

Research Article

Hierarchical Stabilization and Tracking Control of a Flexible-Joint Bipedal Robot Based on Anti-Windup and Adaptive Approximation Control

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Bipedal robotic mechanisms are unstable due to the unilateral contact passive joint between the sole and the ground. Hierarchical control layers are crucial for creating walking patterns, stabilizing locomotion, and ensuring correct angular trajectories for bipedal joints due to the system's various degrees of freedom. This work provides a hierarchical control scheme for a bipedal robot that focuses on balance (stabilization) and low-level tracking control while considering flexible joints. The stabilization control method uses the Newton–Euler formulation to establish a mathematical relationship between the zero-moment point (ZMP) and the center of mass (COM), resulting in highly nonlinear and coupled dynamic equations. Adaptive approximation-based feedback linearization control (so-called adaptive computed torque control) combined with an anti-windup compensator is designed to track the desired COM produced by the high-level command. Along the length of the support sole, the ZMP with physical restrictions serves as the control input signal. The viability of the suggested controller is established using Lyapunov's theory. The low-level control tracks the intended joint movements for a bipedal mechanism with flexible joints. We use two control strategies: position-based adaptive approximation control and cascaded position-torque adaptive approximation control (cascaded PTAAC). The interesting point is that the cascaded PTAAC can be extended to deal with variable impedance robotic joints by using the required velocity concept, including the desired velocity and terms related to control errors such as position, force, torque, or impedance errors if needed. A 6-link bipedal robot is used in simulation and validation experiments to demonstrate the viability of the suggested control errors such as position, force, torque, or impedance errors if needed. A 6-link bipedal robot is used in simulation and validation experiments to demonstrate the viability of the suggested control structure.

1. Introduction

A local pattern generator on the spinal cord regulates humans walking on flat ground without brain directives. The control system is complex due to multiple local controllers and cerebellar commands. The central nervous system maintains human equilibrium at various levels. The hierarchy of controls ranges from action planning to low-level reflex or local control. Multiple walking and running modes need a single stability or balance criterion. Still, the literature on balance techniques needs to be revised, making it challenging to provide a coherent background [1, 2]. Four distinct criteria are used as balancing and stability indices for bipedal locomotion:

- (i) The zero-moment point (ZMP);
- (ii) The Poincare map for limit-cycle walking;
- (iii) The angular momentum-based criterion;
- (iv) The footstep-based criterion.

In summary, the first two criteria are required to achieve practical bipedal mobility, and the last two criteria may serve as helpful cues for restoring the biped's equilibrium. The ZMP, center of mass (COM), angular momentum, and footstep are directly related, as detailed in [3–5]. The initial design of bipedal robots relies on static equilibrium; COM is preserved inside the support polygon (footprints). The researchers then attempted to loosen the static constraint by requiring that the center of pressure (COP) be contained within the support polygon, forming the so-called ZMP requirement. The ZMP criterion can produce a nonrotated support foot during the walking phases. Still, it cannot ensure that the upper body of the biped is in an upright position or orientation. Solving the challenging nonlinear equations related to the COM and ZMP is frequently problematic. Investigators often use simple models to estimate gait patterns of bipeds, but linking walking periods with different COM speeds can be challenging, necessitating a control strategy that considers both internal and external modeling errors. In such a situation, regulating the spinning angular momentum around the biped COM has been researched to develop practicable, stable walking patterns. The ZMP-based biped robots may require multilevel control systems to provide the appropriate motion in light of the concerns above. While a low-level control system is offered to track the necessary angular joint trajectories, high-level control consists of online walking patterns and balancing control techniques. This study proposes a hierarchical control system for the bipedal mechanism, which includes designing reference walking patterns and controlling the ZMP or the COM. At the same time, it is being tracked, maintaining balance and low-level control, as described later. The correlation between COM and ZMP trajectories is crucial for creating walking patterns for biped robots. Three methods have been effectively used to create motion for bipedal robots: optimization-based gait, COM-based gait, and interpolation-based gait. The optimization tool manages minimal energy or input control effort, optimal design, and dynamic and kinematic restrictions [6]. The ZMP criteria can be viewed as a constraint or an objective function. The computational complexity allows for frequent offline use. The COMbased gait assumes all masses are centered in the COM, and the support foot's ankle joint experiences a pushing force despite no applied torque. This concept is suggested for the online implementation of bipedal walking patterns. However, it requires modification to account for errors caused by this approximation and other disturbances [7]. Interpolation-based gait requires proper polynomial or spline functions for the COM and foot trajectory to track or satisfy the planned ZMP with compensated ZMP offline or online. Real-time motion planning is challenging due to the need for a single iteration of the walking generator's algorithm and the discontinuous COM velocity in transition instances [8–11]. A multilevel control plan is always required to stabilize bipedal locomotion, comprising trajectory generation for the COM, ZMP, or ZMP tracking control, an inverse kinematics approach, and joint tracking control. This work provides a hierarchical control scheme for a bipedal robot that focuses on balance and low-level tracking control while considering flexible joints. Refer to [12–18] for further information on bipedal mechanism control, balancing criteria, and motion planning.

The following concerns are investigated in this research:

1.1. Balance and Stabilization Control Level. A high-level control regulates the ZMP trajectory due to modeling errors and disturbances or tracks the COM trajectory with bounded ZMP values. The ZMP trajectory can be indirectly manipulated by COM motion using a straightforward linear inverted pendulum model. Consequently, Sugihara et al. [19] created real-time motion generation that indirectly controls the ZMP

to regulate the biped robot's COM. Their algorithm is divided into four sections: local control of joint angles, joint angle localization, referential ZMP planning, and ZMP manipulation. By manipulating the biped COM trajectory, Choi et al. [20] took advantage of indirect ZMP control and demonstrated the stability of its disturbance input-to-state (ISS). After identifying the issue with the one-mass inverted pendulum's non-minimum phase property, Napoleon et al. [21] used a two-mass inverted pendulum as a simplified biped model to eliminate the unstable zeros. They used a linear quadratic regulator (LQR) to track the desired ZMP trajectory. Pole-zero cancelation was employed by Hong et al. [22] as a feedforward controller to eliminate the unstable zeros, and LQR was used as a feedback controller for tracking. Online walking patterns employing the cart-pole system were successfully generated by Kajita et al. [23] using the preview control mechanism of the ZMP. They recommended using the preview control twice. Stage 1 might follow the planned ZMP trajectory. Stage 2 would fix any ZMP inaccuracies brought on by differences between the proposed biped robot model and the actual multibody model. This approach needs help specifying the appropriate ZMP at the heel or toe of the foot to achieve human-like walking [24]. Auxiliary ZMP [25], observer-based preview control [26], and hierarchical preview control [27] have all been improved. The preview control has two drawbacks: (1) it ignores ZMP limitations, and (2) it is affected by perturbation and disturbance. To improve the preview control, Wieber [28] adopted linear predictive control of biped robots in the presence of perturbation without considering the computing complexity, which is well handled in [29]. In addition to using the optimal control method, Miura et al. [30] also tracked the target humanoid's COM and ZMP utilizing a time delay and a PID controller. Generally, most linear control techniques can be used to control an inverted pendulum model to follow the ZMP or COM states. As mentioned previously, the preview control has several issues, and [31] demonstrates how the ordinal control system struggles to track ZMP accurately.

Furthermore, most ZMP/COM tracking control is coupled with high computational complexity. Therefore, utilizing the boundary value problem, researchers have attempted to directly solve the inverted pendulum differential equation. Harada et al. [11] developed an analytical technique for concurrently estimating the COM and ZMP during a change in walking stride. Real-time and quasi-real-time strategies have been successfully proposed for tying the new trajectory to the existing one. In [32], the analytical procedure is extended. See [33–38] for further information on bipedal robot balance or stability control. As a result, the current study proposes a high-level control system to maintain the ZMP trajectory within a stable zone while following the intended COM references. The system uses a bipedal robot's centroidal dynamics to form an equation connecting the output COM to the input ZMP. An input saturation compensator is required due to the constrained system. The adaptive computed torque control (CTC) law is designed for uncertain

bipedal parameters. Details on saturation compensators will be provided later.

Remark 1. Some stabilization strategies are used for balance recovery when subjected to severe disturbances, referred to as balance and push recovery. These strategies can be exploited alternatively for ZMP/COM compensation. Unique balance techniques are used by all four-legged animals to prevent unexpected falls and to preserve their general rotational stability, dynamic stability, and postural stability [39]. Humans can employ sophisticated tactics to maintain equilibrium and prevent potential disturbances. The support base must be adjusted to keep the body's center of gravity while standing, moving, or sprinting. The balance system actively observes the surroundings and anticipates how forces produced by voluntary movements will affect the body. It then makes the required modifications to preserve posture and equilibrium. The reactive balancing response only appears when these adjustments are unsuccessful or unexpected instability occurs [40]. Thus, the reactive mechanism (feedback control) and the proactive mechanism (feedforward control) are used to establish balancing conditions [2]. Al-Shuka et al. [3] classified four possible methods for balance recovery:

- (i) Ankle/hip modification;
- (ii) Whole-body modification;
- (iii) Foot-step strategy;
- (iv) Predictive control strategy using a feedforward technique.

