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Research Article

Iterative Learning Control for Switched Systems with Sensor Saturation Constraints

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To solve trajectory tracking problem of switched system with sensor saturation, an iterative learning control algorithm is proposed. The method uses actual measurement error to modify the control variable of system on the premise that switched rule does not change along iteration axis, but it randomly changes along time axis. Moreover, by dealing with the saturation via diagonal matrix method, the convergence of the algorithm is strictly proved in the sense of λ-norm, and the convergence condition is derived. The algorithm can achieve complete tracking of desired trajectory in the finite time interval under the random switched rule, as iterations increase. The simulation example verifies the validity of the proposed algorithm.

1. Introduction

The switched system is one of the important hybrid systems, which consists of a set of continuous-time or discrete-time subsystems and switched logics on these subsystems [1], because it can describe many complex systems in the engineering field which cannot be described by any single model. Therefore, the switched system has been applied extensively in practical engineering, such as the power system [2], the chemical batch process [3], the traffic control system [4], the and multistage water tank liquid level control system [5].

Up to now, researchers mostly adopt the method of the dwell time and the average dwell time, the multiple Lyapunov function method, and the common Lyapunov function method to analyze the stability for the switched system [6–9]. In many practical engineering, the output tracking control problem exists extensively, such as the trajectory controls of the spacecraft [10] and the robots [11]. Therefore, aiming at the tracking control problem of switched systems, many scholars at home and abroad have proposed many advanced control methods [12–17]. By analyzing these methods, it can be found that these methods are mainly based on the Lyapunov function method to analyze the tracking performance of the switched system. However, for the general switched systems, the common Lyapunov function may not exist, or even though it exists, it is also difficult to construct. Recently, the construction of the common Lyapunov method is only for the specific systems [18]. The application of the multiple Lyapunov function can improve the flexibility of system analysis and design; however, the multiple Lyapunov function method needs to construct one or more Lyapunov functions for each subsystem and compare the Lyapunov function values of switched time points [19, 20], such that it needs some certain solution information of switched system, which is contrary to the thought of the Lyapunov direct method.

The iterative learning control is a control algorithm for realizing complete tracking of the desired trajectory in a finite time interval by using the tracking error of the system to continuously correct the control rule [21]. Because the iterative learning control does not depend on the accurate model information of the controlled system, and the controller has simple structure, it has achieved fruitful results after nearly 30 years of development [22–34]. At present, in the existing iterative learning control research results, most of the results only discuss nonswitched systems [22–26], while the results of switched systems are relatively few [27–30]. For example, in [27, 28], for linear and nonlinear continuous-time switched systems, an iterative learning control algorithm was adopted to completely track the desired trajectory of the system in a finite time interval. In [29] for discrete-time switched systems, the sufficient conditions for exponential stability of iterative learning control algorithm were derived by using the common two-dimensional Lyapunov function and multiple two-dimensional Lyapunov function. The paper in [30] studied the design of iterative learning controller for a class of switched systems with time-varying delays. By selecting the Lyapunov-Krasovskii functional, sufficient conditions for exponential stability of discrete-time switched systems were given.

From the aforementioned results, it is found that although the iterative learning control problem of switched systems is discussed in [27–30], the sensor saturation constraint problem is not considered. However, in the actual control system, due to the limitations of the physical characteristics of the control elements and industrial instruments, the sensor often exists in the phenomenon of saturation. The existence of this saturation phenomenon leads to uncontrollable nonlinear constraints in the system, which increases the difficulty of controller design. In the design of the controller, if the saturation constraint is not considered, it will lead to the deterioration of the system performance and even lead to instability of the system. In order to solve the problem of saturation constraints in practical engineering, some scholars have studied the iterative learning control problem of saturated constraint systems. The paper in [31] used the advantage of the optimal control in processing the input saturation constraint to propose first a kind of optimal iterative learning control algorithm based on the quadratic performance index. For the single-input single-output and multi-input multi-output systems with input saturation constraints, the convergence of the P-type iterative learning control algorithm was analyzed in [32] by using the composite energy function. In [33], the reference speed regulator was introduced into the iterative learning control, and the output tracking problem of the input saturation dynamic system was solved by using the method that the reference speed controller corrected the reference signal. In [34], an iterative learning control algorithm was designed by using the diagonal matrix method to deal with the saturation term, and the trajectory tracking problem of the linear system with sensor saturation constraints was solved. It is worth noting that although the iterative learning control problem of systems with saturation constraints is discussed in [31–34], the systems discussed are nonswitched systems, and the results cannot be directly applied to switched systems. Therefore, it will be a very meaningful problem to study the iterative learning control problem of switched systems with saturation constraints.

Based on the above analyses, for the continuous-time switched systems with sensor saturation constraints, this paper proposes an iterative learning control algorithm on the premise that the switched rule does not change along the iteration axis but it randomly changes along the time axis. The algorithm uses the actual measurement error to modify the control law and diagonal matrix method to deal with the output saturation term. Through rigorous analysis, we can prove that the proposed algorithm can achieve the complete tracking of the desired trajectory in a finite time interval under the condition of arbitrary fixed initial state and sensor saturation constraints and provide the sufficient conditions for the convergence of the algorithm in the sense of the λ-norm. Finally, the effectiveness of the proposed algorithm is verified by the simulation examples.

