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Research Article

Parametric Methodologies for Detecting Changes in Maximum Temperature of Tlaxco, Tlaxcala, México

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In this paper, comparison results of parametric methodologies of change points, applied to maximum temperature records from the municipality of Tlaxco, Tlaxcala, México, are presented. Methodologies considered are likelihood ratio test, score test, and *binary segmentation* (BS), *pruned exact linear time* (PELT), and *segment neighborhood* (SN). In order to compare such methodologies, a quality analysis of the data was performed; in addition, lost data were estimated with linear regression, and finally, SARIMA models were adjusted.

1. Introduction

Global warming is the most evident manifestation of climate change; it refers to the average increase in global, terrestrial, and marine temperatures [1] and is attributed to the increase in greenhouse gases emissions (mainly CO_2, CH_4, and N_2O). Although greenhouse gases are naturally present in the atmosphere, consumption of fossil fuels for the production of energy and supplies, product of human development, causes emissions of such gases faster than in its natural cycle [2, 3]. The Intergovernmental Panel on Climate Change (IPCC) estimates radical impacts that imply increases in temperature and the rise in sea level (derived from the melting of the poles), as well as the destruction of ecosystemic services [4]; therefore, it is important to monitor and analyze the behaviour of climatological data in order to establish policies, in case of being necessary, that contribute to diminish impacts on the environment and society.

Based on the previous remarks, it was considered pertinent to perform an analysis of the behaviour of the maximum temperature in a region of the Mexican state of Tlaxcala, which has a climatological station that registers, among other variables, the maximum temperature; such station is located at municipality Tlaxco-Tlaxcala, and it is the responsibility of the Civil Protection Commission of Tlaxcala State and the Climate Change Research Center of the Autonomous University of Tlaxcala. With aforementioned records, change point methodologies were applied and compared for identifying changes in maximum temperature of the early mentioned municipality; for this, data quality analysis was performed, lost data were completed by linear regression, and SARIMA models were adjusted. In order to contrast the goodness of fit of considered models, the Akaike information criterion (AIC) as well as the Bayesian information criterion (BIC) was considered. In Section 2, applied parametric methodologies are described. In Section 3, characterization of the area under study is presented as well as the results obtained from the applied methodologies. In Section 4, the results obtained are discussed.

2. Parametric Methodologies for Detecting Change Points

Change point problem started in control theory, and it dates back to 1950s decade [5, 6] where the existence of a single change point was tested. In order to detect a single change point, the first case analyzed dealt with independent random variables considering parametric methodologies [7–13]. To improve criteria for identifying and estimating change points, Bayesian and nonparametric methodologies have been used [7, 8, 14–21].

Let $\{\mathbf{X}_i\}_{i=1}^n$ be a sequence of n independent random vectors each one with probability distribution functions F_i, $i = 1, 2, \ldots, n$, respectively. The goal of change point problem is to test

$$H_0 : F_1 = F_2 = \cdots = F_n, \quad (1)$$

versus

$$\begin{aligned} H_1 : F_1 = F_2 = \cdots = F_{k_1} \neq F_{k_1+1} = \cdots = F_{k_2} \neq F_{k_2+1} = \cdots \\ = F_{k_q} \neq F_{k_{q+1}} = \cdots = F_n, \end{aligned} \quad (2)$$

where $1 < k_1 < k_2 < \cdots < k_q < n$, q is the unknown number of change points, and $k_1 < k_2 < \cdots < k_q$ are the respective unknown positions that have to be estimated. When the distributions belong to a common parametric family $F(\boldsymbol{\theta})$, with $\boldsymbol{\theta} \in \mathbb{R}^p$, then the hypothesis tests are about the population parameters:

$$H_0 : \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = \cdots = \boldsymbol{\theta}_n, \quad (3)$$

versus

$$\begin{aligned} H_1 : \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = \cdots = \boldsymbol{\theta}_{k_1} \neq \boldsymbol{\theta}_{k_1+1} = \cdots = \boldsymbol{\theta}_{k_2} \neq \boldsymbol{\theta}_{k_2+1} = \cdots \\ = \boldsymbol{\theta}_{k_q} \neq \boldsymbol{\theta}_{k_{q+1}} = \cdots = \boldsymbol{\theta}_n, \end{aligned} \quad (4)$$

where q and $k_1, k_2, \ldots, k_q$ have to be estimated.

Since methodologies to detect change points in the context of independent random variables cannot be applied directly in time series [22, 23], parametric, nonparametric, and Bayesian methodologies have also been developed [12, 16, 24–31].

Due to the natural disasters recorded in recent decades attributed mainly to anthropogenic activities, there have been an increasing number of studies using change point methods to detect shifts in climate. According to Beaulieu et al. [32], change point detection techniques have been used to detect changes in temperature and in precipitation to detect regime shifts, to detect shifts in aerosol and cloud data, and also to study past changes in the land uptake of carbon. There exist an extensive literature about detecting shifts into environmental data, which apply parametric and nonparametric methodologies [32–38]. The main objective of this work is to compare some parametric methodologies to detect abrupt and multiple changes in the maximum temperature in the municipality of Tlaxco, Tlaxcala, Mexico.

2.1. Parametric Methodologies for a Single Change Point in Time Series. For detecting change points in environmental time series, most studies have to apply Bayesian and nonparametric methodologies; various nonparametric tests, including Mann–Kendall test and Pettit's test, are widely used to detect trend and change point in the historical series of climatic and hydrological variables [35]. In this study, we consider identifying a single change point under the assumption that the data fit an autoregressive model.

Let $X_1, X_2, \ldots, X_n$ be consecutive observations from the model

$$X_t = \begin{cases} \phi_0 + \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p} + \varepsilon_t, & \text{if } -\infty < t < \tau, \\ \alpha_0 + \alpha_1 X_{t-1} + \cdots + \alpha_p X_{t-p} + \varepsilon_t, & \text{if } t \geq \tau + 1, \end{cases} \quad (5)$$

where $p \in \mathbb{N}$, $\tau \in (p, n]$, $\{\varepsilon_t\}$ is a white-noise sequence that has a finite fourth moment, and $\phi(z) = 1 - \phi_1 - \cdots - \phi_p z^p \neq 0$, $\forall\ |z| \leq 1$.

To test

$$H_0 : \tau = n \text{ vs } H_a : \tau = k, \quad p < k \leq n, \quad (6)$$

under H_0 and normality assumption, via likelihood ratio technique conditioned on the first p observations and applying $-2\ \ln$ to the likelihood ratio, is possible to establish

$$\begin{aligned} \Lambda_n(k) := -2 \ln\left(\frac{L(n)}{L(k)}\right) &= \min_{\phi} \sum_{p+1 \leq t \leq n} \left(X_t - \phi_0 - \sum_{1 \leq j \leq p} \phi_j X_{t-j} \right)^2 \\ &- \min_{\phi} \sum_{p+1 \leq t \leq k} \left(X_t - \phi_0 - \sum_{1 \leq j \leq p} \phi_j X_{t-j} \right)^2 \\ &- \min_{\alpha} \sum_{k+1 \leq t \leq n} \left(X_t - \alpha_0 - \sum_{1 \leq j \leq p} \alpha_j X_{t-j} \right)^2. \end{aligned} \quad (7)$$

For studying the asymptotic behaviour of the distribution of the likelihood ratio statistic under H_0, Davis et al. [39] considered a quadratic form and defined the process $Q_n(t) = \Lambda_n([nt])$ over $[0, 1]$ and assumed that $\sup_t E|\varepsilon_t|^{4+\delta} < \infty$, for some $0 \leq \delta \leq 1$. The details are in the following theorem.