One of the most potent control approaches is wholebody control. This technique uses all degrees-of-freedom (DOFs) of the bipedal robot to follow the desired COM trajectory or the desired linear or angular momentum. For further details, see [41–44].

1.2. Tracking Control of Desired Bipedal Joint Trajectories. The low-level control's task is to follow the intended angular joint trajectories of the biped mechanism. As previously stated, the ZMP criterion concept presupposes a fully actuated biped robot in the single-support phase (SSP) to follow the required path of the ZMP or meet it and an over-actuated biped in the double-support phase (DSP). To effectively use the traditional control approaches for manipulators, the stance foot of the biped robot has to be fixed during the SSP. Most of the methods studied in the excellent work by the literatures [45-47] can successfully control ZMP-based gait. However, the researcher should consider the biped humanoid robot's considerable degree of freedom. As a result, most humanoid robots feature decoupled tracking PID control systems. The early attempts at controlling the bipedal robot were centered on linearizing the equation of motion. Golliday and Hemami [48] linearized the three-link biped robot with locking knees around the necessary operational points. Observability, controllability, and stability were then examined. Feedback control systems for controlling the speed and stride length of the investigated bipedal mechanism were suggested. Their inverted pendulum-based biped

robot was controlled by a local PID controller created by Kajita and Toni [49]. The abovementioned linearizationbased control has the drawback that its solution only applies in a limited area surrounding the operational locations (nominal trajectory) [50]. Any moving walking robot will experience several abrupt geometric limitations, such as knee locking, stepping on the ground, etc. These restrictions in every walking machine lead to disturbances that resemble impulses and are very challenging for a standard PID controller to regulate [51]. The typical PID might be unable to keep the gait stable if the uncertainty level (is greater than 80%). Raibert et al. [51] compared a local PD controller, CTC, and sliding mode control (SMC) to simulate a fivelink planar biped robot. They demonstrated the superiority of sliding-mode control for bipedal systems in the presence

of parameter uncertainty. Similar works utilizing SMC were accomplished by the literatures [52–54]. The literature has described various control strategies, such as learning control algorithms [55–57], impedance control [58, 59], and torque control [60, 61].

On the other hand, most researchers avoided using typical adaptive controls in complex bipedal systems because they may depend on a regressor matrix only applicable to six DOFs or fewer. Zhu's virtual decomposition control (VDC) [62] is a potential solution to this problem. Every link in the complex system is broken down, giving each subsystem similar classical adaptive control. It has been used to underactuated bipeds via time-scaling-based adaptive VDC. Decentralized control strategies have been proposed in [63–65]. This paper focuses on fully tracking decentralized adaptive approximation control, considering joint flexibility and input saturation, which will be briefly discussed in subsequent subsections.

1.3. Compliant Actuation Systems. Electrical drives power most robot actuators due to their accessibility and established control mechanisms. Bipedal robots often use transmission units, with harmonic drives being particularly interesting due to their low backlash, high accuracy, small dimensions, and high torque output. High gear ratios in manipulator robots hinder back driving, causing issues with shock absorption. Stiff actuators, which store energy, are preferred over compliant joints for improved tracking accuracy. Therefore, well-known ZMP-based bipedal robots, such as ASIMO [66], WEBIAN [67], HRP series [68], Johnnie [69], and KHR series [8], utilize position control strategies using stiff joints with harmonic drive units. Legged robots may benefit from elastic joint characteristics for shock absorption, energy storage, and reduced control effort, even though their tracking accuracy is less rigorous than overall dynamic stability. As a result, some bipedal robots such as cCUP [70], Valkyrie [71], and COMAN robots [72] use series elastic joint actuators to avoid the shortcomings of stiff actuators. Series elastic actuators feature an elastic element with constant stiffness attached to the robotic joints, offering advantages over stiff actuators like low impedance, impact load absorption, and increased peak power output. Joint flexibility presents challenges in modeling and control due to its additional degrees of

freedom, causing the order of related dynamics to be twice that of rigid robots. This results in more complex dynamic behavior that requires further study and evaluation in mathematical modeling. The control of rigid robots, such as full actuation and passivity, is lost when joint flexibility is included in the dynamic model. Flexible joint robots face control issues due to quick dynamics, vibration, uncertainties in connection dynamics, payload fluctuations, external distances, and drive dynamics, particularly joint stiffness values. Over the past decade, various strategies for controlling flexible joint robots have been proposed, including feedback linearization [73], which offers fast response and large bandwidth but requires higher-order time derivatives. The cascaded control method is a control method for flexible joint robots that involves breaking down a high-order system into multiple lower-order subsystems [74]. This method requires constant reference input to the inner control loop, slowing down the response time and resulting in a lower bandwidth in the state-space approach. The singular perturbation approach [75] and integral manifold approach are popular methods for controlling joint torque subsystems, adding damping to the fast mode. Other flexible joint robot control methods, such as integral backstepping control [76], passivity-based control [77], PD control [78], and so on, primarily focus on position control. The joint torque tracking loop is crucial for robotic systems operating in constrained environments. The torque control loop faces a significant issue with noisy torque derivative time signals, necessitating the use of a low-pass filter for feasible results [79-83]. Two control algorithms are proposed in this paper to address joint flexibility issues effectively.

1.4. Input Saturation. Actuators have practical limits like saturation, dead zone, and hysteresis. These constraints can reduce closed-loop system performance and cause instability. Researchers have concentrated their efforts on developing controllers for systems with saturation restrictions. There are two approaches: changing the control effort signal and building an auxiliary system to specify tracking errors. These ideas are intended to solve these restrictions while also improving the safety and performance of closed-loop systems. See [84–87] for more details.

This study presents a hierarchical control strategy for a bipedal robot, emphasizing balance (stabilization) and lowlevel tracking control while considering flexible joints. The Newton-Euler (N-E) formulation is used in the stabilization control approach to building a mathematical correlation between the COM and the ZMP, leading to highly coupled and nonlinear dynamic equations. Adaptive approximationbased feedback linearization control, also known as adaptive CTC, is paired with an anti-windup compensator to follow the intended COM generated by the high-level command. The ZMP with physical limits acts as the control input signal along the length of the support sole. The Lyapunov theory is used to prove the viability of the proposed controller. The low-level control for a bipedal system with flexible joints follows the planned joint motions. Position-based adaptive approximation control (PAAC) and cascaded positiontorque adaptive approximation control (cascaded PTAAC)



FIGURE 1: The proposed control structure. The focus of the current work is stabilization and tracking joint controls.

are the two control techniques we employ. The intriguing aspect is that by using the necessary velocity concept, which also includes the desired velocity and terms relating to control faults such as position, force, torque, or impedance errors as needed, the cascaded PTAAC may be expanded to cope with variable impedance robotic joints. In simulation and validation studies, a 6-link bipedal robot is employed to show the effectiveness of the recommended control.

The rest of the paper is structured as follows: The difficulties and imposed assumptions of the work are outlined in Section 2. The methodology is described in Section 3, with two control stages to stabilize and track the target robot's mobility. Section 4 introduces the simulation experiments and the validation results, while Section 5 ends.

2. Problem Formulation and Assumptions

This study addresses two control issues using anti-windup compensation principles: tracking the COM trajectory while keeping ZMP control input within the stance sole and following desired joint states, considering joint flexibility and physical torque input saturation. The study proposes a fourlevel control strategy for legged mechanisms, focusing on balance control (COM tracking) and low-level control (flexible joint monitoring). The control architecture, shown in Figure 1, uses the saturation compensator for motion stabilization, addressing physical limitations on joint torques and ground reaction forces. As illustrated in Figure 1, the proposed control structure consists of the following key elements:

- (1) COM planner. It is beyond the scope of this paper; however, it is responsible for designing stabilized walking patterns for the target bipedal. The desired references for the COM and the ZMP can be generated using three possible methods: optimizationbased gait, COM-based gait, and interpolation-based gait [6–11]. Among these methods, our motion planning algorithm proposed in [88] is selected.
- (2) Stabilization control. It aims to regulate the constrained ZMP trajectory by tracking the referential COM. It develops an explicit dynamic relationship between the ZMP and the COM and designs an adaptive CTC scheme, integrating it with an antiwindup compensator for motion stabilization.
- (3) Inverse kinematics. This topic is outside the purview of the present work and should be discussed at a later time. The inverse kinematics strategy is used to find desired angular trajectories by calculating the desired COM and referential foot trajectory. This strategy can be integrated with multiple tasks for balancing and stable configurations of the bipedal mechanism. The geometry method is used instead, based on the work of [31]. Please see [4, 89, 90] and the references therein for more details on this topic.
- (4) Tracking control of joint trajectories. It precisely tracks desired angular joint references obtained by inverse kinematics. In this stage, joint flexibility is considered, complicating the control task due to increasing the DOFs to double, under-actuation behavior, and noisy torque signals if needed. Modeling the flexible-joint robots using Euler-Lagrange (E–L) formulation leads to a cascaded system. The control inputs for the link and joint subsystem are the elastic torques and the motor torques, respectively. Using the concept of the AAC, the system can be decomposed into link-joint subsystems, and hence, a decoupled controller for each link-joint subsystem can be developed. Two control schemes are proposed: position-based control and cascaded position-torque control. The interesting point is that the cascaded position-torque control scheme can be extended to control a robotic system with time-varying joint stiffness and damping.

In general, the following assumptions are considered [31, 66].