Compared with the existing results, the main advantages of this paper are summarized as follows:

(i) The algorithm in this paper only needs to use the previous control input and measurement error to obtain the current control input, so compared with the traditional control algorithm, the controller design in this paper does not depend on the mathematical model of the system and has the advantages of simple structure and easy engineering implementation. In addition, the proposed algorithm can enable the system output to completely track the desired trajectory in a finite time interval, rather than asymptotic tracking

(ii) Compared with [27–30], the algorithm in this paper considers the saturation constraint of the sensor in practical engineering and modifies the control input by using the actual measurement error with saturation constraint instead of the tracking error, and thus, the iterative learning control problem of switched systems with sensor saturation constraints is solved

(iii) In [29, 30], the convergence condition of linear matrix inequality form is derived by constructing Lyapunov function, while in this paper, the convergence condition of norm form is obtained by using contraction mapping method based on the idea of [27, 28], which avoids constructing the Lyapunov function and solving linear matrix inequality.

The rests of the paper are arranged as follows. Section 2 describes the problem and designs the controller. In section 3, the convergence of the control algorithm is proved, and the convergence conditions of the algorithm are given. In section 4, the simulation example is used to further verify the effectiveness of the proposed algorithm. Section 5 is the conclusion of this paper.

2. Problem Description

Before describing the problem, the notations used in this article are first explained.

Notations. $\mathbf{R}^n$, $\mathbf{R}^m$, and $\mathbf{R}^r$ are sets of n-dimension, m -dimension, and r-dimension real vectors, respectively. $\mathbf{R}^{m+}$ is a set of m-dimension positive real vectors, $\mathbf{R}$ is a set of 1-dimension real vectors, Z^+ is a set of positive integers, $\mathbf{R}^{n\times n}$ is an $n \times n$-dimension real matrix, max $(\cdot)$ represents the maximum, $\|\cdot\|$ represents the according norm, sup $\{\cdot\}$ represents the supremum, and diag $\{\cdot\}$ represents the diagonal matrix.

Consider the following class of switched system with output saturation, which has repetitive operation characteristic on the iteration axis:

$$\begin{cases} \dot{\boldsymbol{x}}_k(t) = \boldsymbol{A}_{\alpha(t)}\boldsymbol{x}_k(t) + \boldsymbol{B}_{\alpha(t)}\boldsymbol{u}_k(t) \\ \boldsymbol{y}_k(t) = \boldsymbol{C}_{\alpha(t)}\boldsymbol{x}_k(t) + \boldsymbol{D}_{\alpha(t)}\boldsymbol{u}_k(t) \\ \boldsymbol{z}_k(t) = \mathrm{sat}_{\boldsymbol{y}_0}(\boldsymbol{y}_k(t)), \end{cases} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^n$, $\boldsymbol{y}(t) \in \mathbf{R}^m$, $\boldsymbol{u}(t) \in \mathbf{R}^r$, and $\boldsymbol{z}(t) \in \mathbf{R}^m$ are state vector, output vector, control vector, and measurement output, respectively; $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ are system parameter matrices in appropriate dimensions; subscript k denotes iterative learning number; $\alpha(t)$ is random switched sequence of switched system with output saturation, defined by $\alpha(t)$: $(1, 2,\cdots) \rightarrow M = \{1, 2,\cdots,N\}$, i.e., $\alpha(t)$ is chosen value in the finite set $M = \{1, 2,\cdots,N\}$, in which $N > 1$ is number of subsystems contained in the switched system; $\alpha(t) = j$ means the jth subsystem is motivated, in which $t \in [0, T]$ is finite time interval; $\boldsymbol{y}_0 = [y_{01},\cdots,y_{0m}]^{\mathrm{T}} \in \mathbf{R}^{m+}$ is the threshold of saturation function. Then, the saturation function is defined by

$$z_i(t) = \mathrm{sat}_{y_{0i}}(y_i(t)) = \begin{cases} -y_{0i}, \; y_i(t) \le -y_{0i} \\ y_i(t), -y_{0i} < y_i(t) < y_{0i} \\ y_{0i}, y_i(t) \ge y_{0i}, \end{cases} \tag{2}$$

where $y_i(t)$ and $z_i(t)$ are controlled output $\boldsymbol{y}(t)$ and the ith component of measurement output, respectively.

The switched system with output saturation (1) is assumed as follows:

Assumption 1. The initial state of the system is identical when each iterative learning starts, i.e., $\boldsymbol{x}_k(0) = \boldsymbol{x}_0$.

