

## Research Article

# Dynamic Modeling of Unmanned Underwater Vehicles with Online Disturbance Compensation Scheme

Sheikh Izzal Azid<sup>(1)</sup>,<sup>1,2</sup> Utkal Mehta<sup>(1)</sup>,<sup>3</sup> Adarsh Kuar,<sup>3</sup> and Zahid Ali<sup>3</sup>

<sup>1</sup>School of Engineering and Energy, Murdoch University, Perth, Australia <sup>2</sup>North Australia Centre for Autonomous Systems, Charles Darwin University, Darwin, Australia <sup>3</sup>Electrical and Electronics Engineering, University of the South Pacific, Laucala Campus, Suva, Fiji

Correspondence should be addressed to Sheikh Izzal Azid; sheikhizzal@gmail.com

Received 11 October 2023; Revised 7 December 2023; Accepted 29 February 2024; Published 14 March 2024

Academic Editor: Bingxiao Ding

Copyright © 2024 Sheikh Izzal Azid et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

With the advancement in robotics technology over the recent years, underwater robots' design and development are gaining interest. Unmanned underwater vehicles (UUVs) have many applications in aquaculture, deep-sea exploration, research, and enhanced rescue tasks. However, various factors must be considered when developing any underwater vehicle system to explore the deep ends of the underwater world. In this paper, we develop the most suitable model for understanding various system parameters. The new mathematical model considers certain constraints and external disturbances exerted on the system. Also, a control strategy is suggested for the UUV's stability and robustness. The suggested observer and model are simple, allowing for accurate estimations of all system states and the global impacts of unknown limped perturbations with a minimal computational cost.

### 1. Introduction

The robotics platform has made great strides in developing advanced technology over recent years, especially in underwater vehicles. Underwater vehicles can be classified into two categories, i.e., remotely operated vehicles (ROVs) and unmanned underwater vehicles (UUVs), commonly known as autonomous underwater vehicles (AUVs). The ROV is tethered to the buoy via cable and operated using a remote control. The UUV system operates independently in water (i.e., no direct human input is required). Science, the environment, the maritime industry, national defense, and underwater surveillance use AUVs. Autonomous underwater drones help humans explore the ocean. Thus, various control schemes are being carried out most recently to improve the performance of the traditional strategies [1]. The challenges with AUVs, in particular, operate in unfamiliar and dangerous habitats, and their research and implementation have been identified as one of human's primary aims and difficulties. As a result, it is essential to learn more about the AUV's role and context to respond correctly to any unanticipated circumstances [2].

In 1958, the US Navy built the first UUV. Then, this technology was used to find offshore oil and gas reserves in

the North Sea. This was the first time the UUV was used for business. The discovery was made with a remotely operated vehicle or ROV. ROVs are still used in the offshore industry, but autonomous vehicles are becoming increasingly popular [3]. In recent years, there have been significant advances in advanced robotic systems, particularly in the evolution of unmanned systems. On the other hand, the ocean still has a lot of uncharted territory. An underwater vehicle must travel with steadiness in all degrees of freedom in the underwater environment. Some mechanical, electrical, and software criteria must be met to achieve robust movement.

Even while research into AUVs has made great strides in recent years, some AUVs, such as those used to dock with other underwater objects, precise target tracking, or continuous long-distance navigation, still lack adequate mobility and precision control [4]. These vehicles might not accomplish the required jobs effectively. However, UUVs have limitations due to the environments in which they have been deployed [5]. To this end, numerous UAVs with fixed-wing, multirotor, and flapping wings and those with other architectural configurations and mathematical description methods have been developed and researched. Multirotor vehicles benefit from a more reliable and effective control system than conventional aircraft. Therefore, multirotor vehicles significantly benefit in vertical take-off and landing (VTOL), hovering accuracy, and evasive maneuverability [2].

The literature has shown that the quadrotor system is the most promising option for UUVs. Most researchers modeled the UUV as a quadrotor with four independent thrusters or propellers. The rotation of one set of propellers is counterclockwise to the rotation of the other set. The UUV's maneuverability and stability come from the fact that the vehicle's motion is controlled solely by the rotor speeds and the force created by the rotors. The use of an open structure (low extra masses), a high payload/volume ratio, and excellent maneuverability are all bonuses of this portrayal [6].

Over the years, experts have developed various control mechanisms to bend the behavior of any system to meet the intended specifications. Proportional–integral–derivative (PID) control is popular among the many available control methods due to its straightforward design and valuable practical applications. The PID was proposed for attitude stabilization and orientation control in [7, 8]. Since the PID values serve as valuable input to the system, calibration of these parameters is essential. The PID control approach is suitable for autonomous vehicles since applying and altering the gain parameters is simple. It is not usually feasible to directly implement the PID control for the UUVs since the quadrotors are a nonlinear, underactuated system [9]. The performance can be improved for drones using a fractional-order controlling scheme [10], recently shown better results [11].