Assumption 1: The stance foot is fixed without rotation. This assumption is crucial for ZMP-based locomotion to maintain the ZMP location within the support polygon (stance sole) in the SSP.

Assumption 2: The bipedal robot is fully actuated to apply the ZMP criteria.

This assumption coincides with Assumption 1, which includes a fixed-stance foot. If a rotating stance foot is selected, then the ZMP criteria cannot be held, and the bipedal system is underactuated; see [91, 92] for more details.

Assumption 3. The gait cycle consists solely of the SSP due to the short duration of the DSP.

Assumption 4. The dynamic coefficients of the system equation involving the COM and ZMP can be linearly parameterized using orthogonal functions.

This assumption is necessary for the application of adaptive approximation control [83].

Assumption 5. The COM velocity and displacement are all measurable.

Assumption 6. The physical parameter values for the link and joint dynamics are unknown and can be linearly parameterized in terms of orthogonal functions.

The implementation of adaptive approximation control requires Assumption 7 [83].

Assumption 7. The elastic torque is measurable up to the second order.

3. Methodology

This section comprehensively describes balance and tracking control using various control scenarios.

3.1. Stabilization Control. Balance, stabilization, or postural control compensate for modeling errors and disturbances in a bipedal robot. It aims to track the COM trajectory while maintaining the ZMP within a safe range. The relationship between the COM and ZMP is determined using N-E dynamics, which is nonlinearly coupled. Adaptive control is based on the function approximation technique and windup strategy, with an anti-windup compensator integrated to deal with the limits of the control input, the ZMP trajectory. The output and input variables are the COM and ZMP trajectories, respectively. Saturation restriction is a common challenge in actuator design due to physical properties and safety issues. It can improve closed-loop system efficiency and potentially cause instability. In our case, however, the ZMP signals are the control input and should be positioned within the stance sole of the SSP. Researchers have developed controllers for systems with saturation constraints using two techniques: readjusting the control effort signal and building an auxiliary system. The method involves readjusting the control input signal using the function approximation technique based on Chebyshev orthogonal polynomials.

Using the N–E formula, the following explicit expression can be obtained relating the ZMP–COM trajectories; see [4] for details.

$$I_c \ddot{c} + \alpha_c = u_c, u_c \le u_c \le \overline{u_c}, \tag{1}$$

with

$$I_{c} = \begin{bmatrix} \frac{-c_{z}}{\ddot{c}_{z} + g} & 0\\ 0 & \frac{-c_{z}}{\ddot{c}_{z} + g} \end{bmatrix}, c = \begin{bmatrix} c_{x} & c_{y} \end{bmatrix}^{T},$$

$$\alpha_{c} = \begin{bmatrix} c_{x} - \frac{\tau_{y}}{\dot{P}_{z} + Mg}\\ c_{y} + \frac{\tau_{x}}{\dot{P}_{z} + Mg} \end{bmatrix}, \text{ and } u_{c} = \begin{bmatrix} p_{x} & p_{y} \end{bmatrix}^{T},$$
(2)



FIGURE 2: A bipedal robot with the definition of the ZMP. The force f is the ground reaction force while the torque (moment) vector at the ZMP has only component in *z*-axis. By taking a moment about point O and using the N–E formulation, Equation (1) is obtained for a multibody robot.

where, P_z is the total linear momentum in *z*-dimension, $\tau = \begin{bmatrix} \tau_x & \tau_y & \tau_z \end{bmatrix}^T$ is the moment vector about the COM, and $p_{(.)}$ is the ZMP location. The control input signals represented by u_c has upper and lower bounds denoted by $\overline{u_c}$ and u_c , respectively.

Figure 2 shows the external force and moments affected on a bipedal robot. The ZMP moments (torques) in the *x* and *y* axes should be small, approximately zero, to ensure stability. The bipedal robot maintains a constant height for the COM or hip.

Adaptive feedback linearization, or so-called CTC based on the function approximation technique, is suggested to stabilize the nonlinear system in Equation (1). The feedback linearization control technique aims to create a nonlinear control law that makes the dynamics of the closed-loop linear. Equation (1) can be recast as Equation (3) to address saturated control input [93].

$$I_c \ddot{c} + \alpha_c = u_c = v + \delta, \tag{3}$$

where

$$u_{c_i} = \operatorname{sat}(v_i) = \begin{cases} u_{c_i \max}, v_i \ge u_{c_i \max} \\ u_i, u_{c_i \min} < v_i < u_{c_i \max}, i = 1, 2. \\ u_{c_i \min}, v_i \le u_{c_i \min} \end{cases}$$
(4)

The control tolerance value (δ_i , i = 1, 2) with $\delta = [\delta_1 \delta_2]^T$ can be written as follows:

$$\delta_{i} = \begin{cases} u_{c_{i}\max} - v_{i}, v_{i} \ge u_{c_{i}\max} \\ 0, u_{c_{i}\min} < v_{i} < u_{c_{i}\max} \\ u_{c_{i}\min} - v_{i}, v_{i} \le u_{c_{i}\min} \end{cases}$$
(5)

The proposed control structure accounts for nonlinear control saturation by tracking desired references using adaptive feedback linearization control and function approximation technique, with the chosen control law being as follows:

$$v = \widehat{I}_c (\ddot{c}_d - K_d (\dot{c} - \dot{c}_d) - K_p (c - c_d)) - \kappa \operatorname{sgn}(B^T P^T x) + \widehat{\alpha}_c - \widehat{\delta},$$
(6)

where c_d is the desired reference for the COM trajectory, the estimation is represented by the symbol, $K_d \in R^{2\times 2}$ and $K_p \in R^{2\times 2}$ are both diagonal positive definite matrices, $\kappa \in R^{2\times 2}$ is a robust sliding gain, $B = \begin{bmatrix} 0 \\ I_2 \end{bmatrix} \in R^{4\times 2}$, $x = \begin{bmatrix} e^T & \dot{e}^T \end{bmatrix}^T \in R^4$, and, $P = P^T \in R^{4\times 4}$ is a symmetric positive definite matrix meeting the Lyapunov equation as follows:

$$A^T P + P A = -Q, (7)$$

with

$$\mathbf{A} = \begin{bmatrix} 0 & I_2 \\ -K_p & -K_d \end{bmatrix} \in \mathbb{R}^{4 \times 4},\tag{8}$$

and $Q = Q^T \in \mathbb{R}^{4\times 4}$ is also a symmetric positive definite matrix. Notice that if $\hat{\delta} = \delta$ in the controller (Equation (6)), then it results an algebraic loop. The compensation of the dead-zone function estimation $\hat{\delta}$ only works in a compact set of the state space, resulting is local stability.

Substituting Equation (6) into Equation (3) leads to the following closed-loop dynamics:

$$\ddot{e} + K_d \dot{e} + K_p e + \kappa \operatorname{sgn}(B^T P^T x) = -\widehat{I}_c^{-1} \left(\widetilde{I}_c \, \ddot{c} + \widetilde{\alpha}_c - \widetilde{\delta} \right) + \varepsilon.$$
(9)

Equation (9) is basically a linear closed-loop dynamics with residual error, $\varepsilon \in R^2$, and $(\sim) = (.) - (^{\wedge})$. However due to the robust sliding term, $\kappa \operatorname{sgn}(B^T P^T x)$, it's no longer linear. The FAT can represent mass and nonlinear matrices and vectors as follows:

$$I_{c} = W_{I}^{T} \varphi_{I} + \varepsilon_{I}$$

$$\alpha_{c} = W_{\alpha}^{T} \varphi_{\alpha} + \varepsilon_{\alpha},$$

$$\delta = W_{\delta}^{T} \varphi_{\delta} + \varepsilon_{\delta}$$
(10)

where the weighting matrices are $W_I \in R^{2\beta \times 2}$, $W_\alpha \in R^{2\beta \times 2}$, and $W_\delta \in R^{2\beta \times 2}$, the basis function matrices are $W_I \in R^{2\beta \times 2}$, $\varphi_{\alpha} \in \mathbb{R}^{2\beta}$, and $\varphi_{\delta} \in \mathbb{R}^{2\beta}$, and β denotes the number of basis function terms. The estimated matrices using the same set of basis functions are as follows:

$$\widehat{I}_{c} = \widehat{W}_{I}^{T} \varphi_{I}
\widehat{\alpha}_{c} = \widehat{W}_{a}^{T} \varphi_{a}
\widehat{\delta} = \widehat{W}_{b}^{T} \varphi_{\delta}.$$
(11)

Consequently, the control law (Equation (6)) is formulated as follows:

$$u = \widehat{W}_{I}^{T} \varphi_{I} (\ddot{c}_{d} - K_{d} (\dot{c} - \dot{c}_{d}) - K_{p} (c - c_{d})) - \kappa \operatorname{sgn}(B^{T} P^{T} x) + \widehat{W}_{a}^{T} \varphi_{a} - \widehat{W}_{\delta}^{T} \varphi_{\delta},$$
(12)

then the closed-loop dynamics (Equation (9)) becomes the following:

$$\ddot{e} + K_{d}\dot{e} + K_{p}e + \kappa \text{sgn}(B^{T}P^{T}x)$$

$$= -\left(\widehat{W}_{I}^{T}\varphi_{I}\right)^{-1}\left(\widetilde{W}_{I}^{T}\varphi_{I} \ddot{q} + \widetilde{W}_{\alpha}^{T}\varphi_{\alpha} - \widetilde{W}_{\delta}^{T}\varphi_{\delta}\right) + \varepsilon.$$
(13)

Expressing Equation (13) in a state space form as follows:

$$\dot{x} = Ax - B \left(\widehat{I}^{-1} \left(\widetilde{W}_{I}^{T} \varphi_{I} \, \ddot{c} + \widetilde{W}_{\alpha}^{T} \varphi_{\alpha} - \widetilde{W}_{\delta}^{T} \varphi_{\delta} \right) - \varepsilon + \kappa \operatorname{sgn}(B^{T} P^{T} x) \right).$$
(14)

Selecting the relevant updated laws as follows:

$$\begin{split} \hat{W}_{I} &= -\Psi_{I}\varphi_{I}\,\ddot{c}\left(x^{T}PB\widehat{I}^{-1}\right) \\ \hat{W}_{\alpha} &= -\Psi_{\alpha}\varphi_{\alpha}\left(x^{T}PB\widehat{I}^{-1}\right) \\ \hat{W}_{\delta} &= \Psi_{\delta}\varphi_{\delta}\left(x^{T}PB\widehat{I}^{-1}\right), \end{split}$$
(15)

where the adaptation matrix is $\Psi_{(.)} \in \mathbb{R}^{2\beta \times 2\beta}$.