Theorem 2.1. *Let $\{X_t\}$ be the process defined by*

$$X_t = \phi_0 + \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p} + \varepsilon_t, \quad -\infty < t < \infty, \quad (8)$$

and assume that under H_0, $\{\varepsilon_t\}$ satisfies $E(\varepsilon_i \varepsilon_j) = \sigma^2$ (for $i = j$), with $\sup_t E|\varepsilon_t|^{4+\delta} < \infty$, for some $0 < \delta \leq 1$, and $\{X_t\}$ is strongly related with $\rho(n) \ll n^{-(1+\varepsilon)(1+4/\delta)}$ for some $\varepsilon > 0$. Then, under H_0, we have that $\forall\ x \in \mathbb{R}$

$$\mathbb{P}\left\{ \frac{\sigma^{-2} \max_{p<k\leq n} \Lambda_n(k) - b_n(p+1)}{a_n(p+1)} \leq x \right\} \longrightarrow \exp\left(-2e^{-x/2}\right), \quad (9)$$

where $b_n(d) = (2 \ln\ \ln n + (d/2) \ln\ \ln \ln\ n - \ln(\Gamma(d/2)))^2 / 2 \ln \ln n$ and $a_n(d) = \sqrt{b_n(d)/2\ \ln\ \ln n}$ are normalization constants and $\Gamma(\cdot)$ is the gamma function.

Proof 2.1. Details of the proof can be consulted in [39].

The methodology proposed by Davis et al. [39] uses a quadratic form and does not allow identifying which parameter of the autoregressive model has changed. Gombay [40] has proposed a method that establishes one- or two-sided hypothesis tests to identify which parameter of an autoregressive model has changed; for this, the proposed methodology uses the components of the efficient score vector as test statistic.

Consider the following AR(p) process:

$$X_i - \mu = \phi_1 (X_{i-1} - \mu) + \cdots + \phi_p (X_{i-p} - \mu) + \varepsilon_i, \quad i \geq p+1, \tag{10}$$

where $\{\varepsilon_i\}$ is a white noise sequence with $E(\varepsilon_1^2) = \sigma^2$.

To test

$$H_0 : \text{there does not exist a change in } \xi = (\mu, \sigma^2, \phi_1, \ldots, \phi_p)^T, \tag{11}$$

versus

$$H_1 : \text{there exists a change in } \xi = (\mu, \sigma^2, \phi_1, \ldots, \phi_p)^T, \tag{12}$$

components of the efficient score vector under Gaussian assumption are needed.

Let $\nabla_\xi l_k (X_1, \ldots, X_k; \boldsymbol{\xi}) = \nabla_\xi l_k (\boldsymbol{\xi})$, the components are

$$\begin{gathered}\frac{\partial}{\partial \mu} l_k (\boldsymbol{\xi}) = \frac{1 - \sum_{j=1}^{p} \phi_j}{\sigma^2} \sum_{i=1}^{k} \left[X_i - \mu - \sum_{j=1}^{p} \phi_j (X_{i-j} - \mu) \right], \\ \frac{\partial}{\partial \sigma^2} l_k (\boldsymbol{\xi}) = -\frac{k}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{i=1}^{k} \left[X_i - \mu - \sum_{j=1}^{p} \phi_j (X_{i-j} - \mu) \right]^2, \\ \frac{\partial}{\partial \phi_s} l_k (\boldsymbol{\xi}) = \frac{1}{\sigma^2} \sum_{i=1}^{k} \left[X_i - \mu - \sum_{j=1}^{p} \phi_j (X_{i-j} - \mu) \right] (X_{i-s} - \mu), \\ s = 1, \ldots, p,\end{gathered} \tag{13}$$

where l_k denotes the logarithm function of likelihood based on observations $X_1, \ldots, X_k$. Information matrix of size $(p+2) \times (p+2)$ is given by

$$\mathbf{I}(\boldsymbol{\xi}) = \mathbf{I}(\mu, \sigma^2, \phi_1, \ldots, \phi_p) = \begin{bmatrix} \dfrac{\left(1 - \sum_{i=1}^{p} \phi_j\right)^2}{\sigma^2} & 0 & 0 \\ 0 & \dfrac{1}{\sigma^4} & 0 \\ 0 & 0 & \dfrac{1}{\sigma^2} V \end{bmatrix}, \tag{14}$$

with V being the covariance matrix of $(X_1, X_2, \ldots, X_p)$. In addition, let $\widehat{\mu}_n, \widehat{\sigma}_n^2$, and $\widehat{\phi}_n$ be the maximum likelihood estimators of parameters, and consider under H_0

$$\widehat{B}(u) = n^{-1/2} \mathbf{I}(\widehat{\xi}_n) \begin{pmatrix} \dfrac{\partial}{\partial \mu} l_{[nu]} (\widehat{\xi}_n) \\ \dfrac{\partial}{\partial \sigma^2} l_{[nu]} (\widehat{\xi}_n) \\ \nabla_\phi l_{[nu]} (\widehat{\xi}_n) \end{pmatrix}, \tag{15}$$

as test statistic. For testing the change in d parameters with an exact level of significance α, Gombay [40] suggested the use of $\alpha^* = 1 - (1-\alpha)^{1/d}$ on each component, for both unilateral and bilateral tests, as follows:

Unilateral test: if $\sup_{0 \leq u \leq 1} \widehat{B}^{(j)}(u) \geq C_1(\alpha^*)$, then there exists a change in parameter ξ_j $(j = 1, \ldots, p+2)$ over the sequence $X_1, \ldots, X_n$. Critic value $C_1(\alpha^*)$ is obtained from the relation

$$\mathbb{P}\left(\sup_{0 \leq u \leq 1} \mathbf{B}^{(1)}(u) \geq x \right) = e^{-2x}. \tag{16}$$

For unilateral test, the estimator time is $\widehat{\tau}_n = \min_k \left\{ |(\partial/\partial \xi_j) l_k (\widehat{\xi}_n)| = \max_{1<m\leq n} |(\partial/\partial \xi_j) l_m (\widehat{\xi}_n)| \right\}$.

Bilateral test: if $\sup_{0 \leq u \leq 1} \widehat{B}^{(j)}(u) \geq C_2(\alpha^*)$, then there exits a change in parameter ξ_j $(j = 1, \ldots, p+2)$ over the sequence $X_1, \ldots, X_n$. Critic value $C_2(\alpha^*)$ is obtained from the relation

$$\mathbb{P}\left(\sup_{0 \leq u \leq 1} \left| \mathbf{B}^{(1)}(u) \right| > x \right) = \sum_{k \neq 0} (-1)^{k+1} e^{-2k^2 x^2}. \tag{17}$$

Once a change in ξ_j, $j = 1, \ldots, p+2$ over $\tau = [\rho n]$, $0 \leq \rho \leq 1$, is guaranteed, the estimator for τ is

$$\begin{gathered}\widehat{\tau} = \min_k \left\{ \frac{\partial}{\partial \xi_j} l_k (\widehat{\xi}_n) = \max_{1<m\leq n} \frac{\partial}{\partial \xi_j} l_m (\widehat{\xi}_n) \right\}, \\ \text{or } \widehat{\tau} = \min_n \left\{ \frac{\partial}{\partial \xi_j} l_k (\widehat{\xi}_n) = \min_{1<m\leq n} \frac{\partial}{\partial \xi_j} l_m (\widehat{\xi}_n) \right\}.\end{gathered} \tag{18}$$

Proof 2.2. Details of the proof can be consulted in [40].

