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Research Article

Backstepping Sliding Mode Control Algorithm for Unmanned Aerial Vehicles Based on Fractional-Order Theory

Yanzhu Zhang, Bo Zhuang, Chunhao Ma, and Cheng Zhang

School of Automation and Electrical, Shenyang Ligong University, Shenyang, Liaoning, China

Correspondence should be addressed to Bo Zhuang; 2106620724@stu.sylu.edu.cn

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Aiming at the problems of slow convergence speed and low tracking accuracy in attitude control and position tracking of quadrotor unmanned aerial vehicles (UAVs). This paper combines the fractional-order calculus theory with the backstepping sliding mode control algorithm, using the backstepping control to compensate for the nonlinearity of the system and the fractional-order theory to eliminate the jitter brought about by the sliding mode control, and proposes a new fractional-order backstepping sliding mode control strategy for the trajectory tracking control of the quadrotor UAV. The proposed fractional-order sliding mode surface increases the control flexibility and improves the robustness and anti-interference ability of the system to some extent. The stability analysis of the system is carried out using the Lyapunov stability theory, and the results prove the stability of the proposed controller. Finally, the effectiveness and feasibility of the proposed method are verified by comparing it with the traditional backstepping sliding mode controller. The simulation results show that the fractional-order reverse-step sliding mode control algorithm proposed in this paper is significantly better than other control algorithms in terms of convergence speed and also has a certain degree of superiority in terms of error elimination.

1. Introduction

In the past decade, the four-rotor unmanned aerial vehicle (UAV) has attracted more and more attention due to its simple mechanical structure, ability to vertical take-off and landing and hover, and low cost, and it has become an important aircraft. Its application range widely covers agriculture, industry, life, military, commerce, search and rescue, and other fields, and it plays an important role in crop monitoring, fertilization and spraying, aerial terrain survey, power maintenance, cargo transportation, and other tasks [1]. Trajectory tracking control is the basis of UAV to complete specific tasks. However, the nonlinear characteristics and uncertainties of UAV will seriously affect the control response, flight stability, control accuracy, navigation, and position estimation during flight. In view of the above problems, domestic and foreign scholars have conducted a lot of research. At present, there are many effective control technologies for four-rotor UAV, such as PID control, adaptive nonlinear control, feedback linearization control, model predictive control, sliding mode control, and backstepping control.

The backstepping method utilizes the dynamic model and nonlinear characteristics of the system; the complex nonlinear system is decomposed into subsystems that do not exceed the system order, and then the Lyapunov function and intermediate virtual control are designed for each subsystem. Through iterative adjustment, the designed virtual control quantity can gradually eliminate the nonlinear term of the system, so as to obtain better control performance and stability, which is an effective method to solve the control problem of complex nonlinear systems. However, the backstepping method does not have good resistance to the uncertainties and external interference in the model. The sliding mode variable structure control has complete adaptability, robustness, and fast convergence when the system parameters are uncertain and external interference exists. Therefore, it is usually combined with the backstepping control algorithm to improve the anti-interference performance of the system [2, 3]. The combination of sliding mode and backstepping control has been used to develop robust controllers for nonlinear systems [4]. Literature [5] proposes a four-rotor UAV attitude stability control strategy based on the integral backstepping control method, which improves the stability and control accuracy of the nonlinear system in the presence of externally disturbed torque. In literature [6], a feedback linearization controller is proposed for quadrotors with tiltable rotors to realize the trajectory tracking performance of UAV in the presence of gust disturbance. Literature [7] puts forward an improved reverse-step control method to solve the problem that the classical reverse-step control method has many control parameters and difficult parameter adjustments. Aiming at the large tracking error of a class of nonlinear fully driven and underdriven mechanical systems with uncertainty, literature [8] introduces a high-gain design parameter to construct a new reverse-step sliding mode cascade active disturbance rejection control scheme, which improves the tracking control accuracy of nonlinear systems. Literature [9] modified the nonlinear dynamics model of the eight-rotor coaxial UAV system and proposed an adaptive reverse-step sliding mode control scheme for the underactuated, fully actuated, and rotor thrust systems. A new adaptive law was used in the proposed control scheme to obtain good stability and tracking performance. Aiming at the problem of low control accuracy of four-rotor UAV under gust interference, literature [10] proposed a decentralized backstepping control method and then optimized the controller parameters through a differential evolution technique, effectively improving the robustness of the control system. Literature [11] proposes a control method that combines reverse sliding mode control and RBF neural network adaptive algorithm. The approach characteristic of the RBF neural network is used to compensate for the external interference, which effectively reduces the trajectory tracking error and overshoot of the UAV attitude system in the case of external interference torque and improves the anti-interference ability of the system. To solve the problem of slow position tracking convergence of four-rotor UAV under model uncertainty and external disturbance, an adaptive integral terminal sliding mode control method was proposed in literature [12] to achieve trajectory tracking performance of the system under known uncertainty and bounded disturbance.

Although the sliding mode control has excellent control performance against parameter uncertainty and external interference, due to the discontinuity of the sliding mode surface and the fast response of the controller, the control quantity of the system will jitter rapidly near the sliding mode surface, that is, the chattering phenomenon will occur. In order to reduce buffeting, many methods have been studied, and one of the most common methods is to replace the discontinuous sign function of sliding mode control with saturation function, hyperbolic tangent function, or other continuous function to reduce buffeting. Although this change can weaken buffeting to some extent, it increases tracking errors and reduces the robustness of the system, and it also makes the system highly sensitive to unmodeled dynamic models. Another method to solve the buffeting problem is to combine the fractional-order theory with the sliding mode control method in the design of the sliding mode controller and propose a fractional-order sliding mode control method.