In effect, the estimate \hat{I} of the inertia matrix I is not always invertible, even though it is nonsingular for every t > 0. As a result, as the determinant of \hat{I} approaches zero, Equation (15) may experience a singularity issue, and a address these issues. The L_2 and L_{∞} stability of target systems is frequently demonstrated in this paper using the following lemma [62]:

on the passivity design was put out by Slotine and Li [94] to

Lemma 1. Assuming that $\overline{Q} \in \mathbb{R}^{n \times n}$ is a symmetric positivedefinite matrix, $x(t) \in \mathbb{R}^n$, $n \ge 1$, and that V(t) is a nonnegative piecewise continuous function defined as follows:

$$V(t) \ge \frac{1}{2} x(t)^T \overline{Q} x(t).$$
(16)

If the time derivative of V(t) is determined by the following:

$$\dot{V} \le -y(t)^T \overline{P} y(t) - \overline{\sigma}(t) \le -\overline{\sigma}(t), \tag{17}$$

where $y(t) \in \mathbb{R}^m$, $m \ge 1$, $\overline{P} \in \mathbb{R}^{m \times m}$ is a symmetric positivedefinite matrix, and $\overline{\sigma}(t)$ is as follows:

$$\int_{0}^{\infty} \overline{\sigma}(t) dt \ge -\overline{\rho}, \qquad (18)$$

with $0 \leq \overline{\rho} < \infty$, then $V(t) \in L_{\infty}$, $x(t) \in L_{\infty}$, and $y(t) \in L_2$ hold.

The proof of Lemma 1 is described in the appendix.

Theorem 1. In the sense of L_2 and L_{∞} stability introduced in Lemma 1, the ZMP-COM dynamics in Equation (1) are stable when combined with the control law, closed-loop dynamics, and related update laws stated in Equation (12) through Equation (15).

Proof. Choosing the following Lyapunov-like function along the closed-loop dynamics (Equation (14)):

$$V = \frac{1}{2}x^{T}Px + \frac{1}{2}tr\Big(\widetilde{W}_{I}^{T}\Psi_{I}^{-1}\widetilde{W}_{I} + \widetilde{W}_{\alpha}^{T}\Psi_{\alpha}^{-1}\widetilde{W}_{\alpha} + \widetilde{W}_{\delta}^{T}\Psi_{\delta}^{-1}\widetilde{W}_{\delta}\Big).$$
(19)

By substituting Equation (14) for the time-derivative of Equation (19), we obtain the following:

$$\dot{V} = -\frac{1}{2}x^{T}Qx - x^{T}P\Big[B\widehat{M}^{-1}\Big(\widetilde{W}_{I}^{T}\varphi_{I}\ddot{c} + \widetilde{W}_{\alpha}^{T}\varphi_{\alpha} - \widetilde{W}_{\delta}^{T}\varphi_{\delta}\Big) - B\varepsilon + B\kappa \operatorname{sgn}(B^{T}P^{T}x)\Big] - tr\Big(\widetilde{W}_{I}^{T}\Psi_{I}^{-1}\dot{\widehat{W}}_{I}\Big) - tr\Big(\widetilde{W}_{\alpha}^{T}\Psi_{\alpha}^{-1}\dot{\widehat{W}}_{\alpha}\Big) - tr\Big(\widetilde{W}_{\delta}^{T}\Psi_{\delta}^{-1}\dot{\widehat{W}}_{\delta}\Big).$$

$$(20)$$

(25b)

It is possible to rewrite Equation (20) as follows:

$$\dot{V} = -\frac{1}{2}x^{T}Qx - tr\left(\widetilde{W}_{I}^{T}\left(\varphi_{I}\ddot{c}\left(x^{T}PB\widehat{M}^{-1}\right) + \Psi_{I}^{-1}\dot{\widehat{W}}_{I}\right)\right) - tr\left(\widetilde{W}_{\alpha}^{T}\left(\varphi_{\alpha}\left(x^{T}PB\widehat{M}^{-1}\right) + \Psi_{\alpha}^{-1}\dot{\widehat{W}}_{\alpha}\right)\right) - tr\left(\widetilde{W}_{\delta}^{T}\left(\varphi_{\delta}\left(x^{T}PB\widehat{M}^{-1}\right) - \Psi_{\delta}^{-1}\dot{\widehat{W}}_{\delta}\right)\right) - x^{T}PB(-\varepsilon + \kappa \operatorname{sgn}(B^{T}P^{T}x)),$$

$$(21)$$

by substituting Equation (15) into the previous equation, we get the following:

$$\dot{V} = -\frac{1}{2}x^{T}Qx - x^{T}PB(-\varepsilon + \kappa \text{sgn}(B^{T}P^{T}x))$$

$$= -\frac{1}{2}x^{T}Qx + \zeta^{T}\varepsilon - \Sigma\kappa_{i}|\zeta_{i}|,$$
(22)

where $\zeta = B^T P^T x$ and choosing the components κ_i so that

$$\kappa_i \ge |\varepsilon_i| + \chi_i, \tag{23}$$

where χ_i is a positive constant, making Equation (22) become

$$\dot{V} = -\frac{1}{2}x^T Q x - \sum \kappa_i |\zeta_i|.$$
(24)

In the perspective of Lemma 1, Equation (21) implies L_2 and L_{∞} stability.

3.2. Tracking Control of Joint Trajectories. In this section, two tracking control strategies—position control and cascaded position and torque control—are presented for tracking joints with unknown parameters and input saturation. The complexity of the problem is compounded by the inclusion of joint stiffness and damping, as the system will be under-actuated due to the doubling of the DOFs. Furthermore, it is challenging to design control laws that are decoupled while taking into account the uncertainty of joint stiffness and damping. To fix the control issue, you need an adaptive backstepping control system. Figure 3 shows a 6-link bipedal robot with flexible joints.

3.2.1. Position-Based Adaptive Control. This section develops the E–L dynamics for an *n*-DOF bipedal robot with flexible joints and torque input saturation [95, 96]. It is made up of cascading link subsystems coupled by joint impedance. The control law's goal is to apply the AAC's principles to decouple the *n*-DOF bipedal system into link-joint subsystems. An orthogonal basis function with particular terms is used to approximate the nonlinear coupling expressions. Consequently, the E–L formulation for the *n*-DOF bipedal robot is expressed as follows:

$$M\ddot{q} + C\dot{q} + g = \tau_t, \tag{25a}$$

and

$$\tau_t = K_s(\theta - q) + C_v (\dot{\theta} - \dot{q}), \qquad (25c)$$

where $M \in \mathbb{R}^{n \times n}$ is a positive definite and symmetric inertia matrix, $q \in \mathbb{R}^n$ is the angular displacement of links, $C \in \mathbb{R}^{n \times n}$ is the Coriolis and centripetal matrix, $g \in \mathbb{R}^n$ is the gravity vector, $\tau_t \in \mathbb{R}^n$ is the elastic joint torque, $I_m \in \mathbb{R}^{n \times n}$ is a diagonal inertia matrix with a diagonal element $r(r + 1)J_r$ with rbeing gear ratio, J_r is the rotor and gear inertia, $B_m \in \mathbb{R}^{n \times n}$ is the viscous damping matrix, $K_s \in \mathbb{R}^{n \times n}$ is the stiffness matrix for flexible element, $C_v \in \mathbb{R}^{n \times n}$ is damping matrix for the joint flexibility, $\theta \in \mathbb{R}^n$ is the angular displacement of motor, and $u \in \mathbb{R}^n$ is the control input and subjected to the following constraints:

 $I_m \ddot{\theta} + B_m \dot{\theta} + \tau_t = u,$

$$u_{i} = \operatorname{sat}(\mu_{i}) = \begin{cases} u_{i\max}, \mu_{i} \ge u_{i\max} \\ \mu_{i}, u_{i\min} < \mu_{i} < u_{i\max}, i = 1, 2, 3, ..., n. \\ u_{i\min}, \mu_{i} \le u_{i\min} \end{cases}$$
(26)