Assumption 2. The desired trajectory $\boldsymbol{y}_{\mathrm{d}}(t)$ is unrelated with the iteration numbers, and there exists functions $\boldsymbol{x}_{\mathrm{d}}(t)$ and $\boldsymbol{u}_{\mathrm{d}}(t)$ satisfying

$$\begin{cases} \dot{\boldsymbol{x}}_{\mathrm{d}}(t) = \boldsymbol{A}_{\alpha(t)}\boldsymbol{x}_{\mathrm{d}}(t) + \boldsymbol{B}_{\alpha(t)}\boldsymbol{u}_{\mathrm{d}}(t) \\ \boldsymbol{y}_{\mathrm{d}}(t) = \boldsymbol{C}_{\alpha(t)}\boldsymbol{x}_{\mathrm{d}}(t) + \boldsymbol{D}_{\alpha(t)}\boldsymbol{u}_{\mathrm{d}}(t). \end{cases} \tag{3}$$

Assumption 3. Desired target trajectory $\boldsymbol{y}_{\mathrm{d}}(t)$ locates within the measurable range, i.e., $\boldsymbol{y}_{\mathrm{d}}(t)$ satisfies inequality $\max(|y_{\mathrm{d}i}(t)|) < y_{0i}$, such that the conclusion is obtained as

$$0 \le \boldsymbol{e}'(t)^{\mathrm{T}}\boldsymbol{e}(t) \le \boldsymbol{e}(t)^{\mathrm{T}}\boldsymbol{e}(t), \tag{4}$$

where $\boldsymbol{e}(t) = \boldsymbol{y}_{\mathrm{d}}(t) - \boldsymbol{y}(t)$ is the tracking error of the system. Since there exists sensor saturation in the system, the actual measurement error is $\boldsymbol{e}'(t) = \boldsymbol{y}_{\mathrm{d}}(t) - \boldsymbol{z}(t)$. Combined with (2) and (4), $\boldsymbol{e}'(t) = \mathrm{sat}_{\boldsymbol{y}_{\mathrm{d}}(t) \pm \boldsymbol{y}_0}(\boldsymbol{e}(t))$ is derived, where $\boldsymbol{y}_{\mathrm{d}}(t) \pm \boldsymbol{y}_0$ represents the upper and lower bounds of the saturation function, respectively.

For the switched system (1) with sensor saturation satisfying Assumptions 1–3, the following iterative learning control algorithm is designed as

$$\boldsymbol{u}_{k+1}(t) = \boldsymbol{u}_k(t) + \boldsymbol{L}\boldsymbol{e}_k'(t), \tag{5}$$

where k is an iterative learning number, $\boldsymbol{L}$ is a learning gain matrix, $\boldsymbol{e}_k'(t) = \boldsymbol{y}_{\mathrm{d}}(t) - \boldsymbol{z}_k(t)$ is a measurement error at the k th iteration, $\boldsymbol{y}_{\mathrm{d}}(t)$ is a desired signal, $\boldsymbol{z}_k(t) = \mathrm{sat}_{\boldsymbol{y}_0}(\boldsymbol{y}_k(t))$ is an output signal of the sensor with saturation constraints, and $\boldsymbol{y}_0$ is a saturation threshold. There are some following differences between this paper and the traditional iterative learning control algorithm.

(i) Due to the fact that traditional iterative learning control takes no consideration of the saturation constraint problem, the output signal can be fully fed back, such as constructing the tracking error $\boldsymbol{e}_k(t)$. Traditional iterative learning control algorithm is $\boldsymbol{u}_{k+1}(t) = \boldsymbol{u}_k(t) + \boldsymbol{L}\boldsymbol{e}_k(t)$, where $\boldsymbol{e}_k(t) = \boldsymbol{y}_{\mathrm{d}}(t) - \boldsymbol{y}_k(t)$ is the tracking error. $\boldsymbol{y}_{\mathrm{d}}(t)$ is the desired signal, and $\boldsymbol{y}_k(t)$ is the output at the kth iteration

(ii) In this paper, the iterative learning control algorithm takes into account the sensor saturation constraints, and the system output cannot be fully reflected by the sensor, so that we in fact get the measurement error $\boldsymbol{e}_k'(t) = \boldsymbol{y}_{\mathrm{d}}(t) - \boldsymbol{z}_k(t)$. That is, we use the actual measurement errors $\boldsymbol{e}_k'(t)$ affected by saturation constraints to construct an iterative learning control algorithm (5).

Nowadays, our control objective is to find what condition learning gain matrix satisfies to enable actual output of the system to completely track the desired trajectory in the finite time interval as iterations increase, when using algorithm (5) to control the switched system with output saturation.

3. Convergence Analysis

For convenience of the convergence proof of Algorithm (5), we first give the following definition and lemma.

Definition 4. λ-norm of vector function $\boldsymbol{h} : [0, \mathrm{T}] \rightarrow \mathbf{R}^n$ is defined by

$$\|\boldsymbol{h}\|_\lambda = \sup_{t \in [0,T]} \left\{ \mathrm{e}^{-\lambda t} \|\boldsymbol{h}(t)\| \right\}, (\lambda > 0). \tag{6}$$

Lemma 5. *If $\forall i \in Z^+$ satisfies the inequality $|z_{i+1}| \le \theta_i |z_i| + |\delta_i|$, such that $\lim_{i\to\infty} |z_i| = 0$, when $\lim_{i\to\infty} |\delta_i| = 0$, where both $z_i \subset R$ and $\delta_i \subset R$ are real sequences, constant θ is required as $0 < \theta_i \le \theta < 1$, and $i \in Z^+$. See proof in [27].*

Lemma 6. *For matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$, if $\|\boldsymbol{I} - \boldsymbol{P}\|_1 < 1$, $\|\boldsymbol{I} - \boldsymbol{P}\boldsymbol{Q}\|_1 < 1$ holds, where $\boldsymbol{Q} = \mathrm{diag}\{q_1,\cdots,q_n\}$, $\boldsymbol{P} \in \mathbf{R}^{n\times n}$ in which $0 < q_i \le 1$, $i = 1, \cdots, n$, and $\boldsymbol{I}$ denotes n dimensional identity matrix. See proof in [34].*

