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Research Article

Forecasting the Covolatility of Coffee Arabica and Crude Oil Prices: A Multivariate GARCH Approach with High-Frequency Data

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Forecasting the covolatility of asset return series is becoming the subject of extensive research among academics, practitioners, and portfolio managers. This paper estimates a variety of multivariate GARCH models using weekly closing price (in USD/barrel) of Brent crude oil and weekly closing prices (in USD/pound) of Coffee Arabica and compares the forecasting performance of these models based on high-frequency intraday data which allows for a more precise realized volatility measurement. The study used weekly price data to explicitly model covolatility and employed high-frequency intraday data to assess model forecasting performance. The analysis points to the conclusion that the varying conditional correlation (VCC) model with Student's t distributed innovation terms is the most accurate volatility forecasting model in the context of our empirical setting. We recommend and encourage future researchers studying the forecasting performance of MGARCH models to pay particular attention to the measurement of realized volatility and employ high-frequency data whenever feasible.

1. Introduction

Forecasting the covolatility of asset return series is becoming the subject of extensive research among academics, practitioners, and portfolio managers [1]. This has been used in risk management, derivative pricing and hedging, portfolio selection, and policy making. Similarly, the analysis of volatility spillovers between commodity and asset prices has a profound implication for risk management and portfolio maximization by the government and investors [2]. In view of the current bearish behavior of oil price in the international markets, it is arguably of special interest to study the relationship between oil and coffee prices [2]. Recently, a number of papers have studied the comovement of oil prices with equities, agricultural commodities, and precious metals prices.

Prior studies provide evidence on the connectedness between oil and one or more markets. However, the bulk of these studies has focused on price connectedness. Relatively little research has been devoted to volatility comovement. This is surprising since price volatilities are the main driving forces in options and commodity future markets. This paper seeks to bridge this gap in the literature. In particular, it undertakes a systematic analysis of the volatility interconnectedness between two important commodity markets—oil and coffee markets—using multivariate GARCH (MGARCH) family of models with the view of providing some rigorous price covolatility forecasting models.

Naturally, different papers in the literature also tend to focus on different commodity pairs when studying volatility interconnectedness. The focus of this work is on the covolatility between Brent crude oil and Coffee Arabica prices in the commodity futures markets which are markets where one can buy specific quantities of a commodity at a specified price with delivery set at a specified time in the future. The choice of these commodities is motivated by the fact that these are the most actively traded commodities in the world. Existing work appears to focus mostly on in-sample modeling of covolatility

of commodity prices giving less attention to model forecasting performance. This work will also seek to bridge this gap in the literature.

A number of recent papers have investigated the covolatility of energy and commodity price mainly due to increased interest for understanding the drivers and dynamics of such covolatilities in these highly volatile times. Alternative estimation approaches are used in the literature, but thus far there is a dearth of work undertaking a thorough "MGARCH" analysis—a statistical framework which is particularly suited for modeling asset prices covolatility. Moreover, existing work appears to focus mostly on in-sample modeling of covolatility of commodity prices giving less attention to model forecasting performance. Accordingly, this work will seek to contribute to the literature by undertaking a systematic analysis of volatility forecasting performance.

Most of the existing research on volatility spillovers employ statistical models in order to estimate realized volatilities which turned out to be oftentimes poor approximations of true volatilities. An attractive alternative to model-based statistical volatility is to compute realized volatility based on high-frequency intraday or "tick" data. Realized volatility has been found to be more accurate than model-based volatilities in predicting latent volatility [3]. Thus, the results provided by previous studies can be potentially misleading in that they may have underestimated or overestimated the extent of the true volatility.

This work avoids this potential pitfall by using realized volatility based on 30-minute intraday data. We argue that doing so represents an important contribution to the literature of covolatility measurement within commodity markets.

The work makes the following contributions to the literature:

(1) From an applied econometrics stand point, the work provides a robust and systematic investigation of the statistical and econometric properties of the covolatility of the two important commodities.

(2) The direction and magnitude of correlation between the prices of coffee and crude oil which are documented in this work would help the design of a more informed policy making on the part of financial authorities as well as other stakeholders and market participants interested in foreseeing the volatility of these two important commodities.

2. Methods

2.1. Multivariate GARCH Models. A large part of the literature deals with univariate model price series. But markets interact, and therefore a generalization from the univariate model to a multivariate one is important. Multivariate GARCH models can be categorized into four types [4].

(i) Models of the conditional covariance matrix: in these models, the conditional covariance is computed in a direct way. The vector error correction (VEC) and Baba–Engle–Kraft–Kroner (BEKK) models are among this type of models.

(ii) Factor models: the return process is assumed to consist of a small number of unobservable heteroskedastic factors. This approach benefits from that the dimensionality of the problem reduces when the number of factors compared to the dimension of the return vector is small.

(iii) Models of conditional variances and correlations: at first, the univariate conditional variances and correlations are computed and then used to get the conditional covariance matrix. Some models are, for example, the constant conditional correlation (CCC) model and the dynamic conditional correlation (DCC) model.