Fractional calculus theory is a generalization and extension of the traditional integral calculus theory, which has the characteristics of time memory and strong robustness. The earliest research results of fractional calculus theory can be traced back to 1695. With the in-depth progress of research, the fractional calculus theory has been continuously improved and applied more and more widely under the efforts of many scholars. Some scholars have introduced fractional calculus theory into controller design to make its design method more flexible [13]. Fractional-order operators have noninteger derivative and integral operations, which can describe the dynamic characteristics of the system more accurately. Compared with integer order, fractional-order closed-loop characteristics have more obvious advantages and are more conducive to improving the stability and reliability of the controlled object [14, 15], so fractional-order control methods are widely used. Podlubny [16] analyzed the theory of fractional calculus and did corresponding experiments, changed the traditional PID control to a fractional PID control strategy, and made the fractional calculus theory achieve leapfrog development in the field of automatic control. Literature [17] combined fractional-order theory with sliding mode control theory to design a new fractional-order sliding mode controller to eliminate the influence of bounded disturbance and improve the robustness of the system. Literature [18] proposes a class of intelligent robust fractional-order sliding mode control for nonlinear systems, which solves the buffeting problem by replacing the symbolic function in sliding mode control with a fuzzy controller. Literature [19] proposes a new fractional-order nonlinear sliding surface. Based on the fractional-order sliding surface, an adaptive interval 2 type fuzzy compensator is used to estimate the uncertainty and perturbation of the nonlinear system, further reducing the chattering caused by the switching term and enhancing the immunity of the system. Aiming at the problem that the quadrotor slung-load system is easily affected by the swing angle of the suspension load, a robust fractional-order sliding mode control method is proposed in literature [20]. On this basis, an antiswing controller is installed, which effectively reduces the buffeting of the system and improves the convergence accuracy of the system. Literature [21] proposes a new fractional-order sliding mode controller for a fractional-order quadrotor system, which reduces the overshoot and response time of the system and improves the response rate of the system. In literature [22], aiming at the problem that ships cannot accurately track the expected course due to the interference of uncertain factors during the navigation of underdriven ships, a new fractional-order reaching law is constructed by using fractional-order calculus operators instead of integer order calculus operators, and a ship course keeping controller based on fractional-order sliding mode is designed. Thus, the rapidity and accuracy of the system response are improved.

This paper combines the advantages of fractional calculus theory and backward sliding mode control algorithm;

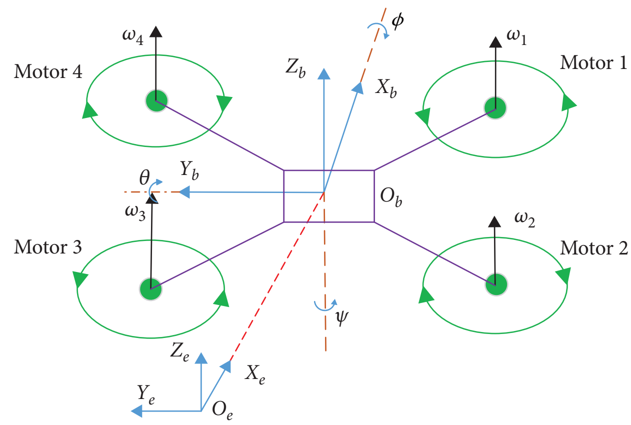

FIGURE 1: Simple model of a quadcopter UAV.

based on Caputo's definition, a new fractional-order reverse-step sliding mode control (FOBSMC) method is proposed to control the four-rotor UAV. The controller can not only track the desired flight path according to a certain accuracy difference but also has a small adjustment time. The structure of this paper is as follows: The first section is about the dynamics model analysis of the four-rotor UAV. The second section is the explanation of relevant preparatory knowledge. The third section is the design of the position subsystem and attitude subsystem controller. Then, the Lyapunov stability theory is used to prove the stability of the nonlinear control method. The fourth section shows the MATLAB simulation results of different flight paths of the four-rotor UAV. The fifth section gives the conclusion.

2. Mathematical Modeling of UAVs

This paper mainly takes the $''X''$ type UAV as the research object and analyzes the forces and moments on the quadrotor through the earth coordinate system O_e and the body coordinate system O_b in Figure 1. In order to meet the constraints of the four-rotor UAV, the maximum speed, attitude, and dynamic system constraints of the UAV are considered in the derivation of the mathematical model, and some hypotheses are proposed for the structure.

Assumption 1. *In this paper, the four-rotor UAV is regarded as a rigid body.*

Assumption 2. *The center of mass of the quadrotor is located at the origin of O_b.*

Assumption 3. *The lift produced by the propeller is proportional to the square of the propeller speed.*

The dynamics model of a typical four-rotor UAV can be expressed by the following equation [23]:

$$\begin{aligned}
\ddot{x} &= (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\frac{U_1}{m}\\
\ddot{y} &= (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\frac{U_1}{m}\\
\ddot{z} &= (\cos\phi\cos\theta)\frac{U_1}{m} - g\\
\ddot{\phi} &= \dot{\theta}\dot{\psi}\frac{(I_y - I_z)}{I_x} + \frac{J_r}{I_x}\dot{\theta}\omega_r + \frac{U_2}{I_x}\\
\ddot{\theta} &= \dot{\phi}\dot{\psi}\frac{(I_z - I_x)}{I_y} - \frac{J_r}{I_y}\dot{\phi}\omega_r + \frac{U_3}{I_y}\\
\ddot{\psi} &= \dot{\phi}\dot{\theta}\frac{(I_x - I_y)}{I_z} + \frac{U_4}{I_z},
\end{aligned} \tag{1}$$