An advanced method, such as a sliding mode controller (SMC), can control nonlinear quadrotor UUVs [2]. It is beneficial, particularly for external forces acting on the system. The SMC technique directs the system states toward the sliding surface, which employs a well-designed law to keep the system states on the body. However, the system must be developed and deployed to mitigate the associated risks. Another improved method with dynamic SMC was proposed to improve robustness overactuated autonomous underwater when exposed to under ocean current and model uncertainties [12, 13]. Other strategies, such as fuzzy logic (FL) [14] and the model predictive controller (MPC) [15], have produced noteworthy outcomes. In [14], FL is integrated with the Kalman filter to assess the position of AUVs and to drive them using a specified route. It is also capable of handling qualitative and unclear decisionmaking issues. During operation, the system was able to calculate its position precisely. Shen et al. [16] developed an MPC controller for the dynamic positioning of the UUV based on the Lyapunov theory. The MPC inherits the characteristics of Lyapunov controllers, such as stability, which makes this scheme desirable for the dynamic placement of UUVs. The result presented in the literature shows better robustness against external shocks and parametric uncertainties. Due to its ability to operate with disturbances and limitations, behavior prediction, tuning simplicity, and advanced performance for systems with numerous variables, the MPC has made significant strides in controller design [9]. Utilizing the plant model to anticipate the future output behavior of the plant is the fundamental aspect of an MPC. Since the UUV system is very nonlinear, the linearization-acquired stability is often impractically ineffective. It is to note [17] that the MPC and FL have been determined as more suitable controllers in a nonlinear system.

It is crucial that the reference model be determined. Recent work in [18] has presented an adaptive approach for dealing with structured and unstructured uncertainties. The intricate design of the compensator necessitated a tracking controller with a high degree of accuracy. As is very likely the case with UUVs, one of the enduring problems with such a controller is that its performance immediately diminishes with diverse physical systems. In fact, due to the complex and changing environment underwater and various potential risks, the variability of the structural parameters and environmental parameters of the underwater vehicle [19], very recently discussed the control of UUV in fixed depth motion in water.

Despite numerous control schemes that has been proposed for underwater vehicles in the literature, designing and implementing autonomous control algorithms for these vehicles to perform real-time marine tasks remains an open challenge. This is due to several factors, these includes internal perturbations, time-varying external disturbances such as ocean current effects, inherent parametric uncertainties in UUVs, unmodeled dynamics, and the unpredictable nature of the underwater environment [13]. These challenges become even more complex when dealing with a fleet of low-cost UUVs [20]. Due to the highly nonlinearity model of UUVs and it being exposed to several disturbances and uncertainties as discussed before, this article proposes a novel method for estimating and compensating for all disturbances at once, regardless of their source or nature, including but not limited to internal perturbations, timevarying external disturbances, inherent parametric uncertainties in UUVs, unmodeled dynamics, and the unpredictable nature of the underwater environment. More specifically:

- A low computation demand is acquired by the simplicity of the proposed modeling and observer, which requires no additional sensors and robustly estimates all the states of the system and the overall effects of the unknown lumped disturbances;
- (2) It is worth mentioning that conventional robust control methods employ intricate controllers and may lack the agility to respond promptly to significant disturbances. Also, these controllers need expensive sensors and real-time information fed to the controller to compensate for the lumped disturbance [20]; in contrast, this article's approach compensates for the exact amount of disturbance which is precisely estimated online as presented in the results section;
- (3) By being independent of the type of controller used to determine the rotor speeds of the UUVs, the developed scheme can provide existing controllers with the additional capacity to better deal with disturbances. This fact is shown in the paper in simulation, which has excellent results when exposed to different types of disturbances with a simple MPC controller. Moreover, this add-on feature can be utilized by any UUV provided the controller has access to rotor speeds.



FIGURE 1: Unmanned underwater vehicle mechanical structure and reference frames.

The rest of this paper is arranged as follows: Dynamic modeling of the UUV is introduced to depict the positions and velocity of the vehicle. Following this section, the proposed control scheme with UUV model is presented with an observer design. The following section discusses simulation results, whereas two scenarios with real-time challenges validate the proposed scheme. Conclusions are presented in the last section.

## 2. Dynamic Modeling of Unmanned Underwater Vehicle (UUV)

To mathematically model the UUV, it is essential to identify the vehicle's reference frame, which will be utilized to depict the equations for the positions and velocities of the vehicle. This is shown in Figure 1. The position (x, y, z) of a UUV center of mass is measured by underwater global positioning system (UGPS) sensors, which provide data in the inertial earth frame  $F_0$ , while the orientation  $(\phi, \theta, \psi)$  is measured by inertial measurement units (IMU) in a body frame  $F_B$ . The state vector of a UUV consists of  $(x, y, z, \phi, \theta, \psi)$  and (u, v, w, p, q, r) variables representing the vehicle's position and velocity, respectively.

The Euler angle describes the vehicle's orientation in 3D Euclidean space. Among many possibilities, the ZXY convention aligns the axes of  $F_0$  and  $F_B$ . The orientation of the aircraft is determined by first rotating about the *z*-axis of  $\psi$  (yaw) radians, then about the *x*-axis of  $\phi$  (roll) radians, and finally about the *y*-axis of  $\theta$  (pitch) radians. Consequently, considering the fundamental rotations as,

$$R_{x}(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\phi} & s_{\phi} \\ 0 & -s_{\phi} & c_{\phi} \end{pmatrix}, R_{y}(\theta) = \begin{pmatrix} c_{\theta} & 0 & -s_{\theta} \\ 0 & 1 & 0 \\ s_{\theta} & 0 & c_{\theta} \end{pmatrix},$$
$$R_{z}(\psi) = \begin{pmatrix} c_{\psi} & s_{\psi} & 0 \\ -s_{\psi} & c_{\psi} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
(1)

where  $s_* = \sin(*)$  and  $c_* = \cos(*)$ , the complete rotation converting body-frame coordinates into inertial ones is:

$$R_{zxy} = \left(R_z(\psi)R_x(\phi)R_y(\theta)\right)^\top = \\ = \begin{pmatrix} c_\theta c_\psi - s_\phi s_\theta s_\psi & -c_\phi s_\psi & s_\theta c_\psi + s_\phi c_\theta s_\psi \\ c_\theta s_\psi + s_\phi s_\theta c_\psi & c_\phi c_\psi & s_\theta s_\psi - s_\phi c_\theta c_\psi \\ -c_\phi s_\theta & s_\theta & c_\phi c_\theta \end{pmatrix}.$$

$$(2)$$

Furthermore, the four rotors provide orthogonal force to the rotation plane of their blades in the system, generating thrust forces  $F_i$  in the negative *z*-axis direction relative to the body frame to support the systems' weight and produce further maneuvering. The propellers of the UUV rotate at an angular velocity  $\Omega_i$  where the subscript *i* represents the rotors 1, 2, 3, and 4. Moreover, due to the rotation, each *i*<sup>th</sup> rotor produces torque ( $M_i$ ) along the orthogonal axis, which can be expressed as;

$$F_i = K_f \Omega_i^2$$
  

$$M_i = K_m \Omega_i^2 , \qquad (3)$$

where  $K_f$  and  $K_m$  are the force constant and torque constant, respectively. The inputs for the UUV depend on the magnitudes of the rotor's angular velocities, which are the combinations of the force and torques generated around an axis in the body-fixed frame. Thus, the input vector is represented by  $U = [U_1U_2U_3U_4]$ . Where  $U_1$  is the overall thrust force  $F_z$ and  $(U_2, U_3, U_4)$  are the components of the torque vector  $(\tau_{\phi}, \tau_{\theta}, \tau_{\psi})$  are linearly coupled with the squares of rotor speeds. All such quantities are grouped in the state and input vectors.

$$U = \begin{bmatrix} F_z \\ \tau_{\phi} \\ \tau_{\theta} \\ \tau_{\psi} \end{bmatrix} = \begin{bmatrix} 2\sqrt{\frac{K_f g}{m}}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ \frac{1}{J_{xx}}\Sigma M_{xw}^b \\ \frac{1}{J_{yy}}\Sigma M_{yw}^b \\ \frac{1}{J_{zz}}\Sigma M_{zw}^b \end{bmatrix}.$$
(4)

Considering the theorem of momentum and using the second law of motion, the dynamic model of the UUV in the body frame can be expressed as;

$$F_b = m\dot{v}_b + m\omega_b \times v_b M_b = J_w \dot{\omega}_b + \omega_b \times J_w \omega_b , \qquad (5)$$

where,  $F_b$  is the resultant force,  $M_b$  is the moment that acts on the UUV, and M is the vehicle's mass. Moreover,  $v_b$  is the linear velocity, and  $\omega_b$  is the angular velocity of the vehicle's body. The UUV model is supposed to be symmetric about each coordinate system. Hence, the inertia around the *x*, *y* and *z* axes can be represented by  $J_w = \text{diag}[J_{xx}J_{yy}J_{zz}]$ . Furthermore, the angular velocity  $(p, q, r)^T$  of the UUV in the body frame is related to the Euler angles via the dynamic relation that depicts the ZXY conventions as:

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + R_{y}(\theta) \begin{pmatrix} \dot{\phi} \\ 0 \\ 0 \end{pmatrix} + R_{y}(\theta)R_{x}(\phi) \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix},$$
(6)

which can be simplified as;

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} c_{\theta} & 0 & -s_{\theta} \\ 0 & 1 & 0 \\ s_{\theta} & 0 & c_{\phi}c_{\theta} \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}.$$
 (7)

Hence, Equation (7) can be rewritten as;

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} pc_{\theta} + rs_{\theta} \\ pt_{\phi}s_{\theta} + q - rt_{\phi}c_{\theta} \\ -p\frac{s_{\theta}}{c\phi} + r\frac{c_{\theta}}{c\phi} \end{bmatrix}.$$
 (8)

Various factors need to be considered for the UUV to successfully navigate and maneuver underwater. As stated in Equation (5), two important factors, which are forces and moments acting on the vehicle's body, must be considered while modeling any underwater vehicle. Moreover, to determine the net force that acts upon the UUV while maneuvering underwater, the thrust of the four propellers, gravity, and the vehicle's buoyancy in the water are considered. The standard principle to express force in the inertial coordinate system is that gravity is always opposite the *z*-axis. On the contrary, the buoyancy acts upwards in the positive *z* direction. Therefore, the following is obtained:

$$\begin{bmatrix} F_g^b F_b^b & \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -mg & \rho_w gV \end{bmatrix},$$
(9)

where  $F_g^b$  is the gravitational force and  $F_b^b$  is buoyancy force. While  $\rho_w$  is the density of water, *V* is the volume of the UUV, and *g* is gravity. Indicating with *m* the aircraft mass and Newton's equations for the translational motion of the center of mass reads,

$$m\begin{bmatrix} \ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = m\begin{bmatrix} 0\\ 0\\ -g\end{bmatrix} + R_{zxy}\begin{bmatrix} 0\\ 0\\ F\end{bmatrix} + \begin{bmatrix} 0\\ 0\\ \rho_w gV\end{bmatrix}.$$
(10)

Equation (10) can be further expanded as;