(iv) Nonparametric and semiparametric approaches: models in this class form an alternative to parametric estimation of the conditional covariance structure. The advantage of these models is that they do not impose a particular distribution (that can be misspecified) on the data.

In this paper, conditional variance and correlation models are used. Before detailed discussion of the MGARCH models, some definitions of the series are presented. This study uses two commodity price series, resulting in a bivariate approach.

$$r_t = \begin{bmatrix} r_{1,t} \\ r_{2,t} \end{bmatrix}, \tag{1}$$

where r_t is the vector of returns which can be decomposed as

$$\begin{aligned} r_t &= \mu_t + \varepsilon_t, \\ \varepsilon_t &= H_t^{1/2} z_t, \\ \mu_t &= E\left(r_t \middle| I_{t-1}\right), \\ z_t &\sim iidN\left(\underline{0}, I_N\right), \\ \operatorname{var}\left(r_t \middle| I_{t-1}\right) &= H_t^{1/2}\left(H_t^{1/2}\right)' = \sum_t, \end{aligned} \tag{2}$$

where r_t is the $N \times 1$ vector of returns; μ_t is the $N \times 1$ vector of expected return at time t given the available information set I_{t-1}; $H_t^{1/2}$ $N \times N$ is the Cholesky factor of time-varying conditional covariance matrix $\sum_t$; and z_t is the $N \times 1$ vector of independent errors with mean zero and variance one.

The variance of the conditional unpredictable component can also be defined as

$$\operatorname{var}\left(r_t \mid I_{t-1}\right) = \sum_t = \begin{bmatrix} h_{11,t} h_{12,t} \\ h_{21,t} h_{22,t} \end{bmatrix}. \tag{3}$$

In equation (3), the matrix obtained should be symmetric. Furthermore, the matrix $\sum_t$ has to be positive definite for all t. The approach of multivariate modeling brings complications. Firstly, it may be hard to ensure that $\sum_t$ is positive definite for all t. Secondly, the number of parameters becomes too large to estimate, and at last, it may be difficult to obtain the stationarity condition for $\sum_t$.

The various MGARCH models proposed in the literature differ in how they trade off flexibility and parsimony in their specifications for $\sum_t$. Increased flexibility allows a model to capture more complex $\sum_t$. The following sections will describe the theory of the models, where the VEC and DVEC models which are the predecessors to the CCC, VCC, and DCC models are first described.

2.1.1. The Constant Conditional Correlation (CCC) Model. The constant conditional correlation model was suggested by Bollerslev [5], where the time-varying covariance matrix $\sum_t$ at time t is expressed as

$$\sum_t = \mathrm{Dt}\, R_t \mathrm{Dt}. \tag{4}$$

The right hand side consists of the conditional correlation matrix R that is time invariant, i.e., $R_t = R.D$ is a diagonal matrix of $(h_{1t,} h_{2t}, \ldots, h_{kt})$ such as

$$D_t = \begin{bmatrix} \sqrt{h_{1,t}} \\ \sqrt{h_{2,t}} \\ \sqrt{h_{2,t}} \\ \vdots \\ \sqrt{h_{k-1,t}} \\ \sqrt{h_{k,t}} \end{bmatrix}, \tag{5}$$

where each $h_{i,t}$ follows a univariate GARCH process. The conditional correlation matrix is given by $R = [\rho_{i,j}]$, and the nondiagonal elements of $\sum_t$ are

$$\sum_{ij,t} = h_{i,t}^{1/2} h_{j,t}^{1/2} \rho_{i,j}, \quad \forall i \neq j. \tag{6}$$

The desired conditional variances can be expressed in vector form:

$$\sum_t = C + \sum_{i=1}^{q} A_i \varepsilon_{t-j} \odot \varepsilon'_{t-j} + \sum_{j=1}^{p} B_j \sum_{t-j} \odot \sum'_{t-j.} \tag{7}$$

The first term C is a vector of the intercepts with a dimension of $n \times 1$ and the matrices of the coefficients are $\times n$.

The advantage of the CCC model is that the computational procedure is more easily performed since the correlation matrix R is constant. However, this means that the model may be too restrictive [6].

2.1.2. The Dynamic Conditional Correlation (DCC) Model. The dynamic conditional correlation model is given as

$$\sum_t = Dt\, Rt\, Dt, \tag{8}$$

where $\sum_t$ is the covariance matrix and Rt is $n \times n$ matrix of the conditional correlation of the returns which is symmetric by definition. The diagonal matrix Dt is expressed as

$$D_t = \mathrm{diag}\left(h_{1t}, h_{2t,} \ldots, h_{kt}\right), \tag{9}$$

where each $h_{i,t}$ follows a univariate GARCH process. Furthermore, $\sum_t$ has to be positive definite, which is automatically obtained while Rt is a correlation matrix that is symmetric by definition. When this matrix is defined, two requirements are needed. Firstly, $\sum_t$ needs to be positive definite, since it is a covariance matrix. Secondly, the parts that belong to Rt need to be less than one. These requirements are met through decomposition:

$$R_t = \mathrm{diag}\left(q_{iit}\right)^{-1} Q_t \mathrm{diag}\left(q_{iit}\right)^{-1} \quad \text{for}, ii = 1, \&, n, \tag{10}$$

where

$$\begin{aligned} Q_t &= \left(1 - \pi_1 - \pi_2\right)\overline{Q} + \pi_1 \varepsilon_{t-1} \varepsilon'_{t-1} + \pi_2 Q_{t-1,} \\ \overline{Q} &= \mathrm{cov}\left[\varepsilon_t \varepsilon'_t\right] = E\left[\varepsilon_t \varepsilon'_t\right], \end{aligned} \tag{11}$$

where $\overline{Q}$ is the unconditional covariance and will be finite as the model contains finite parameters and the vector ε_t has finite variance. Additionally, the parameters π_1 and π_2 are scalars and diag (Q) is used to rescale the parts of Qt in order to fulfill that $|\rho_{ij}| = |q_{ijt}/\sqrt{q_{iit} q_{jjt}}| \leq 1$, where q_{ij} is the element of the matrix diag (Q). An estimate of $\overline{Q}$ is

$$\widehat{Q} = \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t \varepsilon'_t. \tag{12}$$

Moreover, the scalars π_1 and π_2 must be larger than zero, but the sum has to be less than one. One may note that these are conditions of the univariate GARCH to be stationary, but which is applied in the DCC model [7].

2.1.3. The Varying Conditional Correlation (VCC) Model. The varying conditional correlation (VCC) is a multivariate generalized autoregressive conditionally heteroskedastic (MGARCH) model in which the conditional variances are modeled as univariate GARCH models and the conditional covariance is modeled as nonlinear functions of the conditional variances. The conditional correlation parameters that weight the nonlinear combinations of the conditional variance follow the GARCH-like process specified by Tse and Tsui [6]. The VCC MGARCH model is about as flexible as the closely related dynamic conditional correlation MGARCH model and more flexible than the constant conditional correlation model and more parsimonious than the diagonal vector error conditionally heteroskedastic (DVECH) models.

MGARCH models differ in the parsimony and flexibility of their specifications for a time-varying conditional covariance matrix of the disturbances, denoted by $\sum_t$. In the conditional correlation family of MGARCH models, the diagonal elements of $\sum_t$ are modeled as univariate GARCH models, whereas the off-diagonal elements are modeled as nonlinear functions of the diagonal terms. In the VCC MGARCH model,

$$h_{ij,t} = \rho_{ij,t} \sqrt{h_{ii,t} h_{jj,t}}, \tag{13}$$

where the diagonal elements hii,t and hjj,t follow univariate GARCH processes and $\rho ij,t$ follows the dynamic process specified in Tse and Tsui [6] and discussed below. Because $\rho ij,t$ varies with time, this model is known as the VCC GARCH model.

The VCC-GARCH(1, 1) model proposed by Tse and Tsui [6] can be written as

$$\begin{aligned} r_t &= \theta x_t + \epsilon_t, \\ \epsilon_t &= H_t^{1/2} z_t, \\ \sum_t &= D_t R_t D_t, \\ R_t &= (1 - \pi_1 - \pi_2) R + \pi_1 \epsilon_{t-1} + \pi_2 R_{t-1}, \end{aligned} \tag{14}$$

where x_t is a $k \times 1$ vector of independent variables, which may contain lags of r_t; θ is an $m \times k$ matrix of parameters; r_t is an $m \times 1$ vector of dependent variables; $H_t^{1/2}$ is the Cholesky factor of the time-varying conditional covariance matrix $\sum_t$; z_t is an $m \times 1$ vector of independent and identically distributed innovations; and D_t is a diagonal matrix of conditional standard deviation. $D_t = \mathrm{diag}(h_{1t}, h_{2t}, \ldots, h_{\mathrm{kt}})$. R_t is a matrix of conditional correlations, $R_t = \begin{bmatrix} 1 \rho_{12,t} & \cdots & \rho_{1m,t} \\ \rho_{21,t} 1 & \cdots & \rho_{2m,t} \\ \vdots & \vdots & \vdots \\ \rho_{m1,t} & \cdots & \rho_{m2,t} 1 \end{bmatrix}$.

R is the matrix of means to which the dynamic process in equation (14) reverts; π_1 and π_2 are parameters that govern the dynamics of conditional correlations, and they are nonnegative and satisfy $0 \le \pi_1 + \pi_2 < 1$.

2.2. Determining the Conditional Distribution. When fitting a GARCH model based on financial data, the conditional distribution of the returns has to be defined. Studies, for example Bollerslev [8], illustrate that returns are not normally distributed. Instead, the Student t distribution captures the observed kurtosis in empirical returns in a more sufficient way than the normal distribution. Returns have excess kurtosis and fatter tails than the normal distribution. Therefore, the Student t distribution is more suitable [9]. There are three assumptions about the conditional distribution of the error term commonly employed when working with GARCH models: normal (Gaussian) distribution, Student's t distribution, and the generalized error distribution (GED).