where m is the total mass of the quadrotor, x, y, z, respectively, represent the linear position of the UAV in the inertial coordinate system. ϕ represents the roll angle about the x axis, θ represents the pitch angle about the y axis, ψ represents the yaw angle about the $z$axis. J_r is rotor inertia. I_x, I_y, I_z is the moment of inertia of the x, y, z axis. k_{dx}, k_{dy}, k_{dz} are translational air resistance coefficients, k_{ax}, k_{ay}, k_{az} are frictional air resistance coefficients. $\omega_r = \sum(-1)^i\omega_i, i = 1, 2, 3, 4.$ ω_i $(i = 1, 2, 3, 4)$ is defined as the speed of the i motor. U_i $(i = 1, 2, 3, 4)$ is the control input, which is related to ω_i as follows:

$$\begin{aligned}
U_1 &= b\omega_1^2 + b\omega_2^2 + b\omega_3^2 + b\omega_4^2\\
U_2 &= bl(-\omega_2^2 + \omega_4^2)\\
U_3 &= bl(\omega_1^2 - \omega_3^2)\\
U_4 &= d(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2).
\end{aligned} \tag{2}$$

Since the six outputs of the quadrotor UAV are only controlled by four input signals, it is necessary to introduce virtual control quantity$u = (u_x, u_y, u_z)^T$ to solve the problem of underdrive of horizontal position. Virtual control quantity is defined as follows:

$$\begin{aligned}
u_x &= (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)U_1\\
u_y &= (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)U_1\\
u_z &= (\cos\phi\cos\theta)U_1.
\end{aligned} \tag{3}$$

Then the required angles ϕ_d, θ_d and the control input U_1 can be expressed as follows:

$$\begin{aligned}
U_1 &= \sqrt{u_x^2 + u_y^2 + (u_z + g)^2}\\
\phi_d &= \arcsin\left(\frac{u_x\sin\psi_d - u_y\cos\psi_d}{U_1}\right)\\
\theta_d &= \arctan\left(\frac{u_x\cos\psi_d + u_y\sin\psi_d}{u_z}\right).
\end{aligned} \tag{4}$$

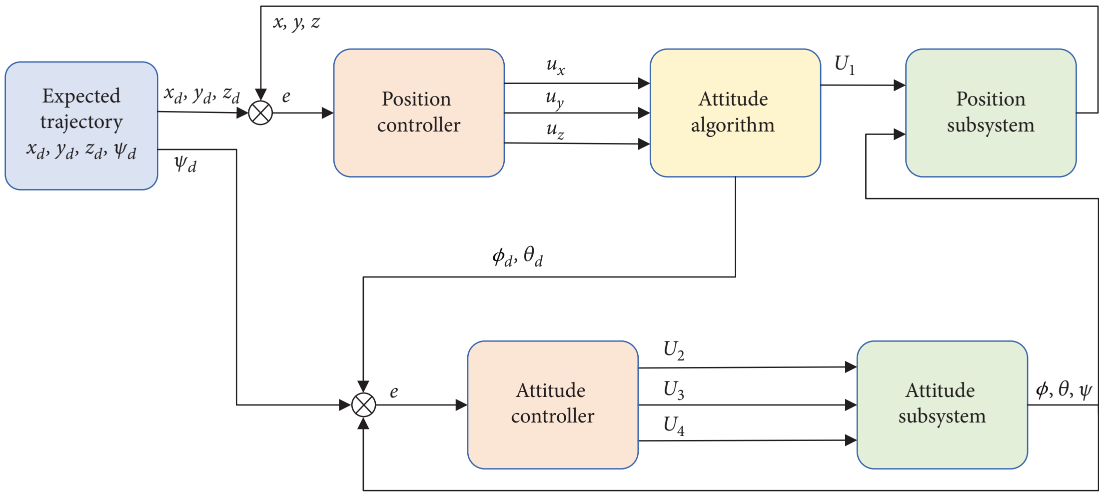

Figure 2: Block diagram of the controller structure of the FOBSMC.

3. Preliminary Knowledge Statement

In this paper, the sliding mode control plane under the definition of fractional order is used for controller design, and the related used fractional order theory is explained as follows. There are various definitions of fractional order calculus in the development process, and there are three commonly used definitions at present. They are the Riemann–Liouville (R–L) type, the Grunwald–Letnikov (G–L) type, and the Caputo type. Among them, Caputo is defined in such a way that it has the same form as the integer order differential equations under the initial conditions; therefore, Caputo's definition is widely used in engineering applications.

Definition 1 [13]. Caputo fractional-order differential:

$$ {}_{t0}^{C}D_t^{\alpha}y(t) = \frac{1}{\Gamma(m-\alpha)}\int_{t_0}^{t}\frac{y^{(m)}(\tau)}{(t-\tau)^{1+\alpha-m}}d\tau, \tag{5}$$

where $m = \alpha$ is an integer. $\alpha \in (0, 1)$.

Definition 2 [13]. Caputo fractional-order integral:

$$ {}_{t0}^{C}D_t^{-\gamma}y(t) = \frac{1}{\Gamma(\gamma)}\int_{t_0}^{t}\frac{y(\tau)}{(t-\tau)^{1-\gamma}}d\tau. \tag{6}$$

For convenience of expression, when the upper and lower limits of fractional-order operators are not involved, the symbol ${}_{t0}^{C}D_t^{\alpha}y(t)$is abbreviated as $D^{\alpha}y(t), \alpha \in (-1, 1)$. When $\alpha > 0$ denotes differentiation, and $\alpha < 0$ denotes integration.