And for the sake of convenience, we can provide each subsystem of the switched system (1) that runs only once in whole time interval $[0, T]$, and the random switched rule is

$$\alpha(t) = j = \begin{cases} 1, t \in [0, t_1] \\ 2, t \in [t_1, t_2] \\ \vdots \\ N, t \in [t_{N-1}, T]. \end{cases} \tag{7}$$

Here, the switched system (1) with sensor saturation is rewritten as

$$\begin{cases} \dot{\boldsymbol{x}}_k(t) = \boldsymbol{A}_j \boldsymbol{x}_k(t) + \boldsymbol{B}_j \boldsymbol{u}_k(t) \\ \boldsymbol{y}_k(t) = \boldsymbol{C}_j \boldsymbol{x}_k(t) + \boldsymbol{D}_j \boldsymbol{u}_k(t) \\ \boldsymbol{z}_k(t) = \mathrm{sat}_{y_0}(\boldsymbol{y}_k(t)), \end{cases} \tag{8}$$

where $j \in \{1, 2, \cdots, N\}$. Then, the state response of system (8) is shown as the following N stages:

$$\begin{cases} \boldsymbol{x}_k(t) = \mathrm{e}^{\boldsymbol{A}_1(t-t_0)} \boldsymbol{x}_k(t_0) \\ + \int_{t_0}^{t} \mathrm{e}^{\boldsymbol{A}_1(t-\tau)} \boldsymbol{B}_1 \boldsymbol{u}_k(\tau) \mathrm{d}\tau, t \in [0, t_1] \\ \boldsymbol{x}_k(t) = \mathrm{e}^{\boldsymbol{A}_2(t-t_1)} \boldsymbol{x}_k(t_1) \\ + \int_{t_1}^{t} \mathrm{e}^{\boldsymbol{A}_2(t-\tau)} \boldsymbol{B}_2 \boldsymbol{u}_k(\tau) \mathrm{d}\tau, t \in [t_1, t_2] \\ \vdots \\ \boldsymbol{x}_k(t) = \mathrm{e}^{\boldsymbol{A}_N(t-t_{N-1})} \boldsymbol{x}_k(t_{N-1}) \\ + \int_{t_{N-1}}^{t} \mathrm{e}^{\boldsymbol{A}_N(t-\tau)} \boldsymbol{B}_N \boldsymbol{u}_k(\tau) \mathrm{d}\tau, t \in [t_{N-1}, T]. \end{cases} \tag{9}$$

If $t_0 = 0$ and $t_N = T$, (9) is rewritten as

$$\boldsymbol{x}_k(t) = \mathrm{e}^{\boldsymbol{A}_j(t-t_{j-1})} \boldsymbol{x}_k(t_{j-1}) + \int_{t_{j-1}}^{t} \mathrm{e}^{\boldsymbol{A}_j(t-\tau)} \boldsymbol{B}_j \boldsymbol{u}_k(\tau) \mathrm{d}\tau, t \in [t_{j-1}, t_j], \ j = 1, 2, \cdots, N. \tag{10}$$

Remark 7. From (9), we may know that two differences in iterative learning control of switched system and non-switched system lie in the following: first, the state response of the switched system not only depends on the parameters of system matrix and initial condition but also relates to the switched sequence of subsystems and time; second, the initial condition in Assumption 1 is applicable to only the first subsystem rather than other subsystems.

Remark 8. Although (10) is obtained provided that switched rule $\alpha(t)$ of switched system is (7), it is easy to find from (9) that state response of the switched system is still described by (10), when $\alpha(t)$ is any other switched rule, only different in the parameters value of system matrix in the corresponding time.

According to above analysis, we can draw following conclusions.

Theorem 9. *For the switched system (1) with sensor saturation, under the condition satisfying Assumptions 1–3, we use iterative learning control algorithm (5) to control system (1). If learning gain satisfies*

$$\left\| \boldsymbol{I} - \boldsymbol{D}_j \boldsymbol{L} \boldsymbol{\Gamma}_k(t) \right\|_1 = \rho_j < 1, \ (j = 1, 2, \cdots, N), \tag{11}$$

then system output is uniformly convergent to desired trajectory in the finite time interval $[0, T]$ as iterations increase, i.e., $\boldsymbol{y}_k(t) \to \boldsymbol{y}_d(t)$ holds as $k \to \infty$.

Proof. For convenience, the diagonal matrix is used to handle the measurement error, i.e., actual measurement error $\boldsymbol{e}'_k(t)$ denotes $\boldsymbol{e}'_k(t) = \boldsymbol{\Gamma}_k(t) \boldsymbol{e}_k(t)$, where $\boldsymbol{\Gamma}_k(t) = \mathrm{diag}\{\gamma_{k1}, \cdots, \gamma_{km}\}$ is a diagonal matrix, and elements of the diagonal matrix are

$$\gamma_{ki}(t) = \begin{cases} \dfrac{y_{\mathrm{d}i}(t) - y_{0i}}{e_{ki}(t)}, e_{ki}(t) < y_{\mathrm{d}i} - y_{0i} \\ 1, y_{\mathrm{d}i} - y_{0i} \le e_{ki}(t) \le y_{\mathrm{d}i} + y_{0i} \\ \dfrac{y_{\mathrm{d}i}(t) + y_{0i}}{e_{ki}(t)}, e_{ki}(t) > y_{\mathrm{d}i} + y_{0i}. \end{cases} \tag{12}$$

From Assumption 3, it is easy to find $0 < \gamma_{ki}(t) \le 1$ and $1 \le i \le m$.

