Research Article

Using ORRT Models for Mean Estimation under Nonresponse and Measurement Errors in Stratified Successive Sampling

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In the context of a sample survey, the collection of information on a sensitive variable is difficult, which may cause nonresponse and measurement errors. Due to this, the estimates can be biased and the variation may increase. To overcome this difficulty, we propose an estimator for the estimation of a sensitive variable by using auxiliary information in the presence of nonresponse and measurement errors simultaneously. The properties of the proposed estimators have been studied, and the results have been compared with those of the usual complete response estimator. Theoretical results have been verified through a simulation study using an artificial population and two real-life applications. With the outcomes of the proposed estimator, a suitable recommendation has been made to the survey statisticians for their real-life application.

1. Introduction

The utilization of auxiliary information in sample research intends to improve the precision of estimators. The estimation in this research article is accomplished through stratified successive sampling. To reduce population heterogeneity, a possible sampling strategy would be stratified by random sampling. Researchers use stratified random sampling when they are already aware of subdivisions within a population that need to be accounted for in their research. For example, in socioeconomic surveys, a group of people living in rural, sub-urban, and urban regions may need to be examined. It is understood that the nature of the study variable may differ for different parts of the population and each component or part should be considered as separate strata. In addition to the overall estimate, it may also require to estimate particular strata parameters of the population. This can be done only through stratified random sampling. Thus, in stratified sampling, we first divide the entire population into homogeneous subpopulations or subgroups, which are known as strata. Then, random samples are selected independently from each stratum, such that each subgroup of a population is best represented by the entire population being studied. Kadilar and Cingi [1] developed ratio estimators in stratified random sampling and Bouza et al. [2] have utilized auxiliary information to develop new ratio estimators for population parameters in stratified random sampling.

In longitudinal surveys, successive sampling is a well-known approach for estimating population parameters and measuring differences or changes in a study variable. In this type of sampling, some units drawn on the first occasion are preserved and utilized on the second occasion, while the rest of the units are replaced by new ones drawn on the current occasion. This partial unit replacement lowers survey costs. Jessen's [3] study was the beginning of the theory of successive sampling. Furthermore, the work is extended by several authors, including Singh et al. [4] who suggested a logarithmic type estimator to estimate the population mean in successive sampling in the presence of random

nonresponse and measurement errors. Moreover, Singh et al. [5] proposed some imputation methods to deal with the problem of missing data in two-occasion successive sampling. Furthermore, Khalid and Singh [6] investigated some imputation methods to deal with the issue of missing data problems due to random nonresponse in two-occasion successive sampling among others.

Among surveys on sensitive variables, it is usual to encounter that some people do not respond, whereas others respond but often prefer to disguise the true value to avoid social disapproval. For example, estimating the number of drug addicts in a town or students who have cheated on an exam would provide false answers. In such cases, the responses are clearly sensitive. This situation is present in many surveys. Many respondents not responding correctly may cause a social desirability bias. To avoid social desirability response bias, Warner [7] developed a data collection procedure, termed as randomized response technique (RRT), that allows researchers to evoke sensitive questions. Furthermore, it is extended by the work of Diana and Perri [8] who proposed a class of estimators for quantitatively sensitive data. Likewise, Gupta et al. [9] suggested a unified measure of respondent privacy and model efficiency in quantitative RRT models; Zhang et al. [10] proposed a ratio estimation of the mean under RRT models and many others. Nonresponse and measurement errors may be observed in a socioeconomic survey on sensitive data. For instance, in a survey of annual income and expenditure per household, people may attempt to suppress or distort the amount of income or expenditure. Similarly, when assessing the number of illegal abortions performed in a city each year, both nonresponse and measurement errors may occur.

It is impossible to obtain complete information about sample units in surveys where sensitive questions are present. Incomplete information is known as nonresponse, which is a source of nonsampling error. To deal with this issue of nonresponse in sample surveys, Hansen and Hurwitz [11] suggested techniques by taking a subsample from nonrespondent groups. Furthermore, Bouza [12] also developed the problem of the subsampling fraction in the case of nonresponse. Diana et al. [13] proposed a Hansen and Hurwitz estimator with scrambled responses on the second call if the survey is sensitive in nature. Singh et al. [14] used a calibration method for the estimation of population variance in stratified successive sampling with random nonresponse. Moreover, Mukhopadhyay et al. [15] have worked on a general technique for estimating the population means under stratified successive sampling in the presence of random scrambled responses and nonresponses. Apart from nonresponse, measurement error is also a serious issue in sample surveys due to the lack of proper data. A measurement error happens when the real value of the sample units differs from what is observed. Singh et al. [16] investigated difference-type estimators for the estimation of mean in the presence of measurement errors. Zhang et al. [17] have suggested a mean estimation in the simultaneous presence of measurement errors and nonresponse using optional RRT models under stratified sampling. Furthermore, Zahid et al. [18] have also been proposed a generalized class of estimators for a sensitive variable in the presence of nonresponse and measurement errors under stratified sampling. Furthermore, Tiwari et al. [19, 20], Kumar and Kour [21], and others have addressed the issue of nonresponse and measurement error in various sampling strategies, using prior studies as inspiration and realizing the essence of how to handle these flaws in a sample survey. Furthermore, the exponential estimator(s) proposed by Bahl and Tuteja [22] are known to perform better than the corresponding usual ratio and product-type estimators under certain efficiency conditions. But, if these conditions are not readily satisfied, then we propose a logarithmic-type estimator as it is the inverse operation to exponentiation, where both the study and the auxiliary variables are sensitive in nature under ORRT models to estimate the population mean of the sensitive variable in the presence of nonresponse and measurement error simultaneously under stratified successive sampling.

2. Sample Structure and Notations

Let us consider a finite stratified population $U = \{1, 2, \ldots, N\}$ with N distinct units divided into L homogeneous subgroups, known as strata with N_h ($\sum_{h=1}^{L} N_h = N$) units in the hth ($h = 1, 2, \ldots, L$) stratum. On the first(-second) occasion, the character under study is represented by $x_h(y_h)$. We further assume that information on an auxiliary variable z_h is available on both occasions and is correlated with the study variable. Let a simple random sample (without replacement) of size n_h unit is selected on the first occasion. From the first occasion, a sample of size $m_h = n_h p_h$, $(0 < p_h < 1)$ units is retained (matched) on the second occasion. Again, on the second occasion, an independent sample of size $u_h = n_h q_h = n_h - m_h$, $(q_h = 1 - p_h)$ units (i.e., unmatched units with the first occasion) is drawn from the entire population, so that the sample size on the second (current) occasion is also n_h ($n_h = uh + m_h$). To deal with nonresponses, we divide the whole population into two groups, i.e., respondents and nonrespondents. Let N_{1h} and N_{2h} be the number of responding and nonresponding units in the hth stratum, respectively. From sample units, n_{1h} units respond and n_{2h} units do not respond. Similarly, in the unmatched sample, u_{1h} units respond and u_{2h} do not respond and from matched units m_{1h} respond and m_{2h} do not respond. Again, a subsample of size $n_{2h} (= n_{sh}/k_h)$; $k_h > 1$ units is drawn from the nonrespondent class of the sample units. Furthermore, we take a subsample from the nonrespondent group of size $m_{sh} = (m_{2h}/k_h)$; $k_h > 1$, and $u_{sh} = (u_{2h}/k_h)$; $k_h > 1$ units are drawn from the matched and unmatched portions, respectively, where k_h is the inverse sampling ratio. Along with nonresponses, the measurement error is also associated with these sample units, i.e., $V_{ih} = x_{ih} - X_{ih}, U_{ih} = y_{ih} - Y_{ih}$ and $W_{ih} = z_{ih} - Z_{ih}$; for both occasions, which are random in nature with a mean zero and population variance S_{Uh}^2, S_{Vh}^2, and S_{Wh}^2. Hence, the following notations are used for their further use:

$x_h(y_h)$: the sensitive study variable on the first (second) occasion

z_h: the sensitive auxiliary variable for both occasions

$\overline{X}_h (\overline{Y}_h)$: means of the hth stratum on the first and second occasion, respectively

$\overline{Z}_h$: means of the auxiliary variable for h^{th} stratum on both occasions

$\overline{X} (\overline{Y})$: the population means of the sensitive study variables on the first and second occasion, respectively

$\overline{Z}$: the population mean of the auxiliary variable for both occasions

$\overline{y}_{mh}, \overline{y}_{uh}, \overline{x}_{mh}, \overline{x}_{nh}, \overline{z}_{nh}$, and $\overline{z}_{uh}$: the sample means of the variables y_h, x_h, and z_h, based on the respective sample size shown in their subscripts

$\rho_{y_h x_h}, \rho_{y_h z_h}$, and $\rho_{x_h z_h}$: the correlation coefficients between their respective variables, as shown in their subscripts

S^2_{xh}, S^2_{yh}, and S^2_{zh}: the population variances of the variable x_h, y_h, and z_h, respectively

$\widehat{\overline{y}}^*_{mh}, \widehat{\overline{y}}^*_{uh}, \widehat{\overline{x}}^*_{mh}, \widehat{\overline{x}}^*_{nh}, \widehat{\overline{z}}^*_{nh}$, and $\widehat{\overline{z}}^*_{uh}$: the sample means of the variables y_h, x_h, and z_h, when there is a presence of sensitivity with nonresponse and measurement error simultaneously, based on the respective sample size shown in their subscripts

3. Optional Randomized Response Technique (ORRT)

In this section, let S_{1h} and S_{2h} be two scrambled variables with mean $\overline{S}_{1h}$ and $\overline{S}_{2h}$ and known variances $S^2_{S_{1h}}$ and $S^2_{S_{2h}}$, respectively. Let W_h represent the probability that the respondent will find the question sensitive. Since the ORRT model is more efficient, we add it optionally to the Diana and Perri [8] model. In the ORRT version, the respondent may answer in the two ways given in equation (1), depending on whether the respondent considers the question sensitive or not. As a result, for the matched and unmatched portion, we may use the general scrambling model for the sensitive study variable Y_{ah} is given as follows:

$$Z_{1ah} = \begin{cases} Y_{ah}, & \text{with probability } (1 - W_h), \\ S_{1h} Y_{ah} + S_{2h}, & \text{with probability } W_h, \end{cases} \tag{1}$$

where $a = (u, m)$.

The mean and variance of Z_{1ah} are given by the following expression:

$$E(Z_{1ah}) = E(Y_{ah})(1 - W_h) + E(S_{1h} Y_{ah} + S_{2h}) W_h = E(Y_{ah}),$$

$$\text{Var}(Z_{1ah}) = E(Z^2_{1ah}) - E^2(Z_{1ah}) = \sum_{h=1}^{L} S^2_{yah} + S^2_{S_{2h}} W_h + S^2_{S_{1h}} (S^2_{yah} + \overline{Y}^2_{ah}) W_h. \tag{2}$$

We write the randomized linear model as follows:

$$Z_{1ah} = (S_{1h} Y_{ah} + S_{2h}) J_h + Y_{ah}(1 - J_h), \tag{3}$$

and the expectation and variance of the randomized mechanism are as follows:

$$E_R(Z_{1ah}) = (\overline{S}_{1h} W_h + 1 - W_h) Y_{ah} + \overline{S}_{2h} W_h,$$

$$V_R(Z_{1ah}) = \sum_{h=1}^{L} (Y^2_{ah} S^2_{S_{1h}} + S^2_{S_{2h}}) W_h. \tag{4}$$

For sensitive study variable X_{bh} for the unmatched portion, the general scrambling model is given as follows:

$$Z_{2bh} = \begin{cases} X_{bh}, & \text{with probability } (1 - W_h), \\ S_{1h} X_{bh} + S_{2h}, & \text{with probability } W_h, \end{cases} \tag{5}$$

where $b = (m, n)$.