Lemma 1 [24]. *Like integer derivatives, fractional derivatives have linear properties, namely:*

$$D^{\alpha}\left[af(t) + bg(t)\right] = aD^{\alpha}f(t) + bD^{\alpha}g(t). \tag{7}$$

Lemma 2 [14]. *The following inequality holds:*

$$-Ke(t)D^{-\alpha}e(t) \le 0, \forall \alpha \in (0, 1). \tag{8}$$

4. Controller Design

The flight control of four-rotor UAV is generally positioning tracking, and according to the flight principle of four-rotor UAV, the key to keep the vehicle stable in the air is attitude control. Therefore, in this paper, a new fractional-order backstepping sliding mode control method is proposed by designing a double closed-loop control loop with the inner loop for attitude control and the outer loop for position control. Compared with the traditional backstepping control, the backstepping sliding mode control proposed in this paper is combined with the fractional-order theory, which increases the control flexibility while avoiding the discontinuity of the control signals and improves the robustness and tracking accuracy of the system to a certain extent. The control system is divided into two parts: position controller and attitude controller. The position controller is designed by the given desired trajectory and the actual position feedback by the position subsystem. The attitude-solving module can obtain the desired attitude angle of UAV. The attitude controller is designed based on the expected angle obtained from the attitude-solving module and the actual angle feedback from the attitude subsystem, resulting in control quantities U_2, U_3, and U_4. The block diagram scheme of the controller of FOBSMC is shown in Figure 2.

4.1. Position Controller Design. The design idea of the position controller of the four-rotor UAV is to obtain U_1 through the control input in three directions and use the control input U_1 to adjust the thrust of the UAV, so as to control the acceleration of the UAV in all directions, so that the UAV can reach the expected trajectory. The following takes altitude control as an example to introduce the design of the position controller of four-rotor UAV. The dynamic model system in the altitude direction is as follows:

$$\ddot{z} = (\cos\phi\cos\theta)\frac{U_1}{m} - g. \tag{9}$$

The height error is defined as follows:

$$e_{z1} = z - z_d. \tag{10}$$

The virtual control term is defined as follows:

$$\alpha_1 = k_1 e_{z1}. \tag{11}$$

The auxiliary tracking error variable is defined as follows:

$$e_{z2} = \dot{e}_{z1} + \alpha_1, \tag{12}$$

where the parameter k_1 is assumed to be positive and known constant.

Define the Lyapunov function as follows:

$$V_{z1} = \frac{1}{2}e_{z1}^2. \tag{13}$$

Then:

$$\dot{V}_{z1} = e_{z1}\dot{e}_{z1} = e_{z1}(e_{z2} - k_1 e_{z1}). \tag{14}$$

Combined with the theory of fractional-order calculus, a new fractional-order sliding mode surface is designed as follows:

$$S_z = k_1 e_{z1} + \dot{e}_{z1} + k_2 D^{-\alpha} e_{z2}. \tag{15}$$

Taking the derivative of the sliding mode surface S_z, get the following:

$$\dot{S}_z = k_1\dot{e}_{z1} + \ddot{e}_{z1} + k_2 D^{-\alpha}\dot{e}_{z2}. \tag{16}$$

The fractional-order backstepping sliding mode equivalent control law is designed as follows:

$$u_{eq_z} = m\left(\ddot{z}_d + g - k_1\dot{e}_{z1} - \frac{1}{k_2}D^{\alpha}\dot{e}_{z2}\right). \tag{17}$$

In traditional sliding mode controllers, the sliding mode surface is defined by embedding a symbolic function that causes jumping and chattering of the control signal. With the introduction of fractional calculus theory, fractional operators can transform the instantaneous response of symbolic function into a gradual response, which can allow the control signal to change gently and reduce the phenomenon of jumping and buffeting. Therefore, the exponential reaching law of the fractional-order operator D^{α} is adopted in this paper.

The switching control law is defined as follows:

$$u_{sw_z} = -\eta D^{\alpha}\mathrm{sgn}\,(S_z) - h_1 D^{\alpha} S_z. \tag{18}$$

The total control input is written as follows:

$$u_z = m\left[\ddot{z}_d + g - k_1\dot{e}_{z1} - \frac{1}{k_2}D^{\alpha}\dot{e}_{z2} - \eta D^{\alpha}\mathrm{sgn}(S_z) - h_1 D^{\alpha} S_z\right]. \tag{19}$$

Stability proof:

Define the Lyapunov function as follows:

$$V_{z2} = V_{z1} + \frac{1}{2}S_z^2. \tag{20}$$

Taking the derivative of V_{z2} yields the following:

$$\dot{V}_{z2} = \dot{V}_{z1} + S_z\dot{S}_z. \tag{21}$$

Substituting (16) gives the following:

$$\dot{V}_{z2} = \dot{V}_{z1} + S_z(k_1\dot{e}_{z1} + \ddot{e}_{z1} + k_2 D^{-\alpha}\dot{e}_{z2}). \tag{22}$$

Substituting Equations (9) and (10) into (22) can obtain the following:

$$\begin{aligned}\dot{V}_{z2} = {} & e_{z1}(e_{z2} - k_1 e_{z1}) + S_z\dot{e}_{z2} \\ & + k_2 S_z D^{-\alpha}\left(\frac{u_z}{m} - g - \ddot{z}_d + k_1\dot{e}_{z1}\right).\end{aligned} \tag{23}$$

Substituting Equations (15) and (19) into (23) can obtain the following:

$$\begin{aligned}\dot{V}_{z2} = {} & -e_z^T F_z e_z - k_2\eta|S_z| - h_1 k_2(k_2 D^{-\alpha} e_{z2})^2 \\ & - 2h_1 k_2^2 e_{z2} D^{-\alpha} e_{z2},\end{aligned} \tag{24}$$

where $e_z = [e_{z1}e_{z2}]^T$ and F_z is a symmetric matrix with the form.

$$F_z = \begin{bmatrix} k_1 & -0.5 \\ -0.5 & h_1 k_2 \end{bmatrix}. \tag{25}$$

By adjusting the values of h_1, k_1, and k_2, we can make $h_1 k_1 k_2 \geq 0.25$, which ensures that F_z is a positive definite matrix. By Lemma 2, it can be shown that $-2h_1 k_2^2 e_{z2} D^{-\alpha} e_{z2}$

is negative definite. Thus $\dot{V}_{z2} \leq 0$ and the system will converge asymptotically.