Consider the tracking error at $k+1$th iteration as

$$\begin{aligned} \boldsymbol{e}_{k+1}(t) &= \boldsymbol{y}_{\mathrm{d}}(t) - \boldsymbol{y}_{k+1}(t) = \boldsymbol{e}_k(t) - [\boldsymbol{y}_{k+1}(t) - \boldsymbol{y}_k(t)] \\ &= \boldsymbol{e}_k(t) - \boldsymbol{C}_j(\boldsymbol{x}_{k+1}(t) - \boldsymbol{x}_k(t)) - \boldsymbol{D}_j(\boldsymbol{u}_{k+1}(t) - \boldsymbol{u}_k(t)) \\ &= \boldsymbol{e}_k(t) - \boldsymbol{C}_j(\boldsymbol{x}_{k+1}(t) - \boldsymbol{x}_k(t)) - \boldsymbol{D}_j \boldsymbol{L} \boldsymbol{e}'_k(t) \\ &= (\boldsymbol{I} - \boldsymbol{D}_j \boldsymbol{L} \boldsymbol{\Gamma}_k(t)) \boldsymbol{e}_k(t) - \boldsymbol{C}_j(\boldsymbol{x}_{k+1}(t) - \boldsymbol{x}_k(t)) \\ &= (\boldsymbol{I} - \boldsymbol{D}_j \boldsymbol{L} \boldsymbol{\Gamma}_k(t)) \boldsymbol{e}_k(t) - \boldsymbol{C}_j \delta \boldsymbol{x}_k(t). \end{aligned} \tag{13}$$

where $\delta \boldsymbol{x}_k(t) = \boldsymbol{x}_{k+1}(t) - \boldsymbol{x}_k(t)$.

Next, we individually discuss the convergence in the time intervals $[0, t_1]$, $[t_1, t_2]$, $\cdots$, $[t_{N-1}, T]$.

(1) Running the first subsystem of the switched system in the run time $t \in [0, t_1]$, from (10), we can get

$$\boldsymbol{x}_k(t) = \mathrm{e}^{\boldsymbol{A}_1(t)} \boldsymbol{x}_k(0) + \int_0^t \mathrm{e}^{\boldsymbol{A}_1(t-\tau)} \boldsymbol{B}_1 \boldsymbol{u}_k(\tau) \mathrm{d}\tau, \tag{14}$$

$$\boldsymbol{x}_{k+1}(t) = \mathrm{e}^{\boldsymbol{A}_1(t)} \boldsymbol{x}_{k+1}(0) + \int_0^t \mathrm{e}^{\boldsymbol{A}_1(t-\tau)} \boldsymbol{B}_1 \boldsymbol{u}_{k+1}(\tau) \mathrm{d}\tau. \tag{15}$$

Subtracting (14) from (15), and again according to

Assumption 1, we obtain

$$\begin{aligned}\delta \boldsymbol{x}_k(t) &= \boldsymbol{x}_{k+1}(t) - \boldsymbol{x}_k(t) = \int_0^t \mathrm{e}^{\boldsymbol{A}_1(t-\tau)}\boldsymbol{B}_1(\boldsymbol{u}_{k+1}(\tau) - \boldsymbol{u}_k(\tau))\mathrm{d}\tau \\ &= \int_0^t \mathrm{e}^{\boldsymbol{A}_1(t-\tau)}\boldsymbol{B}_1\boldsymbol{L}\boldsymbol{\Gamma}_k(\tau)\boldsymbol{e}_k(\tau)\mathrm{d}\tau.\end{aligned} \tag{16}$$

Substituting (16) into (13) yields

$$\boldsymbol{e}_{k+1}(t) = (\boldsymbol{I} - \boldsymbol{D}_1\boldsymbol{L}\boldsymbol{\Gamma}_k(t))\boldsymbol{e}_k(t) - \int_0^t \boldsymbol{C}_1\mathrm{e}^{\boldsymbol{A}_1(t-\tau)}\boldsymbol{B}_1\boldsymbol{L}\boldsymbol{\Gamma}_k(\tau)\boldsymbol{e}_k(\tau)\mathrm{d}\tau. \tag{17}$$