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} \left( \frac{F}{m} \right) \left( s_{\theta} c_{\phi} + s_{\theta} c_{\theta} s_{\psi} \right) \\ \left( \frac{F}{m} \right) \left( s_{\theta} c_{\phi} - s_{\theta} c_{\theta} s_{\psi} \right) \\ \left( \frac{F}{m} \right) \left( c_{\phi} c_{\theta} \right) + g + \left( \frac{\rho_{w} g V}{m} \right) \end{bmatrix}.$$
(11)

Since the origin of the body coordinate frame is located at the center of gravity of UUV, gravity does not generate a moment. To add on, the buoyancy does generate moments, and the buoyancy center coordinates in the inertial frame are denoted as  $[x_f y_f z_f]^T$ , it is considered to be acting in the positive *z*-axis of the body frame. Thus, the buoyancy moment is expressed as;

$$M_b^b = -F_b^b \begin{bmatrix} z_f s_\phi c_\theta - y_f c_\phi c_\theta \\ x_f c_\phi c_\theta + z_f s_\theta \\ -y_f s_\theta - x_f s_\phi c_\theta \end{bmatrix}.$$
 (12)

The gyro moment of the vehicle is produced when there is a change in the attitude of the UUV and can be expressed as;

$$M_{\rm gyro}^b = - \begin{bmatrix} -J_{rw}q(\Omega_1 - \Omega_2 + \Omega_3 - \Omega_4) \\ J_{rw}p(\Omega_1 - \Omega_2 + \Omega_3 - \Omega_4) \\ 0 \end{bmatrix},$$
(13)

where  $J_{rw}$  is the moment of inertia of the rotors. Thus adding up, the total external moment of the UUV is expressed as;

$$M_{w}^{b} = M_{b}^{b} + M_{gyro}^{b} = \begin{bmatrix} -F_{b}^{b}(z_{f}s_{\phi}c_{\theta} - y_{f}c_{\phi}c_{\theta}) - J_{rw}q(\Omega_{1} - \Omega_{2} + \Omega_{3} - \Omega_{4}) \\ -F_{b}(x_{f}c_{\phi}c_{\theta} + z_{f}s_{\theta}) + J_{rw}p(\Omega_{1} - \Omega_{2} + \Omega_{3} - \Omega_{4}) \\ -F_{b}(-y_{f}s_{\theta} - x_{f}s_{\phi}c_{\theta}) \end{bmatrix}.$$

$$(14)$$

Summing up, the dynamic model of the UUV can be obtained using with inertia matrix (i.e.,  $J_w$ ) along with the equations derived for the external force and moments of the vehicle. Thus, the nonlinear dynamic mathematical model of the UUV can be represented as,  $\dot{\zeta} = [x, y, z, \phi, \theta, \psi, u, v, w, p, q, r]^T \in \mathbb{R}^{12}$ .

$$\begin{aligned} \dot{x} &= u \\ \dot{y} &= v \\ \dot{z} &= w \\ \dot{\phi} &= p c_{\theta} + r s_{\theta} \\ \dot{\theta} &= \frac{s_{\phi}}{c_{\phi}} s_{\theta} p + q - \frac{s_{\phi}}{c_{\phi}} c_{\theta} r \\ \dot{\theta} &= \frac{s_{\theta}}{c_{\phi}} s_{\theta} p + q - \frac{s_{\phi}}{c_{\phi}} c_{\theta} r \\ \dot{\psi} &= -\frac{s_{\theta}}{c_{\phi}} p + \frac{c_{\theta}}{c_{\phi}} r \\ \dot{\psi} &= (s_{\theta} c_{\psi} + s_{\phi} c_{\theta} s_{\psi}) \frac{F}{m} \\ \dot{v} &= (s_{\theta} c_{\psi} - s_{\phi} c_{\theta} s_{\psi}) \frac{F}{m} \\ \dot{w} &= c_{\phi} c_{\theta} \frac{F}{m} + g + \left(\frac{\rho_{w} g V}{m}\right) \\ \dot{\psi} &= -\frac{J_{zz} - J_{yy}}{J_{xx}} q r + \frac{1}{J_{xx}} \Sigma M_{xw}^{b} \\ \dot{q} &= -\frac{J_{xx} - J_{xx}}{J_{yy}} p r + \frac{1}{J_{yy}} \Sigma M_{yw}^{b} \\ \dot{r} &= -\frac{J_{yy} - J_{zz}}{J_{zz}} p q + \frac{1}{J_{zz}} \Sigma M_{zw}^{b} \end{aligned}$$
(15)

In the following section, using the proposed control strategy, Equation (15) is used for precision and robust control in the presence of disturbances, internal perturbations, and the unpredictable nature of the underwater environment.

## 3. Control Strategy

The nonlinear dynamic model discussed in the earlier section shows that it is purely an integrating system and highly complex. Moreover, the UUV is underactuated since the control inputs are the four rotors while its outputs are the positions  $x, y, z, \phi, \theta, \psi$ . Therefore, roll  $\phi$  and pitch  $\theta$  angles are kept zero. Thus, the UUV would have four inputs which are  $U = [F_z, \tau_{\phi}, \tau_{\theta}, \tau_{\psi}]$  from Equation (4) and four outputs  $Y = [x, y, z, \psi]$  enabling the UUV having four degrees of freedom. Now, in the following subsections, we design an observer with a proposed model, considering no additional sensors however, can robustly estimate all the states of the system and the minimize effects of the unknown limped disturbances.

