Research Article

Optimal Connected Cruise Control Design with Time-Varying Leader Velocity and Delays

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With the development of intelligent transportation system (ITS), owing to its flexible connectivity structures and communication network topologies, connected cruise control (CCC), increasing the situation awareness of the autonomous vehicle without redesigning the other vehicles, is an advanced cruise control technology attracted extensive attention. However, due to the uncertain traffic environment and the movement of the connected vehicles, the leader speed is typically highly dynamic. In this paper, taking the uncertain time-varying leading vehicle velocity and communication delays into consideration, an optimal CCC algorithm is proposed for both near-static case and general dynamic control cases. First, the analysis for discrete-time error dynamics model of the longitudinal vehicle platoon is performed. Then, in order to minimize the error between the desired and actual states, a linear quadratic optimization problem is formulated. Subsequently, in near-static control case, an efficient algorithm is proposed to derive the solution of the optimization problem by two steps. Specifically, the online step calculates the optimal control scheme according to the current states and previous control signals, and the off-line step calculates the corresponding control gain through backward recursion. Then, the results are further extended to the general dynamic control case where the leader vehicle moves at an uncertain time-varying velocity. Finally, simulation results verify the effectiveness of the proposed CCC algorithm.

1. Introduction

In recent years, the increasing number of vehicles has led to serious problems of congestion, air pollution, and traffic accidents. Intelligent transportation system (ITS), as a system integrating key technologies in the fields of electronics, computing, sensing, robotics, control, signal processing, and communications, will provide a more coordinated and safer traffic network by reducing driving load, increasing traffic information, and strengthening route management [1–3]. One of the typical applications of ITS is cruise control system, which plays an important role in improving on-road safety by relieving drivers' burden [4–8].

Connected cruise control (CCC) has been recognized as an emerging cruise control technology that is significantly improving the reliability, stability, and safety of vehicles and

has recently attracted considerable attention in both academia and industry [9–11]. In particular, CCC allows the autonomous vehicle to benefit from V2V information broadcast by multiple vehicles in front without requiring those vehicles to be equipped with sensors and actuators. Human-driven vehicles in a platoon only need to broadcast their motion information to the communication network, without the need to install other equipment. Furthermore, the CCC system has a flexible network topology that allows each vehicle to communicate with nearby members using vehicle-to-vehicle (V2V) communications [12–14]. CCC vehicles can exploit V2V communications to collect real-time status information such as the distance, speed, and acceleration of multiple vehicles from vehicles ahead [15, 16]. Then, a suitable control strategy is proposed to provide safe and stable driving according to the information received. In general, there is evidence that CCC is

capable to meet the high requirements of future intelligent transportations, due to its flexible communication network topologies and connectivity structures [17–19].

Despite the merits mentioned above, various factors induced by wireless communication such as communication delays and connectivity topologies will cause the significant stability degradation of CCC system, which makes the controller design more challenging [20–22]. In [23–25], the influence of heterogeneous connectivity topologies on the dynamics of connected automated vehicles is investigated, and analytical conditions for stability are derived. To prevent the feasibility and efficiency of the CCC system from decreasing due to communication delays, the control design has been extensively concerned. In [15, 26], the authors propose relevant approaches to compensate for the communication delays of the CCC system. Molnár et al. [15] propose two predictor feedback control methods to deal with the destabilizing effect of the delay induced by the packet losses and sample-and-hold unit. In order to compensate for time delays including state delays and control input delays, a comprehensive predictive control method for automated vehicles is proposed by Liu and Li [26]. However, the works in [15, 26] are aimed at dealing with the specific delays. To promote the control performance and enhance the system stability, studies focus on the optimal control strategy of CCC system in the presence of communication delays [27, 28]. The work [27] derives the optimal feedback strategy to minimize the cost function in the infinite horizon considering both driver's reaction time and communication delays. In [28], in order to address the disturbances caused by stochastic communication delays, an optimal CCC control algorithm based on reverse iterative calculation is proposed. Unfortunately, although the above optimal control approaches take the uncertainty of wireless communication into consideration, they focus on near-static scenarios. However, the dynamic traffic environment and the movement of the controlled vehicle make the CCC system parameters a time-varying characteristic [29, 30]. Studies have shown that since the platoon is an interconnected coupling system, even if a small disturbance, the changing state of the leading vehicle may affect other vehicle or even amplify the error, which may trigger string instability of the system [31]. In other words, the traditional optimal control approaches which focus on static scenarios may lead to system instability and even safety issues when dealing with dynamic and complex traffic environments [32].