2.2. Parametric Methodologies for Multiple Change Points. Let $\{x_t,\ t = 1, \ldots, n\}$ be an ordered data sequence, assuming that there are m ordered change points in random times $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_m)$, with $\tau_0 = 1$, $\tau_{m+1} = n$; hence, such change points induce a partition over the data into $m+1$ segments, where the ith segment will contain observations $(x_{\tau_{i-1}+1}, \ldots, x_{\tau_i})$. Something commonly used to identify multiple points is to minimize

$$\sum_{1 \leq i \leq m+1} \left[C\left(x_{\tau_{i-1}+1, \ldots, \tau_i}\right) \right] + \beta f(m), \tag{19}$$

with respect to m and $\tau_1, \tau_2, \ldots, \tau_m$. Here, $C(x_{\tau_{i-1}+1,\ldots,\tau_i})$ is a cost function for a segment and $\beta f(m)$ is a penalty term in order to avoid over fitting [19]. Some commonly used cost functions are the additive inverse of the log likelihood function, accumulated sums [12], and other methods which include applying entropy information, for the penalization of $\beta f(m)$ [24]. The most common choice of penalty is one which is linear in the number of change points, that is, $\beta f(m) = \beta m$. Examples of such penalties include Akaike information criterion (AIC) and Schwarz information criterion (BIC) [41].

For the following methods, the change point hypothesis test is

$$H_0: \text{there does not exist a change point in the data,} \tag{20}$$

versus

$$H_1: \text{there exists a change point in the data.} \tag{21}$$

2.2.1. Pruned Exact Linear Time Method. Pruned exact linear time (PELT) method is based on the method proposed by Jackson et al. [42] and Yao [43]. Let $\{x_t, t = 1, \ldots, n\}$ be a set of observations; the idea of the method is the search for an optimal partition of data given on a time interval I whose elements are sets of data cells, resulting in no significant loss of information.

A partition $\mathbf{P}$ of an interval I is a set of $\{\tau_1, \ldots, \tau_m\}$ blocks, $\mathbf{P}(I) = \{B_k, k \in \mathscr{M}\}$, $\mathscr{M} \equiv \{1, 2, \ldots, m\}$, where the blocks are sets of data cells defined by index sets $\mathscr{N}_k$: $B_k = \{x_t, t \in \mathscr{N}_k\}$ satisfying the usual conditions $\cup_k B_k = I$ and $B_k \cap B_{k'} = \varnothing$ if $k \neq k'$, and the number of blocks must satisfy $0 \leq m \leq n$. Let P^* be the (finite) set of all partitions of I into blocks. Take as given an additive fitness function that assigns a value to any partition $\mathbf{P} \in P^*$ in the form

$$V(\mathbf{P}) = \sum_{1 \leq i \leq m+1} \left[C\left(x_{\tau_{i-1}+1,\ldots,\tau_i}\right) + \beta\right], \tag{22}$$

where $C(x_{\tau_{i-1}+1,\ldots,\tau_i})$ is the fitness of block B_k and has to be minimized in order to detect a change point. According to [19], the PELT method proposes a pruning step, which increases the computational efficiency of the previous method while ensuring a global minimum of the cost function which is linear in n (22). The optimal segmentation is $F(n)$, where

$$F(n) = \min_{\tau} \left\{ \sum_{1 \leq i \leq m+1} \left[C\left(x_{\tau_{i-1}+1}, \ldots, x_{\tau_i}\right) + \beta\right] \right\}. \tag{23}$$

The pseudocode of the PELT method can be consulted in [19, 44] and proofs about the optimal partition in [42].

2.2.2. Binary Segmentation Method. In the *binary segmentation* (BS) method, consider the model $x_t = f_t + \xi_t$, $t = 1, \ldots, n$, where f_t is a deterministic, one-dimensional, piecewise-constant signal with change points whose number N and locations $\tau_1, \ldots, \tau_n$ are unknown, and the random sequence $\{\xi_t\}_1^n$ is i.i.d. Gaussian with mean zero and variance one.

The method uses the CUSUM statistic defined by the inner product between the vector $(x_s, \ldots, x_e)$ and a particular vector of contrast weights given by

$$\tilde{x}_{s,e}^b = \sqrt{\frac{e-b}{n(b-s+1)}} \sum_{t=s}^{b} x_t - \sqrt{\frac{b-s+1}{n-(e-b)}} \sum_{t=b+1}^{e} x_t, \tag{24}$$

where $s \leq b < e$, with $n = e - s + 1$. In its first step, the BS algorithm computes $\tilde{x}_{1,n}^b$ and then takes $b_{1,1} = \arg\max_{b:1 \leq b < n} |\tilde{x}_{1,n}^b|$ to be the first change point candidate, whose significance is to be judged against a certain criterion. If it is considered significant, the domain $[1, n]$ is split into two subintervals to the left and to the right of $b_{1,1}$, and the recursion continues by computing $\tilde{x}_{1,b_{1,1}}^n$ and $\tilde{x}_{b_{1,1}+1,n}^b$ possibly resulting in further splits [44].

This method extends any single change point method for multiple change points by iteratively repeating the method on different subsets of the sequence for this. It begins by initially applying the single change point method to the entire data and prove if there exists τ such that

$$C(x_{1:\tau}) + C(x_{\tau+1:n}) + \beta < C(x_{1:n}). \tag{25}$$

If (25) does not hold, it is guaranteed that there is no change point along $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, and the process ends; otherwise, the data are partitioned into two sets, which consist of the data sequence before and after the change point, τ_α; the process is applied again in each partition until no more change points are found (see pseudocode in [44, 45]).

2.2.3. Segment Neighborhood Method. Segment neighborhood (SN) method uses dynamic programming to detect change point; it begins by setting an upper limit on the size of the segmentation space (i.e., the maximum number of change points) that is required, and this method continues by computing the cost function for all possible segments.

It takes the constrained case which segments the data up to t, for $t \geq m+1$, into $m+1$ segments (using k change points) and denote the minimum value of the cost by $C_{m,t}$. The idea of segment neighborhood search is to derive a relationship between $C_{t,m}$ and for $C_{s,m-1}$ for $s < t$:

$$\begin{aligned} C_{m,t} &= \min_{\tau} \sum_{1 \leq j \leq m+1} C\left(\mathbf{x}_{\tau_{j-1}:\tau_j}\right), \\ &= \min_{\tau \in \{m,\ldots,t-1\}} \left[C_{m-1,\tau} + C\left(\mathbf{x}_{\tau:t}\right)\right]. \end{aligned} \tag{26}$$

If this is run for all values of t up to n and for $j = 2, \ldots, m$, then the exact segmentations with $1, \ldots, m$ segments can be acquired. To extract the exact segmentation, we first let $\tau_l^*(t)$ to denote the optimal position of the last change point if we segment data $y_{1:t}$ using l change points. This can be calculated as

$$\tau_l^*(t) \arg\min_{\tau \in \{l,\ldots,t-1\}} \left[C_{l-1,\tau} + C\left(x_{\tau+1:t}\right)\right]. \tag{27}$$

The pseudocode of the SN method can be consulted in [14].

PELT, BS, and SN methods are implemented in *R* within the library *changepoint*; according to [46], the library *changepoint* creates class objects *.cpt*. The structure in *R* to detect change in the mean and variance is as follows:

$$\text{cpt.mean}\left(\text{data, method} = \text{“method”, test.stat} = \text{“Normal or CUSUM”}\right), \tag{28}$$

where data = a vector of objects that contain the data where a change in mean will be tested; method = choice for a single change point (AMOC) or multiple change points (PELT, SN, or BS); test.stat = statistic test, i.e., a distribution is assumed; choice “Normal” or “CUSUM” = see [11] or [6] for the normality assumption or the CUSUM test, respectively.

