	**3198**	**3666**	**4167**	**4703**	5273	5877	6516	7188	7895
$(l, p) = (3, 2)$									
$SMSE(\hat{y}_{MRE})$	14936	14936	14936	14936	14936	14936	14936	14936	14936
$SMSE(\hat{y}_{SRRE})$	12511	10657	9207	8051	7114	6344	5702	5161	4702
$SMSE(\hat{y}_{SRAURE})$	14707	14172	13489	12748	12002	11277	10590	9945	9347
$SMSE(\hat{y}_{SRLE})$	5072	5903	6800	7764	8794	9890	11052	12280	13575
$SMSE(\hat{y}_{SRAULE})$	9833	10814	11719	12534	13245	13842	14316	14659	14867
$SMSE(\hat{y}_{SRPCR})$	5972	5972	5972	5972	5972	5972	5972	5972	5972
$SMSE(\hat{y}_{SRrk})$	5054	4350	3799	3358	**3000**	**2705**	**2458**	**2250**	**2073**
$SMSE(\hat{y}_{SRrd})$	**2216**	**2536**	**2880**	**3248**	3641	4059	4500	4966	5457

Table B8. Estimated SMSE values of the predictors when $\mathbf{n = 50}$ and $\boldsymbol{\rho = 0.99}$

k/d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(l, p) = (5, 0)$									
$SMSE(\hat{y}_{MRE})$	20010	20010	20010	20010	20010	20010	20010	20010	20010
$SMSE(\hat{y}_{SRRE})$	16798	14337	12410	10871	9620	8591	7731	7007	6390
$SMSE(\hat{y}_{SRAURE})$	19707	18999	18094	17112	16122	15161	14248	13392	12596
$SMSE(\hat{y}_{SRLE})$	6887	8001	9201	10487	11859	13318	14862	16492	18208
$SMSE(\hat{y}_{SRAULE})$	13243	14546	15748	16828	17771	18562	19189	19643	19918
$SMSE(\hat{y}_{SRPCR})$	19969	19969	19969	19969	19969	19969	19969	19969	19969
$SMSE(\hat{y}_{SRrk})$	16763	14309	12386	10849	**9602**	**8574**	**7717**	**6994**	**6378**
$SMSE(\hat{y}_{SRrd})$	**6875**	**7986**	**9184**	**10467**	11836	13291	14832	16458	18171
$(l, p) = (4, 1)$									
$SMSE(\hat{y}_{MRE})$	17211	17211	17211	17211	17211	17211	17211	17211	17211
$SMSE(\hat{y}_{SRRE})$	14481	12389	10748	9436	8369	7490	6756	6135	5607
$SMSE(\hat{y}_{SRAURE})$	16953	16351	15583	14749	13907	13089	12313	11584	10906
$SMSE(\hat{y}_{SRLE})$	6033	6986	8012	9109	10279	11521	12835	14221	15680
$SMSE(\hat{y}_{SRAULE})$	11457	12566	13589	14507	15308	15980	16513	16899	17132
$SMSE(\hat{y}_{SRPCR})$	10589	10589	10589	10589	10589	10589	10589	10589	10589
$SMSE(\hat{y}_{SRrk})$	8960	7709	6725	5937	**5295**	**4765**	**4321**	**3946**	**3625**
$SMSE(\hat{y}_{SRrd})$	**3884**	**4461**	**5080**	**5741**	6444	7189	7976	8805	9676
$(l, p) = (3, 2)$									
$SMSE(\hat{y}_{MRE})$	15859	15859	15859	15859	15859	15859	15859	15859	15859
$SMSE(\hat{y}_{SRRE})$	13285	11318	9779	8552	7556	6738	6056	5482	4994
$SMSE(\hat{y}_{SRAURE})$	15615	15048	14323	13537	12745	11976	11246	10562	9927
$SMSE(\hat{y}_{SRLE})$	5388	6270	7223	8246	9340	10503	11737	13040	14414
$SMSE(\hat{y}_{SRAULE})$	10443	11484	12445	13310	14064	14698	15200	15564	15785
$SMSE(\hat{y}_{SRPCR})$	7664	7664	7664	7664	7664	7664	7664	7664	7664
$SMSE(\hat{y}_{SRrk})$	6471	5557	4840	4267	**3802**	**3418**	**3098**	**2828**	**2597**
$SMSE(\hat{y}_{SRrd})$	**2783**	**3198**	**3646**	**4124**	4635	5177	5751	6357	6995

Table B9. Estimated SMSE values of the predictors when $n = 50$ and $\rho = 0.999$

k/d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(l, p) = (5, 0)$									
$SMSE(\hat{y}_{MRE})$	16692	16692	16692	16692	16692	16692	16692	16692	16692
$SMSE(\hat{y}_{SRRE})$	14034	11998	10401	9125	8088	7234	6520	5918	5405
$SMSE(\hat{y}_{SRAURE})$	16441	15855	15107	14295	13475	12679	11923	11214	10555
$SMSE(\hat{y}_{SRLE})$	5819	6744	7741	8807	9945	11153	12432	13781	15201
$SMSE(\hat{y}_{SRAULE})$	11091	12170	13165	14060	14840	15494	16013	16388	16616
$SMSE(\hat{y}_{SRPCR})$	16689	16689	16689	16689	16689	16689	16689	16689	16689
$SMSE(\hat{y}_{SRrk})$	14031	11995	10399	9123	**8087**	**7232**	**6519**	**5917**	**5404**
$SMSE(\hat{y}_{SRrd})$	**5818**	**6743**	**7739**	**8805**	9943	11151	12429	13778	15198
$(l, p) = (4, 1)$									
$SMSE(\hat{y}_{MRE})$	16427	16427	16427	16427	16427	16427	16427	16427	16427
$SMSE(\hat{y}_{SRRE})$	13834	11845	10284	9036	8021	7184	6484	5893	5389
$SMSE(\hat{y}_{SRAURE})$	16182	15610	14880	14088	13288	12511	11772	11080	10435
$SMSE(\hat{y}_{SRLE})$	5795	6704	7680	8725	9838	11019	12269	13587	14973
$SMSE(\hat{y}_{SRAULE})$	10959	12014	12985	13858	14620	15258	15764	16130	16352
$SMSE(\hat{y}_{SRPCR})$	10765	10765	10765	10765	10765	10765	10765	10765	10765
$SMSE(\hat{y}_{SRrk})$	9110	7839	6840	6039	**5387**	**4848**	**4396**	**4015**	**3689**
$SMSE(\hat{y}_{SRrd})$	**3952**	**4538**	**5168**	**5840**	6554	7311	8111	8953	9837
$(l, p) = (3, 2)$									
$SMSE(\hat{y}_{MRE})$	15149	15149	15149	15149	15149	15149	15149	15149	15149
$SMSE(\hat{y}_{SRRE})$	12696	10820	9352	8182	7233	6452	5801	5253	4787
$SMSE(\hat{y}_{SRAURE})$	14917	14376	13685	12936	12181	11447	10752	10100	9494
$SMSE(\hat{y}_{SRLE})$	5163	6006	6915	7891	8934	10043	11219	12463	13772
$SMSE(\hat{y}_{SRAULE})$	9986	10979	11895	12719	13439	14042	14521	14868	15079
$SMSE(\hat{y}_{SRPCR})$	7736	7736	7736	7736	7736	7736	7736	7736	7736
$SMSE(\hat{y}_{SRrk})$	6527	5600	4874	4294	**3823**	**3435**	**3111**	**2838**	**2605**
$SMSE(\hat{y}_{SRrd})$	**2793**	**3213**	**3665**	**4150**	4667	5216	5798	6412	7058