2.3. Estimation Evaluation. The evaluation is done with several methods. Firstly, the estimations are compared using the method of three measures: MAE, RMSE, and the explanatory power from an ordinary least squares regression. Secondly, an attempt to determine the goodness of fit of the residuals is done through univariate approach consisting of an analysis with the Ljung–Box test.

2.3.1. Defining a Proxy. In evaluating volatility forecasts, the usual proxy for "true" volatility is "ex post" squared returns or the squared errors. However, as noted by Andersen and Bollerslev [5], although the use of squared returns or the squared errors is justifiable as an unbiased estimate of the volatility process, it provides a very noisy measure due to a large idiosyncratic term. Specifically, the returns innovation can be written as $\varepsilon_t = \sigma_t z_t$ with z_t an independent zero mean and unit variance stochastic process and σ_t is the volatility process. If the model for σ_t^2 is correctly specified, then the conditional expectation $E(E(\varepsilon_t^2 \mid I_{t-1}) = E(\sigma_t^2 z_t^2 \mid I_{t-1}) = \sigma_t^2$, and the squared error is an unbiased estimate of the volatility process; however, it still contains the noisy idiosyncratic term, z_t^2. This typically resulted in a poor performance, which instigated a discussion of the practical relevance of volatility models.

Hansen et al. [10] provided another important argument for using the realized variance rather than the squared return. They showed that substituting the squared returns for the conditional variance can severely distort the comparison, in the sense that the empirical ranking of models may be inconsistent for the true (population) ranking. So, an evaluation that is based on squared returns may select an inferior model as the "best" with a probability that converges to one as the sample size increases. For this reason, our evaluation is based on the realized variance. Building upon this line of research, Andersen et al. [11] defined the so-called "realized volatility" on day t as

$$h_t^{rv} = \sum_{i=1}^{N} r_{t,i}^2, \tag{15}$$

where N is the number of equally spaced intervals within a day and $r_{t,i}$ is a logarithmic return on day t at time interval i with $i = 1, 2\ldots, N$ and $t = 1,\ldots,T$.

Thus, realized volatility is the sum of squared intraday returns. In principle, letting N tend to large, i.e., continuous time sampling, the measure approaches to the true integrated volatility of the underlying continuous time process and is theoretically free from measurement error. Further, this measure allows a market participant to essentially treat volatility as an observed variable and to allow direct estimation.

2.4. Forecast Evaluation

2.4.1. The MZ Regression. A popular way to evaluate volatility models' out-of-sample forecasting performance is in terms of R^2 from a Mincer–Zarnowitz (MZ) regression,

$$\sigma_t^2 = a + b h_t^2 + v_t, \tag{16}$$

that is, squared returns are regressed on the model forecasts of σ_t^2 and a constant. Here, σ_t^2 is ex post volatility (e.g., realized volatility) at time t, h_t^2 is estimated (in-sample) or forecasted (out-of sample) volatility at time t, and v_t is the error term which is independent and identically distributed; $v_i \sim N(0,1)$. a and b are parameters to be estimated.

If the model for conditional variance is well specified, we should have $a = 0$, $b = 1$. According to specific features of financial data series, the value of R^2 is usually low (even less than 5%) [11].

2.4.2. Mean Absolute Error (MAE). Another way of determining the goodness of the estimations and forecasts is calculating the MAE. The approach is to measure how the received conditional covariance is close to its corresponding realized value. The formula is

$$\mathrm{MAE} = \frac{1}{n}\sum_{t=1}^{n}\left|\sigma_t^2 - h_t^2\right|, \tag{17}$$

where σ_t^2 is used as a proxy and h_t^2 is the estimated covariance. By comparing the MAE between the estimated models, it can give an indication of which model that makes the best estimations.

2.4.3. Root Mean Square Error (RMSE). The third measure is the root mean square error (RMSE), which is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\sigma_t^2 - h_t^2\right)^2}. \tag{18}$$

Using these methods, the estimated models can be compared, and using the same measurements for estimations and forecasts, one can determine if the relatively best estimation model also makes the best forecast.

3. Results and Discussion

3.1. Data. The data considered in this paper were the weekly time series of Brent crude oil and Coffee Arabica futures market closing price, given in US dollars per barrel and US dollars per pound, respectively. Both series contain data spanning between first week of January 2005 and last week of October 2016 extracted from Bloomberg database. The full sample is split in to two parts: in-sample data, in order to estimate the parameters of models, and out-of-sample data, in order to make forecasts. The in-sample period spans from first week of January 2005 up to last week of December 2015 and the out-of-sample period spans from first week of January 2016 through last week of October 2016. For the out-of-sample period, we have also extracted 30-minute intraday data for realized volatility measuring purpose.

3.2. Test of Stationarity and Features of Log Return Series. Our data consist of two commodity prices: crude oil and Arabica coffee. The two commodity prices present the same trend and direction during the entire study period. Prices for both of the commodities have risen from 2005 up to 2008 and fallen from 2009 up to 2010 with a similar pattern. Another impression is that both commodity level series are nonstationary (see Figure 1).