Taking the 1-norm, we get

$$\begin{aligned}\|\boldsymbol{e}_{k+1}(t)\|_1 &\le \|\boldsymbol{I} - \boldsymbol{D}_1\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1\|\boldsymbol{e}_k(t)\|_1 \\ &\quad + \int_0^t \left\|\boldsymbol{C}_1\mathrm{e}^{\boldsymbol{A}_1(t-\tau)}\boldsymbol{B}_1\boldsymbol{L}\boldsymbol{\Gamma}_k(\tau)\right\|_1 \|\boldsymbol{e}_k(\tau)\|_1\mathrm{d}\tau \\ &\le \|\boldsymbol{I} - \boldsymbol{D}_1\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1\|\boldsymbol{e}_k(t)\|_1 + b_1\int_0^t \|\boldsymbol{e}_k(\tau)\|_1\mathrm{d}\tau,\end{aligned} \tag{18}$$

where $b_1 = \sup_{t\in[0,t_1]} \|\boldsymbol{C}_1\mathrm{e}^{\boldsymbol{A}_1 t}\boldsymbol{B}_1\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1$. Multiplying the two sides of (18) by $\mathrm{e}^{-\lambda t}$, we obtain

$$\begin{aligned}\mathrm{e}^{-\lambda t}\|\boldsymbol{e}_{k+1}(t)\|_1 &\le \|\boldsymbol{I} - \boldsymbol{D}_1\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1\mathrm{e}^{-\lambda t}\|\boldsymbol{e}_k(t)\|_1 \\ &\quad + b_1\int_0^t \mathrm{e}^{-\lambda(t-\tau)}\mathrm{e}^{-\lambda\tau}\|\boldsymbol{e}_k(\tau)\|_1\mathrm{d}\tau.\end{aligned} \tag{19}$$

Again from the definition of λ-norm, we know

$$\|\boldsymbol{e}_{k+1}\|_\lambda \le \left(\|\boldsymbol{I} - \boldsymbol{D}_1\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1 + b_1\frac{1-\mathrm{e}^{-\lambda t_1}}{\lambda}\right)\|\boldsymbol{e}_k\|_\lambda. \tag{20}$$

From Condition (11) of Theorem 9, when λ is chosen sufficiently large, we have

$$\|\boldsymbol{I} - \boldsymbol{D}_1\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1 + b_1\frac{1-\mathrm{e}^{-\lambda t_1}}{\lambda} = \bar{\rho}_1 < 1. \tag{21}$$

Therefore, according to (20) and (21), we get

$$\lim_{k\to\infty}\|\boldsymbol{e}_k\|_\lambda = 0. \tag{22}$$

$\forall t \in [0, t_1]$ then yields

$$\|\boldsymbol{e}_k(t)\| = \mathrm{e}^{-\lambda t}\mathrm{e}^{\lambda t}\|\boldsymbol{e}_k(t)\| \le \mathrm{e}^{\lambda t}\|\boldsymbol{e}_k\|_\lambda \le \mathrm{e}^{\lambda t_1}\|\boldsymbol{e}_k\|_\lambda. \tag{23}$$

Further, from

$$\sup_{t\in[0,t_1]}\|\boldsymbol{e}_k(t)\|_1 \le \mathrm{e}^{\lambda t_1}\|\boldsymbol{e}_k\|_\lambda, \tag{24}$$

we can derive

$$\lim_{k\to\infty}\sup_{t\in[0,t_1]}\|\boldsymbol{e}_k(t)\|_1 = 0. \tag{25}$$

It implies that for subsystem 1 of the switched system, $\lim_{k\to\infty}\boldsymbol{y}_k(t) = \boldsymbol{y}_\mathrm{d}(t)$ holds in the time interval $[0, t_1]$, if learning gain of algorithm (5) satisfies $\|\boldsymbol{I} - \boldsymbol{D}_1\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1 = \rho_1 < 1$.

(2) Running the second subsystem in the run time $t \in [t_1, t_2]$, from (10), we can also get

$$\boldsymbol{x}_k(t) = \mathrm{e}^{\boldsymbol{A}_2(t-t_1)}\boldsymbol{x}_k(t_1) + \int_{t_1}^t \mathrm{e}^{\boldsymbol{A}_2(t-\tau)}\boldsymbol{B}_2\boldsymbol{u}_k(\tau)\mathrm{d}\tau, \tag{26}$$

$$\boldsymbol{x}_{k+1}(t) = \mathrm{e}^{\boldsymbol{A}_2(t-t_1)}\boldsymbol{x}_{k+1}(t_1) + \int_{t_1}^t \mathrm{e}^{\boldsymbol{A}_2(t-\tau)}\boldsymbol{B}_2\boldsymbol{u}_{k+1}(\tau)\mathrm{d}\tau. \tag{27}$$

Subtracting (26) from (27) yields

$$\begin{aligned}\delta\boldsymbol{x}_k(t) &= \boldsymbol{x}_{k+1}(t) - \boldsymbol{x}_k(t) = \mathrm{e}^{\boldsymbol{A}_2(t-t_1)}(\boldsymbol{x}_{k+1}(t_1) - \boldsymbol{x}_k(t_1)) \\ &\quad + \int_{t_1}^t \mathrm{e}^{\boldsymbol{A}_2(t-\tau)}\boldsymbol{B}_2(\boldsymbol{u}_{k+1}(\tau) - \boldsymbol{u}_k(\tau))\mathrm{d}\tau = \mathrm{e}^{\boldsymbol{A}_2(t-t_1)}(\boldsymbol{x}_{k+1}(t_1) \\ &\quad - \boldsymbol{x}_k(t_1)) + \int_{t_1}^t \mathrm{e}^{\boldsymbol{A}_2(t-\tau)}\boldsymbol{B}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(\tau)\boldsymbol{e}_k(\tau)\mathrm{d}\tau.\end{aligned} \tag{28}$$