Appendix C: Corollaries

Corollary C1:

a) $\hat{\gamma}_{SRRE}$ is superior to $\hat{\gamma}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$(\tau A - k\gamma)'(k(2+k)\sigma^2\tau + (1+k)^2(\tau A)(\tau A)')^{-1}(\tau A - k\gamma) \leq 1$$

Proof: Consider $D_{(i,j)} = D(\hat{\gamma}_{MRE}) - D(\hat{\gamma}_{SRRE}) = \sigma^2\tau - (1+k)^{-2}\sigma^2\tau$

$$= ((1+k)^2 - 1)(1+k)^{-2}\sigma^2\tau$$

$$= k(2+k)(1+k)^{-2}\sigma^2\tau$$

Since $k > 0$ and $\tau > 0$, hence $D_{(i,j)} > 0$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRRE}$ is superior to $\hat{y}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*(k(2+k)(1+k)^{-2}\sigma^2\tau + (\tau A)(\tau A)' - (1+k)^{-2}(\tau A - k\gamma)(\tau A - k\gamma)' + k^2(1+k)^{-2}(\gamma + \tau A)(\gamma + \tau A)')X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + k(1+k)^{-1}X_*(\gamma + \tau A)$.

Corollary C2:

a) $\hat{\gamma}_{SRAURE}$ is superior to $\hat{\gamma}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$\left((1+2k)\tau A - k^2\gamma\right)'(k^2(k^2+4k+2)\sigma^2\tau + (1+k)^4(\tau A)(\tau A)')^{-1}\left((1+2k)\tau A - k^2\gamma\right) \leq 1$$

Proof: Consider $D_{(i,j)} = D(\hat{\gamma}_{MRE}) - D(\hat{\gamma}_{SRAURE}) = \sigma^2\tau - (1+k)^{-4}(1+2k)^2\sigma^2\tau$

$$= ((1+k)^4 - (1+2k)^2)(1+k)^{-4}\sigma^2\tau$$

$$= k^2(k^2+4k+2)(1+k)^{-4}\sigma^2\tau$$

Since $k > 0$ and $\tau > 0$, hence $D_{(i,j)} > 0$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRAURE}$ is superior to $\hat{y}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(k^2(k^2+4k+2)(1+k)^{-4}\sigma^2\tau + (\tau A)(\tau A)' - (1+k)^{-4}\left((1+2k)\tau A - k^2\gamma\right)\left((1+2k)\tau A - k^2\gamma\right)' + k^4(1+k)^{-4}(\gamma + \tau A)(\gamma + \tau A)'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + k^2(1+k)^{-2}X_*(\gamma + \tau A)$.

Corollary C3:

a) $\hat{\gamma}_{SRLE}$ is superior to $\hat{\gamma}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$\left((1+d)\tau A - (1-d)\gamma\right)'((3+d)(1-d)\sigma^2\tau + 2^2(\tau A)(\tau A)')^{-1}\left((1+d)\tau A - (1-d)\gamma\right) \leq 1$$

Proof: Consider $D_{(i,j)} = D(\hat{\gamma}_{MRE}) - D(\hat{\gamma}_{SRLE}) = \sigma^2\tau - 2^{-2}(1+d)^2\sigma^2\tau$

$$= (4 - (1+d)^2)2^{-2}\sigma^2\tau$$

$$= 2^{-2}(3+d)(1-d)\sigma^2\tau$$

Since $0 < d < 1$ and $\tau > 0$, hence $D_{(i,j)} > 0$. This completes the proof.

b) If $A \geq 0$, $\hat{\gamma}_{SRLE}$ is superior to $\hat{\gamma}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(2^{-2}(3+d)(1-d)\sigma^2\tau + (\tau A)(\tau A)' - 2^{-2}\big((1+d)\tau A - (1-d)\gamma\big)\big((1+d)\tau A - (1-d)\gamma\big)' + 2^{-2}(1-d)^2(\gamma + \tau A)(\gamma+\tau A)'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-1}(1-d)\, X_*(\gamma + \tau A)$.

Corollary C4:

a) $\hat{\gamma}_{SRAULE}$ is superior to $\hat{\gamma}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)'\big((7+2d-d^2)(1-d)^2\sigma^2\tau + 2^4(\tau A)(\tau A)'\big)^{-1}((1+d)(3-d)\tau A - (1-d)^2\gamma) \leq 1$$

Proof: Consider $D_{(i,j)} = D(\hat{\gamma}_{MRE}) - D(\hat{\gamma}_{SRAULE}) = \sigma^2\tau - 2^{-4}(1+d)^2(3-d)^2\sigma^2\tau$

$$= (2^{-4} - (1+d)^2(3-d)^2)2^{-4}\sigma^2\tau$$

$$= 2^{-4}(7+2d-d^2)(1-d)^2\sigma^2\tau$$

Since $0 < d < 1$ and $\tau > 0$, hence $D_{(i,j)} > 0$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRAULE}$ is superior to $\hat{y}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*(2^{-4}(7+2d-d^2)(1-d)^2\sigma^2\tau + (\tau A)(\tau A)' - 2^{-4}((1+d)(3-d)\tau A - (1-d)^2\gamma)((1+d)(3-d)\tau A - (1-d)^2\gamma)' + 2^{-4}(1-d)^4(\gamma+\tau A)(\gamma+\tau A)')X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-2}(1-d)^2\, X_*(\gamma + \tau A)$.

Corollary C5:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRPCR}$ is superior to $\hat{\gamma}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$\big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big)'(\sigma^2(\tau - T_hT_h'\tau T_h'T_h) + (\tau A)(\tau A)')^{-1}\big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big) \leq 1$$

where λ_* is the largest eigenvalue of $T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider $D_{(i,j)} = D(\hat{\gamma}_{MRE}) - D(\hat{\gamma}_{SRPCR}) = \sigma^2\tau - \sigma^2 T_hT_h'\tau T_h'T_h$

$$= \sigma^2(\tau - T_hT_h'\tau T_h'T_h)$$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRPCR}$ is superior to $\hat{y}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(\sigma^2(\tau - T_hT_h'\tau T_h'T_h) + (\tau A)(\tau A)' - \big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big)' + (I - T_hT_h')(\gamma + \tau A)(\gamma+\tau A)'(I - T_hT_h')'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + X_*(I - T_hT_h')(\gamma + \tau A)$.

Corollary C6:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRrk}$ is superior to $\hat{\gamma}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$(1+k)^{-2}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)'\left(\sigma^2(\tau - (1+k)^{-2}T_hT_h'\tau T_h'T_h) + (\tau A)(\tau A)'\right)^{-1}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big) \leq 1$$

where λ_* is the largest eigenvalue of $(1+k)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider $D_{(i,j)} = D(\hat{\gamma}_{MRE}) - D(\hat{\gamma}_{SRrk}) = \sigma^2\tau - (1+k)^{-2}\sigma^2 T_hT_h'\tau T_h'T_h$
$= \sigma^2(\tau - (1+k)^{-2}T_hT_h'\tau T_h'T_h)$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $(1+k)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRrk}$ is superior to $\hat{y}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(\sigma^2(\tau - (1+k)^{-2}T_hT_h'\tau T_h'T_h) + (\tau A)(\tau A)' - (1+k)^{-2}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)' + (1+k)^{-2}\big((1+k)I - T_hT_h'\big)(\gamma + \tau A)(\gamma + \tau A)'\big((1+k)I - T_hT_h'\big)'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + (1+k)^{-1}X_*\big((1+k)I - T_hT_h'\big)(\gamma + \tau A)$.