Then, the mean and variance of Z_{2bh} are given by the following expression:

$$E(Z_{2bh}) = E(X_{bh})(1 - W_h) + E(S_{1h} X_{bh} + S_{2h}) W_h = E(X_{bh}),$$

$$\text{Var}(Z_{2bh}) = E(Z^2_{2bh}) - E^2(Z_{2bh}) = \sum_{h=1}^{L} S^2_{xbh} + S^2_{S_{2h}} W_h + S^2_{S_{1h}} (S^2_{xbh} + \overline{X}^2_{bh}) W_h. \tag{6}$$

Therefore,

$$Z_{2bh} = (S_{1h} X_{bh} + S_{2h}) J_h + X_{bh}(1 - J_h), \tag{7}$$

and the mean and variance are given as follows:

$$E_R(Z_{2bh}) = (\overline{S}_{1h} W_h + 1 - W_h) X_{bh} + \overline{S}_{2h} W_h,$$

$$V_R(Z_{2bh}) = \sum_{h=1}^{L} (X^2_{bh} S^2_{S_{1h}} + S^2_{S_{2h}}) W_h. \tag{8}$$

Similarly, for auxiliary variable Z_{ch}, we write as follows:

$$Z_{3ch} = \begin{cases} Z_{ch}, & \text{with probability } (1 - W_h), \\ S_{1h}Z_{ch} + S_{2h}, & \text{with probability } W_h, \end{cases} \quad (9)$$

where $c = (u, m, n)$.

Then, the mean and variance of Z_{3ch} are given by the following expressions:

$$E(Z_{3ch}) = E(Z_{ch})(1 - W_h) + E(S_{1h}Z_{ch} + S_{2h})W_h = E(Z_{ch}),$$

$$\text{Var}(Z_{3ch}) = E(Z_{3ch}^2) - E^2(Z_{3ch}) = \sum_{h=1}^{L} S_{zch}^2 + S_{S_{2h}}^2 W_h + S_{S_{1h}}^2 \left(S_{zch}^2 + \overline{Z}_{ch}^2\right)W_h. \quad (10)$$

Therefore,

$$Z_{3ch} = (S_{1h}Z_{ch} + S_{2h})J_h + Z_{ch}(1 - J_h),$$

$$E_R(Z_{3ch}) = (\overline{S}_{1h}W_h + 1 - W_h)Z_{ch} + \overline{S}_{2h}W_h,$$

$$V_R(Z_{3ch}) = \sum_{h=1}^{L} \left(Z_{ch}^2 S_{S_{1h}}^2 + S_{S_{2h}}^2\right)W_h, \quad (11)$$

where S_{1h} and S_{2h} follow a normal distribution with mean $(1, 0)$ and variances $(S_{S_{1h}}^2, S_{S_{2h}}^2)$, i.e., $S_{1h} \sim N(1, S_{S_{1h}}^2)$ and $S_{2h} \sim N(0, S_{S_{2h}}^2)$, $J_h \sim \text{Bernoulli }(W_h)$ with

$$E(J_h) = W_h, \text{Var}(J_h) = W_h(1 - W_h),$$

$$E(J_h^2) = \text{Var}(J_h) + E^2(J_h) = W_h. \quad (12)$$

When face-to-face interview of subsampled units of nonrespondents is performed in the second phase of the Hansen and Hurwitz [11] procedure, we give respondents the opportunity to scramble their responses using ORRT. In this case, we used Hansen and Hurwitz's technique by assuming that the respondent group provides direct responses in the first phase, and then, the ORRT model is used to provide responses from a sample of nonrespondents in the second phase.

Let $\widehat{y}_{iah}$ denotes a transformation of the randomized response on the ith block, the expectation of which is a real response y_{iah} and is given as follows:

$$\widehat{y}_{iah} = \frac{z_{1iah} - \overline{S}_{2h}}{\overline{S}_{1h}W_h + 1 - W_h}, \quad (13)$$

with

$$E_R(\widehat{y}_{iah}) = y_{iah},$$

$$\text{Var}_R(\widehat{y}_{iah}) = \sum_{h=1}^{L} \frac{V_R(z_{1iah})}{(\overline{S}_{1h}W_h + 1 - W_h)^2} = \sum_{h=1}^{L} \frac{\left(y_{iah}^2 S_{S_{1h}}^2 + S_{S_{2h}}^2\right)W_h}{(\overline{S}_{1h}W_h + 1 - W_h)^2} = \sum_{h=1}^{L} \kappa_{1iah}. \quad (14)$$

Also, $\widehat{x}_{ibh}$ denotes a transformation of the randomized response on the ith block, whose expectation equals to x_{ibh} and is defined as follows:

$$\widehat{x}_{ibh} = \frac{z_{2ibh} - \overline{S}_{2h}}{\overline{S}_{1h}W_h + 1 - W_h}, \quad (15)$$

with

$$E_R(\widehat{x}_{ibh}) = x_{ibh},$$

$$\text{Var}_R(\widehat{x}_{ibh}) = \sum_{h=1}^{L} \frac{V_R(z_{2ibh})}{(\overline{S}_{1h}W_h + 1 - W_h)^2} = \sum_{h=1}^{L} \frac{\left(x_{ibh}^2 S_{S_{1h}}^2 + S_{S_{2h}}^2\right)W_h}{(\overline{S}_{1h}W_h + 1 - W_h)^2} = \sum_{h=1}^{L} \kappa_{2ibh},$$

$$\text{Var}_R(\widehat{x}_{ibh}) = \sum_{h=1}^{L} \frac{V_R(z_{2ibh})}{(\overline{S}_{1h}W_h + 1 - W_h)^2} = \sum_{h=1}^{L} \frac{\left(x_{ibh}^2 S_{S_{1h}}^2 + S_{S_{2h}}^2\right)W_h}{(\overline{S}_{1h}W_h + 1 - W_h)^2} = \sum_{h=1}^{L} \kappa_{2ibh}. \quad (16)$$

Likewise, $\widehat{z}_{ich}$ denotes a transformation of the randomized response on the ith block, whose expectation equals to z_{ich} and is given as follows:

$$\widehat{z}_{ich} = \frac{z_{3ich} - \overline{S}_{2h}}{\overline{S}_{1h}W_h + 1 - W_h}, \qquad (17)$$

$$E_R\left(\widehat{z}_{ich}\right) = z_{ich},$$

$$\operatorname{Var}_R\left(\widehat{z}_{ich}\right) = \sum_{h=1}^{L} \frac{V_R\left(z_{3ich}\right)}{\left(\overline{S}_{1h}W_h + 1 - W_h\right)^2} = \sum_{h=1}^{L} \frac{\left(z_{ich}^2 S_{S_{1h}}^2 + S_{S_{2h}}^2\right)W_h}{\left(\overline{S}_{1h}W_h + 1 - W_h\right)^2} = \sum_{h=1}^{L} \kappa_{3ich}. \qquad (18)$$

Now, the Hansen and Hurwitz [11] estimator in the presence of nonresponse by using ORRT is represented as follows:

$$\widehat{\overline{y}}_{ah} = w_{1h}\overline{y}_{1ah} + w_{2h}\widehat{\overline{y}}_{2ah},$$

$$\widehat{\overline{x}}_{bh} = w_{1h}\overline{x}_{1bh} + w_{2h}\widehat{\overline{x}}_{2bh}, \qquad (19)$$

$$\widehat{\overline{z}}_{ch} = w_{1h}\overline{z}_{1ch} + w_{2h}\widehat{\overline{z}}_{2ch},$$

where

$$\widehat{\overline{y}}_{2ah} = \sum_{i=1}^{n_{sh}}\left(\frac{\widehat{y}_{iah}}{n_{sh}}\right),$$

$$\widehat{\overline{x}}_{2bh} = \sum_{i=1}^{n_{sh}}\left(\frac{\widehat{x}_{ibh}}{n_{sh}}\right), \qquad (20)$$

$$\widehat{\overline{z}}_{2ch} = \sum_{i=1}^{n_{sh}}\left(\frac{\widehat{z}_{ich}}{n_{sh}}\right).$$

with

It is simple to illustrate that

$$E\left(\widehat{\overline{y}}_{ah}\right) = \overline{Y}_{ah}; E\left(\widehat{\overline{x}}_{bh}\right) = \overline{X}_{bh}; E\left(\widehat{\overline{z}}_{ch}\right) = \overline{Z}_{ch}, \qquad (21)$$

i.e., $\widehat{\overline{y}}_{ah}$, $\widehat{\overline{x}}_{bh}$, and $\widehat{\overline{z}}_{ch}$ are usual unbiased estimators.
The variance of $\widehat{\overline{y}}_{ah}$, $\widehat{\overline{x}}_{bh}$, and $\widehat{\overline{z}}_{ch}$ are as follows:

$$\operatorname{Var}\left(\widehat{\overline{y}}_{ah}\right) = \sum_{h=1}^{L} \lambda_{ah}S_{yah}^2 + \lambda_{ah}^* S_{yah(2)}^2 + \frac{W_{2h}k_h}{n_h}\left(\frac{\left\{\left(S_{yah(2)}^2 + \overline{y}_{ah(2)}^2\right)S_{S_{1h}}^2 + S_{S_{2h}}^2\right\}W_h}{\left(\overline{S}_{1h}W_h + 1 - W_h\right)^2}\right),$$

$$\operatorname{Var}\left(\widehat{\overline{x}}_{bh}\right) = \sum_{h=1}^{L} \lambda_{bh}S_{xbh}^2 + \lambda_{bh}^* S_{xbh(2)}^2 + \frac{W_{2h}k_h}{n_h}\left(\frac{\left(S_{xbh(2)}^2 + \overline{x}_{bh(2)}^2\right)\left\{S_{S_{1h}}^2 + S_{S_{2h}}^2\right\}W_h}{\left(\overline{S}_{1h}W_h + 1 - W_h\right)^2}\right), \qquad (22)$$

$$\operatorname{Var}\left(\widehat{\overline{z}}_{ch}\right) = \sum_{h=1}^{L} \lambda_{ch}S_{zch}^2 + \lambda_{ch}^* S_{zch(2)}^2 + \frac{W_{2h}k_h}{n_h}\left(\frac{\left(S_{zch(2)}^2 + \overline{z}_{ch(2)}^2\right)S_{S_{1h}}^2 + S_{S_{2h}}^2\right\}W_h}{\left(\overline{S}_{1h}W_h + 1 - W_h\right)^2}\right),$$

where

$$\lambda_{ah} = \frac{(a_h - n_h)}{a_h n_h},$$

$$\lambda_{bh} = \frac{(b_h - n_h)}{b_h n_h},$$

$$\lambda_{lh}^* = \frac{W_{2h}(k_h - 1)}{n_h}, \qquad (23)$$

$$l = a, b, c.$$

Let the measurement error associated with the sensitive variable(s) (Z_{1ah}, Z_{2bh}, and Z_{3ch}) in a face-to-face interview be given as $P_{iah} = z_{1iah} - Z_{1iah}$, $Q_{ibh} = z_{2ibh} - Z_{2ibh}$, and $R_{ich} = z_{3ich} - Z_{3ich}$ with mean zero and variances S^2_{pah}, S^2_{qbh}, and S^2_{rch}, respectively.