3.1. Linearization of the UUV. Firstly, Equation (15) is linearized around as equilibrium point  $(\phi, \theta \approx 0)$  so that the UUV model can be represented in a fully controllable form. The small angle approximation uses  $\sin(\text{angle}) \approx 0$  and  $\cos(\text{angle}) \approx 1$ , using Taylor series expansion to the new state space representation. Hence, the linearization is written in the following form.

$$\dot{\xi}(t) = A\xi(t) + Bu(t) + Bdf(\xi(t), d(t)), \qquad (16)$$
  
$$\eta(t) = C\xi(t)$$

where  $\xi \in \mathbb{R}^n$  is the state vector,  $u \in \mathbb{R}^m$  is a known input vector,  $w \in \mathbb{R}^k$  is the vector of the unknown input, and

 $\eta \in \mathbb{R}^p$  is an output vector. Moreover, *A* is the state matrix, *C* is the output matrix, *Bd* is the disturbance matrix of suitable sizes, and  $g: \mathbb{R}^{m+p} \to \mathbb{R}^n$  and  $f: \mathbb{R}^n \to \mathbb{R}^n$  are nonlinear functions. Then, the nonlinear quadrotor model Equation (15) can be written as by defining:

$$A = \begin{bmatrix} 0_{(3\times3)} & 0_{(3\times3)} & I_{(3\times3)} & 0_{(3\times3)} \\ 0_{(3\times3)} & 0_{(3\times3)} & 0_{(3\times3)} & I_{(3\times3)} \\ & \begin{pmatrix} g.s(\phi) & g.c(\phi) & 0 \\ -g.c(\phi) & g.s(\phi) & 0 \\ 0 & 0 & 0 \end{pmatrix} & 0_{(3\times3)} & 0_{(3\times3)} \\ 0_{(3\times3)} & 0_{(3\times3)} & 0_{(3\times3)} & 0_{(3\times3)} \end{bmatrix},$$
(17)

$$B = \begin{bmatrix} 0_{8\times4} & & \\ \frac{K_f}{m} & \frac{K_f}{m} & \frac{K_f}{m} & \frac{K_f}{m} \\ 0 & \frac{J_{rw} - \sqrt{\frac{LK_f}{2}}}{J_{xx}} & 0 & \frac{-J_{rw} - \sqrt{\frac{LK_f}{2}}}{J_{xx}} \\ \frac{J_{rw} - \sqrt{\frac{LK_f}{2}}}{J_{yy}} & 0 & \frac{J_{rw} - \sqrt{\frac{LK_f}{2}}}{J_{yy}} & 0 \\ \frac{K_m}{J_{zz}} & \frac{K_m}{J_{zz}} & \frac{K_m}{J_{zz}} & \frac{K_m}{J_{zz}} \end{bmatrix},$$
(18)

$$Bd = \begin{bmatrix} 0_{(4\times4)} \\ 0_{(4\times4)} \\ I_{(4\times4)} \end{bmatrix},$$
 (19)

where all the external moments and other unmodelled dynamics are compiled as disturbances, that is,  $Bd_1 = (\rho g V)/m$ ,  $Bd_2 = \frac{1}{J_{xw}} \Sigma M_{xw}^b + d_1$ ,  $Bd_3 = \frac{1}{J_{yy}} \Sigma M_{yw}^b + d_2$ , and  $Bd_4 = \frac{1}{J_{zz}} + + d_3 \Sigma M_{zw}^{b}$ . Also, it is noted that we can only measure six outputs which are  $x, y, z, \phi, \theta, \psi$ , the matrix *C* becomes  $[I_{(6\times 6)}0_{(6\times 6)}]$ .

3.2. Observer Design. From Equation (16), function  $f(\xi, d(t), t)$  denotes the lumped disturbance, which includes external moments, unmodelled dynamics, other external disturbances, parameter variations, friction, and complex nonlinear dynamics, which may be difficult for the modeling and controller to deal with. Hence, all of this can be estimated using an observer and then fed back to any chosen controller for compensation. The observer would all the lump highly unknown disturbances and the parameters that are difficult to the state matrix A, which then becomes  $\overline{A}$ . The observer will be designed as follows.

Firstly, an extended variable is added to the states, which is:

$$\xi_{(n+1)} = d = f(\xi(t), d(t)).$$
(20)

Then, the extended state space can be obtained as;

$$\dot{\xi}(t) = \bar{A}\xi(t) + \bar{B}u(t) + Eh(t) \quad y = \bar{C}\xi(t), \tag{21}$$

where the extended state variables are now,  $\bar{\xi} = \begin{bmatrix} \xi \\ \xi_{n+1} \end{bmatrix}$  and  $h(t) = \frac{f(\xi(t), d(t))}{dt}$  and the matrices.

$$\bar{A} = \begin{bmatrix} A_{n \times n} & D_{n \times k} \\ 0_{k \times n} & 0_{d \times k} \end{bmatrix} \widehat{B} = \begin{bmatrix} B \\ 0_{k \times m} \end{bmatrix} E = \begin{bmatrix} 0_{n \times 1} \\ 1_{k \times 1} \end{bmatrix} \bar{C} = \begin{bmatrix} C_{p \times n} 0_{p \times k} \end{bmatrix}.$$
(22)

Moreover, the following assumptions are required.

- (1) The controllable matrix for the pair A, B and the observability matrix for the pair  $A, \bar{C}$  are fully ranked;
- (2)  $D.D^T$  should be invertible;
- (3) The lumped disturbances satisfy the following conditions d(t) = f(x, d(t), t) is bounded and a constant value in steady state  $\lim_{t\to\infty} d(t) = \lim_{t\to\infty} h(t) = 0$ .