For variance

$$\text{cpt.var}\left(\text{data, know.mean} = \text{FALSE, method, test.stat} = \text{“Normal or CSS”}\right), \tag{29}$$

where data = vector of objects that contain the data where a change in variance will be tested. know.mean = logical argument required only under the normality assumption; if the logical value is TRUE, then the mean is assumed known, and in the other case, mean is estimated via maximum likelihood. test.stat = choice normal or CUSUM test.

3. Study Area and Application of Methodologies

3.1. Study Area and Data Quality Analysis. To detect change points in temperature data applying the methodologies described in the last section, the Civil Protection Commission of the State of Tlaxcala and the Climate Change Research Center of the Autonomous University of Tlaxcala provided information of 6 years (September 2, 2011, to September 2, 2017,) from a climatological station, located at the center of the municipality of Tlaxco, Tlaxcala, México.

For the quality analysis, the missing data were estimated with regression models; also, to understanding the behaviour of the maximum temperature of the region studied, a box-plot of the data was made (see Figure 1), where the maximum temperature per week was considered; for this, it was taken as the first week of the year, the week 1 of January (the week that begins with the first Sunday of the month) and week 52 as the last week of December. We verify that the assumptions of the methodologies studied were fulfilled to apply the studied theory and finally analyzed the results.

In Figure 1, the increase in maximum temperature is observed until it reaches its maximum values in weeks 11 to 23 (the spring season); with the arrival of rainy season (month of June, week 24), it can be seen a decrease in temperature in the box-plot. It also highlights the effect of the heat or drought intraestival (climatic event which consists of a decrease in the amount of precipitation in the middle of the rainy season which occurs mainly in the warm months of July and August); this event is also characterized by the decrease in rainfall due to an increase in temperature (weeks 28 to 33).

3.2. Application of Methodologies

3.2.1. Detection of a Single Change Point. In order to apply the methodologies of Davis et al., as well as Gombay, ARIMA models were fitted applying the Box–Jenkins methodology [47]. To verify stationarity and seasonality, autocorrelation function (*acf*) and partial autocorrelation function (*apf*) were analyzed. Figure 2 shows the time series for maximum temperature; according to the behaviour of the autocorrelation function (Figure 2(b)), it is observed that the time series is not stationary, with a difference the acf decreases in a fast way guaranteeing the stationarity of the series.

Via the analysis of the *acf*, ARIMA models were fitted; we consider as the best model the one with the lowest AIC value. Table 1 shows all fitted models and its corresponding AIC values. According to the AIC criterion, the best model to fit the data was the ARIMA $(3, 1, 0)$ model, and coefficients of this model are shown in Table 2. After fitting the model, a residual analysis was made. Noncorrelation of the residuals was verified with the Ljung-Box test. In addition, the Shapiro test was used for verifying normality (Figure 3).

With presented coefficients in Table 2, we have that the ARIMA $(3, 1, 0)$ model for fitting the maximum temperature is

$$\begin{aligned}\text{Tmax}_t &= 0.0036 + \text{Tmax}_{t-1} - 0.4872\left(\text{Tmax}_{t-1} - \text{Tmax}_{t-2}\right)\\ &\quad - 0.2981\left(\text{Tmax}_{t-2} - \text{Tmax}_{t-3}\right)\\ &\quad - 0.1795\left(\text{Tmax}_{t-3} - \text{Tmax}_{t-4}\right) + \varepsilon_t, \quad \text{for } t \in \mathbb{R}^+,\end{aligned} \tag{30}$$

where Tmax_t denotes the maximum temperature at time t and $\{\varepsilon_t\}$ is a white noise process.

After verifying assumptions of Theorem 2.1, values of statistic $\Lambda_n(k)$ were calculated for $k = 1, \ldots, 313$ (Figure 4), and the maximum value $\Lambda_n(k)$ was 29.17452 with $k = 36$. Considering $\alpha = 0.05$, critic value in order to accept or reject a change in parameters of model expressed by (3.2.1) is 15.76829; therefore, there exists sufficient evidence to guarantee a change in some parameters.

For applying Gombay's methodology, each component of vector $\widehat{\mathbf{B}}(u)$ was estimated and plotted (Figure 5). In addition, in Table 3, maximum values of $\widehat{\mathbf{B}}(u)$ with $0 < u < 1$ as well as the value of α were presented; hence, it may be concluded that there is not considered enough evidence to guarantee a change in any parameter of the model given in Section 3.2.1.

Applying the library *changepoint* of *R*, was detected a single change point (AMOC) in the mean and variance of database (Table 4 Figure 6).

3.2.2. Detection of Multiple Change Points. With the library *changepoint* of *R*, change point analysis of our data was

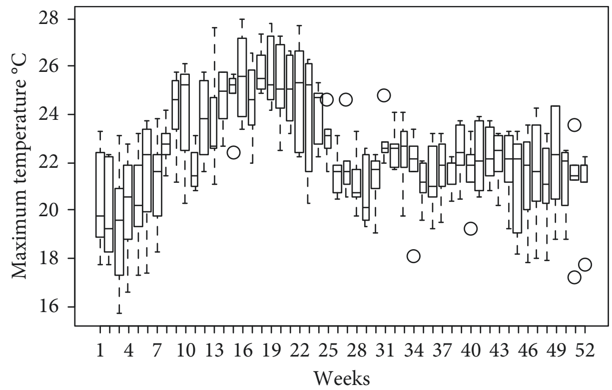

Figure 1: Box-plot for weekly maximum temperature registers, period: 2/09/2011–2/09/2017.

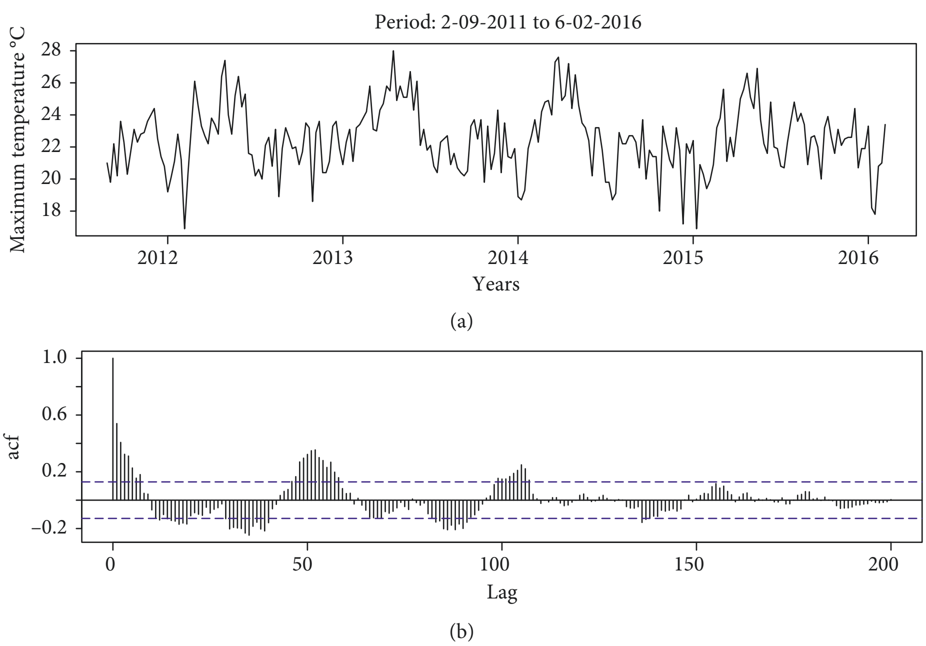

Figure 2: (a) Time series for weekly registers. (b) Autocorrelation function of the time series.

Table 1: Models fitted to the data.