The elements of the control tolerance vector, γ , can be written as follows:

$$\gamma_{i} = \begin{cases} u_{i\max} - \mu_{i}, \mu_{i} \ge u_{i\max} \\ 0, u_{i\min} < \mu_{i} < u_{i\max} \\ u_{i\min} - \mu_{i}, \mu_{i} \le u_{i\min} \end{cases}$$
(27)

Equations (25a-25c)-(27) can be reformulated for the *i*th link-joint subsystems as follows:

$$m_{ii}(q) \ddot{q}_i + c_{ii}(q, \dot{q}) \dot{q}_i + \Delta_i(q, \dot{q}) = \tau_{t_i}, \qquad (28a)$$

$$I_{m_i}\ddot{\theta}_i + B_{m_i}\dot{\theta}_i + \tau_{t_i} = u_i, \qquad (28b)$$

with

$$\Delta_i(q, \dot{q}) = \sum_{\substack{j=1\\j\neq i}}^n m_{ij}(q) \ \ddot{q}_j + \sum_{\substack{j=1\\j\neq i}}^n c_{ij}(q, \dot{q}) \dot{q}_j + g_i(q),$$
(28c)



FIGURE 3: A 6-link bipedal robot with flexible joints.

$$k_{s_i}(\theta_i - q_i) + c_{v_i}(\theta_i - \dot{q}_i) = \tau_{t_i}.$$
(28d)

Equations (28a)–(28d) describe a cascade system where the elastic transmission torque serves as a virtual control input for the ith link, with suitable control laws selected as follows:

$$\widehat{m}_{ii}(q) \, \ddot{q}_{r_i} + \widehat{c}_{ii}(q, \dot{q}) \dot{q}_{r_i} + k_i s_i + \widehat{\Delta}_i(q, \dot{q}) + \sigma_i \operatorname{sgn}(s_i) = \tau_{td_i},$$
(29a)

$$\widehat{k}_{s_i}(\theta_{r_i} - q_{r_i}) + \widehat{c}_{v_i}(\dot{\theta}_{r_i} - \dot{q}_{r_i}) = \tau_{td_i},$$
(29b)

$$\widehat{I}_{m_i}\ddot{\theta}_{r_i} + \widehat{B}_{m_i}\dot{\theta}_{r_i} + \tau_{td_i} + k_{\theta i}s_{\theta i} + \sigma_{\theta i}\mathrm{sgn}(s_{\theta i}) = u_i = \mu_i + \gamma_i,$$
(29c)

where

$$\dot{q}_{r_i} = \dot{q}_{d_i} + \lambda_i (q_{d_i} - q_i) \dot{\theta}_{r_i} = \dot{\theta}_{d_i} + \lambda_{\theta_i} (\theta_{d_i} - \theta_i),$$

$$(30)$$

where the subscript (*d*) refers to desired reference, and λ_i is a positive parameter denoting the time constant. The FATbased adaptive control is adopted to estimate the uncertainty in Equations (29a)–(29c). The uncertain dynamic matrices and vectors are assumed as functions of time, and then selecting an orthogonal polynomial approximator to estimate the uncertainty. Thus, the control law in Equations (29a)–(29c) becomes the following:

$$\widehat{\boldsymbol{w}}_{m_i}^T \varphi_{m_i}(\boldsymbol{q}) \, \ddot{\boldsymbol{q}}_{r_i} + \widehat{\boldsymbol{w}}_{c_i}^T \varphi_{c_i} \dot{\boldsymbol{q}}_{r_i} + k_i s_i + \widehat{\boldsymbol{w}}_{\Delta_i}^T \varphi_{\Delta_i} + \sigma_i \mathrm{sgn}(s_i) = \tau_{td_i},$$
(31a)

$$\widehat{\omega}_{I_{mi}}^{T}\varphi_{I_{mi}}\ddot{\theta}_{r_{i}}+\widehat{\omega}_{b_{i}}^{T}\varphi_{b_{i}}\dot{\theta}_{r_{i}}-\widehat{\omega}_{\gamma_{i}}^{T}\varphi_{\gamma_{i}}\tau_{td_{i}}+k_{\theta_{i}}s_{\theta_{i}}+\sigma_{\theta_{i}}\mathrm{sgn}\left(s_{\theta_{i}}\right)=\mu_{d_{i}},$$
(31b)

where $w_{(.)} \in \mathbb{R}^{\beta}$ and $\varphi_{(.)} \in \mathbb{R}^{\beta}$ are the weighting-coefficients and orthogonal basis function vectors, respectively. Subtracting Equations (31a) and (31b) from Equations (28a)–(28d), we obtain the following closed-loop dynamics:

$$\begin{split} m_{ii}\dot{s}_{i} + c_{ii}s_{i} + k_{i}s_{i} + \sigma_{i}\text{sgn}(s_{i}) \\ &= -\widetilde{w}_{m_{i}}^{T}\varphi_{m_{i}} \,\ddot{q}_{r} - \widetilde{w}_{c_{i}}^{T}\varphi_{c_{i}}\dot{q}_{r} - \widetilde{w}_{\Delta_{i}}^{T}\varphi_{\Delta_{i}} + \left(\tau_{td_{i}} - \tau_{t_{i}}\right) + \varepsilon_{i}, \\ (32a) \end{split}$$

$$I_{m_{ii}}\dot{s}_{\theta_{i}} + B_{m_{ii}}s_{i} + k_{i}s_{\theta_{i}} + \sigma_{\theta_{i}}\mathrm{sgn}(s_{\theta_{i}})$$

= $-\widetilde{w}_{I_{mi}}^{T}\varphi_{I_{mi}}\ddot{\theta}_{r_{i}} - \widetilde{w}_{b_{i}}^{T}\varphi_{b_{i}}\dot{\theta}_{r_{i}} + \widetilde{w}_{\gamma_{i}}^{T}\varphi_{\gamma_{i}}$
 $-(\tau_{td_{i}} - \tau_{t_{i}}) + (\mu_{i} - \mu_{d_{i}}) + \varepsilon_{\theta_{i}}.$ (32b)

The following update adaptive laws are chosen to achieve stable closed-loop dynamics:

$$\widehat{w}_{m_{i}} = \Phi_{m_{i}}\varphi_{m_{i}} \, \ddot{q}_{r_{i}}s_{i}
\widehat{w}_{c_{i}} = \Phi_{c_{i}}\varphi_{c_{i}}\dot{q}_{r_{i}}s_{i}
\widehat{w}_{\Delta_{i}} = \Phi_{\Delta_{i}}\varphi_{\Delta_{i}}s_{i},
\widehat{w}_{I_{mi}} = \Phi_{I_{mi}}\varphi_{I_{mi}}\ddot{\theta}_{r_{i}}s_{\theta_{i}}
\widehat{w}_{b_{i}} = \Phi_{b_{i}}\varphi_{b_{i}}\dot{\theta}_{r_{i}}s_{\theta_{i}}
\widehat{w}_{\gamma_{i}} = -\Phi_{\gamma_{i}}\varphi_{\gamma_{i}}s_{\theta_{i}},$$
(33a)

where $\Phi_{(.)} \in R^{\beta \times \beta}$ is positive-definite gain matrix for adaptation. The following theorem proves the stability of the proposed control structure:

Theorem 2. The dynamics of link-joint subsystems in Equations (28a)–(28d), and the control laws described in Equations (31a) and (31b) with associated update adaptive laws in Equations (33a) and (33b) and the corresponding closed-loop dynamics presented in Equations (32a) and (32b) is stable in view of L_2 and L_{∞} stability introduced in Lemma 1 if

$$\mu_{d_i} = \mu_i, \tag{34a}$$

and

$$\widehat{k}_{s_{i}} = \alpha_{k_{i}} (\theta_{r_{i}} - q_{r_{i}}) \left[\left(\dot{\theta}_{r_{i}} - \dot{\theta}_{i} \right) - \left(\dot{q}_{r_{i}} - \dot{q}_{i} \right) \right]
\widehat{c}_{v_{i}} = \alpha_{c_{i}} \left(\dot{\theta}_{r_{i}} - \dot{q}_{r_{i}} \right) \left[\left(\dot{\theta}_{r_{i}} - \dot{\theta}_{i} \right) - \left(\dot{q}_{r_{i}} - \dot{q}_{i} \right) \right],$$
(34b)

where $\alpha_{(.)}$ is a scalar adaptation gain.