For the platoon control strategy under high dynamic characteristics, relevant research has been carried out [33–35]. In [36], the authors address the impact of external disturbances by introducing the stability of disturbance string, and design a controller track which proposed a control structure by estimating the dynamic parameters to make the heterogeneous CACC problem change into an error dynamics problem for each vehicle. Velocity disturbances are typically caused by environmental circumstances such as gust, rolling resistance, and ground friction [31, 37]. In [31], two distributed adaptive strategies are proposed for string stability of nonlinear vehicle following which take the uncertainty of nonlinear acceleration into consideration.

The authors in [37] propose a sampled-data adaptive dynamic programming strategy to handle the optimal control problem facing both the communication delays and the unpredictable changes in speed of the leading vehicle due to the driver behaviour and wind conditions. To handle the situation that the leading vehicle is moving at a time-varying velocity, Sun et al. [38] develop two coordinated tracking control schemes, integrating the distributed leader velocity observers, which is concerning with both constant and time-varying leader velocity cases, and then, the stability is proved by Lyapunov-Krasovskii method. Unfortunately, the research on optimal control design of discrete-time CCC systems in high dynamic scenarios in the presence of time delays is still not yet sufficient.

In this article, to tackle the above problems and challenges, for the discrete time domain multivehicle platoon with dynamic velocity of leading vehicle and communication delays, the optimal CCC design is comprehensively studied. We first formulate a linear discrete-time model of vehicle's error dynamics. Then, we solve the optimal longitudinal control problem of near-static case. Finally, we extend the results to the general time-varying velocity case. To summarize, this article makes the following contributions.

- (i) According to the analysis of the vehicle platoon error dynamics considering the high dynamic characteristics including both uncertain time-varying leader vehicle velocity and communication delays, we formulate a CCC system model in discrete time domain.
- (ii) In order to make the platoon reach the desired headway and velocity, we propose an optimization longitudinal control problem as a linear quadratic cost function according to error dynamics and then investigate the effects of both time-varying leader velocity and network-induced delays.
- (iii) To address both near-static case and general dynamic control case, we formulate an optimal control algorithm. In near-static case, we derive a linear optimal control strategy, in which the function coefficient can be obtained off-line by using a backward recursion. Then, we extend the results to the general time-varying velocity case where the leading vehicle moves with an uncertain time-varying velocity.
- (iv) Numerical simulation results corroborate the system effectiveness considering the effects of velocity uncertainty of leading vehicle and communication delay.

2. System Modeling and Optimization Problem Formulation

2.1. Vehicle Dynamics Analysis. In general, vehicles in the connected vehicular platoon can be classified into two categories, namely, the human-driven vehicles and the autonomous CCC vehicles. The leading vehicle of the platoon is a tracking target. The other human-driven vehicles can transmit motion

FIGURE 1: The multivehicle longitudinal platoon structure.

information to the CCC vehicle. The autonomous CCC vehicle equipped with a controller which can obtain information of headway and velocity from other vehicles using V2V communication.

As shown in Figure 1, consider total p vehicles in the whole longitudinal control system, including multiple q autonomous CCC vehicles, and then, the whole system can be split into q individual vehicular platoons. In particular, the first platoon is from the leading vehicle to the first CCC vehicle, which is consist of $m+1$ vehicles. We denote the first CCC vehicle as $\#m+1$ and the preceding human-driven vehicles as $m, \dots, 2, 1$, and the leading vehicle $\#1$ is moving at the determined desired speed. In the rest of $q-1$ platoons, there are two CCC vehicles in each platoon. The last CCC vehicle in the j th platoon is the leading vehicle of the $j+1$ th platoon. For example, as shown in Figure 1, the $m+1$ th CCC vehicle becomes the leading vehicle of the second platoon, and the vehicle $\#m+n+1$ is the last vehicle of the second platoon, which also is the leading vehicle of the third platoon.

For a given vehicular platoon, the dynamic equation for the headway and velocity of human-driven vehicles can be modelled as

$$\begin{aligned} \dot{p}_{j+1}(t) &= \gamma_{j+1} \left(p^d(h_j(t)) - p_{j+1}(t) \right) + \eta_{j+1} \left(p_j(t) - p_{j+1}(t) \right), \\ \dot{h}_j(t) &= p_j(t) - p_{j+1}(t), \end{aligned} \quad (1)$$

where p_j represents the velocity of the j th vehicle, h_j represents the headway, $\dot{p}_j$ denotes the derivative with respect to time t , γ_j and η_j are system parameters related to driver's behaviour, and the typical headway-based desired velocity policy $p^d(h)$ is given by

$$p^d(h) = \begin{cases} 0, & \text{if } 0 < h < h_{\min}, \\ \frac{p_{\max}}{2} \left(1 - \cos \left(\pi \frac{h - h_{\min}}{h_{\max} - h_{\min}} \right) \right), & \text{if } h_{\min} \leq h \leq h_{\max}, \\ p_{\max}, & \text{if } h > h_{\max}, \end{cases} \quad (2)$$

where $p_{\max}$ is the maximum velocity and $h_{\min}$ and $h_{\max}$ are the minimum and maximum headways, respectively.