Various notations under measurement error are given as follows:

$$\widehat{\Omega}^*_{uh} = \sum_{h=1}^{L} \left(\sum_{i=1}^{n_{1h}} U_{ih} + \sum_{i=1}^{n_{2h}} P_{ih} \right),$$

$$\widehat{\Omega}^*_{vh} = \sum_{h=1}^{L} \left(\sum_{i=1}^{n_{1h}} V_{ih} + \sum_{i=1}^{n_{2h}} Q_{ih} \right), \qquad (24)$$

$$\widehat{\Omega}^*_{wh} = \sum_{h=1}^{L} \left(\sum_{i=1}^{n_{1h}} W_{ih} + \sum_{i=1}^{n_{2h}} R_{ih} \right),$$

where P_{ih}, Q_{ih}, and R_{ih} are measurement errors on Z_{1ah}, Z_{2bh}, and Z_{3ch}, respectively.

So, the variance of $\widehat{\overline{y}}_{ah}$ and $\widehat{\overline{x}}_{bh}$ ad $\widehat{\overline{z}}_{ch}$ in the presence of measurement error is given by the following expressions:

$$\mathrm{Var}\left(\widehat{\overline{y}}^*_{ah}\right) = \sum_{h=1}^{L} \lambda_{ah}\left(S^2_{yh} + S^2_{uh}\right) + \lambda^*_{ah}\left(S^2_{yh(2)} + S^2_{ph}\right) + K_{1h},$$

$$\mathrm{Var}\left(\widehat{\overline{x}}^*_{bh}\right) = \sum_{h=1}^{L} \lambda_{ah}\left(S^2_{xh} + S^2_{vh}\right) + \lambda^*_{ah}\left(S^2_{xh(2)} + S^2_{qh}\right) + K_{2h},$$

$$\mathrm{Var}\left(\widehat{\overline{z}}^*_{ch}\right) = \sum_{h=1}^{L} \lambda_{ah}\left(S^2_{zch} + S^2_{wh}\right) + \lambda^*_{ah}\left(S^2_{zch(2)} + S^2_{rh}\right) + K_{3h},$$

$$(25)$$

where

$$K_{1h} = \frac{W_{2h}k_h}{n_h}\left(\frac{\left\{\left(S^2_{yah(2)} + \overline{y}^2_{ah(2)}\right)S^2_{S_{1h}} + S^2_{S_{2h}}\right\}W_h}{\left(\overline{S}_{1h}W_h + 1 - W_h\right)^2}\right),$$

$$K_{2h} = \frac{W_{2h}k_h}{n_h}\left(\frac{\left\{\left(S^2_{xbh(2)} + \overline{x}^2_{bh(2)}\right)S^2_{S_{1h}} + S^2_{S_{2h}}\right\}W_h}{\left(\overline{S}_{1h}W_h + 1 - W_h\right)^2}\right),$$

$$K_{3h} = \frac{W_{2h}k_h}{n_h}\left(\frac{\left\{\left(S^2_{zch(2)} + \overline{z}^2_{ch(2)}\right)S^2_{S_{1h}} + S^2_{S_{2h}}\right\}W_h}{\left(\overline{S}_{1h}W_h + 1 - W_h\right)^2}\right), \qquad (26)$$

$$a = (u, m),$$

$$b = (m, n),$$

$$c = (u, m, n).$$

respectively.

4. Proposed Estimator

Under certain efficiency conditions, exponential-type estimators are known to outperform the related customary ratio and product-type estimators in terms of lesser mean square errors. Therefore, the question arises as to what happens when the conditions that favor exponential-type estimators over the customary estimators are not readily met. The answer of course lies in the use of other efficient estimators that would perform better than both the existing exponential and customary estimators. So, we examine the logarithmic-type estimator in our search for such efficient estimators because the logarithm is the inverse operation of exponentiation. The logarithmic function has some helpful qualities and is used extensively in a variety of scientific and nonscientific domains. Motivated by the abovementioned discussions and following the work of Singh et al. [14], we formulate two independent estimators for estimating the population mean of the sensitive variable on the current (second) occasion. The proposed class of estimator $\widehat{T}^{*'}$ is defined as a convex linear combination of two separate classes of estimators $\widehat{T}^{*'}_u$ and $\widehat{T}^{*'}_m$.

$$\widehat{T}^{*'} = \phi\widehat{T}^{*'}_u + (1 - \phi)\widehat{T}^{*'}_m, \qquad (27)$$

where ϕ $(0 \le \phi \le 1)$ is an unknown constant that must be determined, so that the mean squared error (MSE) of the proposed estimator $\widehat{T}^{*'}$ is minimum.

The $\widehat{T}^{*'}_u$ estimator is based on u_h, i.e., unmatched sample, and $\widehat{T}^{*'}_m$ estimator is based on m_h, i.e., matched sample, and is defined as follows:

$$\widehat{T}^{*'}_u = \sum_{h=1}^{L} P_h\left[\widehat{\overline{y}}^{*'}_{uh} + \beta_h\log\left(1 + \frac{\overline{Z}_h - \widehat{\overline{z}}^{*'}_{uh}}{\overline{Z}_h + \widehat{\overline{z}}^{*'}_{uh}}\right)\right],$$

$$\widehat{T}^{*'}_m = \sum_{h=1}^{L} P_h\left[\widehat{\overline{y}}^{*'}_{mh} + \alpha_h\log\left(1 + x^*_{nh} - x^*_{mh}\right)\right], \qquad (28)$$

where

$$x^*_{nh} = \widehat{\overline{x}}^{*'}_{nh} + \log\left(1 + \frac{\overline{Z}_h - \widehat{\overline{z}}^{*'}_{nh}}{\overline{Z}_h + \widehat{\overline{z}}^{*'}_{nh}}\right),$$

$$x^*_{mh} = \widehat{\overline{x}}^{*'}_{mh} + \log\left(1 + \frac{\overline{Z}_h - \widehat{\overline{z}}^{*'}_{mh}}{\overline{Z}_h + \widehat{\overline{z}}^{*'}_{mh}}\right), \qquad (29)$$

$P_h = N_h/N$; β_h and α_h are the constants to be determined by minimizing the mean square errors of the estimators.

Using the following transformations, the bias and mean square errors of the estimators $\widehat{T}^{*}_u$ and $\widehat{T}^{*}_m$ are derived up to the first degree of approximation. Let

$$\widehat{\overline{y}}^{*'}_{uh} = \overline{Y}_h\left(1 + \widehat{e}^*_{0uh}\right),$$

$$\widehat{\overline{y}}^{*'}_{mh} = \overline{Y}_h\left(1 + \widehat{e}^*_{0mh}\right),$$

$$\widehat{\overline{z}}^{*'}_{uh} = \overline{Z}_h\left(1 + \widehat{e}^*_{1uh}\right),$$

$$\widehat{\overline{z}}^{*'}_{nh} = \overline{Z}_h\left(1 + \widehat{e}^*_{1nh}\right), \qquad (30)$$

$$\widehat{\overline{x}}^{*'}_{mh} = \overline{X}_h\left(1 + \widehat{e}^*_{2mh}\right),$$

$$\widehat{\overline{x}}^{*'}_{nh} = \overline{X}_h\left(1 + \widehat{e}^*_{2nh}\right),$$

such that

$$E\left(\widehat{e}^{*}_{\iota uh}\right) = E\left(\widehat{e}^{*}_{\iota mh}\right) = E\left(\widehat{e}^{*}_{\iota nh}\right) = 0; \ (\iota = 0, 1, 2),$$

$$E\left(\widehat{e}^{*2}_{0uh}\right) = \frac{1}{\overline{Y}^2_h}\left[\lambda_{hu}\left(S^2_{yh} + S^2_{Uh}\right) + \lambda^*_{hu}\left(S^2_{yh(2)} + S^2_{ph}\right) + K_{1h}\right] = \frac{1}{\overline{Y}^2_h}\left(\widehat{A}^{*'}_{u} + K_{1h}\right),$$

$$E\left(\widehat{e}^{*2}_{0mh}\right) = \frac{1}{\overline{Y}^2_h}\left[\lambda_{hm}\left(S^2_{yh} + S^2_{Uh}\right) + \lambda^*_{hm}\left(S^2_{yh(2)} + S^2_{ph}\right) + K_{1h}\right] = \frac{1}{\overline{Y}^2_h}\left(\widehat{A}^{*'}_{m} + K_{1h}\right),$$

$$E\left(\widehat{e}^{*2}_{1uh}\right) = \frac{1}{\overline{Z}^2_h}\left[\lambda_{hu}\left(S^2_{zh} + S^2_{Wh}\right) + \lambda^*_{hu}\left(S^2_{zh(2)} + S^2_{rh}\right) + K_{3h}\right] = \frac{1}{\overline{Z}^2_h}\left(\widehat{B}^{*'}_{u} + K_{3h}\right),$$

$$E\left(\widehat{e}^{*2}_{1mh}\right) = \frac{1}{\overline{Z}^2_h}\left[\lambda_{hm}\left(S^2_{zh} + S^2_{Wh}\right) + \lambda^*_{hm}\left(S^2_{zh(2)} + S^2_{rh}\right) + K_{3h}\right] = \frac{1}{\overline{Z}^2_h}\left(\widehat{B}^{*'}_{m} + K_{3h}\right),$$

$$E\left(\widehat{e}^{*2}_{1nh}\right) = \frac{1}{\overline{Z}^2_h}\left[\lambda_{hn}\left(S^2_{zh} + S^2_{Wh}\right) + \lambda^*_{hn}\left(S^2_{zh(2)} + S^2_{rh}\right) + K_{3h}\right] = \frac{1}{\overline{Z}^2_h}\left(\widehat{B}^{*'}_{n} + K_{3h}\right) = E\left(\widehat{e}^*_{1nh}\widehat{e}^*_{1mh}\right),$$

$$E\left(\widehat{e}^{*2}_{2mh}\right) = \frac{1}{\overline{X}^2_h}\left[\lambda_{hm}\left(S^2_{xh} + S^2_{Vh}\right) + \lambda^*_{hm}\left(S^2_{xh(2)} + S^2_{qh}\right) + K_{2h}\right] = \frac{1}{\overline{X}^2_h}\left(\widehat{C}^{*'}_{m} + K_{2h}\right),$$

$$E\left(\widehat{e}^{*2}_{2nh}\right) = \frac{1}{\overline{X}^2_h}\left[\lambda_{hn}\left(S^2_{xh} + S^2_{Vh}\right) + \lambda^*_{hn}\left(S^2_{xh(2)} + S^2_{qh}\right) + K_{2h}\right] = \frac{1}{\overline{X}^2_h}\left(\widehat{C}^{*'}_{n} + K_{2h}\right) = E\left(\widehat{e}^*_{2nh}\widehat{e}^*_{2mh}\right),$$

$$E\left(\widehat{e}^*_{0uh}\widehat{e}^*_{1uh}\right) = \frac{1}{\overline{Y}_h\overline{Z}_h}\left[\lambda_{hu}\left(\rho_{y_hz_h}S_{yh}S_{zh}\right) + \lambda^*_{hu}\left(\rho_{y_hz_h(2)}S_{yh(2)}S_{zh(2)}\right)\right] = \frac{1}{\overline{Y}_h\overline{Z}_h}\left(\widehat{D}^{*'}_{u}\right),$$