Proof. Consider the following Lyapunov's like-function for the link-joint subsystem as follows:

$$V_i = V_{l_i} + V_{j_i}, \tag{35a}$$

with

$$V_{l_{i}} = \frac{1}{2}m_{ii}s_{i}^{2} + \frac{1}{2}\widetilde{w}_{m_{i}}^{T}\varPhi_{m_{i}}^{-1}\widetilde{w}_{m_{i}} + \frac{1}{2}\widetilde{w}_{c_{i}}^{T}\varPhi_{c_{i}}^{-1}\widetilde{w}_{c_{i}} + \frac{1}{2}\widetilde{w}_{\Delta_{i}}^{T}\varPhi_{\Delta_{i}}^{-1}\widetilde{w}_{\Delta_{i}},$$
(35b)

1

$$V_{j_{i}} = \frac{1}{2} I_{mi} s_{\theta i}^{2} + \frac{1}{2} \widetilde{w}_{I_{mi}}^{T} \varPhi_{I_{mi}}^{-1} \widetilde{w}_{I_{mi}} + \frac{1}{2} \widetilde{w}_{b_{i}}^{T} \varPhi_{b_{i}}^{-1} \widetilde{w}_{b_{i}} + \frac{1}{2} \widetilde{w}_{\gamma_{i}}^{T} \varPhi_{\gamma_{i}}^{-1} \widetilde{w}_{\gamma_{i}} \\ + \frac{1}{2} \frac{\widetilde{k}_{s_{i}}^{2}}{\alpha_{k_{i}}} + \frac{1}{2} \frac{\widetilde{c}_{\gamma_{i}}^{2}}{\alpha_{c_{i}}} + \frac{1}{2} k_{s_{i}} [(\theta_{r_{i}} - \theta_{i}) - (q_{r_{i}} - q_{i})]^{2},$$
(35c)

where $\eta_{(.)}$ is a positive scalar adaptation gain. By taking the time derivative of Equation (35b) and substituting Equation (32a) into it, we obtain

$$\dot{V}_{l_{i}} = \frac{1}{2} (\dot{m}_{ii} - 2c_{ii})s_{i}^{2} - k_{i}s_{i}^{2} + s_{i}(\tau_{td_{i}} - \tau_{t_{i}}) - \sigma_{i}s_{i}\text{sgn}(s_{i}) + \varepsilon_{i}s_{i} - \widetilde{w}_{m_{i}}^{T} \left(\Phi_{m_{i}}^{-1}\dot{\hat{w}}_{m_{i}} + \varphi_{m_{i}}\ddot{q}_{r_{i}}s_{i} \right) - \widetilde{w}_{c_{i}}^{T} \left(\varphi_{c_{i}}\dot{q}_{r_{i}}s_{i} + \Phi_{c_{i}}^{-1}\dot{\hat{w}}_{c_{i}} \right) - \widetilde{w}_{\Delta_{i}}^{T} \left(\varphi_{\Delta_{i}}s_{i} + \Phi_{\Delta_{i}}^{-1}\dot{\hat{w}}_{\Delta_{i}} \right).$$
(36)

Using the passivity property with the adaptive laws in Equation (33a), Equation (36) becomes the following:

$$\dot{V}_{l_i} = -k_i s_i^2 + s_i (\tau_{td_i} - \tau_{t_i}) - \sigma_i s_i \operatorname{sgn}(s_i) + \varepsilon_i s_i.$$
(37)

In a similar manner, taking the time derivative for Equation (35c) and substituting Equation (32b) into it to get the following:

$$\dot{V}_{j_{i}} = -b_{i}s_{\theta_{i}}^{2} - k_{\theta_{i}}s_{\theta_{i}}^{2} - s_{\theta_{i}}(\tau_{td_{i}} - \tau_{t_{i}}) - \sigma_{\theta_{i}}s_{\theta_{i}}\mathrm{sgn}(s_{\theta_{i}}) + \varepsilon_{\theta_{i}}s_{\theta_{i}} - \widetilde{w}_{I_{mi}}^{T}\left(\Phi_{I_{mi}}^{-1}\hat{w}_{I_{mi}} + \varphi_{I_{mi}}\ddot{\theta}_{r_{i}}s_{\theta_{i}}\right) - \widetilde{w}_{b_{i}}^{T}\left(\varphi_{b_{i}}\dot{\theta}_{ri}s_{\theta_{i}} + \Phi_{b_{i}}^{-1}\hat{w}_{b_{i}}\right) - \widetilde{w}_{\gamma_{i}}^{T}\left(\varphi_{\gamma_{i}}s_{\theta_{i}} + \Phi_{\gamma_{i}}^{-1}\hat{w}_{\gamma_{i}}\right) - \frac{\widetilde{k}_{s_{i}}\dot{k}_{s_{i}}}{\alpha_{k_{i}}} - \frac{\widetilde{c}_{\nu_{i}}\dot{c}_{\nu_{i}}}{\alpha_{c_{i}}}.$$
(38)

Substituting Equation (33b) into the above equation results in the following:

$$\dot{V}_{j_i} = -b_i s_{\theta i}^2 - k_{\theta i} s_{\theta i}^2 - s_{\theta i} \left(\tau_{t_{d i}} - \tau_{t_i} \right) - \sigma_{\theta i} s_{\theta i} \operatorname{sgn}(s_{\theta i}) + \varepsilon_{\theta i} s_{\theta i}.$$
(39)

Thus, the time-derivative of Equation (35a) becomes the following:

$$\dot{V}_{i} = -k_{i}s_{i}^{2} + s_{i}(\tau_{t_{di}} - \tau_{t_{i}}) + \varepsilon_{i}s_{i} - \sigma_{i}s_{i}\mathrm{sgn}(s_{i}) - b_{i}s_{\theta_{i}}^{2} - k_{\theta_{i}}s_{\theta_{i}}^{2} + s_{\theta_{i}}(\tau_{td_{i}} - \tau_{t_{i}}) - \sigma_{\theta_{i}}s_{\theta_{i}}\mathrm{sgn}(s_{\theta_{i}}) + \varepsilon_{\theta_{i}}s_{\theta_{i}} - \frac{\widetilde{k}_{s_{i}}\dot{k}_{s_{i}}}{\alpha_{k_{i}}} - \frac{\widetilde{c}_{v_{i}}\dot{c}_{v_{i}}}{\alpha_{c_{i}}}.$$

$$(40)$$

Simplifying Equation (40) leads to the following:

$$\dot{V}_{i} = -k_{i}s_{i}^{2} - b_{i}s_{\theta_{i}}^{2} - k_{\theta_{i}}s_{\theta_{i}}^{2} - (s_{\theta_{i}} - s_{i})(\tau_{td_{i}} - \tau_{t_{i}}) - \sigma_{i}s_{i}\mathrm{sgn}(s_{i}) - \sigma_{\theta_{i}}s_{\theta_{i}}\mathrm{sgn}(s_{\theta_{i}}) + \varepsilon_{i}s_{i} + \varepsilon_{\theta_{i}}s_{\theta_{i}} - \frac{\widetilde{k}_{s_{i}}\dot{k}_{s_{i}}}{\alpha_{k_{i}}} - \frac{\widetilde{c}_{v_{i}}\dot{c}_{v_{i}}}{\alpha_{c_{i}}}.$$

$$(41)$$

Using Equations (28d) and (29b), we obtain the following:

$$\begin{aligned} \left(\tau_{td_i} - \tau_{t_i}\right) &= -\widetilde{c}_{\nu_i} \left(\dot{\theta}_i - \dot{q}_i\right) - \widetilde{k}_{s_i} (\theta_i - q_i) + c_{\nu_i} \left(s_{\theta_i} - s_{q_i}\right) \\ &+ k_{s_i} \left(\left(\theta_{r_i} - \theta_i\right) - \left(q_{r_i} - q_i\right) \right). \end{aligned}$$

$$(42)$$

Substituting Equation (42) into Equation (41) and using Equation (34b) results in the following:

$$\dot{V}_{i} = -k_{i}s_{i}^{2} - b_{i}s_{\theta_{i}}^{2} - k_{\theta_{i}}s_{\theta_{i}}^{2} - \sigma_{i}s_{i}\mathrm{sgn}(s_{i}) - \sigma_{\theta_{i}}s_{\theta_{i}}\mathrm{sgn}(s_{\theta_{i}}) + \varepsilon_{i}s_{i} + \varepsilon_{\theta_{i}}s_{\theta_{i}}.$$
(43)

Selecting the robust gains in Equation (43) such that

$$\sigma_{i} \ge |\varepsilon_{i}| + \rho_{i}$$

$$\sigma_{\theta_{i}} \ge |\varepsilon_{\theta_{i}}| + \rho_{\theta_{i}}$$
(44)

Equation (43) becomes the following:

$$\dot{V}_i \le -k_i s_i^2 - b_i s_{\theta_i}^2 - k_{\theta_i} s_{\theta_i}^2 - \rho_i s_i - \rho_{\theta_i} s_{\theta_i}.$$
(45)

This completes the proof.