The dynamic equation for the velocity, headway, and the control signal of the CCC vehicle can be written as

$$\begin{aligned} \dot{p}_j(t) &= u(t - \tau(t)), \\ \dot{h}_{j-1}(t) &= p_{j-1}(t) - p_j(t), \end{aligned} \quad (3)$$

where $u(t)$ represents the control signal to determine the acceleration of CCC vehicle and $\tau(t)$ denotes the communication delays caused by the characteristics of wireless V2V communications.

The target of the control system is to make each vehicle reach the desired headway $h^*(t)$ and desired velocity $p^*(t) = p^d(h^*(t))$. Therefore, the velocity error and the headway error can be formulated as

$$\begin{aligned} \tilde{p}(t) &= p(t) - p^*(t), \\ \tilde{h}(t) &= h(t) - h^*(t). \end{aligned} \quad (4)$$

Then, for the CCC vehicle, the error dynamics can be derived as

$$\begin{aligned} \dot{\tilde{p}}_j(t) &= u(t - \tau(t)), \\ \dot{\tilde{h}}_{j-1}(t) &= \tilde{p}_{j-1}(t) - \tilde{p}_j(t). \end{aligned} \quad (5)$$

In the first platoon, the leading vehicle is moving at the constant speed with a near-static state. In this case, the desired velocity keeps as constant that $p_1 = p^*$, and the desired headway $h^*(t) = h^*$.

$$\begin{aligned} \dot{\tilde{p}}_{j+1}(t) &= \gamma_{j+1} \left(f^* \dot{\tilde{h}}_j(t) - \tilde{p}_{j+1}(t) \right) + \eta_{j+1} \left(\tilde{p}_j(t) - \tilde{p}_{j+1}(t) \right), \\ \dot{\tilde{h}}_j(t) &= \tilde{p}_j(t) - \tilde{p}_{j+1}(t). \end{aligned} \quad (6)$$

In other platoons, the leading vehicle is running in a time-varying velocity with a high dynamic state. Therefore, for the human-driven vehicles, the error dynamics can be derived as

$$\begin{aligned}\dot{\tilde{p}}_{j+1}(t) &= \gamma_{j+1} \left(\dot{p}^d(h^*(t)) \tilde{h}_j(t) - \tilde{p}_{j+1}(t) \right) \\ &\quad + \eta_{j+1} \left(\tilde{p}_j(t) - \tilde{p}_{j+1}(t) \right) + \delta(t), \\ \dot{\tilde{h}}_j(t) &= \tilde{p}_j(t) - \tilde{p}_{j+1}(t),\end{aligned}\quad (7)$$

where $\delta(t)$ is the extra acceleration interference term introduced by the dynamic time-varying characteristics of $h^*(t)$.

The number of vehicles in each platoon independently. Assuming there are total n vehicles in the vehicular platoon and the first one is the leading vehicle, based on equation reference goes here (5) and (7), the error dynamics of the vehicles can be expressed as

$$\dot{\tilde{p}}_{j+1}(t) = \begin{cases} u(t - \tau(t)), & j = n - 1, \\ \gamma_{j+1} \left(\dot{p}^d(h^*(t)) \tilde{h}_j(t) - \tilde{p}_{j+1}(t) \right) \\ \quad + \eta_{j+1} \left(\tilde{p}_j(t) - \tilde{p}_{j+1}(t) \right) + \delta(t), & \text{others.} \end{cases}\quad (8)$$

2.2. Optimization Problem Formulation. In this subsection, the platoon model in discrete-time domain is proposed, and then, the optimal control problem is formulated as minimizing the state errors of CCC vehicle.

A new state vector is defined as

$$\tilde{x}(t) = [\tilde{h}_{n-1}(t), \tilde{p}_n(t), \tilde{h}_{n-2}(t), \tilde{p}_{n-1}(t), \dots, \tilde{h}_1(t), \tilde{p}_2(t)]^T. \quad (9)$$

Then, based on (8), the following vehicle-platoon model can be formulated.