To visualize the returns series for these two markets, we depict the return time series plots in Figure 2. The weekly return series display volatility persistence properties, indicating that large changes tend to be followed by large changes of both signs and small changes tend to be followed by small changes, and the differenced series suggests stationarity. The plots further show that the volatilities of these two commodity returns not only have a volatility clustering phenomenon during the selected sample period but also have certain relation on their return volatility processes. That is, when the fluctuation of the Arabica coffee price grew larger, the volatility of crude oil return also became larger for most of the times. This is the main motive for discussing the relationships of coffee price returns and crude oil price returns.

Table 1 summarizes the DF test results of the two commodity level prices, respectively. As can be seen in the table, p-value is greater than 0.05. Hence, the null hypothesis of unit root would not be rejected, that is, there is a unit root problem in each of the series.

If time series data are nonstationary, it is necessary to look for possible transformations that might bring stationarity. In practice, econometricians usually transform financial prices in to return forms. This is because often return series are found to be stationary such that analysis is possible. The log return series is obtained by

$$r_t = 52 * \left(\log\frac{p_t}{p_{t-1}}\right), \tag{19}$$

where r_t is the log return series of the real price multiplied by 52 (which is simply a scaling factor), to annualize as we are using weekly data and each year contains 52 weeks, and p_t is the originalprice series and log is the natural logarithm in base ten.

Table 2 summarizes the DF test of the log return series for each of the commodities. The table shows that both p-values are too small (p-value <0.05) at 5% level of significance. These indicate that the null hypothesis of unit root would be rejected in both cases. Hence, the log return series are stationary for each of the items as shown by the time series plots.

Looking at Table 3, the average prices for Arabica coffee (per Pound) and crude oil (per Barrel) were \$145.8668 and \$80.9327, respectively, over the entire period. The average log returns were then computed, and they are 0.0458 and 0.0119 for coffee and oil, respectively. The average returns for crude oil price were smaller than those of Arabica coffee price return in the sample period considered. One more impression here is that the two commodities' return is positive, which is an implication of the increasing trend in the two commodity price. The estimate of the standard deviation is 2.229 for coffee and 2.3639 for oil which is nearly similar. Hence, we can say that they exhibit approximately similar variations in their prices.

3.3. Volatility Modeling. To build a volatility model for the log return series, the first step is specifying the mean equation. Once we specify the mean equation, we have to test for ARCH effects using the residuals of the mean equation. If ARCH effects are statistically significant, specifying a volatility model and carrying out a joint estimation of the mean and volatility equations are necessary. The conditional mean specification is, in general, arbitrary for GARCH models of the conditional volatility.

3.3.1. Test for ARCH Effects. Considering a financial return model and on fitting this model, if there is no volatility clustering in each of the returns, the random disturbance

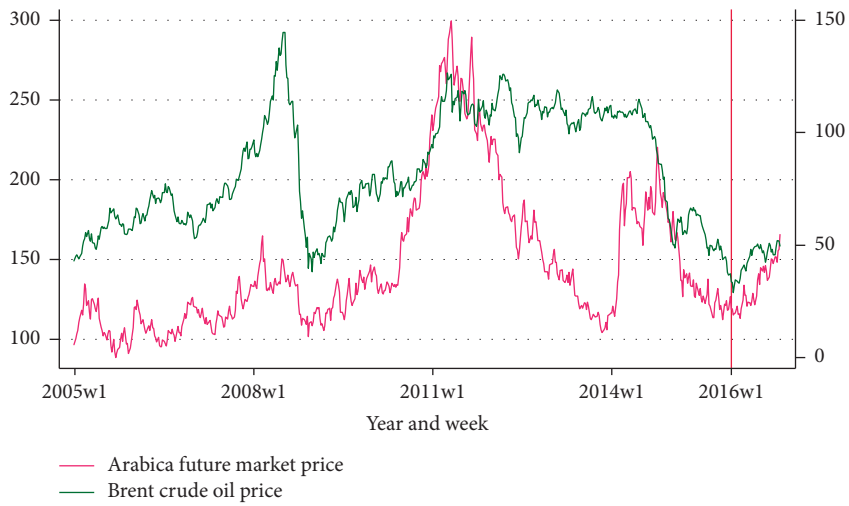

Figure 1: Time series plot for level series, Arabica coffee and crude oil.