Model	AIC	BIC
ARIMA(3,1,0)	1279.85	1.344602
ARIMA(2,1,1)	1281.01	1.348356
ARIMA(3,1,2)	1281.79	1.38138
ARIMA(3,1,1)	1281.79	1.362887
ARIMA(2,1,2)	1282.04	1.363619

Table 2: Coefficients of the ARIMA(3,1,0) model.

	ar1	ar2	ar3	Constant
	−0.4872	−0.2981	−0.1795	0.0036
s.e.	0.0559	0.0600	0.0559	0.0548
σ^2 estimated: 3.563, AIC = 1279.85				

performed with the PELT, BS, and SN methods. Tables 5 and 6 present the number of change points as well as their estimate for mean and variance of the maximum temperature, respectively.

Tables 7 and 8 present all mean and variance change points detected with the PELT method. To have a better conception of the location of change identified points by PELT, BS, and SN methods, and in order to interpret the location of change points, Figures 7 and 8 are presented.

3.3. Fitted Models after Detecting Change Points in the Mean of Maximum Temperature. After applying methods AMOC, BS, PELT, and SN for mean, we observe that PELT method realizes various segmentations on data; hence, an ARIMA fit is not possible. Method BS realizes six partitions to the database; however, last segmentation has only seven data (from weeks 307 to 313) and then is not possible a SARIMA fit in this segment.

In order to model the time series partitions and compare their results, this paper considered partitions from methods AMOC and SN. In the SN method, we

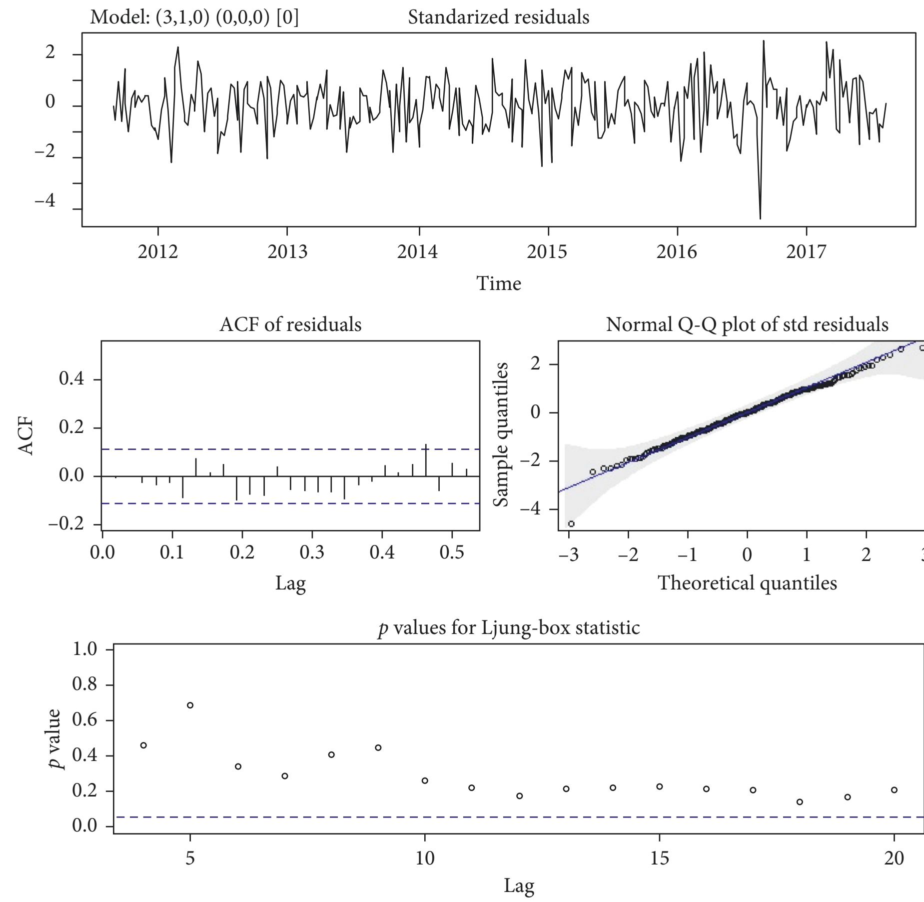

Figure 3: Residual analysis of the ARIMA$(3, 1, 0)$ model.

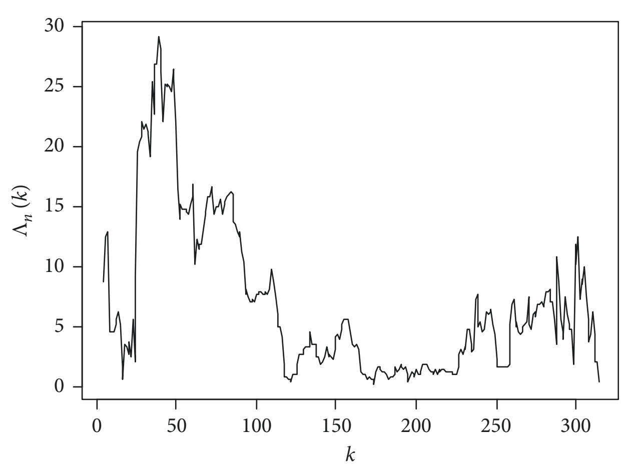

Figure 4: Values of statistic $\Lambda_n(k)$, for $k = 1, \ldots, 313$.

considered, like one segmentation the partitions of weeks 276 to 286 (see Figure 7), three ARIMAs were fitted for the maximum temperature in lapses: weeks 1 to 237, 238 to 286, and 287 to 313 applying the methodology of Box-Jenkins.

Table 9 shows the ARIMA models fitted; Figure 9 presents the partitioned time series and its autocorrelation function for each segment.

Table 10 presents fitted ARIMA models for three partitions considered with the SN method; Figure 10 shows each partition for the time series.

To choose the best model for the partitioned time series, the Akaike information criteria were considered, and after applying the AMOC method, the fit to the time series is presented in (31); equation (32) represents the fit after applying the SN method,

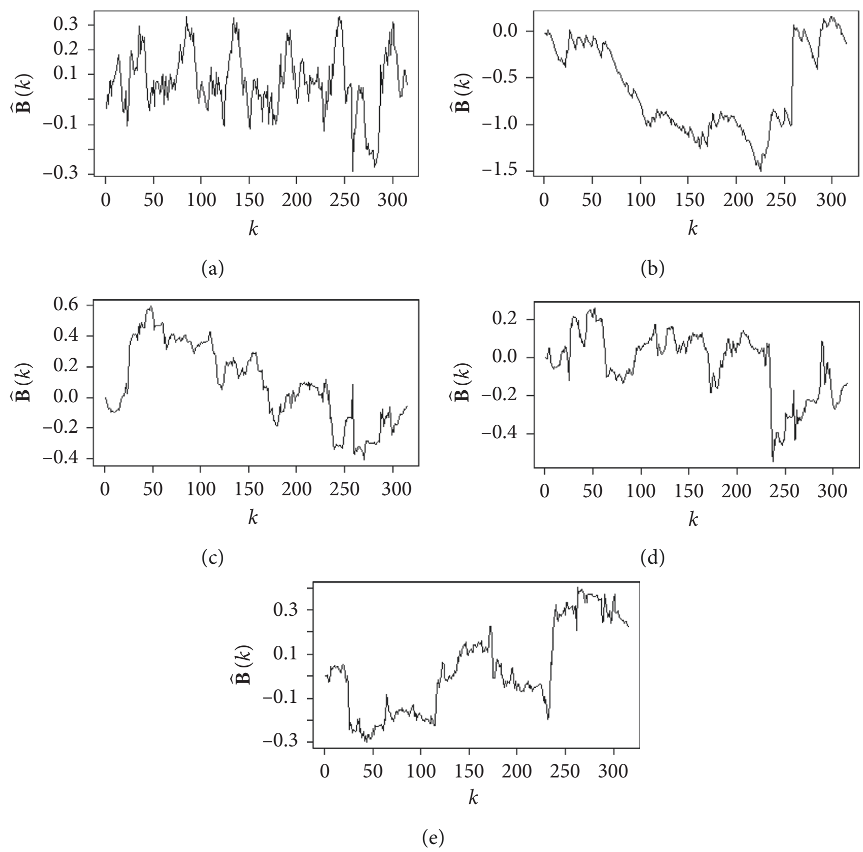

FIGURE 5: Values of $\widehat{\mathbf{B}}(k)$, $0 < k \leq n-1$, for each parameter of model given in Section 3.2.1: (a) μ. (b) σ^2. (c) ϕ_1. (d) ϕ_2. (e) ϕ_3.