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t - \tau(t)) + \delta(t), \quad (10)$$

where

$$A = \begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \alpha_{n-1} & \beta_{n-1} & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & \cdots & \cdots & 0 & \alpha_3 & \beta_3 & 0 & \gamma_3 \\ 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & \alpha_2 & \beta_2 \end{bmatrix}, \quad (11)$$

$$B = [0, 1, 0, \dots, 0]^T,$$

$$\delta(t) = [0, 0, 0, \delta_{n-1}(t), 0, \dots, \delta_2(t)]^T,$$

and here

$$\begin{aligned}\alpha_j &= \gamma_j \dot{p}^d(h^*(t)), \\ \beta_j &= -\gamma_j - \eta_j.\end{aligned}\quad (12)$$

The timing diagram for the CAV control is shown as in Figure 2. In each sampling period, the sampled vehicle states are firstly sent to the controller through the shared wireless communication network, and then, the control signal will be generated to actuate on the CAV to maintain the stability of its tracking. Then, the discrete-time dynamics corresponding to the k th sampling period $[kT, (k+1)T)$ is obtained as

$$x_{k+1} = A_k x_k + B_{1k} u_k + B_{2k} u_{k-1} + G_k, \quad (13)$$

where

$$\begin{aligned}A_k &= e^{A^T T}, \quad B_{1k} = \int_0^{T-\tau_k} e^{A^s} ds B, \quad B_{2k} = \int_{T-\tau_k}^T e^{A^s} ds B, \\ G_k &= \int_{kT}^{(k+1)T} e^{A[(k+1)T-s]} \delta(s) ds.\end{aligned}\quad (14)$$

The objective of the optimal CCC is to minimize the state errors. Thus, subject to the system dynamics in (13), the optimization problem can be formulated into the following form by using the traditional quadratic cost function.

$$\begin{aligned}\min \mathbb{E} &\left[x_N^T Q_0 x_N + \sum_{k=0}^{N-1} (x_k^T Q_0 x_k + u_k^T R u_k) \right], \\ \text{s.t. } x_{k+1} &= A_k x_k + B_{1k} u_k + B_{2k} u_{k-1} + G_k,\end{aligned}\quad (15)$$

where Q_0 and R are system-determined weight matrices, $\mathbb{E}$ is the expectation operator due to the stochastic nature of dynamic leader velocity, and N represents the number of sampling periods in a control length.

In (15), the term $u_k^T R u_k$ is the effect of CCC vehicle's acceleration. The weight R is typically set as $R = 1$, which can maintain the acceleration of the CCC smoothly. The term $x_k^T Q_0 x_k$ is the deviations of velocity and headway, and the diagonal matrix Q_0 is formulated as follow

$$Q_0 = \begin{bmatrix} q_{11} \\ q_{12} \\ q_{21} \\ \ddots \\ q_{nn} \end{bmatrix}. \quad (16)$$

where q_{11} and q_{12} are the CCC vehicle state error weights, and others are the weights on the state errors of human-driven vehicles. This diagonal matrix can be typically simplified as $q_{11} = q_{12} = 1$ and $q_{21} = q_{22} = \dots = q_{nn} = 0$ since only the CCC vehicle is controllable, so that

FIGURE 2: Timing diagram for CAV control.

$$Q_0 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad (17)$$

where I and 0 denote identity and zero matrices, respectively.

3. Optimal Controller Design

In this section, the optimal control strategy for the CCC vehicle is derived. First, a near-static case where the leading vehicle moves in a near-constant velocity is considered. Then, the results are extended to the general case where the leading vehicle that moves with a time-varying velocity.

3.1. Near-Static Control Strategy Design. When the leading vehicle is in a near-constant velocity, this situation can be seen as near-static case. In this case, the desired velocity keeps as constant that $p_1 = p^*$; the acceleration of leading vehicle is equal to zero, so that G_k is equal to zero. Therefore, the corresponding discrete-time system can be simplified as

$$x_{k+1} = A_k x_k + B_{1k} u_k + B_{2k} u_{k-1}, \quad (18)$$

Thus, the optimization problem can be written as the following equivalent form

$$\begin{aligned} \min \quad & x_N^T Q_0 x_N + \sum_{k=0}^{N-1} (x_k^T Q_0 x_k + u_k^T R u_k), \\ \text{s.t.} \quad & x_{k+1} = A_k x_k + B_{1k} u_k + B_{2k} u_{k-1}. \end{aligned} \quad (19)$$

Define

$$z_k = \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix}. \quad (20)$$

Then, the discrete-time dynamics can be derived as

$$z_{k+1} = W_k z_k + Y_k u_k, \quad (21)$$

where

$$W_k = \begin{bmatrix} A_k & B_{2k} \\ 0 & 0 \end{bmatrix}, Y_k = \begin{bmatrix} B_{1k} \\ I \end{bmatrix}. \quad (22)$$