Finally, the observer is designed as follows:

$$\hat{\hat{\xi}} = \bar{A}\hat{\xi}(t) + \bar{B}\,u(t) + L(y-\hat{y})\eta = \bar{C}(\xi)\,,\tag{23}$$

where  $\widehat{\xi} = \begin{bmatrix} \widehat{\xi}, \widehat{\xi_{n+1}} \end{bmatrix}^T$  and *L* is  $n \times m$  gain matrix for the observer. The observer state equations are seen to model the actual state equation, with the true state  $\xi(t)$  replaced by the estimate  $\widehat{\xi}$ , and a correction term which is the difference between the actual measured output  $\eta(t)$  and its estimate  $\widehat{\eta}(t)$ . The output in the second equation is also seen to be a model of the system's output equation, with  $\xi(t)$  replaced by its estimate.

The estimation errors of state and disturbance are defined as;

$$e_x = \hat{\xi} - \xi e_d = \hat{d} - d$$
(24)

where  $\hat{d} = \hat{\xi}_{n+1}$  represents the estimate of system uncertainties. Hence combining Equations (21), (23), and (24), the estimation error is governed by:

$$\dot{e} = A_e e - Eh(t), \tag{25}$$

where  $e = \begin{bmatrix} e_x \\ e_d \end{bmatrix}$  and  $A_e = \widehat{A} - L\widehat{C}$ . The boundedness stability of the observer has been concluded by the following lemma.

**Lemma 1.** Assuming that the observer gain vector L in is chosen such that  $A_e$  is a Hurwitz matrix, then the observer error, e for the observer is bounded for any bounded h(t).

**Lemma 2.** A system is a linear system  $\dot{\xi} = A\xi + Bu$  is asymptotically stable if A is a Hurwitz matrix, u is bounded and satisfies  $\lim_{t\to 0} u(t) = 0$ .

Hence, this method will lump all disturbances, including buoyancy, external moment, and other parameter variations, and can be compensated by any controller.

3.3. Controller Design. The well-adopted control scheme, namely, the model predictive controller, is chosen to verify the new dynamics model of the UUV. This is because the MPC works with nonlinear MIMO systems and is also a favored control strategy due to its versatility in handling complex systems. The MPC employs a predictive approach by using a model of the system to forecast future behavior. This prediction-based methodology allows MPC to proactively plan optimal control actions over a defined time horizon, considering various constraints, objectives, and uncertainties. This capability is particularly advantageous in process control, robotics, and autonomous systems, where intricate dynamics, multivariable interactions, and stringent constraints are common. By incorporating optimization techniques, MPC excels at balancing multiple objectives, ensuring stability, and adapting to changing conditions. Matlab Toolbox [21] is utilized in the process of developing the controller for the proposed idea. It is a powerful tool for real-time decision-making in dynamic and challenging environments. In our example, the predictive controller minimizes the cost function for optimal control, as shown below.

$$J = \int_{t_0}^{t_f} [\xi(t)^T Q \xi(t) + u(t)^T R u(t)] dt, \qquad (26)$$

where matrices R and Q are tuned control variables.

To prove the stability consider Equations (18) and (19) and  $u(t) = u_{\text{MPC}}(t|x(t))$  and  $d(t) = u_{\text{obs}}(t|x(t))$ . To show that the system is stable when controlled by an MPC strategy, the Lyapunov function is derived as follows:

$$V(x) = x^T P x, (27)$$

where *P* is chosen as a positive definite matrix. Computing the derivative of V(x), we get:

$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x}.$$
(28)

After applying the MPC control law,

$$\dot{V}(x) = (Ax + Bu + B_d d)^T P x + x^T P (Ax + Bu + B_d d).$$
(29)

This can be further simplified to:

$$\dot{V}(x) = x^T A^T P x + x^T B^T u + x^T B_d^T d + x^T P A x + x^T P B u + x^T P B_d d$$
(30)

#### Journal of Robotics

Parameter	Symbol	Value	Unit
Mass	т	1.12	kg
Arm length	l	0.141	m
Gravity	9	1.2	$m/s^2$
Inertia in X	J <sub>xx</sub>	$3.8 \times 10^{-2}$	kg/m <sup>2</sup>
Inertia in Y	$J_{\nu\nu}$	$4.6 \times 10^{-2}$	$kg/m^2$
Inertia in Z	$J_{zz}$	$9.8 \times 10^{-2}$	$kg/m^2$
Inertia of the rotors	$J_{rw}$	$8.6 \times 10^{-4}$	$kg/m^2$
Propeller drag constant	$J_{rw}$	$8.6 \times 10^{-2}$	Nm/rad <sup>2</sup>
Propeller thrust constant	$J_{rw}$	$1.6 \times 10^{-2}$	Nm/rad <sup>2</sup>

TABLE 1: Unmanned underwater parameters.



FIGURE 2: Proposed control scheme with UUV model.

Substituting the MPC control and observer estimation gives:

$$\dot{V}(x) = x^T A^T P x + x^T B^T u_{\text{MPC}} + x^T B_d^T u_{\text{obs}} + x^T P A x + x^T P B u_{\text{MPC}} + x^T P B_d u_{\text{obs}}$$
(31)

Factorizing the above gives:

$$\dot{V}(x) = x^T (A^T P + PA)x + x^T (B^T - PB)(u_{\text{MPC}} + u_{\text{obs}}).$$
(32)

For stability validation,  $\dot{V}(x)$  should be negative definite; therefore,  $A^TP + PA$  should be negative definite and  $B^T - PB$  should be chosen appropriately.