$$E\left(\widehat{e}^*_{0mh}\widehat{e}^*_{1nh}\right) = \frac{1}{\overline{Y}_h\overline{Z}_h}\left[\lambda_{hn}\left(\rho_{y_hz_h}S_{yh}S_{zh}\right) + \lambda^*_{hn}\left(\rho_{y_hz_h(2)}S_{yh(2)}S_{zh(2)}\right)\right] = \frac{1}{\overline{Y}_h\overline{Z}_h}\left(\widehat{D}^{*'}_{n}\right),$$

$$E\left(\widehat{e}^*_{0mh}\widehat{e}^*_{1mh}\right) = \frac{1}{\overline{Y}_h\overline{Z}_h}\left[\lambda_{hm}\left(\rho_{y_hz_h}S_{yh}S_{zh}\right) + \lambda^*_{hm}\left(\rho_{y_hz_h(2)}S_{yh(2)}S_{zh(2)}\right)\right] = \frac{1}{\overline{Y}_h\overline{Z}_h}\left(\widehat{D}^{*'}_{m}\right),$$

$$E\left(\widehat{e}^*_{0mh}\widehat{e}^*_{2nh}\right) = \frac{1}{\overline{Y}_h\overline{X}_h}\left[\lambda_{hn}\left(\rho_{y_hx_h}S_{yh}S_{xh}\right) + \lambda^*_{hn}\left(\rho_{y_hx_h(2)}S_{yh(2)}S_{xh(2)}\right)\right] = \frac{1}{\overline{Y}_h\overline{X}_h}\left(\widehat{F}^{*'}_{n}\right),$$

$$E\left(\widehat{e}^*_{0mh}\widehat{e}^*_{2mh}\right) = \frac{1}{\overline{Y}_h\overline{X}_h}\left[\lambda_{hm}\left(\rho_{y_hx_h}S_{yh}S_{xh}\right) + \lambda^*_{hm}\left(\rho_{y_hx_h(2)}S_{yh(2)}S_{xh(2)}\right)\right] = \frac{1}{\overline{Y}_h\overline{X}_h}\left(\widehat{F}^{*'}_{m}\right),$$

$$E\left(\widehat{e}^*_{2mh}\widehat{e}^*_{1mh}\right) = \frac{1}{\overline{X}_h\overline{Z}_h}\left[\lambda_{hm}\left(\rho_{x_hz_h}S_{xh}S_{zh}\right) + \lambda^*_{hm}\left(\rho_{x_hz_h(2)}S_{xh(2)}S_{zh(2)}\right)\right] = \frac{1}{\overline{X}_h\overline{Z}_h}\left(\widehat{E}^{*'}_{m}\right),$$

$$E\left(\widehat{e}^*_{2nh}\widehat{e}^*_{1nh}\right) = \frac{1}{\overline{X}_h\overline{Z}_h}\left[\lambda_{hn}\left(\rho_{x_hz_h}S_{xh}S_{zh}\right) + \lambda^*_{hn}\left(\rho_{x_hz_h(2)}S_{xh(2)}S_{zh(2)}\right)\right] = \frac{1}{\overline{X}_h\overline{Z}_h}\left(\widehat{E}^{*'}_{n}\right),$$

$$E\left(\widehat{e}^*_{2nh}\widehat{e}^*_{1mh}\right) = \frac{1}{\overline{X}_h\overline{Z}_h}\left(\widehat{E}^{*'}_{n}\right) = E\left(\widehat{e}^*_{2mh}\widehat{e}^*_{1nh}\right),$$

$$(31)$$

where

$$\lambda_{hu} = \left(\frac{1}{uh} - \frac{1}{Nh}\right),$$

$$\lambda_{hm} = \left(\frac{1}{mh} - \frac{1}{Nh}\right),$$

$$\lambda_{hn} = \left(\frac{1}{nh} - \frac{1}{Nh}\right),$$

$$\lambda_{hu}^* = \frac{W_2(k-1)}{uh},$$

$$\lambda_{hm}^* = \frac{W_2(k-1)}{mh},$$

$$\lambda_{hn}^* = \frac{W_2(k-1)}{nh}.$$

(32)

Using the abovementioned results, the bias of the estimator $\widehat{T}^{*'}$ to the first degree of approximation is calculated as follows:

$$B\left(\widehat{T}^{*'}\right) = \phi B\left(\widehat{T}_u^{*'}\right) + (1-\phi)B\left(\widehat{T}_m^{*'}\right),$$

(33)

where

$$B\left(\widehat{T}_u^{*'}\right) = E\left(\widehat{T}_u^{*'} - \overline{Y}\right)_h = -\frac{3}{8}\sum_{h=1}^{L} P_h \beta_h \left[\frac{1}{\overline{Z}_h^2}\widehat{B}_1^{*'}\right],$$

(34)

$$B\left(\widehat{T}_m^{*'}\right) = E\left(\widehat{T}_m^{*'} - \overline{Y}\right)_h$$
$$= -\frac{1}{2}\sum_{h=1}^{L} P_h \alpha_h \left[\frac{1}{2\overline{Z}_h^2}\widehat{B}^{*'} - \widehat{C}^{*'} - \frac{1}{\overline{Z}_h}\widehat{E}^{*'}\right],$$

(35)

where

$$\widehat{B}_1^{*'} = \left(\widehat{B}_u^{*'} + K_{3h}\right),$$

$$\widehat{B}^{*'} = \left(\widehat{B}_n^{*'} - \widehat{B}_m^{*'}\right),$$

$$\widehat{C}^{*'} = \left(\widehat{C}_n^{*'} - \widehat{C}_m^{*'}\right),$$

$$\widehat{E}^{*'} = \left(\widehat{E}_n^{*'} - \widehat{E}_m^{*'}\right).$$

(36)

The mean square error (MSE) of the estimator $\widehat{T}^{*'}$ to the first degree of approximation is calculated as follows:

$$MSE\left(\widehat{T}^{*'}\right) = \phi^2 MSE\left(\widehat{T}_u^{*'}\right) + (1-\phi)^2 MSE\left(\widehat{T}_m^{*'}\right),$$

(37)

where

$$MSE\left(\widehat{T}_u^{*'}\right) = E\left(\widehat{T}_u^{*'} - \overline{Y}_h\right)^2 = \sum_{h=1}^{L} P_h^2 \left[\widehat{A}_1^{*'} + \beta_h^2 \frac{\widehat{B}_1^{*'}}{4\overline{Z}_h^2} + \beta_h \frac{\widehat{D}_u^{*'}}{\overline{Z}_h}\right],$$

$$MSE\left(\widehat{T}_m^{*'}\right) = E\left(\widehat{T}_m^{*'} - \overline{Y}_h\right)^2 = \sum_{h=1}^{L} P_h^2 \left[\widehat{A}_2^{*'} - \alpha_h^2 \left\{\widehat{C}^{*'} + \frac{1}{4\overline{Z}_h^2}\widehat{B}^{*'} + \frac{1}{\overline{Z}_h}\widehat{E}^{*'}\right\}\right.$$
$$\left. + \alpha_h \left\{2\widehat{F}^{*'} + \frac{\widehat{D}^{*'}}{\overline{Z}_h}\right\}\right],$$

(38)

where

$$\widehat{A}_1^{*'} = \left(\widehat{A}_u^{*'} + K_{1h}\right),$$

$$\widehat{A}_2^{*'} = \left(\widehat{A}_m^{*'} + K_{1h}\right),$$

$$\widehat{D}^{*'} = \left(\widehat{D}_n^{*'} - \widehat{D}_m^{*'}\right),$$

$$\widehat{F}^{*'} = \left(\widehat{F}_n^{*'} - \widehat{F}_m^{*'}\right).$$

(39)

The mean square errors of $\widehat{T}_u^{*'}$ and $\widehat{T}_m^{*'}$ are functions of the unknown constants β_h and α_h, respectively. So, we minimize the MSEs of $\widehat{T}_u^{*'}$ and $\widehat{T}_m^{*'}$, with respect to β_h and α_h, respectively. The optimal values are obtained as follows:

$$\beta_h = -\frac{2\widehat{D}_u^{*'}\overline{Z}_h}{\widehat{B}_1^{*'}} = \beta_{h(\mathrm{opt})},$$

$$\alpha_h = \frac{2\widehat{F}^{*'} + \left(\widehat{D}^{*'}/\overline{Z}_h\right)}{2\left\{\widehat{C}^{*'} + \left(1/4\overline{Z}_h^2\right)\widehat{B}^{*'} + (1/\overline{Z}_h)\widehat{E}^{*'}\right\}} = \alpha_{h(\mathrm{opt})}.$$

(40)

After substituting the optimum value of $\beta_{h(\mathrm{opt})}$ and $\alpha_{h(\mathrm{opt})}$ in equations (37) and (38), we obtain the minimum MSE of $\widehat{T}_u^{*'}$ and $\widehat{T}_m^{*'}$.

$$\text{min.}MSE\left(\widehat{T}_u^{\acute{*}}\right) = \sum_{h=1}^{L} P_h^2 \left[\widehat{A}_1^{\acute{*}} - \frac{\widehat{D}_u^{2\acute{*}}}{\widehat{B}_1^{\acute{*}}}\right], \tag{41}$$

$$\text{min.}MSE\left(\widehat{T}_m^{\acute{*}}\right) = \sum_{h=1}^{L} P_h^2 \left[\widehat{A}_2^{\acute{*}} + \frac{1}{4}\widehat{\delta}^{*'}\right],$$

where

$$\widehat{\delta}^{*'} = \left[\frac{\left\{2\widehat{F}^{*'} + \left(\widehat{D}^{*'}/\overline{Z}_h\right)\right\}}{\left\{\widehat{C}^{*'} + \left(1/4\overline{Z}_h^2\right)\widehat{B}^{*'} + (1/\overline{Z}_h)\widehat{E}^{*'}\right\}}\right]. \tag{42}$$

The MSE of $\widehat{T}^{*'}$ is dependent on the unknown constant "ϕ;" then, we minimize equation (35) w.r.t "ϕ" and equate it to zero, and we obtained the optimum value of "ϕ" as follows:

$$\phi_{\text{opt}} = \frac{\text{min.}MSE\left(\widehat{T}_m^{\acute{*}}\right)}{\text{min.}MSE\left(\widehat{T}_u^{\acute{*}}\right) + \text{min.}MSE\left(\widehat{T}_m^{\acute{*}}\right)}. \tag{43}$$

Substituting the optimum value of ϕ_{opt} in equation (35), we obtain the optimum mean square error of the estimator $\widehat{T}^{*'}$ as follows:

$$MSE\left(\widehat{T}^{*'}\right)_{\text{opt}} = \frac{\text{min.}MSE\left(\widehat{T}_u^{\acute{*}}\right)\text{min.}MSE\left(\widehat{T}_m^{\acute{*}}\right)}{\text{min.}MSE\left(\widehat{T}_u^{\acute{*}}\right) + \text{min.}MSE\left(\widehat{T}_m^{\acute{*}}\right)}. \tag{44}$$