Then, the optimization problem (15) can be simplified as

$$\begin{aligned} \min \quad & z_N^T \bar{Q} z_N + \sum_{k=0}^{N-1} \begin{bmatrix} z_k \\ u_k \end{bmatrix}^T \begin{bmatrix} \bar{Q} & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} z_k \\ u_k \end{bmatrix}, \\ \text{s.t.} \quad & z_{k+1} = W_k z_k + Y_k u_k. \end{aligned} \quad (23)$$

Theorem 1. For the near-static case, the optimal control problem (23) can be derived as

$$u_k^* = -L_k z_k, \quad k = 0, 1, \dots, N-1, \quad (24)$$

where the optimal control gain L_k is iteratively calculated by

$$\begin{aligned} L_k &= [Y_k^T S_{k+1} Y_k + R]^{-1} Y_k^T S_{k+1} W_k, \\ S_k &= W_k^T S_{k+1} W_k + \bar{Q} - L_k^T Y_k^T S_{k+1} W_k, \\ S_N &= \bar{Q}. \end{aligned} \quad (25)$$

Proof. The residual cost function can be defined as

$$V_k = z_N^T \bar{Q} z_N + \sum_{i=k}^{N-1} \begin{bmatrix} z_i \\ u_i \end{bmatrix}^T \begin{bmatrix} \bar{Q} & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} z_i \\ u_i \end{bmatrix}. \quad (26)$$