## 4. Results and Discussion

The UUV is modeled and simulated in a 3D environment using MATLAB Simulink 2023b. The computer requirement for the simulation is 16 GB RAM with more than 1.80 GHz processing speed. This virtual environment provides an authentic representation of the UUV's behavior, enabling users to introduce time-varying and ramp disturbances to the UUV using sinusoidal and ramp functions. Utilizing the mathematical model presented in the earlier section, the simulation outputs linear and angular positions, as well as velocities. These outputs are then fed into the MATLAB MPC controller toolbox and the observer discussed in the earlier section to compute the correct rotor speed for compensation. The parameters of the nonlinear UUV are detailed in Table 1. We have referred to some recent articles to choose the simulation parameters [2, 22].

The derived mathematical model of the UUV together with the disturbance estimation and compensation scheme have been implemented as per Figure 2. The system parameters are the same as discussed in Table 1. The control system has been tested under two different scenarios:

- (1) Linear trajectory with time-varying disturbances;
- (2) Spiral trajectory with ramp-type disturbances.

It is verified and observed through simulation, that stepor ramp-type disturbances are comparatively simpler to detect and the system can cancel time-varying disturbances.



FIGURE 3: UUV results with Scenario 1 for time-varying disturbances, where  $x_d$ ,  $y_d$  and  $z_d$  is the reference, and x, y and z are the results from proposed method.

4.1. Scenario 1: Straight Trajectory with Time-Varying Disturbances. Firstly, the proposed UUV model, the estimation technique where part of the model is taken as lumped disturbances with the MPC controller is used to verify for linear path where the UUV. The reference linear positions are 1, 2 and -3 m to in  $x_d, y_d, z_d$  directions, respectively. Figure 3 shows the effectiveness of the proposed estimation technique and control methods to the model. As the observer reconstructs the lumped disturbance component and provides the signal values to the controller for compensation, the UUV quickly reaches the desired position. The controller commands the rotor speeds via a signal ensuring that the UUV receives the correct rotor velocities, leading to desired x, y, x directions. The proposed method is compared to MPC

controller only with no state estimation, which means that the output from the UUV are from UGPS and IMU which are positions only. The results show the proposed method outperforms the MPC controller with time varying disturbances. The rotor speed plot in Figure 3 shows how the controller compensates for the disturbances by varying the rotor speeds. This shows that the controller can easily compensate for the effect of disturbances to maintain the precise position of the UUV. It is noted that the estimation of the partial states with unknown parameters as well as disturbances are done online and compensated directly by the controller, this requires no additional sensors, water currents, model uncertainty, or other disturbance measurement devices and it precisely controls the UUV in water.



FIGURE 4: UUV results with Scenario 2 for ramp-type disturbances and spiral trajectories, where  $x_d$ ,  $y_d$  and  $z_d$  is the reference, and x, y and z are the results from proposed method.

4.2. Scenario 2: Helical Trajectory with Time-Varying Disturbances. The subsequent phase involves initiating a helical trajectory for the UUV while subjecting it to higher levels of disturbance. It becomes evident that the UUV exhibits seamless navigation, successfully reaching its intended destination, driven by the pursuit of precision and the effective mitigation of disturbances, achieved through the manipulation of rotor speeds. The UUV's performance is thoughtfully depicted in Figure 4.

In order to comprehensively evaluate the proposed methodology, a rigorous testing regimen is employed. Four distinct disturbances, each in the form of a ramp, are input into the system, with the highest values reaching magnitudes of 1N, 2, 3, and 4rad/ $s^2$ . The proposed method adeptly computes precise estimates for all relevant states seamlessly integrated into MPC. The observer, in turn, calculates the necessary

adjustments to the rotor speeds of the UUV. Given the ramp-like nature of these disturbances, the rotor speeds continuously adapt over time to counteract these perturbations and guide the vehicle toward its predefined positions with exceptional accuracy.

#### 5. Conclusions

In conclusion, this paper presents a mathematical model of an unmanned underwater vehicle. It introduces an innovative approach for accurately estimating and compensating for various disturbances and unknown parameters. These disturbances encompass external moments, unmodeled dynamics, other external interferences, parameter variations, friction, and complex nonlinear dynamics. Dealing with these factors can be challenging during modeling and complex for the controller. Therefore, by utilizing the proposed estimation method, these disturbances are aggregated and effectively compensated for. Additionally, the velocities of the UUV are also estimated, as these values are essential for the controller to compute the rotor speeds. The UUV demonstrates robust performance, accurately following the desired path when subjected to various disturbances, as confirmed through validation in the Simulink environment. This underscores that the proposed method requires no additional sensors, involves low computational complexity, and possesses the capability to respond rapidly to different types of disturbances during its course. Ultimately, this independent and robust control method holds significant potential for various realworld applications in underwater vehicles or drones.

#### **Data Availability**

Data available on request, please contact Sheikh Izzal Azid on sheikhizzal@gmail.com.

## **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

## Acknowledgments

Open access publishing facilitated by Charles Darwin University, as part of the Wiley–Charles Darwin University agreement via the Council of Australian University Librarians.