TABLE 3: Maximum values to test a change in parameters μ, σ^2, ϕ_1, ϕ_2, and ϕ_3.

To test a single change in parameters	k	$\sup_{0\leq u\leq 1}\widehat{\mathbf{B}}(u)$	C_1 (0.01021614)
μ	245	0.3325948	1.5139
σ^2	300	0.1475097	1.5139
ϕ_1	48	0.5952468	1.5139
ϕ_2	52	0.258343	1.5139
ϕ_3	263	0.4006972	1.5139

TABLE 4: Single change point location for the mean and the variance for maximum temperature using the library *changepoint* of *R*.

Estimated change point in the mean	Week	Estimated mean before change point	Estimated mean after change point
	250	22.64560	21.50152
Estimated change point in the variance	Week	Estimated variance before change point	Estimated variance after change point
	27	4.078351	9.970822

$$\mathrm{Tmax}_t = \begin{cases} \mathrm{Tmax}_{t-1} + 0.0213 - 0.7750\left(\mathrm{Tmax}_{t-1} - \mathrm{Tmax}_{t-2}\right) + 0.0732\left(\mathrm{Tmax}_{t-2} - \mathrm{Tmax}_{t-3}\right) - 0.2464\varepsilon_{t-1} + 0.5789\varepsilon_{t-2} + \varepsilon_t, & \text{if } 1 \leq t \leq 250, \\ 0.0102 + \mathrm{Tmax}_{t-1} + 0.0419\left(\mathrm{Tmax}_{t-1} - \mathrm{Tmax}_{t-2}\right) + 0.5162\varepsilon_{t-1} + \varepsilon_t, & \text{if } t \geq 250, \end{cases} \tag{31}$$

$$\mathrm{Tmax}_t = \begin{cases} -0.0008 + \mathrm{Tmax}_{t-1} + 0.4547\left(\mathrm{Tmax}_{t-1} - \mathrm{Tmax}_{t-2}\right) + 0.1698\left(\mathrm{Tmax}_{t-2} - \mathrm{Tmax}_{t-3}\right) - \varepsilon_{t-1} + \varepsilon_t, & \text{if } 1 \leq t \leq 237, \\ -0.2044 + \mathrm{Tmax}_{t-1} + 0.3740\left(\mathrm{Tmax}_{t-1} - \mathrm{Tmax}_{t-2}\right) + \varepsilon_{t-1} + \varepsilon_t, & \text{if } 238 \leq t \leq 286, \\ 23.8714 + 0.3968\varepsilon_{t-1}, & \text{if } t \geq 287. \end{cases} \tag{32}$$

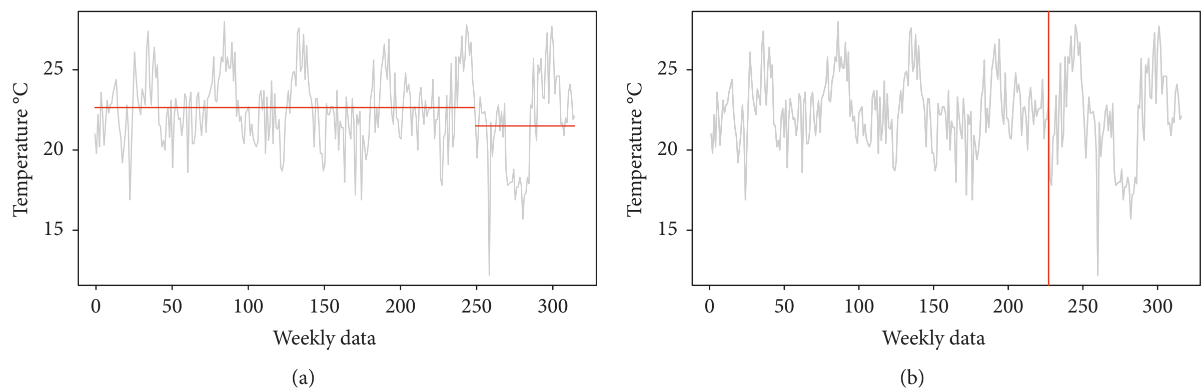

Figure 6: Location of change points for the (a) mean and (b) variance.

Table 5: Number of change points for the mean of maximum temperature.

Method	Number of change points	Estimated change point	Estimated mean
PELT	48	See Table 6	See Table 6
SN	4	237, 249, 270, 286	22.48312, 25.79167, 21.23333, 17.75000, 23.75333
BS	5	237, 250, 270, 286, 306,	22.48312, 25.60769, 21.12500, 17.75000, 24.48500, 22.29000

Table 6: Location of change points in the mean of maximum temperature, determined with the PELT method.

Week	8	16	23	24	25	34	36	42	52
Average	21.38750	23.20000	20.94286	16.90000	20.30000	23.56667	26.90000	24.70000	21.14000
Week	61	62	74	83	93	122	125	129	133
Average	22.32222	18.60000	22.05000	23.94444	25.73000	21.84138	18.96667	22.65000	24.47500
Week	135	141	148	152	164	165	171	172	175
Average	27.45000	25.46667	22.50000	19.35000	22.00000	18.00000	22.06667	17.20000	22.06667
Week	176	181	184	188	194	227	229	234	235
Average	16.90000	20.28000	24.20000	22.07500	25.60000	22.54848	18.00000	21.38000	25.40000
Week	237	249	259	260	270	286	288	289	291
Average	20.60000	25.79167	21.78000	12.20000	21.59000	17.75000	22.70000	25.70000	21.10000
Week	299	302	306						
Average	24.80000	26.90000	24.32500	22.29000					

Table 7: Number of change points in the variance of maximum temperature.

Method	Number of change points	Estimated change points	Estimated variances
PELT	17	See Table 8	See Table 8
SN	4	18 194 227 306	1.847222, 4.804260 1.454451, 11.119685, 1.043222
BS	4	18, 194, 227, 306	1.847222, 4.804260, 1.454451, 11.119685, 1.043222

Table 8: Location of change points determined with the PELT method.