It can be observed that the residual cost function V_k is a part of the cost function. Minimizing V_k is equal to minimizing the cost function. Then, to obtain the optimal control

```

1  Step1: Off-line
2  Initialize  $\tilde{S}_N = \bar{Q}$ 
3  for  $k = N - 1 : -1 : 0$  do
4    Calculate  $\tilde{L}_k$  according to
5     $\tilde{L}_k = [\tilde{Y}_k^T \tilde{S}_{k+1} \tilde{Y}_k + R]^{-1} \tilde{Y}_k^T \tilde{S}_{k+1} \tilde{W}_k$ 
6    Calculate  $\tilde{S}_k$  according to
7     $\tilde{S}_k = \tilde{W}_k^T \tilde{S}_{k+1} \tilde{W}_k + \bar{Q} - \tilde{L}_k^T \tilde{Y}_k^T \tilde{S}_{k+1} \tilde{W}_k$ 
8  end
9  Step2: On-line
10 Initialize  $x_0, \tilde{u}_k^* = 0, k \leq 0$ .
11 for  $k = 0 : 1 : N - 1$  do
12   Update  $x_k$  and  $z_k = [x_k^T, (\tilde{u}_{k-1}^*)^T]^T$ .
13   Calculate  $\tilde{u}_k^*$  according to  $\tilde{u}_k^* = -\tilde{L}_k z_k$ .
14 end

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ALGORITHM 1: Optimal Control Design.

FIGURE 3: Headway of CCC vehicle in near-static control case.

gain L_k , the cost function can be iteratively calculated by the backwards recursion.

- (1) $k = N$: when $k = N$, V_k is the last term of the cost function:

$$V_N = z_N^T \bar{Q} z_N. \quad (27)$$

Set $S_N = \bar{Q}$, it can be obtained

$$V_N = z_N^T \bar{Q} z_N. \quad (28)$$

It is obvious that V_k is a quadratic function of z_N .

FIGURE 4: Velocity of CCC vehicle in near-static control case.

- (2) $k = N - 1$: V_{N-1} can be formulated into the following form:

$$V_{N-1} = V_N + \begin{bmatrix} z_{N-1} \\ u_{N-1} \end{bmatrix}^T \begin{bmatrix} \bar{Q} & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} z_{N-1} \\ u_{N-1} \end{bmatrix} = z_N^T S_N z_N + z_{N-1}^T \bar{Q} z_{N-1} + u_{N-1}^T R u_{N-1}. \quad (29)$$

Based on (23) and (26), V_{N-1} can be derived as

$$\begin{aligned} V_{N-1} &= z_N^T S_N z_N + z_{N-1}^T \bar{Q} z_{N-1} + u_{N-1}^T R u_{N-1} \\ &= [W_{N-1} z_{N-1} + Y_{N-1} u_{N-1}]^T S_N [W_{N-1} z_{N-1} + Y_{N-1} u_{N-1}] \\ &\quad + z_{N-1}^T \bar{Q} z_{N-1} + u_{N-1}^T R u_{N-1} \\ &= z_{N-1}^T [W_{N-1}^T S_N W_{N-1} + \bar{Q}] z_{N-1} + u_{N-1}^T [Y_{N-1}^T S_N W_{N-1} + R] u_{N-1} \\ &\quad + z_{N-1}^T W_{N-1}^T S_N Y_{N-1} u_{N-1} + u_{N-1}^T [Y_{N-1}^T S_N Y_{N-1} + R] u_{N-1}, \end{aligned} \quad (30)$$

Then, V_{N-1} can be further simplified as

$$V_{N-1} = \begin{bmatrix} z_{N-1} \\ u_{N-1} \end{bmatrix}^T \begin{bmatrix} P_{1,1} & P_{1,2} \\ P_{2,1} & P_{2,2} \end{bmatrix} \begin{bmatrix} z_{N-1} \\ u_{N-1} \end{bmatrix}, \quad (31)$$

where

$$\begin{aligned} P_{1,1} &= W_{N-1}^T S_N W_{N-1} + \bar{Q}, \\ P_{2,2} &= Y_{N-1}^T S_N Y_{N-1} + R, \\ P_{1,2} &= W_{N-1}^T S_N Y_{N-1}, \\ P_{2,1} &= P_{1,2}^T. \end{aligned} \quad (33)$$

FIGURE 5: Acceleration of CCC vehicle in near-static control case.

FIGURE 6: Headway of CCC vehicle in general control case.

Evidently, V_{N-1} is determined by u_{N-1} and z_{N-1} . Therefore, Equation (30) can be rewritten as

$$\begin{aligned} V_{N-1} &= z_{N-1}^T P_{1,1} z_{N-1} + u_{N-1}^T P_{2,1} z_{N-1} + z_{N-1}^T P_{1,2} u_{N-1} + u_{N-1}^T P_{2,2} u_{N-1} \\ &= P_{2,2} (u_{N-1} + P_{2,2}^{-1} P_{2,1} z_{N-1})^2 + z_{N-1}^T P_{1,1} z_{N-1} - P_{2,2}^{-1} (P_{2,1} z_{N-1})^2. \end{aligned} \quad (34)$$

To minimizing V_{N-1} , the optimal control strategy u_{N-1}^* can be derived as

$$u_{N-1}^* = -L_{N-1} z_{N-1}, \quad (35)$$

where

$$L_{N-1} = -P_{2,2}^{-1} P_{2,1} = [Y_{N-1}^T S_N Y_{N-1} + R]^{-1} Y_{N-1}^T S_N W_{N-1}. \quad (36)$$

According to u_{N-1}^* in (35), V_{N-1} is given by

$$V_{N-1} = z_{N-1}^T S_{N-1} z_{N-1}, \quad (37)$$

where

$$S_{N-1} = W_{N-1}^T S_N W_{N-1} + \bar{Q} - L_{N-1}^T Y_{N-1}^T S_N W_{N-1}. \quad (38)$$

(3) $k = N-2, N-3, \dots, 0$: when $i > k$, assuming V_i has the same quadratic form that

$$V_i = z_i^T S_i z_i. \quad (39)$$

According to (26) and (39), V_k is given by

$$V_i = z_i^T S_i z_i. \quad (40)$$

Similarly, since the form of (29) and (40) are the same, u_k can be derived as

$$u_k^* = -L_k z_k, \quad (41)$$

FIGURE 7: Velocity of CCC vehicle in general control case.

where

$$L_k = [Y_k^T S_{k+1} Y_k + R]^{-1} Y_k^T S_{k+1} W_k. \quad (42)$$

Then, V_k is formulated by

$$V_k = z_{k+1}^T S_{k+1} z_{k+1} + z_k^T \bar{Q} z_k + u_k^T R u_k = z_k^T S_k z_k. \quad (43)$$

From the above procedures, it is evident that the optimal control signal u_k^* is only determined by z_k , and L_k is derived by using a backward recursion based on (42). $\square$