#### References

- J. Neira, C. Sequeiros, R. Huamani, E. Machaca, P. Fonseca, and W. Nina, "Review on unmanned underwater robotics, structure designs, materials, sensors, actuators, and navigation control," *Journal of Robotics*, vol. 2021, Article ID 5542920, 26 pages, 2021.
- [2] Y. Chen, Y. Liu, Y. Meng, S. Yu, and Y. Zhuang, "System modeling and simulation of an unmanned aerial underwater vehicle," *Journal of Marine Science and Engineering*, vol. 7, no. 12, Article ID 444, 2019.
- [3] J. Vervoort, "Modeling and control of an unmanned underwater vehicle," M.S. thesis, , University of Canterbury, Christchurch, New Zealand, 2008.
- [4] B. Wu, X. Han, and N. Hui, "System identification and controller design of a novel autonomous underwater vehicle," *Machines*, vol. 9, no. 6, Article ID 109, 2021.
- [5] G. Chen, A. Liu, J. Hu, J. Feng, and Z. Ma, "Attitude and altitude control of unmanned aerial-underwater vehicle based on incremental nonlinear dynamic inversion," *IEEE Access*, vol. 8, pp. 156129–156138, 2020.
- [6] M. Moustanir, K. Benkirane, A. Sayouti, and H. Medromi, "Four-propellers underwater drone modelling," E3S Web of Conferences, vol. 229, Article ID 01010, 2021.
- [7] M. M. Maia, D. A. Mercado, and F. J. Diez, "Design and implementation of multirotor aerial-underwater vehicles with experimental results," in 2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), pp. 961–966, IEEE, Vancouver, BC, Canada, September 2017.
- [8] K. M. Gul, C. Kaya, A. Bektas, and Z. Bingul, "Design and control of an unmanned underwater vehicle," in 2020 4th International Symposium on Multidisciplinary Studies and

*Innovative Technologies (ISMSIT)*, pp. 1–9, IEEE, Istanbul, Turkey, October 2020.

- [9] J. Kim, S. A. Gadsden, and S. A. Wilkerson, "A comprehensive survey of control strategies for autonomous quadrotors," *Canadian Journal of Electrical and Computer Engineering*, vol. 43, no. 1, pp. 3–16, 2020.
- [10] K. Kothari, U. Mehta, and R. Prasad, "Fractional-order system modeling and its applications," *Journal of Engineering Science and Technology Review*, vol. 12, no. 6, pp. 1–10, 2019.
- [11] V. P. Shankaran, S. I. Azid, U. Mehta, and A. Fagiolini, "Improved performance in quadrotor trajectory tracking using MIMO PI<sup>2</sup>-D control," *IEEE Access*, vol. 10, pp. 110646– 110660, 2022.
- [12] M. T. Vu, T.-H. Le, H. L. N. N. Thanh et al., "Robust position control of an over-actuated underwater vehicle under model uncertainties and ocean current effects using dynamic sliding mode surface and optimal allocation control," *Sensors*, vol. 21, no. 3, Article ID 747, 2021.
- [13] M. T. Vu, H. N. N. Le Thanh, T.-T. Huynh et al., "Stationkeeping control of a hovering over-actuated autonomous underwater vehicle under ocean current effects and model uncertainties in horizontal plane," *IEEE Access*, vol. 9, pp. 6855– 6867, 2021.
- [14] Z. Ermayanti, E. Apriliani, H. Nurhadi, and T. Herlambang, "Estimate and control position autonomous underwater vehicle based on determined trajectory using fuzzy kalman filter method," in 2015 International Conference on Advanced Mechatronics, Intelligent Manufacture, and Industrial Automation (ICAMIMIA), pp. 156–161, IEEE, Surabaya, Indonesia, October 2015.
- [15] D. A. Nguyen, D. Duy Thanh, N. T. Tien, and P. V. Anh, "Fuzzy controller design for autonomous underwater vehicles path tracking," in 2019 International Conference on System Science and Engineering (ICSSE), pp. 592–598, IEEE, Dong Hoi, Vietnam, July 2019.
- [16] C. Shen, Y. Shi, and B. Buckham, "Lyapunov-based model predictive control for dynamic positioning of autonomous underwater vehicles," in 2017 IEEE International Conference on Unmanned Systems (ICUS), pp. 588–593, IEEE, Beijing, China, October 2017.
- [17] R. Roy, M. Islam, N. Sadman, M. A. P. Mahmud, K. D. Gupta, and M. M. Ahsan, "A review on comparative remarks, performance evaluation and improvement strategies of quadrotor controllers," *Technologies*, vol. 9, no. 2, Article ID 37, 2021.
- [18] N. Kumar and M. Rani, "An efficient hybrid approach for trajectory tracking control of autonomous underwater vehicles," *Applied Ocean Research*, vol. 95, Article ID 102053, 2020.
- [19] Y. Xie, A. Zhu, and Z. Huang, "Research on the control performance of depth-fixed motion of underwater vehicle based on fuzzy-PID," *Journal of Robotics*, vol. 2023, Article ID 4168433, 10 pages, 2023.
- [20] A. S. Tijjani, A. Chemori, and V. Creuze, "A survey on tracking control of unmanned underwater vehicles: Experiments-based approach," *Annual Reviews in Control*, vol. 54, pp. 125–147, 2022.
- [21] MathWorks, "Model predictive control toolbox documentation," MathWorks, September 8, 2023, https://au.mathworks.com/ help/mpc/.
- [22] J. Bian and J. Xiang, "QUUV: a quadrotor-like unmanned underwater vehicle with thrusts configured as X shape," *Applied Ocean Research*, vol. 78, pp. 201–211, 2018.