Again substituting (28) into (13), we have

$$\begin{aligned}\boldsymbol{e}_{k+1}(t) &= (\boldsymbol{I} - \boldsymbol{D}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(t))\boldsymbol{e}_k(t) - \boldsymbol{C}_2(\boldsymbol{x}_{k+1}(t) - \boldsymbol{x}_k(t)) \\ &= (\boldsymbol{I} - \boldsymbol{D}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(t))\boldsymbol{e}_k(t) - \boldsymbol{C}_2\mathrm{e}^{\boldsymbol{A}_2(t-t_1)}(\boldsymbol{x}_{k+1}(t_1) \\ &\quad - \boldsymbol{x}_k(t_1)) - \int_{t_1}^t \boldsymbol{C}_2\mathrm{e}^{\boldsymbol{A}_2(t-\tau)}\boldsymbol{B}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(\tau)\boldsymbol{e}_k(\tau)\mathrm{d}\tau.\end{aligned} \tag{29}$$

Taking the 1-norm for both sides of (29), we get

$$\begin{aligned}\|\boldsymbol{e}_{k+1}(t)\|_1 &\le \|\boldsymbol{I} - \boldsymbol{D}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1\|\boldsymbol{e}_k(t)\|_1 \\ &\quad + \int_{t_1}^t \left\|\boldsymbol{C}_2\mathrm{e}^{\boldsymbol{A}_2(t-\tau)}\boldsymbol{B}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(\tau)\right\|_1\|\boldsymbol{e}_k(\tau)\|_1\mathrm{d}\tau \\ &\quad + \left\|\boldsymbol{C}_2\mathrm{e}^{\boldsymbol{A}_2(t-t_1)}\right\|_1|\delta\boldsymbol{x}_k(t_1)| \le \|\boldsymbol{I} - \boldsymbol{D}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1\|\boldsymbol{e}_k(t)\|_1 \\ &\quad + b_2\int_{t_1}^t \|\boldsymbol{e}_k(\tau)\|_1\mathrm{d}\tau + a_1|\delta\boldsymbol{x}_k(t_1)|,\end{aligned} \tag{30}$$

where $b_2 = \sup_{t\in[t_1,t_2]}\|\boldsymbol{C}_2\mathrm{e}^{\boldsymbol{A}_2 t}\boldsymbol{B}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1$, $a_1 = \|\boldsymbol{C}_2\mathrm{e}^{\boldsymbol{A}_2(t-t_1)}\|_1$.

Multiplying the two sides of (30) by $\mathrm{e}^{-\lambda t}$, and according to the definition of λ-norm, we can obtain

$$\|\boldsymbol{e}_{k+1}\|_\lambda \le \left(\|\boldsymbol{I} - \boldsymbol{D}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1 + b_2\frac{1-\mathrm{e}^{-\lambda t_2}}{\lambda}\right)\|\boldsymbol{e}_k\|_\lambda + a_1|\delta\boldsymbol{x}_k(t_1)|. \tag{31}$$

Similarly, $\|\boldsymbol{I}-\boldsymbol{D}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1 < 1$ holds since $\|\boldsymbol{I}-\boldsymbol{D}_2\boldsymbol{L}\|_1 < 1$. There exists a sufficiently large λ such that

$$\|\boldsymbol{I}-\boldsymbol{D}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1 + b_2\frac{1-\mathrm{e}^{-\lambda t_2}}{\lambda} = \bar{\rho}_2 < 1. \tag{32}$$

Thus, (31) is rewritten as

$$\|\boldsymbol{e}_{k+1}\|_\lambda \le \bar{\rho}_2\|\boldsymbol{e}_k\|_\lambda + a_1|\delta\boldsymbol{x}_k(t_1)|. \tag{33}$$

State $\boldsymbol{x}_k(t_1)$ changes as iterations change, such that $\delta\boldsymbol{x}_k(t_1)=\boldsymbol{x}_{k+1}(t_1)-\boldsymbol{x}_k(t_1)\neq 0$, but from the proof in the time interval of $[0,t_1]$, we know there exists $\lim_{k\to\infty}\boldsymbol{y}_k(t)=\boldsymbol{y}_\mathrm{d}(t)$ in the whole time interval of $[0,t_1]$. Then, in accordance with the Assumption 2, there exists $\lim_{k\to\infty}\boldsymbol{u}_k(t)=\boldsymbol{u}_\mathrm{d}(t)$, such that $\lim_{k\to\infty}\boldsymbol{x}_k(t)=\boldsymbol{x}_\mathrm{d}(t)$. As a result, there exists $\lim_{k\to\infty}|\delta\boldsymbol{x}_k(t_1)|=0$.

According to Lemma 5, from $\lim_{k\to\infty}|\delta\boldsymbol{x}_k(t_1)|=0$ and (33), we can achieve

$$\lim_{k\to\infty}\|\boldsymbol{e}_k\|_\lambda = 0. \tag{34}$$

By analogy of the derivation of (25), we have

$$\lim_{k\to\infty}\sup_{t\in[t_1,t_2]}\|\boldsymbol{e}_k(t)\|_1 = 0. \tag{35}$$

It implies for subsystem 2 of the switched system that $\lim_{k\to\infty}\boldsymbol{y}_k(t)=\boldsymbol{y}_\mathrm{d}(t)$ holds in the time interval of $[t_1,t_2]$, if the learning gain matrix in (5) satisfies $\|\boldsymbol{I}-\boldsymbol{D}_2\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1=\rho_2<1$.