Case 1. If $S_{Uh}^2 = S_{Vh}^2 = S_{Wh}^2 = S_{ph}^2 = S_{qh}^2 = S_{rh}^2 = 0$, then the mean square error of the proposed estimator without measurement error reduces to the following expression:

$$MSE\left(\widehat{T}^*\right) = \phi^2 MSE\left(\widehat{T}_u^*\right) + (1-\phi)^2 MSE\left(\widehat{T}_m^*\right), \tag{45}$$

where

$$MSE\left(\widehat{T}_u^*\right) = E\left(\widehat{T}_u^* - \overline{Y}_h\right)^2 = \sum_{h=1}^{L} P_h^2 \left[\widehat{A}_1^* + \beta_{h1}^2 \frac{\widehat{B}_1^*}{4\overline{Z}_h^2} + \beta_{h1}\frac{\widehat{D}_u^{*'}}{\overline{Z}_h}\right],$$

$$MSE\left(\widehat{T}_m^*\right) = E\left(\widehat{T}_m^* - \overline{Y}_h\right)^2 = \sum_{h=1}^{L} P_h^2 \left[\widehat{A}_2^* - \alpha_{h1}^2 \left\{\widehat{C}^* + \frac{1}{4\overline{Z}_h^2}\widehat{B}^* + \frac{1}{\overline{Z}_h}\widehat{E}^{*'}\right\}\right. \tag{46}$$

$$\left. + \alpha_{h1}\left\{2\widehat{F}^{*'} + \frac{\widehat{D}^{*'}}{\overline{Z}_h}\right\}\right],$$

where

$$\widehat{A}_1^* = \left(\widehat{A}_u^* + K_{1h}\right)\widehat{A}_u^* = \lambda_{hu}S_{yh}^2 + \lambda_{hu}^* S_{yh(2)}^2,$$

$$\widehat{B}_1^* = \left(\widehat{B}_u^* + K_{3h}\right)\widehat{B}_u^* = \lambda_{hu}S_{zh}^2 + \lambda_{hu}^* S_{zh(2)}^2,$$

$$\widehat{A}_2^* = \left(\widehat{A}_m^* + K_{1h}\right)\widehat{A}_m^* = \lambda_{hm}S_{yh}^2 + \lambda_{hm}^* S_{yh(2)}^2 \quad \widehat{B}^* = \left(\widehat{B}_n^* - \widehat{B}_m^*\right)\widehat{C}^* = \left(\widehat{C}_n^* - \widehat{C}_m^*\right)\widehat{B}_n^* = \lambda_{hn}S_{zh}^2 + \lambda_{hn}^* S_{zh(2)}^2, \tag{47}$$

$$\widehat{B}_m^* = \lambda_{hm}S_{zh}^2 + \lambda_{hm}^* S_{zh(2)}^2 \widehat{C}_n^* = \lambda_{hn}S_{xh}^2 + \lambda_{hn}^* S_{xh(2)}^2 \widehat{C}_m^* = \lambda_{hm}S_{xh}^2 + \lambda_{hm}^* S_{xh(2)}^2.$$

The MSE of $\widehat{T}_u^*$ and $\widehat{T}_m^*$ are minimum, when

$$\beta_{h1} = -\frac{2\widehat{D}_u^{*'}\overline{Z}_h}{\widehat{B}_1^*} = \beta_{h1(\text{opt})},$$

$$\alpha_{h1} = \frac{2\widehat{F}^{*'} + \widehat{D}^{*'}/\overline{Z}_h}{2\left\{\widehat{C}^* + 1/4\overline{Z}_h^2\widehat{B}^* + (1/\overline{Z}_h)\widehat{E}^{*'}\right\}} = \alpha_{h1(\text{opt})}. \tag{48}$$

After substituting the optimum value of $\beta_{h1(\text{opt})}$ and $\alpha_{h1(\text{opt})}$ in MSEs, we obtain the minimum MSE of $\widehat{T}_u^*$ and $\widehat{T}_m^*$.

$$\text{min.}MSE\left(\widehat{T}_u^*\right) = \sum_{h=1}^{L} P_h^2 \left[\widehat{A}_1^* - \frac{\widehat{D}_u^{2\acute{*}}}{\widehat{B}_1^*}\right],$$

$$\text{min.}MSE\left(\widehat{T}_m^*\right) = \sum_{h=1}^{L} P_h^2 \left[\widehat{A}_2^* + \frac{1}{4}\widehat{\delta}^*\right], \tag{49}$$

where

$$\widehat{\delta}^* = \left[\frac{\left\{ 2\widehat{F}^{*'} + \widehat{D}^{*'}/\overline{Z}_h \right\}}{\left\{ \widehat{C}^* + 1/4\overline{Z}_h^2 \widehat{B}^* + (1/\overline{Z}_h)\widehat{E}^{*'} \right\}} \right]. \tag{50}$$

To obtain the optimum value of 'ϕ', we minimize equation (35) w.r.t "ϕ" and equate it to zero, as follows:

$$\phi_{\mathrm{opt}} = \frac{\min.MSE\left(\widehat{T}_m^*\right)}{\min.MSE\left(\widehat{T}_u^*\right) + \min.MSE\left(\widehat{T}_m^*\right)}. \tag{51}$$

After substituting the optimum value of ϕ_{opt} in equation (35), we obtain the optimum mean square error of the estimator $\widehat{T}^*$ as follows:

$$MSE\left(\widehat{T}^*\right)_{\mathrm{opt}} = \frac{\min.MSE\left(\widehat{T}_u^*\right)\min.MSE\left(\widehat{T}_m^*\right)}{\min.MSE\left(\widehat{T}_u^*\right) + \min.MSE\left(\widehat{T}_m^*\right)}. \tag{52}$$

Case 2. If $S_{ph}^2 = S_{qh}^2 = S_{rh}^2 = K_{1h} = K_{2h} = K_{3h} = 0$, then the mean square error of the proposed estimator without sensitivity becomes

$$MSE\left(T^{*'}\right) = \phi^2 MSE\left(T_u^{*'}\right) + (1 - \phi)^2 MSE\left(T_m^{*'}\right), \tag{53}$$

where

$$MSE\left(T_u^{*'}\right) = E\left(T_u^{*'} - \overline{Y}_h\right)^2 = \sum_{h=1}^{L} P_h^2 \left[A_u^{*'} + \beta_{h2}^2 \frac{B_u^{*'}}{4\overline{Z}_h^2} + \beta_{h2} \frac{\widehat{D}_u^{*'}}{\overline{Z}_h} \right],$$

$$MSE\left(T_m^{*'}\right) = E\left(T_m^{*'} - \overline{Y}_h\right)^2 = \sum_{h=1}^{L} P_h^2 \Bigg[A_m^{*'} - \alpha_{h2}^2 \left\{ C^{*'} + \frac{1}{4\overline{Z}_h^2} B^{*'} + \frac{1}{\overline{Z}_h}\widehat{E}^{*'} \right\} \tag{54}$$

$$+ \alpha_{h2} \left\{ 2\widehat{F}^{*'} + \frac{\widehat{D}^{*'}}{\overline{Z}_h} \right\} \Bigg],$$

where

$$A_u^{*'} = \lambda_{hu}\left(S_{yh}^2 + S_{Uh}^2\right) + \lambda_{hu}^*\left(S_{yh(2)}^2 + S_{Uh(2)}^2\right),$$

$$B_u^{*'} = \lambda_{hu}\left(S_{zh}^2 + S_{Wh}^2\right) + \lambda_{hu}^*\left(S_{zh(2)}^2 + S_{Wh(2)}^2\right),$$

$$A_m^{*'} = \lambda_{hm}\left(S_{yh}^2 + S_{Uh}^2\right) + \lambda_{hm}^*\left(S_{yh(2)}^2 + S_{Uh(2)}^2\right),$$

$$B^{*'} = \left(B_n^{*'} - B_m^{*'} \right),$$

$$C^{*'} = \left(C_n^{*'} - C_m^{*'} \right), \tag{55}$$

$$B_n^{*'} = \lambda_{hn}\left(S_{zh}^2 + S_{Wh}^2\right) + \lambda_{hn}^*\left(S_{zh(2)}^2 + S_{Wh(2)}^2\right),$$

$$B_m^{*'} = \lambda_{hm}\left(S_{zh}^2 + S_{Wh}^2\right) + \lambda_{hm}^*\left(S_{zh(2)}^2 + S_{Wh(2)}^2\right),$$

$$C_n^{*'} = \lambda_{hn}\left(S_{xh}^2 + S_{Vh}^2\right) + \lambda_{hn}^*\left(S_{xh(2)}^2 + S_{Vh(2)}^2\right),$$

$$C_m^{*'} = \lambda_{hm}\left(S_{xh}^2 + S_{Vh}^2\right) + \lambda_{hm}^*\left(S_{xh(2)}^2 + S_{Vh(2)}^2\right).$$

The MSE of $\widehat{T}_u^*$ and $\widehat{T}_m^*$ are minimum, when

$$\beta_{h2} = -\frac{2\widehat{D}_u^{*'}\overline{Z}_h}{B_u^{*'}} = \beta_{h2(\mathrm{opt})},$$

$$\tag{56}$$

$$\alpha_{h2} = \frac{2\widehat{F}^{*'} + \left(\widehat{D}^{*'}/\overline{Z}_h\right)}{2\left\{ C^{*'} + 1/4\overline{Z}_h^2 B^{*'} + (1/\overline{Z}_h)\widehat{E}^{*'} \right\}} = \alpha_{h2(\mathrm{opt})}.$$

By substituting the optimum value of $\beta_{h2(\mathrm{opt})}$ and $\alpha_{h2(\mathrm{opt})}$ in MSEs, we obtain the minimum MSE of $T_u^{*'}$ and $T_m^{*'}$.

$$\min.MSE\left(T_u^{*'}\right) = \sum_{h=1}^{L} P_h^2 \left[A_u^{*'} - \frac{\widehat{D}_u^{2*'}}{B_u^{*'}} \right],$$

$$\min.MSE\left(T_m^{*'}\right) = \sum_{h=1}^{L} P_h^2 \left[A_m^{*'} + \frac{1}{4}\delta^{*'} \right], \tag{57}$$

where

$$\delta^{*'} = \left[\frac{\left\{ 2\widehat{F}^{*'} + \widehat{D}^{*'}/\overline{Z}_h \right\}}{\left\{ C^{*'} + \left(1/4\overline{Z}_h^2\right)B^{*'} + (1/\overline{Z}_h)\widehat{E}^{*'} \right\}} \right]. \tag{58}$$

To obtain the optimum value of "ϕ," we minimize equation (52) w.r.t "ϕ" and equate it to zero as follows:

$$\phi_{\mathrm{opt}} = \frac{\min.MSE\left(T_m^{*'}\right)}{\min.MSE\left(T_u^{*'}\right) + \min.MSE\left(T_m^{*'}\right)}. \tag{59}$$

After substituting the optimum value of ϕ_{opt} in equation (52), we obtain the optimum mean square error of the estimator $\widehat{T}^*$ as follows:

$$MSE\left(T^{*'}\right)_{\text{opt}} = \frac{\text{min}.MSE\left(T_u^{*'}\right)\text{min}.MSE\left(T_m^{*'}\right)}{\text{min}.MSE\left(T_u^{*'}\right) + \text{min}.MSE\left(T_m^{*'}\right)}. \tag{60}$$