4.2. Attitude Controller Design. According to the dynamics equation of the four-rotor UAV, the attitude is controlled by U_2, U_3, U_4. Taking roll angle ϕ as an example, the design of an attitude controller is introduced. In order to facilitate calculation, this paper rewrites the attitude subsystem of the four-rotor UAV into the following form:

$$\ddot{\phi} = \frac{f_\phi}{I_x} + \frac{U_2}{I_x}, \tag{26}$$

where f_ϕ is defined as follows:

$$f_\phi = \dot{\theta}\dot{\psi}\left(I_y - I_z\right) + J_r\dot{\theta}\omega_r. \tag{27}$$

Define the roll angle error as follows:

$$e_{\phi 1} = \phi - \phi_d. \tag{28}$$

The virtual control term is defined as follows:

$$\alpha_2 = k_3 e_{\phi 1}. \tag{29}$$

The auxiliary tracking error variable is defined as follows:

$$e_{\phi 2} = \dot{e}_{\phi 1} + \alpha_2, \tag{30}$$

where the parameterk_3 is assumed to be positive and known constant.

Define the Lyapunov function as follows:

$$V_{\phi 1} = \frac{1}{2} e_{\phi 1}^2. \tag{31}$$

Then:

$$\dot{V}_{\phi 1} = e_{\phi 1}\dot{e}_{\phi 1} = e_{\phi 1}\left(e_{\phi 2} - k_3 e_{\phi 1}\right). \tag{32}$$

Define the fractional-order sliding mode surface as follows:

$$S_\phi = k_3 e_{\phi 1} + \dot{e}_{\phi 1} + k_4 D^{-\alpha} e_{\phi 2}. \tag{33}$$

Taking the derivative of the sliding mode surface S_ϕ, get the following:

$$\dot{S}_\phi = k_3 \dot{e}_{\phi 1} + \ddot{e}_{\phi 1} + k_4 D^{-\alpha} \dot{e}_{\phi 2}. \tag{34}$$

The fractional-order backstepping sliding mode equivalent control law is designed as follows:

$$u_{eq_\phi} = I_x\left(\ddot{\phi}_d - k_3 \dot{e}_{\phi 1} - \frac{1}{k_4} D^\alpha \dot{e}_{\phi 2} \right) - f_\phi. \tag{35}$$

The switching control law is designed as follows:

$$u_{sw_\phi} = -\eta D^\alpha \operatorname{sgn}\left(S_\phi\right) - h_2 D^\alpha S_\phi. \tag{36}$$

The total control input is as follows:

$$U_2 = I_x\left[\ddot{\phi}_d - k_3 \dot{e}_{\phi 1} - \frac{1}{k_4} D^\alpha \dot{e}_{\phi 2} - \eta D^\alpha \operatorname{sgn}\left(S_\phi\right) - h_2 D^\alpha S_\phi \right] - f_\phi. \tag{37}$$

Stability proof:

Define the Lyapunov function as follows:

$$V_{\phi 2} = V_{\phi 1} + \frac{1}{2} S_\phi^2. \tag{38}$$

Taking the derivative of $V_{\phi 2}$ yields the following:

$$\dot{V}_{\phi 2} = e_{\phi 1}\left(e_{\phi 2} - k_3 e_{\phi 1}\right) + S_\phi \dot{S}_\phi. \tag{39}$$

Substituting Equations (26) and (34) can obtain the following:

$$\begin{aligned} \dot{V}_{\phi 2} = {} & S_\phi\left[\dot{e}_{\phi 2} + k_4 D^{-\alpha}\left(\frac{f_\phi}{I_x} + \frac{U_2}{I_x} - \ddot{\phi}_d + k_3 \dot{e}_{\phi 1} \right) \right] \\ & + e_{\phi 1}\left(e_{\phi 2} - k_3 e_{\phi 1}\right). \end{aligned} \tag{40}$$

Substituting Equation (37) can obtain the following:

$$\begin{aligned} \dot{V}_{\phi 2} = {} & -e_\phi^T F_\phi e_\phi - k_4 \eta \left|S_\phi\right| - h_2 k_4^3 \left(D^{-\alpha} e_{\phi 2}\right)^2 \\ & -2 h_2 k_4^2 e_{\phi 2} D^{-\alpha} e_{\phi 2}, \end{aligned} \tag{41}$$

where $e_\phi = \left[e_{\phi 1} e_{\phi 2} \right]^T$ and F_ϕ is a symmetric matrix with the following form:

$$F_\phi = \begin{bmatrix} k_3 & -0.5 \\ -0.5 & h_2 k_4 \end{bmatrix}. \tag{42}$$

By adjusting the values of h_2, k_3, and k_4 one can make $h_2 k_3 k_4 \geq 0.25$, thus ensuring that F_ϕ is a positive definite matrix. By Lemma 2, it can be shown that $-2 h_2 k_4^2 e_{\phi 2} D^{-\alpha} e_{\phi 2}$ is negative definite. Therefore$\dot{V}_{\phi 2} \leq 0$ and the system will converge asymptotically.

The design of the controllers for θ and ψ is similar to that of ϕ, so we can derive the controllers U_3 and U_4 in the same way.

Table 1: Parameters of the quadrotor UAV model.