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3.2.2. Cascade Position-Joint Torque Control. In this section, a different strategy is developed to track and stabilize the motion of flexible-joint bipedal robots. The same control laws developed in Equations (29a) and (29c) are used; however, the difference lies in computing the required angular velocity of the motor. The key idea is to integrate a torque error in the required velocity of the motor. Thus, the desired angular velocity of the motor is computed via

$$\widehat{c}_{v_i}\left(\dot{\theta}_{d_i} - \dot{q}_{r_i}\right) + \widehat{k}_{s_i}\left(\theta_{d_i} - q_{d_i}\right) = \tau_{td_i},\tag{46}$$

while the required motor velocity is determined as follows:

$$\dot{\theta}_{r_i} = \dot{\theta}_{d_i} + \lambda_{t_i} (\overline{\tau}_{td_i} - \overline{\tau}_{t_i}), \tag{47}$$

where λ_{t_i} is a constant gain and

$$\left(\overline{\tau}_{td_i} - \overline{\tau}_{t_i}\right) = \frac{\tau_{td_i} - \tau_{t_i}}{1 + \omega_i s},\tag{48}$$

with ω_i is a cutoff frequency and s denoting the Laplace variable. The following theorem proves the stability of the proposed control structure:

Theorem 3. The dynamics of link-joint subsystems in Equations (28a)–(28d), and the control laws described in Equations (31a), (46)–(48) with associated update adaptive laws in Equations (33a) and (33b) and the corresponding

closed-loop dynamics presented in Equations (32a) and (32b) is stable in view of L_2 and L_∞ stability introduced in Lemma 1 if

$$\mu_{d_i} = \mu_i, \tag{49a}$$

and

$$\widehat{k}_{s_i} = \Phi_{k_i} \left(\theta_{d_i} - q_{r_i} \right) \left[\left(\dot{\theta}_{d_i} - \dot{\theta}_i \right) - \left(\dot{q}_{r_i} - \dot{q}_i \right) \right]
\widehat{c}_{\nu_i} = \Phi_{c_i} \left(\dot{\theta}_{d_i} - \dot{q}_{r_i} \right) \left[\left(\dot{\theta}_{d_i} - \dot{\theta}_i \right) - \left(\dot{q}_{r_i} - \dot{q}_i \right) \right].$$
(49b)

Proof. Consider the following Lyapunov's like-function for the link-joint subsystem

$$V_i = V_{l_i} + V_{j_i},\tag{50a}$$

with

$$V_{l_{i}} = \frac{1}{2}m_{ii}s_{i}^{2} + \frac{1}{2}\widetilde{w}_{m_{i}}^{T}\boldsymbol{\Phi}_{m_{i}}^{-1}\widetilde{w}_{m_{i}} + \frac{1}{2}\widetilde{w}_{c_{i}}^{T}\boldsymbol{\Phi}_{c_{i}}^{-1}\widetilde{w}_{c_{i}} + \frac{1}{2}\widetilde{w}_{\Delta_{i}}^{T}\boldsymbol{\Phi}_{\Delta_{i}}^{-1}\widetilde{w}_{\Delta_{i}},$$
(50b)

$$V_{j_{i}} = \frac{1}{2} I_{mi} s_{\theta i}^{2} + \frac{1}{2} \widetilde{\omega}_{I_{mi}}^{T} \varPhi_{I_{mi}}^{-1} \widetilde{\omega}_{I_{mi}} + \frac{1}{2} \widetilde{\omega}_{b_{i}}^{T} \varPhi_{b_{i}}^{-1} \widetilde{\omega}_{b_{i}} + \frac{1}{2} \widetilde{\omega}_{\gamma_{i}}^{T} \varPhi_{\gamma_{i}}^{-1} \widetilde{\omega}_{\gamma_{i}} + \frac{\lambda_{ti}}{2} \omega_{i} (\tau_{td_{i}} - \tau_{ti})^{2} + \frac{1}{2} \frac{\widetilde{\kappa}_{c_{i}}^{2}}{\alpha_{c_{i}}} + \frac{1}{2} k_{s_{i}} [(\theta_{d_{i}} - \theta_{i}) - (q_{r_{i}} - q_{i})]^{2}.$$
(50c)

By taking the time derivative of Equation (50b) and substituting Equation (32a) into it, we obtain the following:

$$\dot{V}_{l_i} = \frac{1}{2} (\dot{m}_{ii} - 2c_{ii}) s_i^2 - k_i s_i^2 + s_i (\tau_{t_{di}} - \tau_{t_i}) - \sigma_i s_i \operatorname{sgn}(s_i) + \varepsilon_i s_i - \widetilde{w}_{m_i}^T \left(\Phi_{m_i}^{-1} \dot{\widehat{w}}_{m_i} + \varphi_{m_i} \, \ddot{q}_{r_i} s_i \right) \\ - \widetilde{w}_{c_i}^T \left(\varphi_{c_i} \dot{q}_{r_i} s_i + \Phi_{c_i}^{-1} \dot{\widehat{w}}_{c_i} \right) - \widetilde{w}_{\Delta_i}^T \left(\varphi_{\Delta_i} s_i + \Phi_{\Delta_i}^{-1} \dot{\widehat{w}}_{\Delta_i} \right).$$

$$(51)$$

Using the passivity property with the adaptive laws in Equation (33a), Equation (51) becomes the following:

$$\dot{V}_{l_i} = -k_i s_i^2 + s_i (\tau_{td_i} - \tau_{t_i}) - \sigma_i s_i \operatorname{sgn}(s_i) + \varepsilon_i s_i.$$
(52)

In a similar manner, taking the time derivative for Equation (50c) and substituting Equation (32b) into it to get the following:

$$\dot{V}_{j_{i}} = -b_{i}s_{\theta_{i}}^{2} - k_{\theta_{i}}s_{\theta_{i}}^{2} - s_{\theta_{i}}(\tau_{td_{i}} - \tau_{t_{i}}) - \sigma_{\theta_{i}}s_{\theta_{i}}\operatorname{sgn}(s_{\theta_{i}}) + \varepsilon_{\theta_{i}}s_{\theta_{i}} - \widetilde{w}_{I_{i}}^{T}\left(\Phi_{I_{mi}}^{-1}\dot{w}_{I_{mi}} + \varphi_{I_{mi}}\ddot{\theta}_{r_{i}}s_{\theta_{i}}\right) - \widetilde{w}_{b_{i}}^{T}\left(\varphi_{b_{i}}\dot{\theta}_{r_{i}}s_{\theta_{i}} + \Phi_{b_{i}}^{-1}\dot{\bar{w}}_{b_{i}}\right) - \widetilde{w}_{\delta_{i}}^{T}\left(\varphi_{\gamma_{i}}s_{\theta_{i}} + \Phi_{\gamma_{i}}^{-1}\dot{\bar{w}}_{\gamma_{i}}\right) + \lambda_{t_{i}}\omega_{i}(\tau_{td_{i}} - \tau_{t_{i}})\left(\dot{\tau}_{td_{i}} - \dot{\bar{\tau}}_{ti}\right) - \frac{\widetilde{k}_{s_{i}}\dot{k}_{s_{i}}}{\alpha_{k_{i}}} - \frac{\widetilde{c}_{v_{i}}\dot{\bar{c}}_{v_{i}}}{\alpha_{c_{i}}}.$$
(53)

Substituting Equation (33b) into the above equation results in the following:

Thus, the time-derivative of Equation (50a) becomes the following:

$$\dot{V}_{j_{i}} = -b_{i}s_{\theta_{i}}^{2} - k_{\theta_{i}}s_{\theta_{i}}^{2} - s_{\theta_{i}}\left(\tau_{td_{i}} - \tau_{t_{i}}\right) - \sigma_{\theta_{i}}s_{\theta_{i}}\operatorname{sgn}\left(s_{\theta_{i}}\right) + \varepsilon_{\theta_{i}}s_{\theta_{i}} + \lambda_{t_{i}}\omega_{i}\left(\tau_{td_{i}} - \tau_{t_{i}}\right)\left(\dot{\tau}_{td_{i}} - \dot{\tau}_{ti}\right) - \frac{\widetilde{k}_{s_{i}}\dot{k}_{s_{i}}}{\alpha_{k_{i}}} - \frac{\widetilde{c}_{v_{i}}\dot{c}_{v_{i}}}{\alpha_{c_{i}}}.$$

$$(54)$$

$$\dot{V}_{i} = -k_{i}s_{i}^{2} + s_{i}(\tau_{td_{i}} - \tau_{t_{i}}) + \varepsilon_{i}s_{i} - \sigma_{i}s_{i}\operatorname{sgn}(s_{i}) - b_{i}s_{\theta_{i}}^{2} - k_{\theta_{i}}s_{\theta_{i}}^{2} - s_{\theta_{i}}(\tau_{td_{i}} - \tau_{t_{i}}) - \sigma_{\theta_{i}}s_{\theta_{i}}\operatorname{sgn}(s_{\theta_{i}}) + \varepsilon_{\theta_{i}}s_{\theta_{i}} + \lambda_{t_{i}}\omega_{i}(\tau_{td_{i}} - \tau_{t_{i}})(\dot{\tau}_{td_{i}} - \dot{\tau}_{ti}) - \frac{\widetilde{k}_{s_{i}}\hat{k}_{s_{i}}}{\alpha_{k_{i}}} - \frac{\widetilde{c}_{v_{i}}\hat{c}_{v_{i}}}{\alpha_{c_{i}}}.$$
(55)

However,

$$s_{i}(\tau_{td_{i}} - \tau_{t_{i}}) - s_{\theta_{i}}(\tau_{td_{i}} - \tau_{t_{i}}) = -\lambda_{t_{i}}(\tau_{td_{i}} - \tau_{t_{i}})^{2} - \lambda_{t_{i}}\omega_{i}(\dot{\tau}_{td_{i}} - \dot{\tau}_{t_{i}})(\bar{\tau}_{td_{i}} - \bar{\tau}_{t_{i}}) + \tilde{c}_{\nu_{i}}(\dot{\theta}_{di} - \dot{q}_{r_{i}}) \Big[(\dot{\theta}_{d_{i}} - \dot{\theta}_{i}) - (\dot{q}_{r_{i}} - \dot{q}_{i}) \Big] + \tilde{k}_{s_{i}}(\theta_{d_{i}} - q_{r_{i}}) \Big[(\dot{\theta}_{d_{i}} - \dot{\theta}_{i}) - (\dot{q}_{r_{i}} - \dot{q}_{i}) \Big] - c_{\nu_{i}} \Big[(\dot{\theta}_{d_{i}} - \dot{\theta}_{i}) - (\dot{q}_{r_{i}} - \dot{q}_{i}) \Big]^{2} - k_{s_{i}} \Big[(\theta_{d_{i}} - \theta_{i}) - (q_{r_{i}} - q_{i}) \Big] \Big[(\dot{\theta}_{d_{i}} - \dot{\theta}_{i}) - (\dot{q}_{r_{i}} - \dot{q}_{i}) \Big].$$
(56)