Case 3. If we substitute $S_{Uh}^2 = S_{Vh}^2 = S_{Wh}^2 = S_{ph}^2 = S_{qh}^2 = S_{rh}^2 = K_{1h} = K_{2h} = K_{3h} = 0$ in equation (60), then the mean square

error of the proposed estimator reduces to only the non-response case (i.e., absence of measurement and sensitivity) as follows:

$$MSE\left(T^*\right) = \phi^2 MSE\left(T_u^*\right) + (1 - \phi)^2 MSE\left(T_m^*\right), \tag{61}$$

where

$$MSE\left(T_u^*\right) = E\left(T_u^* - \overline{Y}_h\right)^2 = \sum_{h=1}^{L} P_h^2\left[A_u^* + \beta_{h3}^2\frac{B_u^*}{4\overline{Z}_h^2} + \beta_{h3}\frac{\widehat{D}_u^{'}}{\overline{Z}_h}\right],$$

$$MSE\left(T_m^*\right) = E\left(T_m^* - \overline{Y}_h\right)^2 = \sum_{h=1}^{L} P_h^2\left[A_m^* - \alpha_{h3}^2\left\{C^* + \frac{1}{4\overline{Z}_h^2}B^* + \frac{1}{\overline{Z}_h}\widehat{E}^{*'}\right\}\right.$$

$$\left. + \alpha_{h3}\left\{2\widehat{F}^{*'} + \frac{\widehat{D}^*{}'}{\overline{Z}_h}\right\}\right], \tag{62}$$

where

$$A_u^* = \lambda_{hu}\left(S_{yh}^2\right) + \lambda_{hu}^*\left(S_{yh(2)}^2\right),$$

$$B_u^* = \lambda_{hu}\left(S_{zh}^2\right) + \lambda_{hu}^*\left(S_{zh(2)}^2\right),$$

$$A_m^* = \lambda_{hm}\left(S_{yh}\right) + \lambda_{hm}^*\left(S_{yh(2)}^2\right),$$

$$B^* = \left(B_n^* - B_m^*\right),$$

$$C^* = \left(C_n^* - C_m^*\right), \tag{63}$$

$$B_n^* = \lambda_{hn}\left(S_{zh}^2\right) + \lambda_{hn}^*\left(S_{zh(2)}^2\right),$$

$$B_m^* = \lambda_{hm}\left(S_{zh}^2\right) + \lambda_{hm}^*\left(S_{zh(2)}^2\right),$$

$$C_n^* = \lambda_{hn}\left(S_{xh}^2\right) + \lambda_{hn}^*\left(S_{xh(2)}^2\right),$$

$$C_m^* = \lambda_{hm}\left(S_{xh}^2\right) + \lambda_{hm}^*\left(S_{xh(2)}^2\right),$$

which are minimum, when

$$\beta_{h3} = -\frac{2\widehat{D}_u^{*'}\overline{Z}_h}{B_u^*} = \beta_{h3(\text{opt})},$$

$$\alpha_{h3} = \frac{2\widehat{F}^{*'} + \left(\widehat{D}^{*'}/\overline{Z}_h\right)}{2\left\{C^* + \left(1/4\overline{Z}_h^2\right)B^* + \left(1/\overline{Z}_h\right)\widehat{E}^{*'}\right\}} = \alpha_{h3(\text{opt})}. \tag{64}$$

We obtain the minimum MSE of T_u^* and T_m^*, after substituting the optimum value of $\beta_{h3(\text{opt})}$ and $\alpha_{h3(\text{opt})}$ in abovementioned MSEs as follows:

$$\text{min}.MSE\left(T_u^*\right) = \sum_{h=1}^{L} P_h^2\left[A_u^* - \frac{\widehat{D}_u^{2'*}}{B_u^*}\right], \tag{65}$$

$$\text{min}.MSE\left(T_m^*\right) = \sum_{h=1}^{L} P_h^2\left[A_m^* + \frac{1}{4}\delta^*\right], \tag{66}$$

where

$$\delta^* = \left[\frac{\left\{2\widehat{F}^{*'} + \left(\widehat{D}^{*'}/\overline{Z}_h\right)\right\}}{\left\{C^* + \left(1/4\overline{Z}_h^2\right)B^* + \left(1/\overline{Z}_h\right)\widehat{E}^{*'}\right\}}\right]. \tag{67}$$

To derive the optimum value of the constant "ϕ," we differentiate equation (60) w.r.t "ϕ" and equate it to zero; we obtain as follows:

$$\phi_{\text{opt}} = \frac{\min.MSE\left(T_m^*\right)}{\min.MSE\left(T_u^*\right) + \min.MSE\left(T_m^*\right)}. \tag{68}$$

After substituting the optimum value of ϕ_{opt} from equation (66), the optimum mean square error of $\widehat{T}^*$ is given as follows:

$$MSE\left(T^*\right)_{\text{opt}} = \frac{\min.MSE\left(T_u^*\right)\min.MSE\left(T_m^*\right)}{\min.MSE\left(T_u^*\right) + \min.MSE\left(T_m^*\right)}. \tag{69}$$

Case 4. If we substitute $S_{xh(2)} = S_{yh(2)} = S_{zh(2)} = S_{Uh}^2 = S_{Vh}^2 = S_{Wh}^2 = 0$, then the mean square error of the proposed estimator reduces to only the sensitivity case (i.e., absence of nonresponse and measurement errors) as follows:

$$MSE(\widehat{T}) = \phi^2 MSE\left(\widehat{T}_u\right) + (1 - \phi)^2 MSE\left(\widehat{T}_m\right), \tag{70}$$

where

$$MSE\left(\widehat{T}_u\right) = E\left(\widehat{T}_u - \overline{Y}_h\right)^2 = \sum_{h=1}^{L} P_h^2\left[\widehat{A}_u + \beta_{h4}^2 \frac{\widehat{B}_u}{4\overline{Z}_h^2} + \beta_{h4}\frac{\widehat{D}_u}{\overline{Z}_h}\right],$$

$$MSE\left(\widehat{T}_m\right) = E\left(\widehat{T}_m - \overline{Y}_h\right)^2 = \sum_{h=1}^{L} P_h^2\left[\widehat{A}_m - \alpha_{h4}^2\left\{\widehat{C} + \frac{1}{4\overline{Z}_h^2}\widehat{B} + \frac{1}{\overline{Z}_h}\widehat{E}\right\}\right. \tag{71}$$

$$\left. + \alpha_{h4}\left\{2\widehat{F} + \frac{\widehat{D}}{\overline{Z}_h}\right\}\right],$$

where

$$\widehat{A}_u = \lambda_{hu}\left(S_{yh}^2\right) + K_{1h},$$
$$\widehat{B}_u = \lambda_{hu}\left(S_{zh}^2\right) + K_{3h},$$
$$\widehat{D}_u = \lambda_{hu}\rho_{y_h z_h}S_{yh}S_{zh},$$
$$\widehat{A}_m = \lambda_{hm}\left(S_{yh}^2\right) + K_{1h},$$
$$\widehat{B} = \left(\widehat{B}_n - \widehat{B}_m\right),$$
$$\widehat{B}_n = \lambda_{hn}\left(S_{zh}^2\right) + K_{3h},$$
$$\widehat{B}_m = \lambda_{hm}\left(S_{zh}^2\right) + K_{3h},$$
$$\widehat{C} = \left(\widehat{C}_n - \widehat{C}_m\right),$$
$$\widehat{C}_n = \lambda_{hn}\left(S_{xh}^2\right) + K_{2h},$$
$$\widehat{C}_m = \lambda_{hm}\left(S_{xh}^2\right) + K_{2h}, \tag{72}$$
$$\widehat{D} = \left(\widehat{D}_n - \widehat{D}_m\right),$$
$$\widehat{D}_n = \lambda_{hn}\rho_{y_h z_h}S_{yh}S_{zh},$$
$$\widehat{D}_m = \lambda_{hm}\rho_{y_h z_h}S_{yh}S_{zh},$$
$$\widehat{F} = \left(\widehat{F}_n - \widehat{F}_m\right),$$
$$\widehat{F}_n = \lambda_{hn}\rho_{y_h x_h}S_{yh}S_{xh},$$
$$\widehat{F}_m = \lambda_{hm}\rho_{y_h x_h}S_{yh}S_{xh},$$
$$\widehat{E} = \left(\widehat{E}_n - \widehat{E}_m\right),$$
$$\widehat{E}_n = \lambda_{hn}\rho_{x_h z_h}S_{xh}S_{zh},$$
$$\widehat{E}_m = \lambda_{hm}\rho_{x_h z_h}S_{xh}S_{zh}.$$

The MSEs are minimum, when

$$\beta_{h4} = -\frac{2\widehat{D}_u\overline{Z}_h}{\widehat{B}_u} = \beta_{h4(\text{opt})},$$

$$\alpha_{h4} = \frac{2\widehat{F} + \left(\widehat{D}/\overline{Z}_h\right)}{2\left\{\widehat{C} + 1/4\overline{Z}_h^2\widehat{B} + \left(1/\overline{Z}_h\right)\widehat{E}\right\}} = \alpha_{h4(\text{opt})}. \tag{73}$$

After substituting the optimum value of $\beta_{h4(\text{opt})}$ and $\alpha_{h4(\text{opt})}$ in equations (70) and (71), we obtain the minimum MSE of $\widehat{T}_u$ and $\widehat{T}_m$.

$$\min.MSE\left(\widehat{T}_u\right) = \sum_{h=1}^{L} P_h^2\left[\widehat{A}_u - \frac{\widehat{D}_u^2}{\widehat{B}_u}\right], \tag{74}$$

$$\min.MSE\left(\widehat{T}_m\right) = \sum_{h=1}^{L} P_h^2\left[\widehat{A}_m + \frac{1}{4}\widehat{\delta}\right], \tag{75}$$

where

$$\widehat{\delta} = \left[\frac{\left\{2\widehat{F} + \left(\widehat{D}/\overline{Z}_h\right)\right\}}{\left\{\widehat{C} + \left(1/4\overline{Z}_h^2\widehat{B}\right) + \left(1/\overline{Z}_h\widehat{E}\right)\right\}}\right]. \tag{76}$$

The MSE of $\widehat{T}$ is dependent on the unknown constant "ϕ;" then, we differentiate equation (69) w.r.t "ϕ" and equate it to zero and we obtain the optimum value of "ϕ" as follows:

$$\phi_{\text{opt}} = \frac{\min.MSE\left(\widehat{T}_m\right)}{\min.MSE\left(\widehat{T}_u\right) + \min.MSE\left(\widehat{T}_m\right)}. \tag{77}$$

Substituting the optimum value of ϕ_{opt} from equation (75), we obtain the optimum mean square error of the estimator $\widehat{T}$ as follows:

$$MSE\left(\widehat{T}\right)_{opt} = \frac{\min.MSE\left(\widehat{T}_u\right)\min.MSE\left(\widehat{T}_m\right)}{\min.MSE\left(\widehat{T}_u\right) + \min.MSE\left(\widehat{T}_m\right)}. \tag{78}$$

5. Complete Response Estimator

To evaluate the proposed estimators with complete response situations to see how well they perform. The complete response estimator τ is defined as follows:

$$\tau = \psi\tau_u + (1 - \psi)\tau_m, \tag{79}$$

where $\tau_u = \overline{y}_{uh}$, $\tau_m = \overline{y}_{mh}/\overline{x}_{mh}\overline{x}_{nm}$, and $\psi\,(0 \le \psi \le 1)$ is an unknown constant to be determined by the minimization of the mean square error of the estimator τ.

To the first-order approximation, the minimum mean square error of the estimator τ is given as follows:

$$\min.MSE(\tau) = \frac{V(\tau_u).MSE(\tau_m)}{V(\tau_u) + MSE(\tau_m)}, \tag{80}$$

where

$$V(\tau_u) = \sum_{h=1}^{L} \lambda_{hu}S_{yh}^2,$$
$$MSE(\tau_m) = \sum_{h=1}^{L} \left(\lambda_{hm}S_{yh}^2 + (\lambda_{hm} - \lambda_{hn})\varepsilon_h\right), \tag{81}$$

where

$$R_h = \frac{\overline{Y}_h}{\overline{x}_h},$$
$$\varepsilon_h = \left(R_h^2 S_{xh}^2 - 2\rho_{y_h x_h} R_h S_{yh} S_{xh}\right). \tag{82}$$

6. Simulation Study

In this section, we conduct a simulation study using R software to verify the results of the previous sections. We have generated a hypothetical (artificial) population in Section 6.1 and considered two real populations in Section 6.2. The descriptions of the variables with parametric values are given in Sections 6.1 and 6.2, respectively. The results are given in Tables 1–4, respectively.