Related parameters	Parameter value	Physical unit
m	2	kg
l	0.21	m
$I_x = I_y$	1.22	Ns^2/rad
I_z	2.2	Ns^2/rad
b	5	Ns^2
d	2	N/ms^2
g	9.8	m/s^2
J_r	0.2	Ns^2/rad

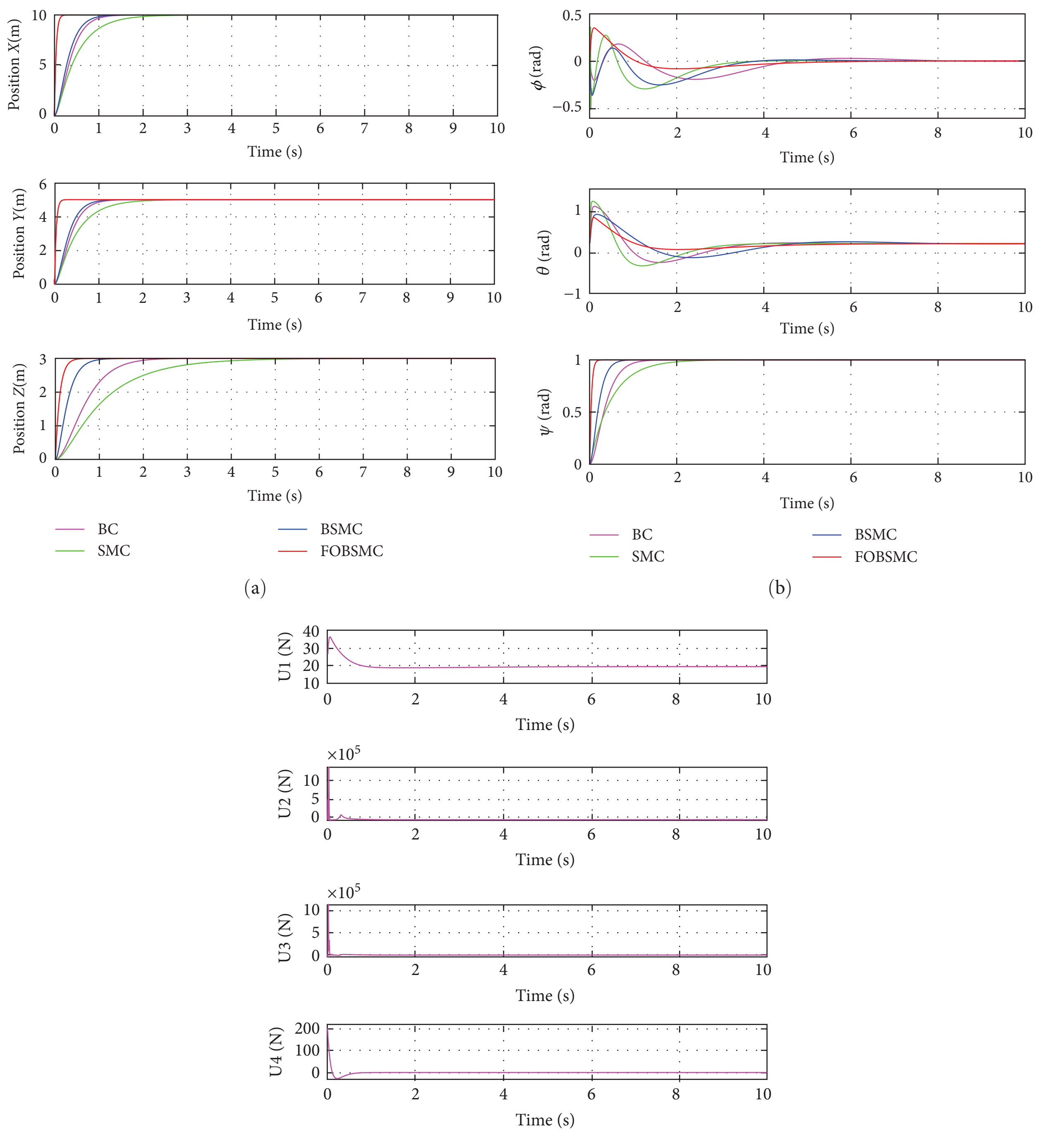

Figure 3: Simulation results of UAV spotting flights: (a) positional response curve; (b) Euler angle response curve; (c) control input signal.

Table 2: Performance indicators of different control algorithms.

Algorithm	Passage	IAE	ISE	Time t_s
BC	Position x	3.8451	2.4223	1.0139
	Position y	1.9226	0.6055	1.0136
	Altitude z	2.1432	0.4113	1.8251
	Yaw angle ψ	0.3846	0.0242	1.0137
SMC	Position x	5.3335	2.9783	1.6957
	Position y	2.6672	0.7445	1.6961
	Altitude z	3.5794	0.6110	3.7176
	Yaw angle ψ	0.5082	0.0258	1.7606
BSMC	Position x	3.3318	2.0825	0.8925
	Position y	1.6656	0.5205	0.8923
	Altitude z	0.8816	0.1622	0.8161
	Yaw angle ψ	0.2212	0.0138	0.5899
FOBSMC	Position x	**0.4321**	**0.2410**	**0.1350**
	Position y	**0.1754**	**0.0495**	**0.1349**
	Altitude z	**0.1051**	**0.0187**	**0.3495**
	Yaw angle ψ	**0.0432**	**0.0024**	**0.1460**

Bold values emphasize that the control effect of the controller designed is better than other controllers.

$$U_3 = I_y\left[\ddot{\theta}_d - k_5\dot{e}_{\theta 1} - \frac{1}{k_6}D^{\alpha}\dot{e}_{\theta 2} - \eta D^{\alpha}\mathrm{sgn}\left(S_\theta\right) - h_3 D^{\alpha} S_\theta\right] - f_\theta. \tag{43}$$

$$U_4 = I_z\left[\ddot{\psi}_d - k_7\dot{e}_{\psi 1} - \frac{1}{k_8}D^{\alpha}\dot{e}_{\psi 2} - \eta D^{\alpha}\mathrm{sgn}\left(S_\psi\right) - h_4 D^{\alpha} S_\psi\right] - f_\psi. \tag{44}$$

So far, the design of fractional-order-based backstepping sliding mode controller for quadrotor UAV is completed.