Corollary C7:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRrd}$ is superior to $\hat{\gamma}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$2^{-2}(1+d)^2\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)'\left(\sigma^2(\tau - 2^{-2}(1+d)^2T_hT_h'\tau T_h'T_h) + (\tau A)(\tau A)'\right)^{-1}\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big) \leq 1$$

where λ_* is the largest eigenvalue of $2^{-2}(1+d)^2T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider $D_{(i,j)} = D(\hat{\gamma}_{MRE}) - D(\hat{\gamma}_{SRrd}) = \sigma^2\tau - 2^{-2}(1+d)^2\sigma^2 T_hT_h'\tau T_h'T_h$
$= \sigma^2(\tau - 2^{-2}(1+d)^2T_hT_h'\tau T_h'T_h)$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $2^{-2}(1+d)^2T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRrd}$ is superior to $\hat{y}_{MRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(\sigma^2(\tau - 2^{-2}(1+d)^2T_hT_h'\tau T_h'T_h) + (\tau A)(\tau A)' - 2^{-2}(1+d)^2\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)' + 2^{-2}(1+d)^2(2(1+d)^{-1}I - T_hT_h')(\gamma + \tau A)(\gamma + \tau A)'(2(1+d)^{-1}I - T_hT_h')'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-1}(1+d)X_*(2(1+d)^{-1}I - T_hT_h')(\gamma + \tau A)$.

Corollary C8:

a) $\hat{\gamma}_{SRRE}$ is superior to $\hat{\gamma}_{SRAURE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$(1+k)^2(\tau A-k\gamma)'\left(k(2+3k)\sigma^2\tau+\ \left((1+2k)\tau A-k^2\gamma\right)\left((1+2k)\tau A-k^2\gamma\right)'\right)^{-1}(\tau A-k\gamma)\leq 1$$

Proof: Consider

$$\begin{aligned}D_{(i,j)}=D(\hat{\gamma}_{SRAURE})-D(\hat{\gamma}_{SRRE})&=(1+k)^{-4}(1+2k)^2\sigma^2\tau-(1+k)^{-2}\sigma^2\tau\\&=((1+2k)^2-(1+k)^2)(1+k)^{-4}\sigma^2\tau\\&=k(2+3k)(1+k)^{-4}\sigma^2\tau\end{aligned}$$

Since $k>0$ and $\tau>0$, hence $D_{(i,j)}>0$. This completes the proof.

b) If $A\geq 0$, $\hat{y}_{SRRE}$ is superior to $\hat{y}_{SRAURE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta\in\mathfrak{R}(A)$ and $\theta'A^{-1}\theta\leq 1$, where $A=X_*\Big(k(2+3k)(1+k)^{-4}\sigma^2\tau+(1+k)^{-4}\left((1+2k)\tau A-k^2\gamma\right)\left((1+2k)\tau A-k^2\gamma\right)'-(1+k)^{-2}(\tau A-k\gamma)(\tau A-k\gamma)'+k^2(1+k)^{-4}(\gamma+\tau A)(\gamma+\tau A)'\Big)X_*'+\delta\delta'$, $\mathfrak{R}(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta=\delta+k(1+k)^{-2}X_*(\gamma+\tau A)$.

Corollary C9:

a) If $k>(1-d)(1+d)^{-1}$, $\hat{\gamma}_{SRRE}$ is superior to $\hat{\gamma}_{SRLE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$2^2(1+k)^{-2}(\tau A-k\gamma)'\left((1+k)^{-2}(k(1+d)+d+3)(k(1+d)+d-1)\sigma^2\tau+\left((1+d)\tau A-(1-d)\gamma\right)\left((1+d)\tau A-(1-d)\gamma\right)'\right)^{-1}(\tau A-k\gamma)\leq 1$$

Proof: Consider $D_{(i,j)}=D(\hat{\gamma}_{SRLE})-D(\hat{\gamma}_{SRRE})=2^{-2}(1+d)^2\sigma^2\tau-(1+k)^{-2}\sigma^2\tau$

$$=((1+k)^2(1+d)^2-2^2)2^{-2}(1+k)^{-2}\sigma^2\tau$$

$$=2^{-2}(1+k)^{-2}(k(1+d)+d+3)(k(1+d)+d-1)\sigma^2\tau$$

Since $k>0$, $0<d<1$ and $\tau>0$. $D_{(i,j)}>0$ if $(k(1+d)+d-1)>0$, which implies $k>(1-d)(1+d)^{-1}$. This complete the proof.

b) If $A\geq 0$, $\hat{y}_{SRRE}$ is superior to $\hat{y}_{SRLE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta\in\mathfrak{R}(A)$ and $\theta'A^{-1}\theta\leq 1$, where $A=X_*\Big(2^{-2}(1+k)^{-2}(k(1+d)+d+3)(k(1+d)+d-1)\sigma^2\tau+2^{-2}\left((1+d)\tau A-(1-d)\gamma\right)\left((1+d)\tau A-(1-d)\gamma\right)'-(1+k)^{-2}(\tau A-k\gamma)(\tau A-k\gamma)'+2^{-2}(1+k)^{-2}(1+k+d+kd)^2(\gamma+\tau A)(\gamma+\tau A)'\Big)X_*'+\delta\delta'$, $\mathfrak{R}(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta=\delta+2^{-1}(1+k)^{-1}(1+k+d+kd)X_*(\gamma+\tau A)$.

Corollary C10:

a) If $k>(1-d)^2(1+d)^{-1}(3-d)^{-1}$, $\hat{\gamma}_{SRRE}$ is superior to $\hat{\gamma}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$2^4(1+k)^{-2}(\tau A-k\gamma)'\left((1+k)^{-2}\big(7+2d-d^2+k(1+d)(3-d)\big)(k(1+d)(3-d)-(1-d)^2)\sigma^2\tau+\big((1+d)(3-d)\tau A-(1-d)^2\gamma\big)\big((1+d)(3-d)\tau A-(1-d)^2\gamma\big)'\right)^{-1}(\tau A-k\gamma)\leq 1$

Proof: Consider

$$D_{(i,j)}=D(\hat{\gamma}_{SRAULE})-D(\hat{\gamma}_{SRRE})=2^{-4}(1+d)^2(3-d)^2\sigma^2\tau-(1+k)^{-2}\sigma^2\tau$$
$$=((1+d)^2(3-d)^2(1+k)^2-2^4)2^{-4}(1+k)^{-2}\sigma^2\tau$$
$$=2^{-4}(1+k)^{-2}\big(7+2d-d^2+k(1+d)(3-d)\big)(k(1+d)(3-d)-(1-d)^2)\sigma^2\tau$$

Since $k>0,\ 0<d<1$ and $\tau>0$. $D_{(i,j)}>0$ if $(k(1+d)(3-d)-(1-d)^2)>0$, which implies $k>(1-d)^2(1+d)^{-1}(3-d)^{-1}$. This completes the proof.