6.1. Population Generated through Simulation Using Normal Distribution. In this section, we investigated the efficiency of our estimator with the help of an artificial population of $N = 18,000$ and generated three strata of equal size, i.e., $N_1 = N_2 = N_3 = 6000$; then, a random sample is drawn from each stratum of size n_h; $h = 1, 2, 3$, i.e., $n_1 = n_2 = n_3 = 2000$, such that $n = \sum_{h=1}^{3} n_h = 6000$. On the first occasion, we generated a sample from n, and on the second occasion, a subsample of size $np = m$, i.e., matched units, is taken from the first occasion and a fresh sample of size $nq = u$, i.e., unmatched units, is drawn

from the remaining population. In the second phase, we take another sample $n_{sh} = n_{2h}/k$; $k > 1$ are drawn from the nonrespondents class. Again, a subsample from the nonrespondent group of size $m_{sh} = m_{2h}/k$; $k > 1$ and $u_{sh} = u_{2h}/k$; $k > 1$ units are drawn by using different values of $k = 2, 3, 4, 5$, respectively. The study variables and auxiliary variables are generated from the normal distribution, i.e., $X = -3.5 + Z + r\text{norm}(N, 20, 5)$, $Y = -3.5 + Z + r\text{norm}(N, 20, 5)$, and $Z = r\text{norm}(N, 20, 5)$, respectively. The scrambling variable S_{1h} is taken from a normal distribution with a mean value of 1 and a variance value of 0, and S_{2h} is also taken from a normal distribution with a mean value of 0 and a variance value of 1. The measurement errors on Y_h, X_h, and Z_h have a normal distribution with a mean value of 0 and a variance value approximately equal to 1. We analyzed the mean squared error (MSE) of the proposed estimator(s) in the cases of presence and absence of measurement errors, respectively, for different values of k, W, and q.

From Tables 1 and 2, we have noted the following observations:

(i) It is investigated from Tables 1 and 2 that when no error is present (i.e., a complete response), the MSE of τ is better than that of the other considered estimators. Furthermore, for the considered estimator, the MSE of $\widehat{T}$ (i.e., presence of sensitivity only) is more efficient than that of the other estimators (i.e., the presence and absence of measurement errors). It represents $\tau < \widehat{T} < T^* < \widehat{T}^* < T^{*'} < \widehat{T}^{*'}$.

(ii) Also, from Tables 1 and 2, it is observed that with the increasing value of k, the MSEs of each estimator, i.e., $\widehat{T}, T^*, \widehat{T}^*, T^{*'}$, and $\widehat{T}^{*'}$, also increase.

(iii) With an increase in the value of W from 0.4 to 0.8, the mean squared error of each estimator also increases, i.e., $\widehat{T}, T^*, \widehat{T}^*, T^{*'}$, and $\widehat{T}^{*'}$, also increases, which is shown in Tables 1 and 2.

(iv) From Table 1, it can be seen that when $\rho_{xy} = 0.9$ and $\rho_{yz} = 0.7$, then for the increasing value of $q = 0.2$ to 0.4, the MSE of $\widehat{T}^*$ (i.e., presence of sensitivity with nonresponse and measurement errors) increases; the MSEs of all the other considered estimators, i.e., $\widehat{T}, T^*, \widehat{T}^*$, and $T^{*'}$, first increase and then decrease.

(v) From Tables 1 and 2, it is observed that for the increasing value of $q = 0.2$ to 0.4, the MSE of $\widehat{T}^*$ increases. But, for $q = 0.2$, the MSEs all the other estimators, i.e., $\widehat{T}, T^*, \widehat{T}^*$, and $T^{*'}$ first increase, then decrease for $q = 0.4$ and increase for $q = 0.6$.

(vi) Higher the values of the correlation coefficient between the study and auxiliary variable, the more effective the suggested technique shown in Tables 1 and 2. This behavior assists us in selecting a population for the application of our strategy in real life.

6.2. Numerical Illustration Using Two Known Natural Population. Two real-life datasets are considered from the work of Singh et al. [14] for numerical comparison, and we have taken into account $\overline{S}_1 = 1$; $\overline{S}_2 = 0$ and $S_{s_1}^2 = 0.05$; $S_{s_2}^2 = 1$

Table 1: MSE of the proposed estimators with nonresponse and measurement errors for $W = 0.4$ and different values of k and q.

		$\widehat{T}^{*\prime}$ With M.E	$\widehat{T}^{*}$ Without M.E	$T^{*\prime}$ With M.E	T^{*} Without M.E	$\widehat{T}$
Q	k			Estimators		

$MSE(\tau)$ is 0.00600

Q	k	$\widehat{T}^{*\prime}$ With M.E	$\widehat{T}^{*}$ Without M.E	$T^{*\prime}$ With M.E	T^{*} Without M.E	$\widehat{T}$
0.2	2	0.05505	0.00713	0.00749	0.00697	0.00598
	3	0.10336	0.00834	0.00898	0.00811	0.00606
	4	0.15158	0.00954	0.01045	0.00924	0.00614
	5	0.19979	0.01073	0.01192	0.01036	0.00621
0.4	2	0.05489	0.00713	0.00736	0.00699	0.00601
	3	0.10303	0.00832	0.00870	0.00810	0.00609
	4	0.15110	0.00950	0.01002	0.00921	0.00616
	5	0.19914	0.01067	0.01134	0.01031	0.00624
0.6	2	0.05476	0.00701	0.00727	0.00686	0.00588
	3	0.10276	0.00821	0.00864	0.00798	0.00596
	4	0.15069	0.00939	0.01000	0.00910	0.00603
	5	0.19859	0.01057	0.01135	0.01020	0.00611

$\rho_{xy} = 0.9$, $\rho_{yz} = \rho_{xz} = 0.7$, and $MSE(\tau)$ is 0.00480

Q	k					
0.2	2	0.05482	0.00616	0.00656	0.00599	0.00491
	3	0.10322	0.00746	0.00814	0.00721	0.00500
	4	0.15149	0.00873	0.00968	0.00840	0.00509
	5	0.19971	0.00997	0.01121	0.00957	0.0051
0.4	2	0.05456	0.00613	0.00638	0.00596	0.00491
	3	0.10288	0.00739	0.00780	0.00715	0.00500
	4	0.15099	0.00863	0.00920	0.00831	0.00508
	5	0.19905	0.00985	0.01057	0.00946	0.00517
0.6	2	0.05457	0.00613	0.00642	0.00596	0.00491
	3	0.10265	0.00739	0.00788	0.00715	0.00500
	4	0.15061	0.00863	0.00930	0.00831	0.00508
	5	0.19852	0.00985	0.01070	0.00946	0.00517

$\rho_{xy} = 0.75$, $\rho_{yz} = \rho_{xz} = 0.55$, and $MSE(\tau)$ is 0.00580

Q	k					
0.2	2	0.05503	0.00706	0.00742	0.00690	0.00590
	3	0.10334	0.00827	0.00892	0.00804	0.00598
	4	0.15158	0.00948	0.01040	0.00918	0.00606
	5	0.19978	0.01067	0.01187	0.01030	0.00614
0.4	2	0.05486	0.00704	0.00727	0.00689	0.00591
	3	0.10302	0.00823	0.00861	0.00801	0.00598
	4	0.15109	0.00941	0.00994	0.00912	0.00606
	5	0.19913	0.01059	0.01126	0.01022	0.00614
0.6	2	0.05477	0.00704	0.00730	0.00689	0.00591
	3	0.10277	0.00823	0.00867	0.00801	0.00599
	4	0.15069	0.00942	0.01003	0.00912	0.00606
	5	0.19859	0.01059	0.01137	0.01023	0.00614

$\rho_{xy} = 0.65$, $\rho_{yz} = \rho_{xz} = 0.45$, and $MSE(\tau)$ is 0.0063

Q	k					
0.2	2	0.05515	0.00747	0.00783	0.00733	0.00635
	3	0.10341	0.00866	0.00929	0.00844	0.00643
	4	0.15162	0.00984	0.01075	0.00955	0.00650
	5	0.19982	0.01102	0.01220	0.01066	0.00657
0.4	2	0.05498	0.00746	0.00768	0.00732	0.00636
	3	0.10309	0.00863	0.00900	0.00841	0.00643
	4	0.15114	0.00979	0.01030	0.00951	0.00650
	5	0.19917	0.01095	0.01161	0.01060	0.00658
0.6	2	0.05487	0.00746	0.00771	0.00732	0.00636
	3	0.10283	0.00863	0.00905	0.00842	0.00643
	4	0.15073	0.00979	0.01038	0.00951	0.00651
	5	0.19862	0.01096	0.01171	0.01060	0.00658

TABLE 2: MSE of the proposed estimators with nonresponse and measurement errors for $W = 0.8$ and different values of k and q.

		$MSE(\tau)$ is 0.00540				
				Estimators		
Q	k	$\widehat{T}^{*'}$	$\widehat{T}^{*}$	$T^{*'}$	T^{*}	$\widehat{T}$
		With M.E	Without M.E	With M.E	Without M.E	
	2	0.10030	0.00642	0.00667	0.00608	0.00524
0.2	3	0.19378	0.00767	0.00816	0.00716	0.00538
	4	0.28724	0.00898	0.00972	0.00832	0.0055
	5	0.38059	0.01022	0.01113	0.00937	0.00563
	2	0.10072	0.00646	0.00670	0.00613	0.00529
0.4	3	0.19466	0.00773	0.00820	0.00724	0.00541
	4	0.28852	0.00899	0.00970	0.00835	0.00553
	5	0.38228	0.01017	0.01111	0.00938	0.00565
	2	0.10082	0.00626	0.00651	0.00592	0.00508
0.6	3	0.19481	0.00752	0.00799	0.00703	0.00521
	4	0.28879	0.00885	0.00960	0.00821	0.00534
	5	0.38265	0.01008	0.01103	0.00929	0.00547
		$\rho_{xy} = 0.9$, $\rho_{yz} = \rho_{xz} = 0.7$, and $MSE(\tau)$ is 0.00480				
	2	0.10027	0.00622	0.00648	0.00586	0.00503
0.2	3	0.19376	0.00748	0.00798	0.00696	0.00517
	4	0.28722	0.00880	0.00955	0.00812	0.00531
	5	0.38059	0.01005	0.01098	0.00922	0.00544
	2	0.10069	0.0062	0.00645	0.00585	0.00502
0.4	3	0.19464	0.00748	0.00796	0.00698	0.00515
	4	0.2885	0.00876	0.00948	0.00811	0.00528
	5	0.38227	0.00994	0.01090	0.00913	0.00541
	2	0.10081	0.00620	0.00645	0.00586	0.00501
0.6	3	0.19480	0.00746	0.00794	0.00697	0.00515
	4	0.28879	0.00880	0.00955	0.00814	0.00528
	5	0.38265	0.01003	0.01098	0.00923	0.00541
		$\rho_{xy} = 0.75$, $\rho_{yz} = \rho_{xz} = 0.55$, and $MSE(\tau)$ is 0.00580				
	2	0.10041	0.00713	0.00736	0.00682	0.00596
0.2	3	0.19384	0.00834	0.00878	0.00787	0.00607
	4	0.28728	0.00961	0.01030	0.00900	0.00617
	5	0.38064	0.01082	0.01167	0.01006	0.00627
	2	0.10082	0.00712	0.00735	0.00682	0.00596
0.4	3	0.19472	0.00836	0.0088	0.00791	0.00607
	4	0.28856	0.00960	0.01026	0.009	0.00616
	5	0.38232	0.01075	0.01165	0.01001	0.00626
	2	0.10093	0.00712	0.00735	0.00682	0.00597
0.6	3	0.19487	0.00834	0.00877	0.00789	0.00607
	4	0.28884	0.00963	0.01033	0.00904	0.00616
	5	0.38268	0.01083	0.01173	0.01009	0.00626
		$\rho_{xy} = 0.65$, $\rho_{yz} = \rho_{xz} = 0.45$, and $MSE(\tau)$ is 0.0063				
	2	0.10048	0.00756	0.00777	0.00726	0.00639
0.2	3	0.19388	0.00875	0.00918	0.00831	0.00648
	4	0.28731	0.01001	0.01067	0.00942	0.00657
	5	0.38066	0.01120	0.01203	0.01047	0.00665
	2	0.10090	0.00756	0.00777	0.00727	0.00639
0.4	3	0.19476	0.00878	0.00920	0.00835	0.00648
	4	0.28858	0.01004	0.01065	0.00943	0.00657
	5	0.38234	0.01116	0.01203	0.01044	0.00665
	2	0.10099	0.00756	0.00777	0.00727	0.00639
0.6	3	0.19491	0.00876	0.00918	0.00833	0.00648
	4	0.28886	0.01004	0.01072	0.00947	0.00657
	5	0.38270	0.01123	0.01210	0.01051	0.00665

TABLE 3: MSE of the proposed estimators with nonresponse and measurement errors for different values of k and W.