3.2. General Control Strategy Design. In general case, the leading vehicle is moving in a time-varying velocity, which is dynamic to the following vehicles. Therefore, the optimization problem should consider the uncertain time-varying term G_k , which is typically assumed to be a stochastic variable with mean value zero and variance matrix σ_ω .

Then, based on the definition in (20), the general control problem can be rewritten as

$$\begin{aligned} \min \quad & \mathbb{E} \left[z_N^T \bar{Q} z_N + \sum_{k=0}^{N-1} \begin{bmatrix} z_k \\ u_k \end{bmatrix}^T \begin{bmatrix} \bar{Q} & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} z_k \\ u_k \end{bmatrix} \right], \\ \text{s.t.} \quad & z_{k+1} = W_k x_k + Y_k u_k + [G_k, 0]^T, \end{aligned} \quad (44)$$

where

$$W_k = \begin{bmatrix} A_k & B_{2k} \\ 0 & 0 \end{bmatrix}, Y_k = \begin{bmatrix} B_{1k} \\ I \end{bmatrix}. \quad (45)$$

Theorem 2. For the general case, the optimal control problem

(44) can be similarly deduced as

$$\tilde{u}_k^* = -\tilde{L}_k z_k, \quad k = 0, 1, \dots, N-1, \quad (46)$$

where $\tilde{L}_k$ is calculated iteratively by

$$\begin{aligned} \tilde{L}_k &= [\tilde{Y}_k^T \tilde{S}_{k+1} \tilde{Y}_k + R]^{-1} \tilde{Y}_k^T \tilde{S}_{k+1} \tilde{W}_k, \\ \tilde{S}_k &= \tilde{W}_k^T \tilde{S}_{k+1} \tilde{W}_k + \bar{Q} - \tilde{L}_k^T \tilde{Y}_k^T \tilde{S}_{k+1} \tilde{W}_k, \\ \tilde{S}_N &= \bar{Q}, \end{aligned} \quad (47)$$

and $\tilde{V}_k$ can be written as

$$\tilde{V}_k = z_k^T \tilde{S}_k z_k + \sum_{i=k+1}^{N-1} \text{tr}(\tilde{S}_i^{1,1} \sigma_\omega), \quad (48)$$

where $\text{tr}(\cdot)$ denotes the trace of the matrix, and $\tilde{S}_i^{1,1}$ represents the (1,1)th block of $\tilde{S}_i$ with the same size of δ_i .

Proof. According to the derivation process in the last subsection from (21) to (30), the results can be extended to the general dynamic case.

(1) $k = N-1$: $\tilde{V}_{N-1}$ can be derived as

$$\begin{aligned} \tilde{V}_{N-1} &= \mathbb{E} \left[z_N^T \tilde{S}_N z_N + z_{N-1}^T \bar{Q} z_{N-1} + u_{N-1}^T R u_{N-1} \right] \\ &= \begin{bmatrix} z_{N-1} \\ u_{N-1} \end{bmatrix}^T \begin{bmatrix} \tilde{P}_{1,1} & \tilde{P}_{1,2} \\ \tilde{P}_{2,1} & \tilde{P}_{2,2} \end{bmatrix} \begin{bmatrix} z_{N-1} \\ u_{N-1} \end{bmatrix} + \mathbb{E} \left[G_{N-1}^T \tilde{S}_N^{1,1} G_{N-1} \right] \\ &= \begin{bmatrix} z_{N-1} \\ u_{N-1} \end{bmatrix}^T \begin{bmatrix} \tilde{P}_{1,1} & \tilde{P}_{1,2} \\ \tilde{P}_{2,1} & \tilde{P}_{2,2} \end{bmatrix} \begin{bmatrix} z_{N-1} \\ u_{N-1} \end{bmatrix} + \text{tr}(\tilde{S}_N^{1,1} \sigma_\omega), \end{aligned} \quad (49)$$

where

$$\begin{aligned} \tilde{P}_{1,1} &= \tilde{W}_{N-1}^T \tilde{S}_N \tilde{W}_{N-1} + \bar{Q}, \\ \tilde{P}_{2,2} &= \tilde{Y}_{N-1}^T \tilde{S}_N \tilde{Y}_{N-1} + R, \\ \tilde{P}_{1,2} &= \tilde{W}_{N-1}^T \tilde{S}_N \tilde{Y}_{N-1}, \\ \tilde{P}_{2,1} &= \tilde{P}_{1,2}^T. \end{aligned} \quad (50)$$

Then, the optimal strategy $\tilde{u}_{N-1}^*$ is formulated by

$$\tilde{u}_{N-1}^* = -\tilde{L}_{N-1} z_{N-1}, \quad (51)$$

where

$$\tilde{L}_{N-1} = \tilde{P}_{2,2}^{-1} \tilde{P}_{2,1} = [\tilde{Y}_{N-1}^T \tilde{S}_N \tilde{Y}_{N-1} + R]^{-1} \tilde{Y}_{N-1}^T \tilde{S}_N \tilde{W}_{N-1}. \quad (52)$$

FIGURE 8: Acceleration of CCC vehicle in general control case.

FIGURE 9: Headway of the CCC vehicle of different algorithms in near-static control case.

FIGURE 10: Velocity of the CCC vehicle of different algorithms in near-static control case.

FIGURE 11: Acceleration of the CCC vehicle of different algorithms in near-static control case.

FIGURE 12: Headway of the CCC vehicle of different algorithms in general control case.

FIGURE 13: Velocity of the CCC vehicle of different algorithms in general control case.

FIGURE 14: Acceleration of the CCC vehicle of different algorithms in general control case.

and V_k can be formulated as

$$\tilde{V}_{N-1} = z_{N-1}^T \tilde{S}_{N-1} z_{N-1} + \sum_{i=k+1}^{N-1} \text{tr}(\tilde{S}_i^{1,1} \sigma_\omega). \quad (53)$$

(2) $k = N-2, \dots, 1, 0$: assuming $\tilde{V}_i$, $i \geq k+1$ also has the quadratic form that

$$\tilde{V}_k = z_k^T \tilde{S}_k z_k + \sum_{i=k+1}^{N-1} \text{tr}(\tilde{S}_i^{1,1} \sigma_\omega). \quad (54)$$

Similar to the derivation from (39) to (43), the optimal control strategy for general dynamic case can be given by

$$\tilde{u}_k^* = -\tilde{L}_k z_k, \quad (55)$$

where $\tilde{L}_k$ can be derived iteratively base on $\tilde{S}_{k+1}$ as in (44), and