b) If $A\geq 0$, $\hat{y}_{SRRE}$ is superior to $\hat{y}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta\in\mathfrak{R}(A)$ and $\theta' A^{-1}\theta\leq 1$, where $A=X_*\Big(2^{-4}(1+k)^{-2}\big(7+2d-d^2+k(1+d)(3-d)\big)(k(1+d)(3-d)-(1-d)^2)\sigma^2\tau+2^{-4}((1+d)(3-d)\tau A-(1-d)^2\gamma)\big((1+d)(3-d)\tau A-(1-d)^2\gamma\big)'-(1+k)^{-2}(\tau A-k\gamma)(\tau A-k\gamma)'+2^{-4}(1+k)^{-2}\big((1+k)d(2-d)+3k-1\big)^2(\gamma+\tau A)(\gamma+\tau A)'\Big)X_*'+\delta\delta'$, $\mathfrak{R}(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta=\delta+2^{-2}(1+k)^{-1}\big((1+k)d(2-d)+3k-1\big)X_*(\gamma+\tau A)$.

Corollary C11:

a) If $\lambda_*<1$, $\hat{\gamma}_{SRPCR}$ is superior to $\hat{\gamma}_{SRRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$\big((T_hT_h'-I)\gamma+T_hT_h'\tau A\big)'\big(((1+k)^{-2}\tau-T_hT_h'\tau T_h'T_h)\sigma^2+(1+k)^{-2}(\tau A-k\gamma)(\tau A-k\gamma)'\big)^{-1}\big((T_hT_h'-I)\gamma+T_hT_h'\tau A\big)\leq 1$$

where λ_* is the largest eigenvalue of $(1+k)^2T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider $D_{(i,j)}=D(\hat{\gamma}_{SRRE})-D(\hat{\gamma}_{SRPCR})=(1+k)^{-2}\sigma^2\tau-\sigma^2T_hT_h'\tau T_h'T_h$

$$=((1+k)^{-2}\tau-T_hT_h'\tau T_h'T_h)\sigma^2$$

Since $\tau>0$, according to Lemma A2 (see Appendix A) $D_{(i,j)}>0$ if $\lambda_*<1$, where λ_* is the largest eigenvalue of $(1+k)^2T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A\geq 0$, $\hat{y}_{SRPCR}$ is superior to $\hat{y}_{SRRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta\in\mathfrak{R}(A)$ and $\theta' A^{-1}\theta\leq 1$, where $A=X_*\Big(((1+k)^{-2}\tau-T_hT_h'\tau T_h'T_h)\sigma^2+(1+k)^{-2}(\tau A-k\gamma)(\tau A-k\gamma)'-\big((T_hT_h'-I)\gamma+T_hT_h'\tau A\big)\big((T_hT_h'-I)\gamma+T_hT_h'\tau A\big)'+(1+k)^{-2}(I-(1+k)T_hT_h')(\gamma+\tau A)(\gamma+\tau A)'(I-(1+k)T_hT_h')'\Big)X_*'+\delta\delta'$, $\mathfrak{R}(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta=\delta+(1+k)^{-1}X_*(I-(1+k)T_hT_h')(\gamma+\tau A)$.

Corollary C12:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRrk}$ is superior to $\hat{\gamma}_{SRRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$\left((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\right)'\left((\tau - T_hT_h'\tau T_h'T_h)\sigma^2 + (\tau A - k\gamma)(\tau A - k\gamma)'\right)^{-1}\left((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\right) \leq 1$$

where λ_* is the largest eigenvalue of $T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider $D_{(i,j)} = D(\hat{\gamma}_{SRRE}) - D(\hat{\gamma}_{SRrk}) = (1+k)^{-2}\sigma^2\tau - (1+k)^{-2}\sigma^2 T_hT_h'\tau T_h'T_h$

$$= (\tau - T_hT_h'\tau T_h'T_h)(1+k)^{-2}\sigma^2$$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRrk}$ is superior to $\hat{y}_{SRRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big((\tau - T_hT_h'\tau T_h'T_h)(1+k)^{-2}\sigma^2 + (1+k)^{-2}(\tau A - k\gamma)(\tau A - k\gamma)' - (1+k)^{-2}\left((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\right)\left((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\right)' + (1+k)^{-2}(I - T_hT_h')(\gamma + \tau A)(\gamma + \tau A)'(I - T_hT_h')'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + (1+k)^{-1}X_*(I - T_hT_h')(\gamma + \tau A)$.

Corollary C13:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRrd}$ is superior to $\hat{\gamma}_{SRRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$2^{-2}(1+d)^2\left((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\right)'\left(((1+k)^{-2}\tau - 2^{-2}(1+d)^2T_hT_h'\tau T_h'T_h)\sigma^2 + (1+k)^{-2}(\tau A - k\gamma)(\tau A - k\gamma)'\right)^{-1}\left((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\right) \leq 1$$

where λ_* is the largest eigenvalue of $2^{-2}(1+d)^2(1+k)^2T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider

$$D_{(i,j)} = D(\hat{\gamma}_{SRRE}) - D(\hat{\gamma}_{SRrd}) = (1+k)^{-2}\sigma^2\tau - 2^{-2}(1+d)^2\sigma^2T_hT_h'\tau T_h'T_h$$
$$= ((1+k)^{-2}\tau - 2^{-2}(1+d)^2T_hT_h'\tau T_h'T_h)\sigma^2$$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $2^{-2}(1+d)^2(1+k)^2T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRrd}$ is superior to $\hat{y}_{SRRE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(((1+k)^{-2}\tau - 2^{-2}(1+d)^2T_hT_h'\tau T_h'T_h)\sigma^2 + (1+k)^{-2}(\tau A - k\gamma)(\tau A - k\gamma)' - 2^{-2}(1+d)^2\left((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\right)\left((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\right)' + 2^{-2}(1+k)^{-2}(2I - (1+k)(1+d)T_hT_h')(\gamma + \tau A)(\gamma + \tau A)'(2I - (1+k)(1+d)T_hT_h')'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of

A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-1}(1+k)^{-1}X_*(2I - (1+k)(1+d)T_hT_h')(\gamma + \tau A)$.