W	τ	k	$\widehat{T}^{*'}$ With M.E	$\widehat{T}^{*}$ Without M.E	$T^{*'}$ With M.E	T^{*} Without M.E	$\widehat{T}$
					Estimators		
0.4	3.76111	2	16.30700	2.00065	9.04458	1.95015	1.82873
		3	23.61560	2.18979	9.21859	2.11666	1.85261
		4	30.89430	2.37457	9.39241	2.28008	1.87617
		5	38.16080	2.55565	9.56606	2.44084	1.89943
0.8	3.95862	2	13.47750	2.87943	9.26937	2.81292	2.74132
		3	17.80520	3.03436	9.43512	2.93475	2.76591
		4	22.12540	3.18914	9.60084	3.05650	2.79014
		5	26.44180	3.34378	9.76655	3.17818	2.81401

TABLE 4: MSE of the proposed estimators with nonresponse and measurement errors for different values of k and W.

W	τ	k	$\widehat{T}^{*'}$ With M.E	$\widehat{T}^{*}$ Without M.E	$T^{*'}$ With M.E	T^{*} Without M.E	$\widehat{T}$
					Estimators		
0.4	2.47430	2	15.5900	1.76780	7.27640	1.69680	1.65580
		3	24.35600	1.90850	7.53100	1.80350	1.68800
		4	32.92700	2.04800	7.78550	1.90980	1.71970
		5	41.48800	2.18660	8.03990	2.01570	1.75080
0.8	3.49550	2	8.06550	2.69570	7.50100	2.60270	2.56070
		3	8.84530	2.85230	7.76100	2.71250	2.59370
		4	9.62280	3.00870	8.02080	2.82210	2.62610
		5	10.39870	3.16500	8.28040	2.93150	2.65780

in order to minimize the effect of scrambling on the real data. The MSE of the estimators is calculated as follows:

$$MSE(*) = \operatorname{Var}(*) + [\operatorname{Bias}(*)]^2, \tag{83}$$

where "$*$" means τ, $\widehat{T}^{*'}$, $\widehat{T}^{*}$, $T^{*'}$, T^{*}, and $\widehat{T}$.

The details of two real populations are as follows.

The first dataset is based on the Census 2011 literacy rates in India. The data are of $N = 35$ Indian states and union territories, and they generated three strata of unequal size, i.e., $N_1 = 13$, $N_2 = 11$, and $N_3 = 10$; then, a random sample is drawn from each stratum of size n_h; $h = 1, 2, 3$, i.e., $n_1 = 8$, $n_2 = 7$, and $n_3 = 5$. The literacy rate is spread across the major parameters overall, rural, and urban. Let y, x, and z denote the number of literates (people) in 2001, 2011, and the total literacy rate (2011), respectively.

The second dataset is based on abortion rates from Statistical Abstract of the United States (2011) to clarify the performance of our proposed estimator (free access to the data by the Statistical Abstracts of the United States). The population consists of $N = 51$ states of the U.S. and generates three strata of unequal size, i.e., $N_1 = 10$, $N_2 = 15$, and $N_3 = 10$; then, a random sample is drawn from each stratum of size n_h; $h = 1, 2, 3$, i.e., $n_1 = 8$, $n_2 = 7$, $n_3 = 5$. Furthermore, let y, x, and z denote the number of abortions reported in the state of the U.S. during the years 2008, 2007, and 2005, respectively.

The results are shown in Tables 3 and 4 for the different choices of the nonresponse, i.e., $k = 2, 3, 4, 5$ and different probability levels of sensitive variables, i.e., $W = 0.4$ and 0.8, are used.

Tables 3 and 4 display the mean squared error of the proposed estimator(s) in the presence of nonresponse and measurement errors using ORRT for different values of k and W. It is envisaged from Tables 3 and 4 that in the situation of a moderate level of the sensitive variable with different levels of nonresponse and measurement errors, our proposed estimators are more efficient. We also observed that for an increase in the value of k, the MSE of all other estimators increases.

It is noted from Tables 1–4 that the proposed estimator "$\widehat{T}$" is always better than the other considered estimators in terms of having a minimum MSE.

7. Conclusion

Through this article, we have proposed a logarithmic type estimator for the estimation of the population mean of a sensitive variable under stratified successive sampling in the presence of nonresponse and measurement errors simultaneously by utilizing the ORRT model. Up to the first degree of approximation, the bias and MSE are derived. The properties of the proposed estimator have been studied and we compared the results with respect to the complete response situation. A simulation study has been performed for both natural population and an artificial generated population, and as a result of the simulation study, we have demonstrated that our proposed estimator is most efficient in the absence of both nonresponse and measurement error situations and least efficient in the case when there is a

presence of both nonresponse and measurement errors at the same time. Thus, we recommend our suggested estimator, i.e., $\widehat{T}$, for further use in practice.

Data Availability

The data used to support the findings of the study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

References

[1] C. Kadilar and H. Cingi, "Ratio estimators in stratified random sampling," *Biometrical Journal*, vol. 45, no. 2, pp. 218–225, 2003.

[2] C. N. Bouza, C. Herrera, and P. G. Mitra, "A review of randomized responses procedures: the qualitative variable case," *Investigación Operacional*, vol. 31, no. 3, pp. 240–247, 2010.

[3] R. J. Jessen, "Statistical investigation of a sample survey for obtaining farm facts," *Research bulletin*, vol. 26, no. 304, 1942.

[4] G. N. Singh, D. Bhattacharyya, and A. Bandyopadhyay, "Formulation of logarithmic type estimators to estimate population mean in successive sampling in presence of random non-response and measurement errors," *Communications in Statistics - Simulation and Computation*, vol. 51, no. 3, pp. 901–923, 2019.

[5] G. N. Singh, M. Khalid, and J. M. Kim, "Some imputation methods to deal with the problems of missing data in two-occasion successive sampling," *Communications in Statistics - Simulation and Computation*, vol. 50, no. 2, pp. 557–580, 2019.

[6] M. Khalid and G. N. Singh, "Some imputation methods to deal with the issue of missing data problems due to random non-response in two-occasion successive sampling," *Communications in Statistics - Simulation and Computation*, vol. 51, no. 12, pp. 7266–7286, 2020.

[7] S. L. Warner, "Randomized response: a survey technique for eliminating evasive answer bias," *Journal of the American Statistical Association*, vol. 60, no. 309, pp. 63–69, 1965.

[8] G. Diana and P. F. Perri, "A class of estimators for quantitative sensitive data," *Statistical Papers*, vol. 52, no. 3, pp. 633–650, 2011.

[9] S. Gupta, S. Mehta, J. Shabbir, and S. Khalil, "A unified measure of respondent privacy and model efficiency in quantitative rrt models," *Journal of Statistical Theory and Practice*, vol. 12, no. 3, pp. 506–511, 2018.

[10] Q. Zhang, S. Gupta, G. Kalucha, and S. Khalil, "Ratio estimation of the mean under rrt models," *Journal of Statistics & Management Systems*, vol. 22, no. 1, pp. 97–113, 2019.

[11] M. H. Hansen and W. N. Hurwitz, "The problem of nonresponse in sample surveys," *Journal of the American Statistical Association*, vol. 41, no. 236, pp. 517–529, 1946.

[12] C. N. Bouza, "Sobre el problema de la fracción de submuestreo para el caso de las no respuestas," *Trabajos de Estadistica y de Investigacion Operativa*, vol. 32, no. 2, pp. 30–36, 1981.

[13] G. Diana, S. Riaz, and J. Shabbir, "Hansen and hurwitz estimator with scrambled response on the second call," *Journal of Applied Statistics*, vol. 41, no. 3, pp. 596–611, 2014.

[14] G. N. Singh, D. Bhattacharyya, and A. Bandyopadhyay, "Calibration estimation of population variance under stratified successive sampling in presence of random non response," *Communications in Statistics - Theory and Methods*, vol. 50, no. 19, pp. 4487–4509, 2021.

[15] P. Mukhopadhyay, G. N. Singh, and A. Bandyopadhyay, "A general estimation technique of population mean under stratified successive sampling in presence of random scrambled response and non-response," *Communications in Statistics - Simulation and Computation*, pp. 1–21, 2021.

[16] V. K. Singh, R. Singh, and F. Smarandache, "Different-type estimators for estimation of mean in the presence of measurement error," 2014, https://arxiv.org/ftp/arxiv/papers/1410/1410.0279.pdf.

[17] Q. Zhang, S. Khalil, and S. Gupta, "Mean estimation in the simultaneous presence of measurement errors and non-response using optional rrt models under stratified sampling," *Journal of Statistical Computation and Simulation*, vol. 91, no. 17, pp. 3492–3504, 2021.

[18] I. Zahid, J. Shabbir, and O. A. Alamri, "A generalized class of estimators for sensitive variable in the presence of nonresponse and measurement error under stratified sampling," *Journal of King Saud University Science*, vol. 34, pp. 1–10, 2022.

[19] K. K. Tiwari, S. Bhougal, S. Kumar, and K. U. I. Rather, "Using randomized response to estimate the population mean of a sensitive variable under the influence of measurement error," *Journal of Statistical Theory and Practice*, vol. 16, no. 2, p. 28, 2022.

[20] K. K. Tiwari, S. Bhougal, S. Kumar, and R. Onyango, "Assessing the effect of nonresponse and measurement error using a novel class of efficient estimators," *Journal of Mathematics*, vol. 2022, Article ID 4946265, 9 pages, 2022.

[21] S. Kumar and S. P. Kour, "The joint influence of estimation of sensitive variable under measurement error and non-response using orrt models," *Journal of Statistical Computation and Simulation*, vol. 92, no. 17, pp. 3583–3604, 2022.

[22] S. Bahl and R. K. Tuteja, "Ratio and product type estimator," *Information and Optimization Sciences*, vol. 12, pp. 159–163, 1991.