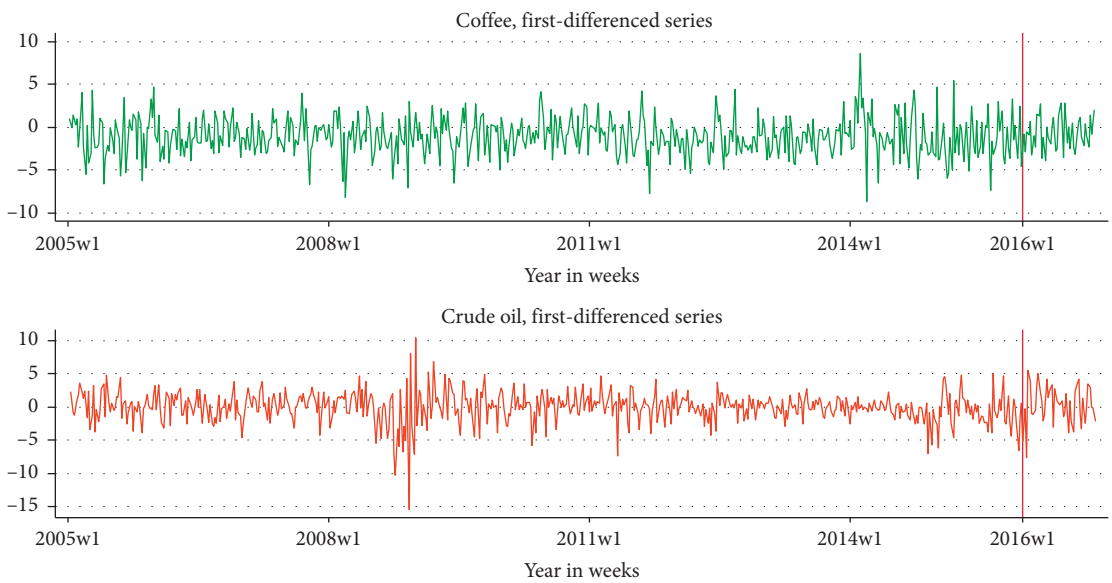

Figure 2: Time series plot for differenced series, Arabica coffee and crude oil.

Table 1: DF unit root test of stationarity for level prices, with trend.

Series	t-statistic	MacKinnon approximate p-value
Crude oil	−1.675	0.4562
Arabica coffee	−2.053	0.3896

Table 2: DF unit root test of stationarity for log returns.

Series	t-statistic	MacKinnon approximate p-value
Crude oil	−5.791	0.001
Arabica coffee	−5.877	0.001

term ε_t should be a white noise process. The standardized residual plot can be an initial insight to judge the heteroskedastic characteristics of the error term. The standardized residual plot from each commodity return series is shown in Figure 3. The figure depicts the residual plots of the two commodity return series generated from the mean equation.

We see from the figure that for both of the series, there is a prolonged period of low volatility and prolonged period of high volatility. For example, for Arabica coffee return, there is a long period of low volatility from the first week of 2005 to the end of 2008 and there exist long periods of high volatility from first week of 2014 to the end week of 2016. Crude oil return also exhibits a prolonged period of low volatility than the coffee series, which is from the first week of 2005 to the first week of 2009. In other words, periods of high volatility are followed by periods of high volatility and periods of low volatility tend to be followed by periods of low volatility, which is known as volatility clustering. This suggests that the residuals or error terms are conditionally heteroskedastic and can be represented by GARCH models.

Table 3: General summary statistics for Arabica coffee and crude oil series.

Statistics	Coffee market		Crude oil market	
	Pt	r_t	Pt	r_t
Mean	145.8668	0.0458	80.9327	0.0119
Median	132.7250	0	76.1950	0.1327
Maximum	299.3500	9.6800	144.4900	10.449
Minimum	88.2000	−7.538	28.9400	−15.451
Std.dev.	43.3636	2.229	26.1545	2.3639
Skewness	1.4120	0.0359	0.1347	−0.6890
Kurtosis	4.4545	3.6941	1.7781	7.2470
Jarque–Bera	144.8500 (0.001)	8.2600 (0.016)	488.0200 (0.001)	107.4100 (0.001)

**Values below the Jarque–Bera statistics are p-values at 5% of significance level.

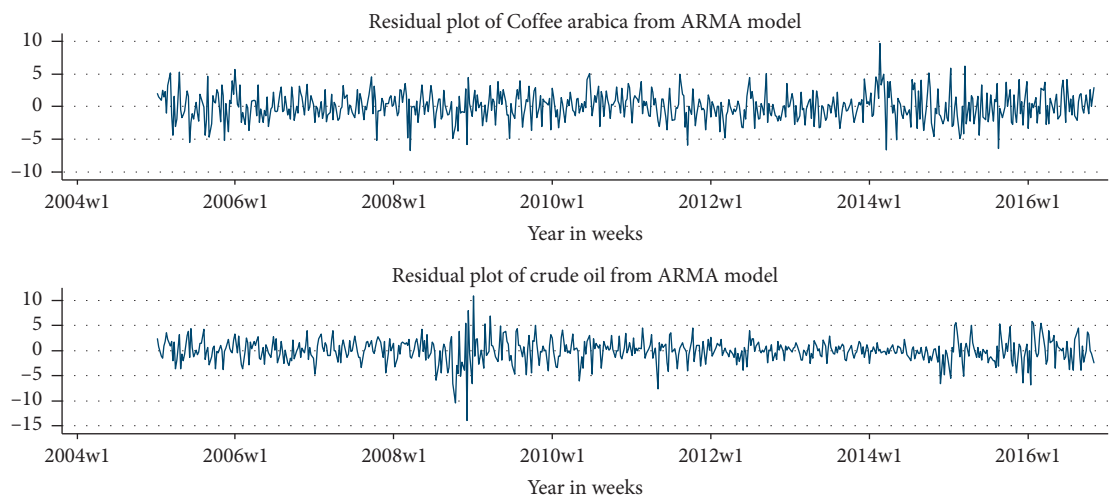

Figure 3: Residual plots of the Arabica coffee and crude oil return series from the mean.