5. Simulation Results Analysis

In this section, two simulation examples are given to prove the performance of the proposed control scheme. In the experimental part, the system in Equation (1) is selected as the experimental object. MATLAB/Simulink simulation experiment platform is used to build the UAV simulation model based on fractional-order backstepping sliding mode, including the input module, translation dynamics module, rotation dynamics module, and controller module, and the performance is verified experimentally. The platform has powerful simulation and modeling capabilities to simulate and analyze complex control systems. In the simulation environment, the fractional-order toolbox FOMCON is selected for fractional-order partial calculation. The initial simulation conditions are $x = 0, y = 0, z = 0$. Table 1 lists the parameter values of the four-rotor UAV model in this study. The FOBSMC algorithm designed in this paper is compared with the traditional backward step algorithm, sliding mode algorithm, and simple backward step sliding mode control algorithm to prove the superiority of the proposed algorithm.

5.1. Fixed-Point Flight Simulation. In the fixed-point flight, the tracking target is set to $x_d = 10, y_d = 5, z_d = 3, \psi_d = 1$, comparing the step tracking effect of the four controllers, the simulation results of the UAV fixed-point flight are shown in Figure 3.

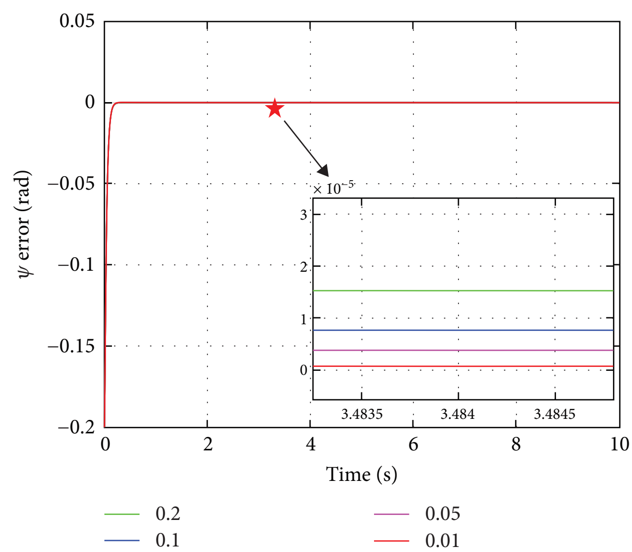

Figure 4: Yaw angle error curves of different fractional order operators.

The simulation results show that the output of the system reaches the desired level in finite time with smooth transient and steady-state responses. As shown in Figure 3(a), the algorithm proposed in this paper can significantly improve the convergence speed of trajectory tracking. The simulation results in Figure 3(b) show that compared with several other control algorithms, the fractional-order backstepping sliding mode control algorithm proposed in this paper can effectively reduce the overshooting phenomenon during the convergence process of pitch and roll angles.

Absolute error integral and square error integral are selected as the evaluation conditions for the tracking