Week	18	42	82	93	122	152
Variance	1.847222222	6.236503623	2.305583333	1.078545455	1.564655172	7.142298851
Week	158	194	218	222	254	257
Variance	0.092000000	5.700571429	1.654184783	0.056666667	7.507006048	0.003333333
Week	260	270	286	288	306	
Variance	25.270000000	1.105444444	0.548000000	0.020000000	3.610882353	1.043222222

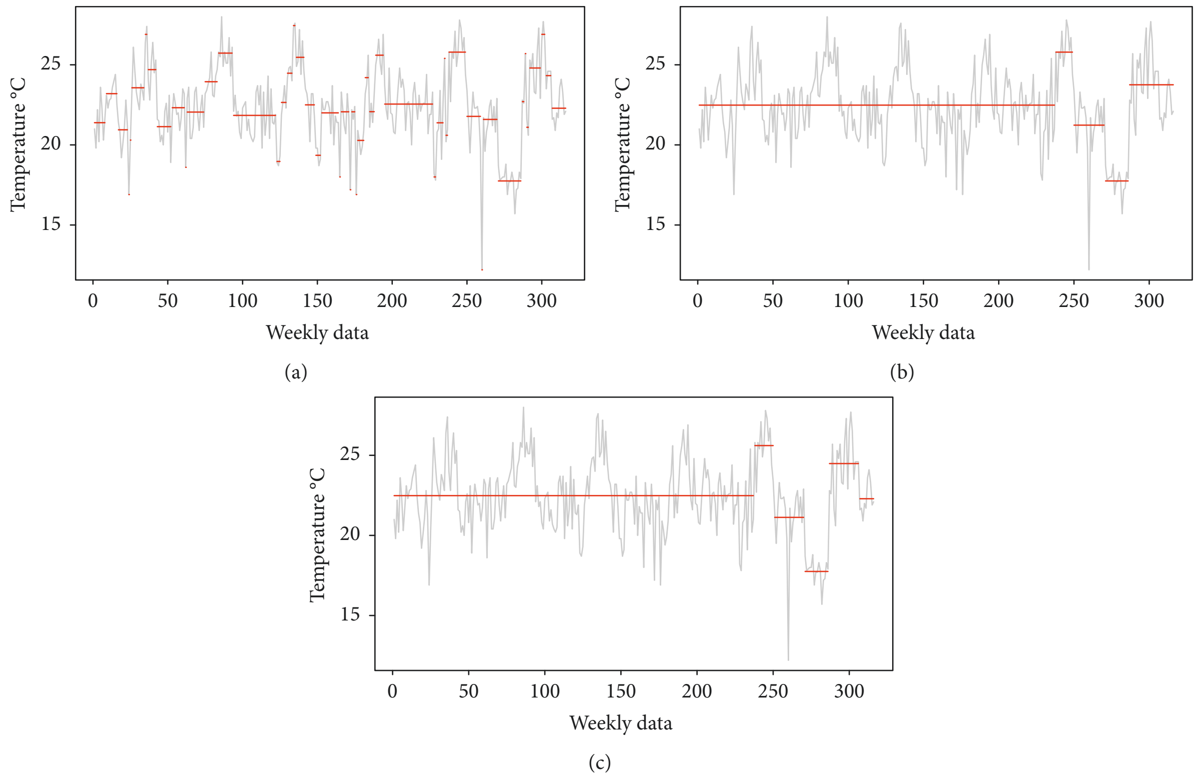

Figure 7: Location of change points for the mean of maximum temperature. (a) PELT method. (b) SN method. (c) BS method.

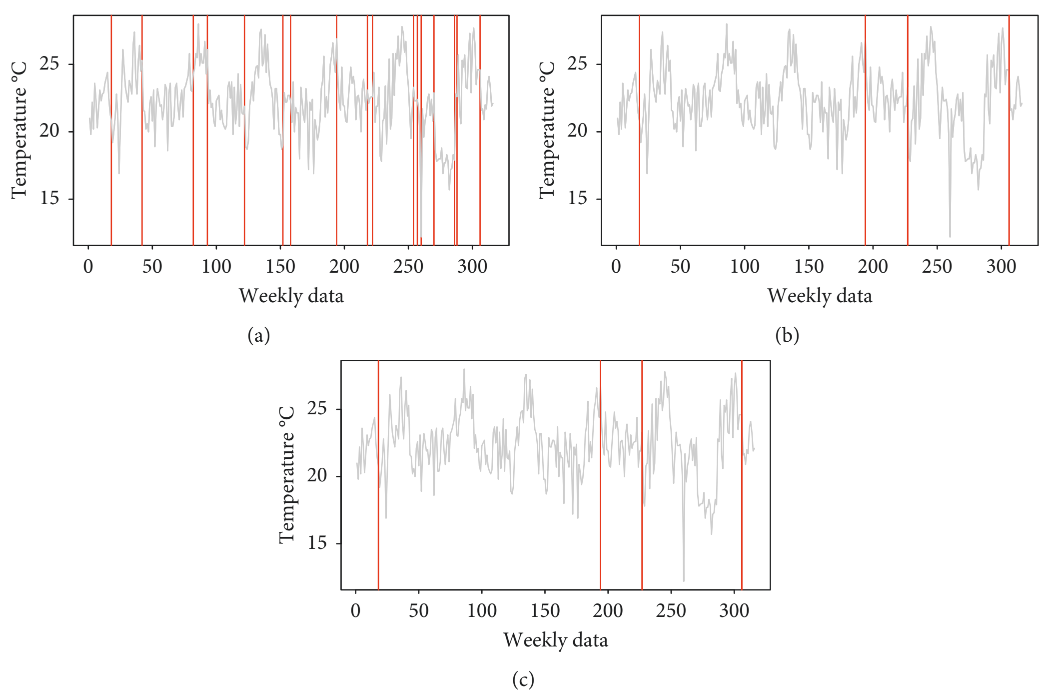

Figure 8: Location of change points for the variance of maximum temperature. (a) PELT method. (b) SN method. (c) BS method.

4. Comparison of Methodologies and Conclusion

The methodologies of Davis et al. [39] and Gombay [40] allow us to test if there exists a single change point or not in some parameter of the autoregressive model of order p; however, the methodology of Davis et al. does not allow us to estimate the time in which it occurs. When applying these methodologies in the database of maximum temperature, found results show with the methodology of Davis et al.,

Table 9: Models fitted after applying the AMOC method.

	Fitted model First data block	AIC	Fitted model Second data block	AIC
AMOC method	ARIMA(2,1,1)	987.06	ARIMA(1,1,1)	273.61
	ARIMA(3,1,0)	987.74	ARIMA(2,1,0)	274.79
	ARIMA(2,1,2)	988.47	ARIMA(2,1,1)	275.61
	ARIMA(3,1,1)	988.85	ARIMA(2,1,2)	277.52
	ARIMA(3,1,2)	990.47		
	ARIMA(2,2,2)	991.34		
	ARIMA(2,1,0)	994.72		

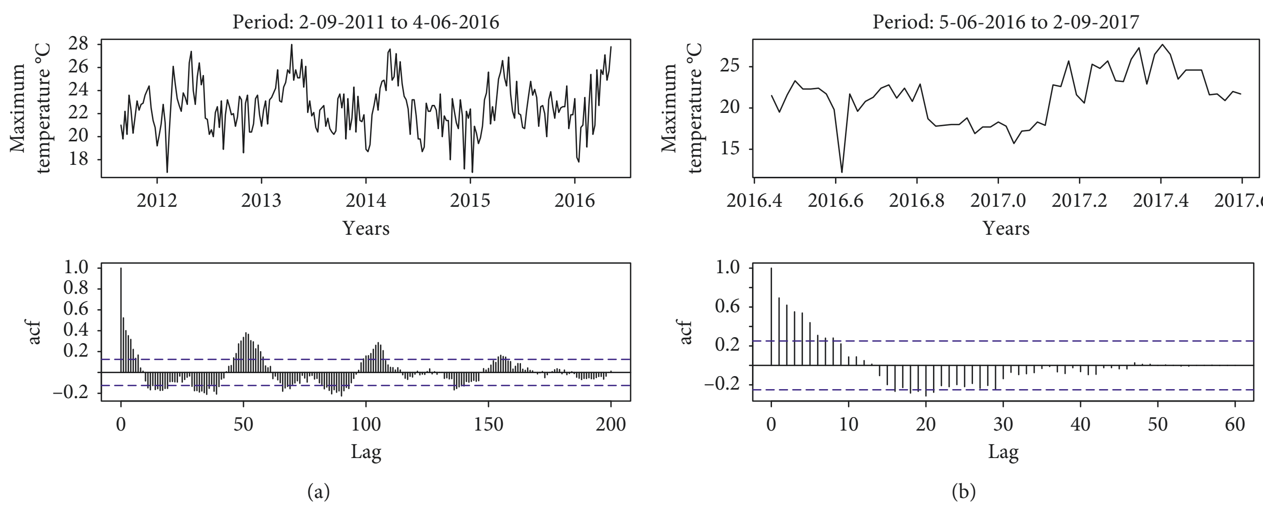

Figure 9: Partitioned time series after applying the AMOC method. (a) First data block. (b) Second data block.