Corollary C14:

a) If $d > (1+2k-k^2)(1+k)^{-2}$, $\hat{\gamma}_{SRAURE}$ is superior to $\hat{\gamma}_{SRLE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$2^2\big((1+2k)\tau A - k^2\gamma\big)'\Big((3+6k+k^2+d(1+k)^2)(k^2-2k-1+d(1+k)^2)\sigma^2\tau + (1+k)^4\big((1+d)\tau A-(1-d)\gamma\big)\big((1+d)\tau A-(1-d)\gamma\big)'\Big)^{-1}\big((1+2k)\tau A-k^2\gamma\big) \le 1$$

Proof: Consider

$$\begin{aligned} D_{(i,j)} &= D(\hat{\gamma}_{SRLE}) - D(\hat{\gamma}_{SRAURE}) = 2^{-2}(1+d)^2\sigma^2\tau - (1+k)^{-4}(1+2k)^2\sigma^2\tau \\ &= ((1+d)^2(1+k)^4 - 2^2(1+2k)^2)2^{-2}(1+k)^{-4}\sigma^2\tau \\ &= (3+6k+k^2+d(1+k)^2)(k^2-2k-1+d(1+k)^2)2^{-2}(1+k)^{-4}\sigma^2\tau \end{aligned}$$

Since $k > 0$, $0 < d < 1$ and $\tau > 0$. $D_{(i,j)} > 0$ if $(k^2-2k-1+d(1+k)^2) > 0$, which implies $d > (1+2k-k^2)(1+k)^{-2}$. This completes the proof.

b) If $A \ge 0$, $\hat{y}_{SRAURE}$ is superior to $\hat{y}_{SRLE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \le 1$, where $A = X_*\Big((3+6k+k^2+d(1+k)^2)(k^2-2k-1+d(1+k)^2)2^{-2}(1+k)^{-4}\sigma^2\tau + 2^{-2}\big((1+d)\tau A-(1-d)\gamma\big)\big((1+d)\tau A-(1-d)\gamma\big)' - (1+k)^{-4}\big((1+2k)\tau A-k^2\gamma\big)\big((1+2k)\tau A-k^2\gamma\big)' + 2^{-2}(1+k)^{-4}\big((1+d)k^2+(2k+1)(d-1)\big)^2(\gamma+\tau A)(\gamma+\tau A)'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-1}(1+k)^{-2}\big((1+d)k^2+(2k+1)(d-1)\big)X_*(\gamma+\tau A)$.

Corollary C15:

a) If $(1+d)(3-d) > 4(1+2k)(1+k)^{-2}$, $\hat{\gamma}_{SRAURE}$ is superior to $\hat{\gamma}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$2^4\big((1+2k)\tau A-k^2\gamma\big)'\Big(\big((1+k)^2(1+d)(3-d)+4(1+2k)\big)\big((1+k)^2(1+d)(3-d)-4(1+2k)\big)\sigma^2\tau + (1+k)^4\big((1+d)(3-d)\tau A-(1-d)^2\gamma\big)\big((1+d)(3-d)\tau A-(1-d)^2\gamma\big)'\Big)^{-1}\big((1+2k)\tau A-k^2\gamma\big) \le 1$$

Proof: Consider

$$\begin{aligned} D_{(i,j)} &= D(\hat{\gamma}_{SRAULE}) - D(\hat{\gamma}_{SRAURE}) = 2^{-4}(1+d)^2(3-d)^2\sigma^2\tau - (1+k)^{-4}(1+2k)^2\sigma^2\tau \\ &= ((1+k)^4(1+d)^2(3-d)^2 - 2^4(1+2k)^2)2^{-4}(1+k)^{-4}\sigma^2\tau \\ &= \big((1+k)^2(1+d)(3-d)+4(1+2k)\big)\big((1+k)^2(1+d)(3-d)-4(1+2k)\big) \\ &\quad 2^{-4}(1+k)^{-4}\sigma^2\tau \end{aligned}$$

Since $k > 0$, $0 < d < 1$ and $\tau > 0$. $D_{(i,j)} > 0$ if $\left((1+k)^2(1+d)(3-d) - 4(1+2k)\right) > 0$, which implies $(1+d)(3-d) > 4(1+2k)(1+k)^{-2}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRAURE}$ is superior to $\hat{y}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(\left((1+k)^2(1+d)(3-d)+4(1+2k)\right)\left((1+k)^2(1+d)(3-d)-4(1+2k)\right)2^{-4}(1+k)^{-4}\sigma^2\tau + 2^{-4}((1+d)(3-d)\tau A - (1-d)^2\gamma)\left((1+d)(3-d)\tau A-(1-d)^2\gamma\right)' - (1+k)^{-4}\left((1+2k)\tau A - k^2\gamma\right)\left((1+2k)\tau A-k^2\gamma\right)' + 2^{-4}(1+k)^{-4}\left((k-1)(3k+1)+d(2-d)(1+k)^2\right)^2(\gamma+\tau A)(\gamma+\tau A)'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-2}(1+k)^{-2}\left((k-1)(3k+1)+d(2-d)(1+k)^2\right)X_*(\gamma+\tau A)$.

Corollary C16:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRPCR}$ is superior to $\hat{\gamma}_{SRAURE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$\left((T_hT_h' - I)\gamma + T_hT_h'\tau A\right)'\left(((1+k)^{-4}(1+2k)^2\tau - T_hT_h'\tau T_h'T_h)\sigma^2 + (1+k)^{-4}\left((1+2k)\tau A - k^2\gamma\right)\left((1+2k)\tau A - k^2\gamma\right)'\right)^{-1}\left((T_hT_h'-I)\gamma + T_hT_h'\tau A\right) \leq 1$$

where λ_* is the largest eigenvalue of $(1+k)^4(1+2k)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider

$$\begin{aligned} D_{(i,j)} = D(\hat{\gamma}_{SRAURE}) - D(\hat{\gamma}_{SRPCR}) &= (1+k)^{-4}(1+2k)^2\sigma^2\tau - \sigma^2 T_hT_h'\tau T_h'T_h \\ &= ((1+k)^{-4}(1+2k)^2\tau - T_hT_h'\tau T_h'T_h)\sigma^2 \end{aligned}$$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $(1+k)^4(1+2k)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRPCR}$ is superior to $\hat{y}_{SRAURE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(((1+k)^{-4}(1+2k)^2\tau - T_hT_h'\tau T_h'T_h)\sigma^2 + (1+k)^{-4}\left((1+2k)\tau A - k^2\gamma\right)\left((1+2k)\tau A - k^2\gamma\right)' - \left((T_hT_h'-I)\gamma + T_hT_h'\tau A\right)\left((T_hT_h'-I)\gamma+T_hT_h'\tau A\right)' + (1+k)^{-4}\left((1+2k)I - (1+k)^2T_hT_h'\right)(\gamma+\tau A)(\gamma+\tau A)'\left((1+2k)I-(1+k)^2T_hT_h'\right)'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + (1+k)^{-2}X_*\left((1+2k)I - (1+k)^2T_hT_h'\right)(\gamma+\tau A)$.

Corollary C17:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRrk}$ is superior to $\hat{\gamma}_{SRAURE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$(1+k)^2\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)' \Big(((1+2k)^2\tau - (1+k)^2 T_hT_h'\tau T_h'T_h)\sigma^2 +$

$\big((1+2k)\tau A - k^2\gamma\big)\big((1+2k)\tau A - k^2\gamma\big)'\Big)^{-1}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big) \leq 1$

where λ_* is the largest eigenvalue of $(1+2k)^{-2}(1+k)^2 T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider

$$
\begin{aligned}
D_{(i,j)} = D(\hat{\gamma}_{SRAURE}) - D(\hat{\gamma}_{SRrk}) &= (1+k)^{-4}(1+2k)^2\sigma^2\tau - (1+k)^{-2}\sigma^2 T_hT_h'\tau T_h'T_h \\
&= ((1+2k)^2\tau - (1+k)^2 T_hT_h'\tau T_h'T_h)(1+k)^{-4}\sigma^2
\end{aligned}
$$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $(1+2k)^{-2}(1+k)^2 T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRrk}$ is superior to $\hat{y}_{SRAURE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(((1+2k)^2\tau - (1+k)^2 T_hT_h'\tau T_h'T_h)(1+k)^{-4}\sigma^2 + (1+k)^{-4}\big((1+2k)\tau A - k^2\gamma\big)\big((1+2k)\tau A - k^2\gamma\big)' - (1+k)^{-2}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)' + (1+k)^{-4}\big((1+2k)I - (1+k)T_hT_h'\big)(\gamma + \tau A)(\gamma + \tau A)'\big((1+2k)I - (1+k)T_hT_h'\big)'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + (1+k)^{-2}X_*\big((1+2k)I - (1+k)T_hT_h'\big)(\gamma + \tau A)$.