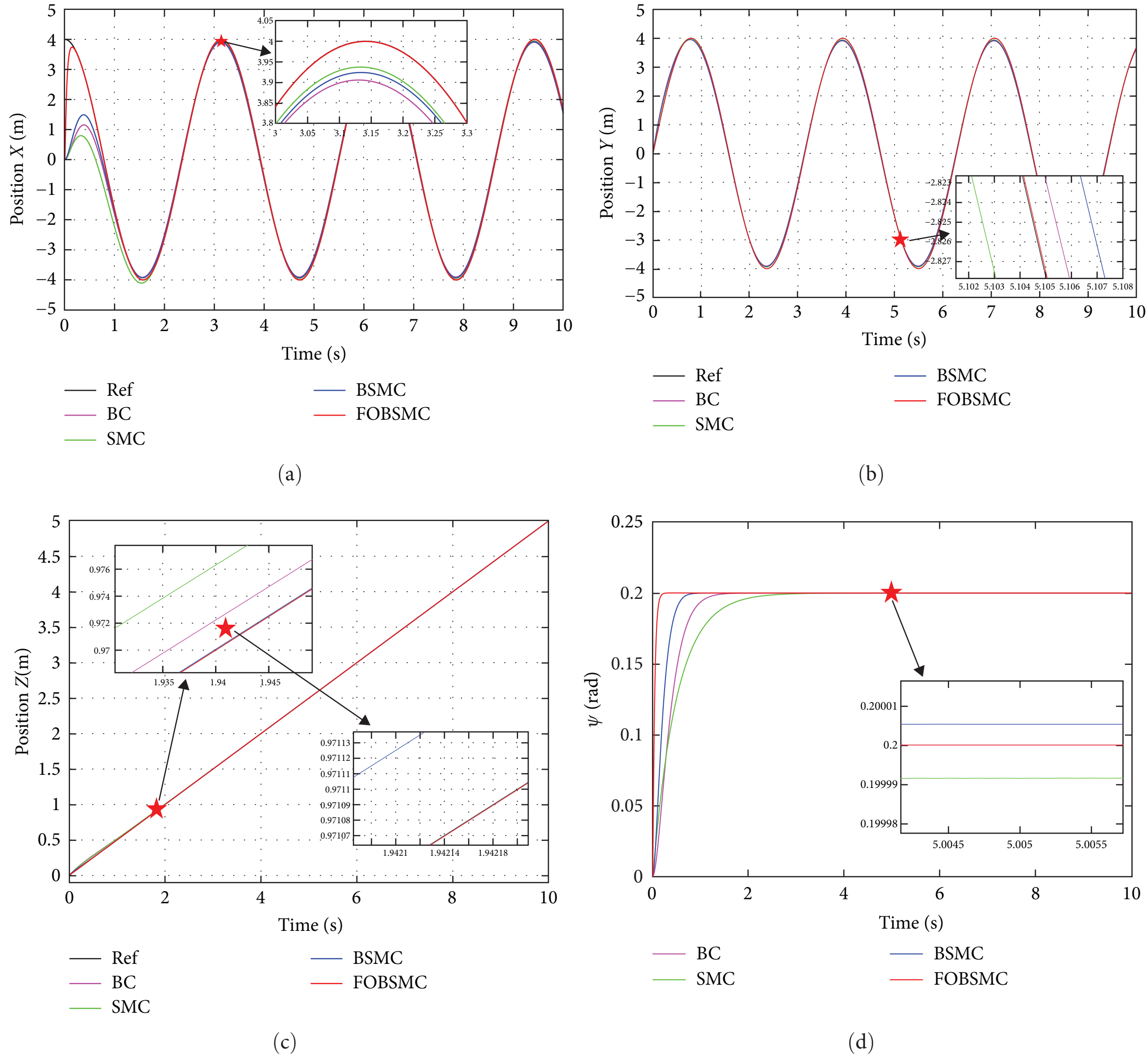

Figure 5: UAV spiral curve tracking simulation results: (a) position response curve in the X direction; (b) position response curve in the Y direction; (c) position response curve in the Z direction; (d) yaw angle response curve.

accuracy of the controller system, and the adjustment time of 3% error band is selected as the evaluation conditions for the convergence speed of the controller system. The absolute error integral and square error integral are defined as follows:

The integral of the absolute value of error (IAE):

$$I_{\mathrm{IAE}} = \int_0^{tf} |e(t)| dt. \tag{45}$$

The integral of squared value of error (ISE):

$$I_{\mathrm{ISE}} = \int_0^{tf} e^2(t) dt. \tag{46}$$

The changes of absolute error integral, square error integral, and adjustment time of different control algorithms are shown in Table 2.

As can be seen from Table 1, compared with other control algorithms, the fraction-order reverse-step sliding mode control algorithm designed in this paper has a smaller error integral and faster response speed.

5.2. Spiral Curve Tracking Simulation. Spiral ascent is a common flight mode of UAVs. This section assumes that the expected trajectory of quad-rotor UAVs is a spiral ascent curve, and the desired trajectory is designed as $x_d = 4\cos(2t), y_d = 4\sin(2t), z_d = 0.5t, \psi_d = 0.2$.

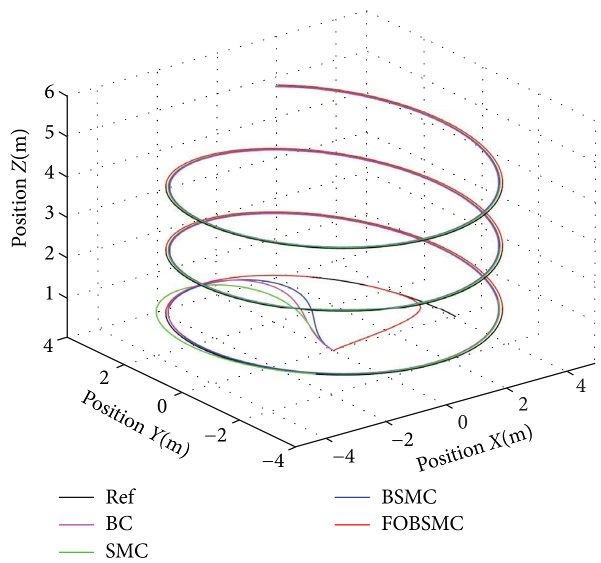

FIGURE 6: Spiral trajectory tracking diagram of a four-rotor UAV.

The choice of fractional order is the key factor affecting the controller effect, and the different order values will directly affect the control performance. In order to obtain the best fractional-order operator values, fractional-order operators 0.01, 0.05, 0.1, and 0.2 were selected, respectively, to compare yaw Angle errors, and the results are shown in Figure 4.

As can be seen from Figure 4, under the condition that other parameters are the same, the larger the value of the fractional order α, the larger the yaw angle error. Therefore, the fractional order of this paper is chosen as $\alpha = 0.01$.

The model parameters of the simulation experiment are the same as those of the fixed-point flight simulation. The simulation results are depicted in Figure 5.

As shown in Figure 5, compared with other control methods, the fractional-order reverse-step sliding mode controller designed increases the flexibility of control parameters, effectively improves the control accuracy and convergence speed of the system, and has a good trajectory tracking effect. Figure 6 shows the 3D trajectory tracking of the four-rotor UAV system under different controllers. It can be seen from the figure that the controller proposed in this paper can reach the desired trajectory in a shorter time when the tracking error is almost the same.

6. Conclusion

In order to solve the problems of slow convergence and low tracking accuracy of four-rotor UAV, a new fractional-order sliding mode controller is proposed by combining the fractional-order calculus theory based on Caputo and the reverse sliding mode control algorithm (BSMC). The proposed algorithm takes into account the dynamic constraints of UAV and the constraints in practical applications and can ensure the safety and operability of UAV flight. The backstepping sliding mode controller with fractional-order operator can not only improve the flexibility of the system but also reduce the chattering caused by the sliding mode control, so that the tracking trajectory of the system is smoother and the control effect is more accurate. Finally, the stability and effectiveness of the controller are verified by two methods. The Lyapunov stability theorem is used to provide the stability analysis of the complete system, and the simulation experiment is carried out by MATLAB/Simulink. The simulation results show that the proposed control algorithm has high trajectory tracking accuracy and fast response speed under the premise of satisfying the dynamic characteristics, which can make the UAV reach the target accurately and adapt to the changing flight conditions and task requirements quickly. In the future work, we plan to carry out real experiments to further verify the effectiveness and feasibility of the proposed method.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest. The funding details provided are only used to support research and will not create any conflicts of interest between authors.

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