Table 10: Models fitted after applying the SN method.

	Fitted model First data block	AIC	Fitted model Second data block	AIC	Fitted model Third data block	AIC
Método SN	ARIMA(2,1,1)	914.29	ARIMA(1,1,1)	191.99	MA(1)	108.29
	ARIMA(3,1,2)	917.13	ARIMA(2,1,2)	194.44	ARMA(1,1)	110.21
	ARIMA(2,1,2)	924.6	ARIMA(1,1,0)	194.84	MA(2)	110.23
	ARIMA(3,1,1)	924.64	ARIMA(1,1,2)	197.58	ARMA(1,2)	111.71
					ARMA(2,2)	113.5

evidence of a change in parameters of the proposed autoregressive model; on the other hand, the methodology of Gombay does not provide sufficient evidence to consider a change; this is attributed to the fact that for fitting the autoregressive model to database, a difference had to be made.

The other applied methodologies do not consider the structure of data, apart from being designed to estimate change points in mean and variance. The AMOC method estimates a single change point, while the PELT method estimates 48 change points in the considered database. Segmentation of the latter methodology makes it hard to propose models to fit the database.

For change in the mean of maximum temperature, methods AMOC, PELT, SN, and BS coincided with the conclusion of a change in week 250 when comparing this with methodology of Gombay, which proposes a possible change in week 245 is very close to that obtained by the other methods.

Gombay's methodology does not have enough evidence to decide a change in variance; the AMOC method estimates a change in variance in week 27, and the PELT, SN, and BS methods coincide with a change in weeks 18, 194, 227, and 306. With the AMOC method, the change is reported in week 250, which represents the third week of June 2016, reviewing the cold and warm episodes per season reported by the Climate Prediction Center [48]; in the June month of 2016, it is observed that the ocean changed from a heating to a cooling, and this allows us to validate the change reflected with the AMOC method; similarly, for the SN method, estimated change points coincide with the transition from a heating to a cooling of the ocean (week 237, which represents

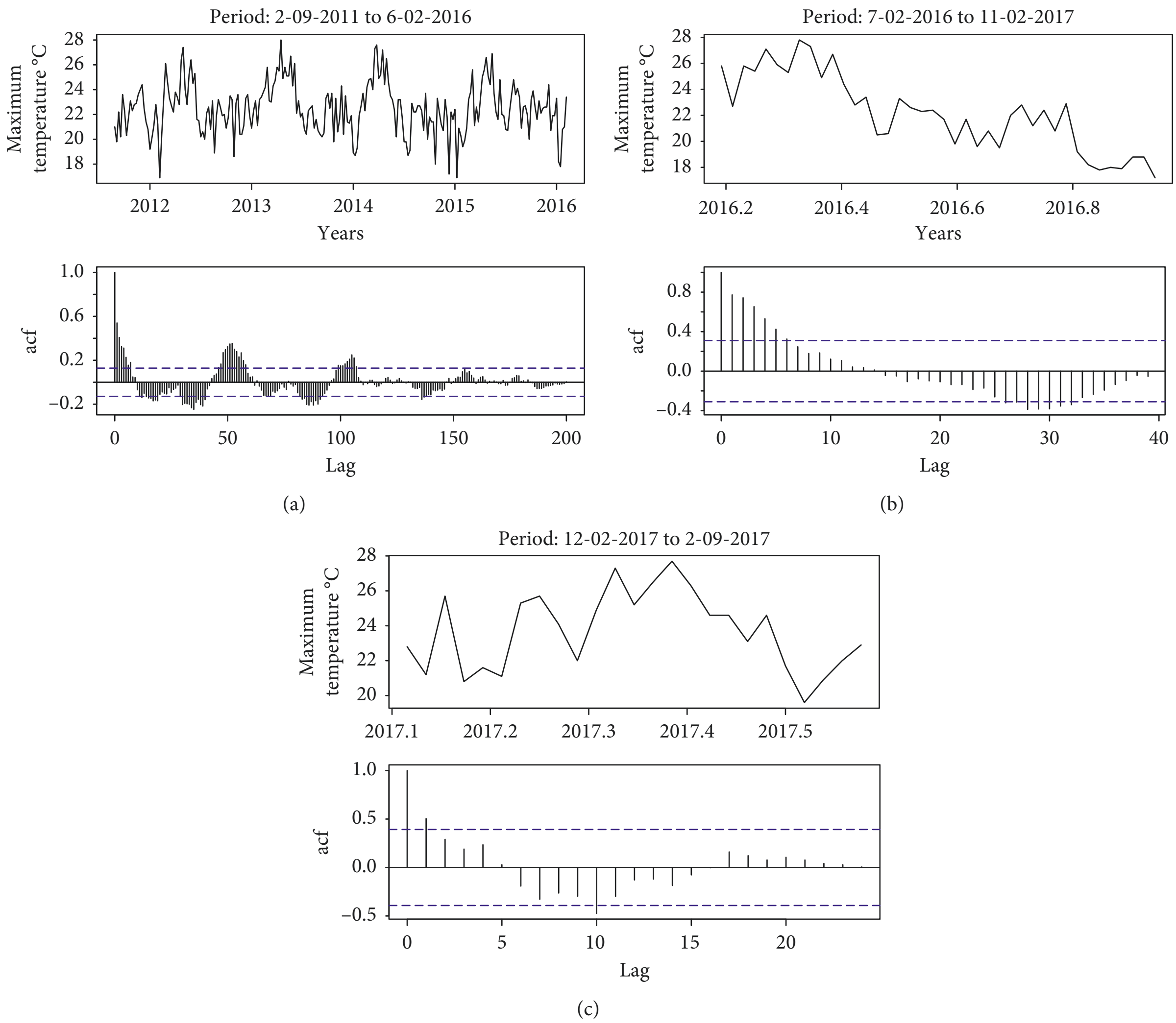

Figure 10: Partitioned time series after applying the SN method. (a) First data block. (b) Second data block. (c) Third data block.

the last week of February month, 2016) and a period without heating of the ocean to a cooling (week 287, which represents June month, 2017).

Although PELT and BS methods detect a change in week 306 (Tables 5–8) which is almost at the end, this week is in the latest record; therefore, it might be an unreliable change point.

The study of the points of change in climate data at the regional level is essential for decision-makers and for the local development of the communities to be analyzed, since the level of impact on their economic growth can be dimensioned at the same time as they will be able to establish measures to adapt to these changes with society and political-economic sectors.

For the subsequent studies, we intend to consider seasonal models as well as multivariate models that may be related to describe the climate of some regions.

Data Availability

The data from maximum temperature used to support the findings of this study were provided by the Climate Change Research of the Autonomous University of Tlaxcala and the State Civil Protection Commission. Requests for access to these data should be made to Rogelio Bernal Morales, M.S. (rbernal07@hotmail.com) or Tomás Morales Acoltzi, Ph.D. (acoltzi@atmosfera.unam.mx).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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