Corollary C18:

a) If $\lambda_* < 1$, $\hat{y}_{SRrd}$ is superior to $\hat{y}_{SRAURE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$2^{-2}(1+d)^2\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)'\Big(((1+k)^{-4}(1+2k)^2\tau - 2^{-2}(1+d)^2 T_hT_h'\tau T_h'T_h)\sigma^2 + (1+k)^{-4}\big((1+2k)\tau A - k^2\gamma\big)\big((1+2k)\tau A - k^2\gamma\big)'\Big)^{-1}\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big) \leq 1$

where λ_* is the largest eigenvalue of $2^{-2}(1+d)^2(1+k)^4(1+2k)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider

$$
\begin{aligned}
D_{(i,j)} = D(\hat{\gamma}_{SRAURE}) - D(\hat{\gamma}_{SRrk}) &= (1+k)^{-4}(1+2k)^2\sigma^2\tau - 2^{-2}(1+d)^2\sigma^2 T_hT_h'\tau T_h'T_h \\
&= ((1+k)^{-4}(1+2k)^2\tau - 2^{-2}(1+d)^2 T_hT_h'\tau T_h'T_h)\sigma^2
\end{aligned}
$$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $2^{-2}(1+d)^2(1+k)^4(1+2k)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRrd}$ is superior to $\hat{y}_{SRAURE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(((1+k)^{-4}(1+2k)^2\tau - 2^{-2}(1+d)^2 T_hT_h'\tau T_h'T_h)\sigma^2 + (1+k)^{-4}\big((1+2k)\tau A - k^2\gamma\big)\big((1+2k)\tau A - k^2\gamma\big)' - 2^{-2}(1+d)^2\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)' + 2^{-2}(1+k)^{-4}(2(1+2k)I - (1+k)^2(1+d)T_hT_h')(\gamma + \tau A)(\gamma + \tau A)'(2(1+2k)I - (1+k)^2(1+$

$d)T_hT_h')'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-1}(1+k)^{-2}X_*(2(1+2k)I - (1+k)^2(1+d)T_hT_h')(\gamma + \tau A)$.

Corollary C19:

a) $\hat{\gamma}_{SRLE}$ is superior to $\hat{\gamma}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$2^2\big((1+d)\tau A - (1-d)\gamma\big)'\Big((5-d)(1-d)(1+d)^2\sigma^2\tau + \big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)'\Big)^{-1}\big((1+d)\tau A - (1-d)\gamma\big) \leq 1$$

Proof: Consider
$$D_{(i,j)} = D(\hat{\gamma}_{SRAULE}) - D(\hat{\gamma}_{SRLE}) = 2^{-4}(1+d)^2(3-d)^2\sigma^2\tau - 2^{-2}(1+d)^2\sigma^2\tau$$
$$= ((3-d)^2 - 2^2)2^{-4}(1+d)^2\sigma^2\tau$$
$$= (5-d)(1-d)2^{-4}(1+d)^2\sigma^2\tau$$
Since $0 < d < 1$ and $\tau > 0$, hence $D_{(i,j)} > 0$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRLE}$ is superior to $\hat{y}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big((5-d)(1-d)2^{-4}(1+d)^2\sigma^2\tau + 2^{-4}((1+d)(3-d)\tau A - (1-d)^2\gamma)\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)' - 2^{-2}\big((1+d)\tau A - (1-d)\gamma\big)\big((1+d)\tau A - (1-d)\gamma\big)' + 2^{-4}(1+d)^2(1-d)^2(\gamma + \tau A)(\gamma + \tau A)'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-2}(1+d)(1-d)X_*(\gamma + \tau A)$.

Corollary C20:

a) If $\lambda_* < 1$, $\hat{y}_{SRPCR}$ is superior to $\hat{y}_{SRLE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$\big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big)'\Big((2^{-2}(1+d)^2\tau - T_hT_h'\tau T_h'T_h)\sigma^2 + 2^{-2}\big((1+d)\tau A - (1-d)\gamma\big)\big((1+d)\tau A - (1-d)\gamma\big)'\Big)^{-1}\big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big) \leq 1$$

where λ_* is the largest eigenvalue of $2^2(1+d)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider
$$D_{(i,j)} = D(\hat{\gamma}_{SRLE}) - D(\hat{\gamma}_{SRPCR}) = 2^{-2}(1+d)^2\sigma^2\tau - \sigma^2 T_hT_h'\tau T_h'T_h$$
$$= (2^{-2}(1+d)^2\tau - T_hT_h'\tau T_h'T_h)\sigma^2$$
Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $2^2(1+d)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRPCR}$ is superior to $\hat{y}_{SRLE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big((2^{-2}(1+d)^2\tau - T_hT_h'\tau T_h'T_h)\sigma^2 + 2^{-2}\big((1+d)\tau A - (1-d)\gamma\big)\big((1+d)\tau A - (1-d)\gamma\big)' - ((T_hT_h' - I)\gamma + T_hT_h'\tau A)\big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big)' + 2^{-2}\big((1+d)I - 2T_hT_h'\big)(\gamma + \tau A)(\gamma + \tau A)'\big((1+d)I -$

$2T_hT_h')'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-1}X_*\big((1+d)I - 2T_hT_h'\big)(\gamma + \tau A)$.

Corollary C21:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRrk}$ is superior to $\hat{\gamma}_{SRLE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$(1+k)^{-2}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)'\Big((2^{-2}(1+d)^2\tau - (1+k)^{-2}T_hT_h'\tau T_h'T_h)\sigma^2 + 2^{-2}\big((1+d)\tau A - (1-d)\gamma\big)\big((1+d)\tau A - (1-d)\gamma\big)'\Big)^{-1}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big) \leq 1$$

where λ_* is the largest eigenvalue of $2^2(1+d)^{-2}(1+k)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider $D_{(i,j)} = D(\hat{\gamma}_{SRLE}) - D(\hat{\gamma}_{SRrk}) = 2^{-2}(1+d)^2\sigma^2\tau - (1+k)^{-2}\sigma^2T_hT_h'\tau T_h'T_h$
$= (2^{-2}(1+d)^2\tau - (1+k)^{-2}T_hT_h'\tau T_h'T_h)\sigma^2$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $2^2(1+d)^{-2}(1+k)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRrk}$ is superior to $\hat{y}_{SRLE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big((2^{-2}(1+d)^2\tau - (1+k)^{-2}T_hT_h'\tau T_h'T_h)\sigma^2 + 2^{-2}\big((1+d)\tau A - (1-d)\gamma\big)\big((1+d)\tau A - (1-d)\gamma\big)' - (1+k)^{-2}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)' + 2^{-2}(1+k)^{-2}((1+k)(1+d)I - 2T_hT_h')(\gamma + \tau A)(\gamma + \tau A)'((1+k)(1+d)I - 2T_hT_h')'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-1}(1+k)^{-1}X_*((1+k)(1+d)I - 2T_hT_h')(\gamma + \tau A)$.