Substituting Equation (56) into Equation (55) and using Equation (34b) results in the following:

$$\dot{V}_{i} = -k_{i}s_{i}^{2} - b_{i}s_{\theta_{i}}^{2} - k_{\theta_{i}}s_{\theta_{i}}^{2} - \sigma_{i}s_{i}\mathrm{sgn}(s_{i}) - \sigma_{\theta_{i}}s_{\theta_{i}}\mathrm{sgn}(s_{\theta_{i}}) + \varepsilon_{i}s_{i} + \varepsilon_{\theta_{i}}s_{\theta_{i}}.$$
(57)

Selecting the robust gains in Equation (57) such that

$$\sigma_{i} \ge |\varepsilon_{i}| + \rho_{i}$$

$$\sigma_{\theta_{i}} \ge |\varepsilon_{\theta_{i}}| + \rho_{\theta_{i}}.$$
(58)

Equation (57) becomes the following:

$$\dot{V}_i \leq -k_i s_i^2 - b_i s_{\theta_i}^2 - k_{\theta_i} s_{\theta_i}^2 - \rho_i s_i - \rho_{\theta_i} s_{\theta_i}.$$
(59)

This completes the proof. \Box

4. Results and Discussions

The section focuses on making several simulation experiments to investigate the validation of the proposed control architecture on a planar 6-link bipedal robot depicted in Figure 3 with physical parameters shown in Table 1. Four control levels are designed to stabilize bipedal walking: design of desired walking patterns, stabilization controller, inverse-kinematics control, and tracking control of desired joint trajectories. The desired walking patterns are selected based on a previous work [88] that proposes an algorithm to tune walking parameters to satisfy kinematic and dynamic constraints, such as singularity conditions at the knee joint, ZMP, and unilateral contact constraints, whereas an algebraic inverse kinematics algorithm is used for capturing the desired joint trajectories [31]. The simulation experiments are focused on stabilization and tracking controls. Figure 4 shows the stick diagram for the target biped in the SSP, where the desired COM and swing foot trajectories are developed according to Al-shuka et al. [88].

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TABLE 1: Physical parameters of the bipedal robot.

Bipedal parts	Moments of inertia (kgm ²)	Lengths (m)	Masses (kg)
Foot	0.016	0.3	2
Shank	0.06	0.45	3.6
Thigh	0.06	0.45	3.7
Trunk	0.145	0.45	10



FIGURE 4: Stick diagram for the simulated biped with desired COM and swing foot references.

4.1. Stabilization Control. In this subsection, several experiments are implemented to investigate the effectiveness of the proposed stabilization controller suggested in Section 3.1. As was already indicated, the COM serves as the stabilization control law's output variable and the ZMP serves as the control input. A constrained control should be considered due to the limits of the ZMP within the stance sole in the SSP. To highlight the strength of the proposed adaptive CTC, a comparison study is performed with the classical PID controller, considering the input saturation (Figure 5). The feedback gains used are selected, as shown in Table 2.

The desired COM reference with COM position error is shown in Figure 5. The control algorithm proposed in Section 3.1 is performed considering the control input saturation. The control input signal for the ZMP is shown in Figure 6 with and without saturation. The evolution of the ZMP signal with saturation input is necessary to avoid exceeding the stability margin limited by the support sole in the SSP. A simple comparative study with the PID considering the saturation effect is performed, and the superior of our control algorithm is clear concerning the COM position error.

4.2. Tracking Low-Level Control. This subsection is focused on low-level tracking control for the trajectories of the biped joints. A 6-link bipedal robot with full actuation is tested. The biped is provided with flexible joints that complicate the control problem. As discussed in Section 3.2, two control



FIGURE 5: Position error for the COM trajectory using two control strategies.

TABLE 2: Gains parameters used for simulation of stabilization control.

Control method	Gain values	
СТС	$\beta = 11, K_{p} = 20, K_{d} = 70, \Psi_{I} = I_{11}, \Psi_{\alpha} = 50I_{11}, \Psi_{\delta} = 50I_{11}, Q = I_{2}, P = \begin{bmatrix} 1.9179 & 0.0071 \\ 0.0071 & 0.0254 \end{bmatrix}$	
PID	Proportional gain $(p) = 14$, integral gain (I) = 60, derivative gain $(D) = 1$, filter coefficient $(N) = 754^*$	

*A low-pass filter with a time constant of (1/N) is added to the derivative term to filter out the noise.

methods are proposed: AAC-based position control and AAC-based cascade position-torque control. A comparison study is implemented to test the features of the proposed two control methods. The control gains are listed in Table 3. Figure 7 shows the position error for the proposed control methods, while Figure 8 shows the control inputs with saturated signals with torque limits ± 150 N.m. The AAC-based position control provides more precise tracking than the cascade position-torque method. This occurs since the torque control has noisy torque signals with higher orders.

5. Conclusions

This work proposes a multilevel control architecture for a bipedal robot governed by the ZMP balance criteria. The joint flexibility is considered complicating the control problem. Two-level control scheme is proposed: stabilization (balance control) and tracking joint control. The proposed stabilizer includes designing an adaptive CTC based on the function approximation technique, whereas two control methods are proposed for tracking the motion of the



FIGURE 6: The control ZMP input shows the stability region represented by the stance sole length.



FIGURE 7: Position error.

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TABLE 3: Control gains.

Control method*	Gain values
AAC-based position control	$k_i = 200, \ k_{\theta i} = 150, \ \Phi_{mi} = \Phi_{c_i} = \Phi_{I_{mi}} = \Phi_{b_i} = I_{11}, \ \Phi_{\Delta_i} = 100I_{11}, \ \Phi_{\gamma_i} = 75I_{11}, \ \lambda_i = 5, \ \lambda_{\theta i} = 200, \ \Phi_{k_i} = 50$
AAC-based cascade position-torque control	$k_i = 70, \ k_{\theta i} = 150, \ \Phi_{mi} = \Phi_{c_i} = \Phi_{I_{mi}} = \Phi_{b_i} = I_{11}, \ \Phi_{\Delta_i} = 100I_{11}, \ \Phi_{\gamma_i} = 75I_{11}, \ \lambda_i = 5, \ \lambda_{ti} = 500, \ \omega_i = 0.05, \ \Phi_{k_i} = 50$

*1. The approximation modeling error is assumed to be zero, and hence, the robust gains are neglected. 2. The damping coefficient in the flexible elements is set to zero.





flexible-joint bipedal robot: position control and cascade position-torque control. The control architecture can be extended to deal with high-level motion planning and inverse kinematic control. The following points are investigated in future work:

- (1) Future research will focus on 3D locomotion, inverse kinematics, and motion planning, incorporating camera sensor integration, obstacle avoidance, and redundancy in high-level control with real-time experiments.
- (2) Whole-body control with input saturations.
- (3) Flexible joints with variable impedance or stiffness.
- (4) Integrating the angular momentum with the stabilization controller.
- (5) The effect of trunk and upper-limb parts on balancing the robot.

Appendix

Proof of Lemma 1. Reformulate Equation (17) as follows:

$$y(t)^T \overline{P} y(t) \le -\dot{V} - \overline{\sigma}(t). \tag{A.1}$$

Integrate (A1) results in the following:

$$\int_{0}^{\infty} y(t)^{T} \overline{P} y(t) dt \leq -\int_{0}^{\infty} \dot{V} dt - \int_{0}^{\infty} \overline{\sigma}(t) dt, \qquad (A.2)$$

$$\leq V(0) - V(\infty) + \overline{\rho}, \tag{A.3}$$

$$\leq V(0) + \overline{\rho},\tag{A.4}$$

which indicates $y(t) \in L_2$.

By integrating Equation (17) and applying Equation (18), it yields $V(t) \leq V(0) + \overline{\rho}, \forall t > 0$, which indicates that $V(t) \in L_{\infty}$ holds. Define $a = \lambda_{\min}(\overline{Q})$. Based on Equation (16), it can be concluded that

$$\|\boldsymbol{x}(t)\|^2 \le \frac{2V}{a} < \infty. \tag{A.5}$$

 \square

holds for t > 0, yielding $x(t) \in L_{\infty}$.

Abbreviations

AAC:	Adaptive approximation control
COM:	Center of mass
COP:	Center of pressure
CTC:	Computed torque control
DOF:	Degrees-of-freedom
DSP:	Double-support phase
E–L:	Euler–Lagrange
ISS:	Input-to-state stability
LQR:	Linear quadratic regulator
PAAC:	Position-based adaptive approximation control
PD:	Proportional derivative control
PID:	Proportional-integral-derivative control
PTAAC:	Position-torque adaptive approximation control
SMC:	Sliding mode control
SSP:	Single-support phase
VDC:	Virtual decomposition control
ZMP:	Zero moment point.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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