Table 4: ARCH LM test summary statistics.

Series	Obs* R^2	χ^2 (4)	Lag (p)	Prob.
Arabica coffee	12.6477	9.488	4	0.004
Crude oil	105.6419	9.488	4	0.001

Table 4 shows the results of ARCH LM test for the two commodity returns. The last column of the table includes the p-values that indicate rejection of the null hypothesis that "there is no ARCH effect" up to the fourth lag at 5% level of significance. The results indicate that the two commodities' price log return series are volatile and need to be modeled using GARCH models.

3.4. Multivariate GARCH Modeling. At the beginning of reviewing different formulations of MGARCH models, one should consider what specification of a MGARCH model should be imposed in contrast to the univariate case. On the one hand, it should be flexible enough to state the dynamics of the conditional variances and covariance. On the other hand, as the number of parameters in a MGARCH model increases rapidly along with the dimension of the model, the specification should be parsimonious to simplify the model estimation and also reach the purpose of easy interpretation of the model parameters. However, parsimony may reduce the number of parameters, in which situation the relevant dynamics in the covariance matrix cannot be captured.

A number of alternative MGARCH formulations for modeling long-run conditional heteroskedasticity are extensively used in relevant literature. The most widely used are the time-varying conditional correlation (VCC), the constant conditional correlation (CCC), and the dynamic conditional correlation (DCC) models [5]. A more elaborate discussion of these models is presented in section2.1 of chapter two . One complexity of MGARCH models is that the number of parameters grows rapidly and researchers often use a single ARCH and GARCH term in the model specification, and often it is enough in capturing the variation. So, in this paper, we use bivariate-GARCH (1,1) models.

One of the major objectives of this study is choosing an appropriate multivariate GARCH model from the competing models. Using the Mincer-Zarnowitz regression and the forecast error measures, a model with minimum forecast error measure value and maximum MZ R^2 value is selected as an appropriate model. We can chose a single bivariate GARCH model and then estimate the parameters of the selected model. For the distribution of the innovation term, the multivariate normal distribution and the multivariate version of Student's t distribution with eight degree of

Table 5: Bivariate GARCH model comparison under normal distribution for covolatility of coffee and oil returns.

Statistics	Realized volatility measure					
	Weekly returns squared			30-minute interval returns (high freq.)		
	CCC	DCC	VCC	CCC	DCC	VCC
MZ.reg.R2	0.0145	0.0037	0.0070	0.2575	0.1740	0.3929
MSE	29.2503	29.9553	29.313629	0.7865	0.9161	0.8590
RMSE	5.1442	5.4730	5.4083	0.8868	0.9571	0.9268
MAE	3.7900	3.7452	3.7307	0.879	0.8517	0.8000
AIC	4965.981	4957.583	4962.655	4965.981	4962.655	4957.583
BIC	5022.474	5022.768	5027.84	5022.474	5022.768	5027.84
LL	−2469.99	−2463.79	−2466.32	−2469.99	−2463.79	−2466.32

Table 6: Bivariate GARCH model comparison under Student's t distribution for covolatility of coffee and oil returns.

Statistics	Realized volatility measure					
	Weekly return square			High frequency		
	CCC	DCC	VCC	CCC	DCC	VCC
MZ.reg.R2	0.0937	0.0747	0.060	0.2372	0.1614	0.4014
MSE	29.36	29.99	29.28	0.7876	0.8877	0.8634
RMSE	5.4184	5.4763	5.4110	0.8874	0.9421	0.9291
MAE	3.7953	3.7532	3.742	0.8819	0.8423	0.8372
AIC	4945.74	4940.492	4943.54	4945.74	4943.54	4940.492
BIC	5002.24	5005.677	5008.725	5002.24	5005.677	5008.725
LL	−2459.84	−2455.24	−2456.77	−2459.84	−2455.24	−2456.77

Table 7: Variance parameter estimates from time-varying conditional correlation (VCC-GARCH) model under t distribution ($v = 8$).

Parameters	Coefficients	z-statistics	p-value
$\widehat{\omega}_{\text{coffee}}$	0.1647	1.04	0.300
$\widehat{\alpha}_{\text{coffee}}$	0.0305	1.60	0.109
$\widehat{\beta}_{\text{coffee}}$	0.9393	22.16	0.001
$\widehat{\omega}_{\text{oil}}$	0.0835	1.60	0.109
$\widehat{\alpha}_{\text{oil}}$	0.0774	3.60	0.001
$\widehat{\beta}_{\text{oil}}$	0.9092	38.48	0.001
$\widehat{\rho}_{\text{coffee,oil}}$	0.2886	1.60	0.001
$\widehat{\pi}_1$	0.02919	1.70	0.034
$\widehat{\pi}_2$	0.919	15.88	0.001
$\widehat{\alpha}_{\text{coff.}} + \widehat{\beta}_{\text{coff.}}$	0.9698	—	—
$\widehat{\alpha}_{\text{oil}} + \widehat{\beta}_{\text{oil}}$	0.9866	—	—

freedom are considered. Like the previous analysis, we also consider two volatility measures, that is, the weekly return square and the 30-minute intraday return square.