Corollary C22:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRrd}$ is superior to $\hat{\gamma}_{SRLE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)'\Big((\tau - T_hT_h'\tau T_h'T_h)\sigma^2 + (1+d)^{-2}\big((1+d)\tau A - (1-d)\gamma\big)\big((1+d)\tau A - (1-d)\gamma\big)'\Big)^{-1}\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big) \leq 1$$

where λ_* is the largest eigenvalue of $T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider $D_{(i,j)} = D(\hat{\gamma}_{SRLE}) - D(\hat{\gamma}_{SRrd}) = 2^{-2}(1+d)^2\sigma^2\tau - 2^{-2}(1+d)^2\sigma^2T_hT_h'\tau T_h'T_h$
$= (\tau - T_hT_h'\tau T_h'T_h)2^{-2}(1+d)^2\sigma^2$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRrd}$ is superior to $\hat{y}_{SRLE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big((\tau -$

$T_hT_h'\tau T_h'T_h)2^{-2}(1+d)^2\sigma^2 + 2^{-2}\big((1+d)\tau A-(1-d)\gamma\big)\big((1+d)\tau A-(1-d)\gamma\big)' - 2^{-2}(1+d)^2\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)' + 2^{-2}(1+d)^2(I - T_hT_h')(\gamma+\tau A)(\gamma+\tau A)'(I-T_hT_h')'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-1}(1+d)X_*(I-T_hT_h')(\gamma+\tau A)$.

Corollary C23:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRPCR}$ is superior to $\hat{\gamma}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$\big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big)'\Big((2^{-4}(1+d)^2(3-d)^2\tau - T_hT_h'\tau T_h'T_h)\sigma^2 + 2^{-4}\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)'\Big)^{-1}\big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big) \le 1$$

where λ_* is the largest eigenvalue of $2^4(1+d)^{-2}(3-d)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider

$$D_{(i,j)} = D(\hat{\gamma}_{SRLAUE}) - D(\hat{\gamma}_{SRPCR}) = 2^{-4}(1+d)^2(3-d)^2\sigma^2\tau - \sigma^2 T_hT_h'\tau T_h'T_h = (2^{-4}(1+d)^2(3-d)^2\tau - T_hT_h'\tau T_h'T_h)\sigma^2$$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $2^4(1+d)^{-2}(3-d)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \ge 0$, $\hat{\gamma}_{SRPCR}$ is superior to $\hat{\gamma}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \le 1$, where $A = X_*\Big((2^{-4}(1+d)^2(3-d)^2\tau - T_hT_h'\tau T_h'T_h)\sigma^2 + 2^{-4}((1+d)(3-d)\tau A - (1-d)^2\gamma)\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)' - ((T_hT_h' - I)\gamma + T_hT_h'\tau A)\big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big)' + 2^{-4}\big((1+d)(3-d)I - 2^2T_hT_h'\big)(\gamma+\tau A)(\gamma+\tau A)'\big((1+d)(3-d)I - 2^2T_hT_h'\big)'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-2}X_*\big((1+d)(3-d)I - 2^2T_hT_h'\big)(\gamma+\tau A)$.

Corollary C24:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRrk}$ is superior to $\hat{\gamma}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$(1+k)^{-2}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)'\Big((2^{-4}(1+d)^2(3-d)^2\tau - (1+k)^{-2}T_hT_h'\tau T_h'T_h)\sigma^2 + 2^{-4}\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)'\Big)^{-1}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big) \le 1$$

where λ_* is the largest eigenvalue of $2^4(1+d)^{-2}(3-d)^{-2}(1+k)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider $D_{(i,j)} = D(\hat{\gamma}_{SRLAUE}) - D(\hat{\gamma}_{SRrk}) = 2^{-4}(1+d)^2(3-d)^2\sigma^2\tau - (1+k)^{-2}\sigma^2T_hT_h'\tau T_h'T_h = (2^{-4}(1+d)^2(3-d)^2\tau - (1+k)^{-2}T_hT_h'\tau T_h'T_h)\sigma^2$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $2^4(1+d)^{-2}(3-d)^{-2}(1+k)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{\gamma}_{SRrk}$ is superior to $\hat{\gamma}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big((2^{-4}(1+d)^2(3-d)^2\tau - (1+k)^{-2}T_hT_h'\tau T_h'T_h)\sigma^2 + 2^{-4}((1+d)(3-d)\tau A - (1-d)^2\gamma)\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)' - (1+k)^{-2}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)' + 2^{-4}(1+k)^{-2}((1+k)(1+d)(3-d)I - 2^2T_hT_h')(\gamma+\tau A)(\gamma+\tau A)'((1+k)(1+d)(3-d)I - 2^2T_hT_h')'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-2}(1+k)^{-1}X_*((1+k)(1+d)(3-d)I - 2^2T_hT_h')(\gamma+\tau A)$.

Corollary C25:

a) If $\lambda_* < 1$, $\hat{\gamma}_{SRrd}$ is superior to $\hat{\gamma}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$2^2(1+d)^2\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)'(((3-d)^2\tau - 2^2T_hT_h'\tau T_h'T_h)(1+d)^2\sigma^2 + \big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)')^{-1}\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big) \leq 1$$

where λ_* is the largest eigenvalue of $2^2(3-d)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$.

Proof: Consider

$$\begin{aligned} D_{(i,j)} = D(\hat{\gamma}_{SRLAUE}) - D(\hat{\gamma}_{SRrd}) &= 2^{-4}(1+d)^2(3-d)^2\sigma^2\tau - 2^{-2}(1+d)^2\sigma^2T_hT_h'\tau T_h'T_h \\ &= ((3-d)^2\tau - 2^2T_hT_h'\tau T_h'T_h)2^{-4}(1+d)^2\sigma^2 \end{aligned}$$

Since $\tau > 0$, according to Lemma A2 (see Appendix A) $D_{(i,j)} > 0$ if $\lambda_* < 1$, where λ_* is the largest eigenvalue of $2^2(3-d)^{-2}T_hT_h'\tau T_h'T_h\tau^{-1}$. This completes the proof.

b) If $A \geq 0$, $\hat{\gamma}_{SRrd}$ is superior to $\hat{\gamma}_{SRAULE}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(((3-d)^2\tau - 2^2T_hT_h'\tau T_h'T_h)2^{-4}(1+d)^2\sigma^2 + 2^{-4}((1+d)(3-d)\tau A - (1-d)^2\gamma)\big((1+d)(3-d)\tau A - (1-d)^2\gamma\big)' - 2^{-2}(1+d)^2\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)' + 2^{-4}(1+d)^2((3-d)I - 2T_hT_h')(\gamma+\tau A)(\gamma+\tau A)'((3-d)I - 2T_hT_h')'\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-2}(1+d)X_*((3-d)I - 2T_hT_h')(\gamma+\tau A)$.