$$\tilde{V}_k = z_k^T \tilde{S}_k z_k + \sum_{i=k+1}^{N-1} \text{tr}(\tilde{S}_i^{1,1} \sigma_\omega). \quad (56)$$

□

Thus, considering the uncertain item δ_k , the residual cost function has an extra cost item $\sum_{i=k+1}^N \text{tr}(\tilde{S}_i^{1,1} \sigma_\delta)$. Although time-varying items $\tilde{W}_k$, $\tilde{Y}_k$, and δ_k introduced by high dynamic features of leading vehicle exist, the control strategy

can also be acquired through backward recursion, which is summarized as Algorithm 1.

3.3. Simulation Results. In this section, simulation results of the connected vehicle system have been provided to demonstrate the effectiveness of the proposed backward iteration algorithm in different situations. As described in Figure 1, we consider a three-vehicle platoon. The initial states for the vehicles are set as $p_1(0) = 10(\text{m/s})$, $p_2(0) = 10(\text{m/s})$, $p_3(0) = 10(\text{m/s})$, $h_1(0) = 15(\text{m})$, and $h_2(0) = 15(\text{m})$. The sampling period T are chosen as 0.4 s and 0.2 s , and the communication delays are set as $\tau = 0.5T$. The other parameters are set as $p_{\max} = 35(\text{m/s})$, $h_{\min} = 5(\text{m})$, $h_{\max} = 35(\text{m})$, and $\gamma = \eta = 0.6$.

First, the headway, velocity, and acceleration of CCC vehicle in near-static control case with different network-induced delays are shown in Figures 3–5, respectively. We observe that the headway and velocity of CCC vehicle can be gradually close to the desired headway and velocity that the proposed two-step optimal control algorithm can ensure the system stability. It also can be seen that the network-induced delay causes the performance degradation of CCC system, because the time required for CCC vehicle to successfully catch up with the leading vehicle becomes larger with the network-induced delay increasing.

Then, the results are extended to the general control case that the leader velocity is time-varying. The headway h and velocity trajectories p of the CCC vehicle in this case are shown from Figures 6–8. It can be seen that the proposed optimal control algorithm is able to maintain the system stability with different network-induced delays. Similarly, when the time delay caused by the network becomes larger, it is much more difficult for CCC vehicles to achieve the required

□

headway and velocity. It indicates that the network-induced delays cause the degradation of system performance.

The performance comparisons between the proposed CCC algorithm and existing algorithm in [28] that ignores the dynamic feature of leading vehicle are shown from Figures 9–14. It can be observed from figures that the proposed CCC algorithm has faster convergence and better stability, which effectively compensates the influence of network-induced delays and dynamic leading velocity.

4. Conclusions

In this paper, we investigate an optimal control strategy design for the CCC system considering high dynamic characteristics including the uncertain time-varying acceleration of leading vehicle and network-induced delay. To control the CCC vehicle achieving the desired states, we propose a linear quadratic optimization problem with the objective of minimizing the errors between actual and desired states. Then, a backward recursion algorithm is developed to address this challenge in near-static case by two steps, and then, the algorithm is extended to general case. Simulation results have demonstrated the effectiveness of the proposed algorithm. More importantly, the simulation results provide that even if the leading vehicle moves as an uncertain time-varying acceleration, the control strategy can make the CCC vehicle quickly reach the desired states. Moreover, the network-induced delay will result in slower reactions of CCC vehicle.

In this paper, both the dynamic leader velocity and the communication delay are considered, and an interesting future work is to address the other constraints such as packet losses, channel interference, and communication bandwidth in the optimal CCC design. In addition, the cooperation between automated vehicles is not investigated in the proposed algorithm that only one CCC vehicle needs to be controlled in each individual longitudinal platoon. The cooperative adaptive cruise control for platoon merging with uncertain dynamics is another promising research topic in our future works [39, 40].

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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