Looking at Tables 5 and 6 for each of the models, the R^2 obtained from Mincer-Zarnowitz regression (R^2) value is larger for the high-frequency data. Moreover, the R^2 is larger under Student t distribution assumption of the innovation term. Now combining the high-frequency data and Student t distribution, the time-varying conditional correlation (VCC) model is a better model for the variance covariance estimation of the Arabica coffee return and crude oil returns.

The results in Table 7 of the variance equations actually indicated volatility effects in the returns series across the sample period considered. With regard to the estimates of the time-varying correlation ($\widehat{\rho}_{\text{coffee,oil}}$), the correlation was found to be positive and significant. This implies positive relationship between the conditional volatility series of the two commodity series and indicates that the returns on these commodities rise or fall together.

The estimates of the parameters $\widehat{\pi}_1$ and $\widehat{\pi}_2$, that is, the parameters dealing with the effects of previous shocks and previous dynamic conditional correlations on the current dynamic conditional correlation, were significantly different from zero, with $\widehat{\pi}_1 = 0.0291$ and $\widehat{\pi}_2 = 0.919$. The time-varying conditional correlation (VCC) model reduces to constant conditional correlation (CCC) model when $\widehat{\pi}_1 = \widehat{\pi}_2 = 0$ [1]. Hence, employing CCC is too restrictive for these series.

Concerning the estimates of persistence ($\widehat{\alpha}_i + \widehat{\beta}_i$), the persistence estimates were 0.9698 and 0.9866 at coffee and oil markets, respectively, which is closer to one, implying that volatility is persistent in these markets. The sufficient conditions for the variance matrix to be positive definite are

the usual GARCH restrictions, i.e., ω, α, $\beta > 0$ for the univariate GARCH (1,1) model. Also, this univariate GARCH process needs to be stationary, i.e., $1 - \alpha - \beta > 0$. These conditions are satisfied in the above output table.

3.5. Discussion. The aim has been modeling and forecasting the covolatility dynamics of weekly time series of Brent crude oil and Coffee Arabica futures market closing price using bivariate GARCH models. From the preliminary analysis over the time period considered, both of the price series show an increasing trend. To determine whether the series are stationary or not, the augmented Dickey–Fuller (ADF) test was carried out. Often, raw data of commodity prices are nonstationary, which was also the case in this study. For both level time series, the tests indicate to the existence of unit root I (1). The first log difference of each time series was considered as stationary (Table 3).

In the two commodity returns, it appears that the ARCH term coefficients were significant at 5% level of significance. The result is an implication of the presence of volatility clustering, i.e., large changes followed by large changes and small changes followed by small changes.

Noemi and Junior [12] investigated the volatility of two tropical commodities, i.e., world coffee and crude oil, and found that the two commodities were volatile and concluded that fluctuations in coffee and cocoa price follow the oil price fluctuation. Our results shows that Arabica coffee and crude oil volatilities are correlated (in fact, not highly). In addition, the covariance of Arabica coffee and crude oil is found to be similar to the variance of crude oil which indicates that the variation in the Arabica coffee price follows crude oil price, a similar finding with Noemi and Junior.

Overall, the positive relationship between these commodity returns is consistent with previous research studies on energy and commodity covolatility (see for example, [13, 14]).

4. Conclusion and Recommendations

Forecasting the volatility dynamics of asset returns has been the subject of extensive research among academics, practitioners, and portfolio managers. This thesis estimates a variety of multivariate GARCH models using weekly closing price (in USD/barrel) of Brent crude oil and weekly closing prices (in USD/per pound) of Coffee Arabica and compares the forecasting performance of these models based on high-frequency intraday data which allows for a more precise realized volatility measurement. The analysis points to the conclusion that varying conditional correlation (VCC) model with Student's t distributed innovation terms is the most accurate volatility forecasting model in the context of our empirical setting.

We recommend and encourage future researchers studying the forecasting performance of MGARCH models to pay particular attention to the measurement of realized volatility and employ high-frequency data whenever feasible.

We also recommend that public policy makers interested in foreseeing the price volatility of these two major commodities in the context of the Ethiopian economy consider using the information documented in this study as input in their deliberations given that it is based on some robust econometric work and highly appropriate data.

The scope of the analysis in this study has been limited to the covolatility between the two commodities. In order to overcome this limitation and provide a more nuanced analysis, it might be profitable for future researchers to consider incorporating stock, currency, and bond market volatilities into the analysis. The potential complexity of such a research agenda notwithstanding, the results are likely to be rewarding in light of the deeper integration between global financial and commodity markets in recent years, a phenomenon which came to be known as the financialization of commodities.

Data Availability

The data used to support the findings of the study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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