Corollary C26:

a) If $T_hT_h'\tau T_h'T_h$ is positive definite, $\hat{\gamma}_{SRrk}$ is superior to $\hat{\gamma}_{SRPCR}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$(1+k)^{-2}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)'\Big(k(2+k)(1+k)^{-2}\sigma^2T_hT_h'\tau T_h'T_h + \big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - I)\gamma + T_hT_h'\tau A\big)'\Big)^{-1}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big) \leq 1$$

Proof: Consider
$$\begin{aligned} D_{(i,j)} = D(\hat{\gamma}_{SRPCR}) - D(\hat{\gamma}_{SRrk}) &= \sigma^2T_hT_h'\tau T_h'T_h - (1+k)^{-2}\sigma^2T_hT_h'\tau T_h'T_h \\ &= \sigma^2T_hT_h'(\tau - (1+k)^{-2}\tau)T_h'T_h \end{aligned}$$

$$= \sigma^2 T_h T_h' k(2+k)(1+k)^{-2} \tau T_h' T_h$$
$$= k(2+k)(1+k)^{-2} \sigma^2 T_h T_h' \tau T_h' T_h$$

Since $k > 0$ and $\tau > 0$. $D_{(i,j)} > 0$ if $T_h T_h' \tau T_h' T_h$ is positive definite. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRrk}$ is superior to $\hat{y}_{SRPCR}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1} \theta \leq 1$, where $A = X_* \Big(k(2+k)(1+k)^{-2} \sigma^2 T_h T_h' \tau T_h' T_h + \big((T_h T_h' - I)\gamma + T_h T_h' \tau A\big)\big((T_h T_h' - I)\gamma + T_h T_h' \tau A\big)' - (1+k)^{-2}\big((T_h T_h' - (1+k)I)\gamma + T_h T_h' \tau A\big)\big((T_h T_h' - (1+k)I)\gamma + T_h T_h' \tau A\big)' + k^2 (1+k)^{-2} T_h T_h' (\gamma + \tau A)(\gamma + \tau A)' T_h' T_h \Big) X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + k(1+k)^{-1} X_* T_h T_h' (\gamma + \tau A)$.

Corollary C27:

a) If $T_h T_h' \tau T_h' T_h$ is positive definite, $\hat{y}_{SRrd}$ is superior to $\hat{y}_{SRPCR}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$2^{-2}(1+d)\big((T_h T_h' - 2(1+d)^{-1}I)\gamma + T_h T_h' \tau A\big)' \big(2^{-2}(3+d)(1-d)\sigma^2 T_h T_h' \tau T_h' T_h + \big((T_h T_h' - I)\gamma + T_h T_h' \tau A\big)\big((T_h T_h' - I)\gamma + T_h T_h' \tau A\big)'\big)^{-1} (1+d)\big((T_h T_h' - 2(1+d)^{-1}I)\gamma + T_h T_h' \tau A\big) \leq 1$$

Proof: Consider $D_{(i,j)} = D(\hat{y}_{SRPCR}) - D(\hat{y}_{SRrk}) = \sigma^2 T_h T_h' \tau T_h' T_h - 2^{-2}(1+d)^2 \sigma^2 T_h T_h' \tau T_h' T_h$
$$= \sigma^2 T_h T_h' (\tau - 2^{-2}(1+d)^2 \tau) T_h' T_h$$
$$= \sigma^2 T_h T_h' 2^{-2}(3+d)(1-d) \tau T_h' T_h$$
$$= 2^{-2}(3+d)(1-d) \sigma^2 T_h T_h' \tau T_h' T_h$$

Since $0 < d < 1$ and $\tau > 0$. $D_{(i,j)} > 0$ if $T_h T_h' \tau T_h' T_h$ is positive definite. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRrd}$ is superior to $\hat{y}_{SRPCR}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1} \theta \leq 1$, where $A = X_* \Big(2^{-2}(3+d)(1-d)\sigma^2 T_h T_h' \tau T_h' T_h + \big((T_h T_h' - I)\gamma + T_h T_h' \tau A\big)\big((T_h T_h' - I)\gamma + T_h T_h' \tau A\big)' - 2^{-2}(1+d)^2\big((T_h T_h' - 2(1+d)^{-1}I)\gamma + T_h T_h' \tau A\big)\big((T_h T_h' - 2(1+d)^{-1}I)\gamma + T_h T_h' \tau A\big)' + 2^{-2}(1-d)^2 T_h T_h' (\gamma + \tau A)(\gamma + \tau A)' T_h' T_h \Big) X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-1}(1-d) X_* T_h T_h' (\gamma + \tau A)$.

Corollary C28:

a) If $(k(1+d) + d - 1) T_h T_h' \tau T_h' T_h$ is positive definite, $\hat{y}_{SRrk}$ is superior to $\hat{y}_{SRrd}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if

$$(1+k)^{-2}\big((T_h T_h' - (1+k)I)\gamma + T_h T_h' \tau A\big)' \Big(2^{-2}(1+k)^{-2}(k(1+d) + d + 3)(k(1+d) + d - 1)\sigma^2 T_h T_h' \tau T_h' T_h + 2^{-2}(1+d)^2\big((T_h T_h' - 2(1+d)^{-1}I)\gamma + T_h T_h' \tau A\big)\big((T_h T_h' - 2(1+d)^{-1}I)\gamma + T_h T_h' \tau A\big)'\Big)^{-1} \big((T_h T_h' - (1+k)I)\gamma + T_h T_h' \tau A\big) \leq 1$$

Proof: Consider $D_{(i,j)} = D(\hat{y}_{SRrd}) - D(\hat{y}_{SRrk}) = 2^{-2}(1+d)^2 \sigma^2 T_h T_h' \tau T_h' T_h - (1+k)^{-2} \sigma^2 T_h T_h' \tau T_h' T_h$
$$= \sigma^2 T_h T_h' (2^{-2}(1+d)^2 - (1+k)^{-2}) \tau T_h' T_h$$
$$= \sigma^2 T_h T_h' 2^{-2}(k(1+d) + d + 3)(k(1+d) + d - 1)(1+k)^{-2} \tau T_h' T_h$$

$$= 2^{-2}(1+k)^{-2}\sigma^2(k(1+d)+d+3)(k(1+d)+d-1)T_hT_h'\tau T_h'T_h$$

Since $0 < d < 1$, $k > 0$ and $\tau > 0$. $D_{(i,j)} > 0$ if $(k(1+d)+d-1)T_hT_h'\tau T_h'T_h$ is positive definite. This completes the proof.

b) If $A \geq 0$, $\hat{y}_{SRrk}$ is superior to $\hat{y}_{SRrd}$ in MSEM sense when the regression model is misspecified due to excluding relevant variables if and only if $\theta \in \Re(A)$ and $\theta' A^{-1}\theta \leq 1$, where $A = X_*\Big(2^{-2}(1+k)^{-2}\sigma^2(k(1+d)+d+3)(k(1+d)+d-1)T_hT_h'\tau T_h'T_h + 2^{-2}(1+d)^2\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - 2(1+d)^{-1}I)\gamma + T_hT_h'\tau A\big)' - (1+k)^{-2}\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)\big((T_hT_h' - (1+k)I)\gamma + T_hT_h'\tau A\big)' + 2^{-2}(1+k)^{-2}\big((1+d)(1+k)-2\big)^2 T_hT_h'(\gamma+\tau A)(\gamma+\tau A)'T_h'T_h\Big)X_*' + \delta\delta'$, $\Re(A)$ stands for column space of A and A^{-1} is an independent choice of g-inverse of A and $\theta = \delta + 2^{-1}(1+k)^{-1}((1+d)(1+k)-2)X_*T_hT_h'(\gamma+\tau A)$.