By analogy, the algorithm (5) is proved to be convergent in the time interval of $[t_2,t_3]$, $\cdots$, $[t_{N-1},T]$. Therefore, the condition $\|\boldsymbol{I}-\boldsymbol{D}_j\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1=\rho_j<1$, $j=1,2,\cdots,N$, in Theorem 9 is satisfied; there exists $\lim_{k\to\infty}\boldsymbol{y}_k(t)=\boldsymbol{y}_\mathrm{d}(t)$ in the whole time interval of $[0,T]$.

The proof is completed.

From above analysis, we can know that if the error signal is always bounded in the process of controlling, i.e., $\delta>0$ satisfies

$$0<\delta\le\min_{t\in[0,T],i\in[1,m]}(\gamma_{ki}(t)), \tag{36}$$

then $\|\boldsymbol{I}-\boldsymbol{D}_j\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1<1$ holds based on $\|\boldsymbol{I}-\boldsymbol{D}_j\boldsymbol{L}\|_1<1$ and Lemma 6. Therefore, the convergence condition of Theorem 9 may be further simplified.

Theorem 10. *For the switched system (1) with sensor saturation, under the condition satisfying Assumptions 1–3, we use the iterative learning control algorithm (5) to control system (1). If learning gain satisfies*

$$\|\boldsymbol{I}-\boldsymbol{D}_j\boldsymbol{L}\|_1=\rho_j<1,\ (j=1,2,\cdots,N), \tag{37}$$

then the system output is uniformly convergent to desired trajectory in the finite time interval $[0,T]$ as iterations increase, i.e., $\lim_{k\to\infty}\boldsymbol{e}_k(t)=0$.

Proof. We further discuss whether $\gamma_{ki}(t)$ satisfies the bounded requirement of (36) during the iteration process or not.

When the system is first run, and we let $\boldsymbol{u}_1(t)=0$, it is easy to know there exists $M_1<\infty$ in the time interval $t\in[0,T]$ such that

$$\sup_{t\in[0,T],i\in[1,m]}|y_{1i}(t)|\le M_1. \tag{38}$$

And thus, we can derive

$$\sup_{t\in[0,T]}\|\boldsymbol{e}_1(t)\|_1\le m\left(M_1+\max_{t\in[0,T],i\in[1,m]}(|y_{\mathrm{d}i}(t)|)\right). \tag{39}$$

Here, if we choose

$$\delta_1=\frac{\min_{t\in[0,T],i\in[1,m]}(|y_{\mathrm{d}i}(t)\pm y_{0i}(t)|)}{\max_{t\in[0,T],i\in[1,m]}(|y_{\mathrm{d}i}(t)|)+M_1}, \tag{40}$$

then it is easy to know $\gamma_{ki}(t)\ge\delta_1$, $i=1,2,\cdots,m$, $t\in[0,T]$. Therefore, the convergence condition required by $\|\boldsymbol{I}-\boldsymbol{D}_j\boldsymbol{L}\boldsymbol{\Gamma}_k(t)\|_1+b_j((1-\mathrm{e}^{-\lambda t_j})/\lambda)=\bar{\rho}_j<1$ also holds at the second iteration, and then we yield

$$\|\boldsymbol{e}_2\|_\lambda<\|\boldsymbol{e}_1(t)\|_1\le\sup_{t\in[0,T]}\|\boldsymbol{e}_1(t)\|_1. \tag{41}$$

From $\sup_{t\in[t_{j-1},t_j]}\|\boldsymbol{e}_k(t)\|_1\le\mathrm{e}^{\lambda t_j}\|\boldsymbol{e}_k\|_\lambda$, $\|\boldsymbol{e}_2(t)\|_1$ is bounded, and thus, there exists $\delta_2>0$, such that $\gamma_{2i}(t)\ge\delta_2$ holds. By analogy, we have

$$\gamma_{ki}(t)\ge\delta_2, i\in[1,m], t\in[0,T], k=2,3,\cdots. \tag{42}$$

Let $\delta=\min\{\delta_1,\delta_2\}$, and condition (36) holds, i.e., (36) is bounded. Thus, from the above derivation, we obtain the following convergence conclusion.

$$\lim_{k\to\infty}\boldsymbol{e}_k(t)=0, t\in[0,T], \tag{43}$$

i.e., system error tends to zero within the iteration domain.

The proof is completed.

Remark 11. From Remark 8 and the proof of Theorem 9, we find Theorem 9 also holds for the switched system with any other switched sequence as well as each subsystem implemented repeatedly in the interval of $[0,T]$.

4. Numerical Example

To demonstrate the effectiveness of the proposed algorithm (5), consider the following class of the switched system with sensor saturation:

$$\begin{cases} \dot{\boldsymbol{x}}_k(t) = \boldsymbol{A}_{\alpha(t)}\boldsymbol{x}_k(t) + \boldsymbol{B}_{\alpha(t)}\boldsymbol{u}_k(t) \\ \boldsymbol{y}_k(t) = \boldsymbol{C}_{\alpha(t)}\boldsymbol{x}_k(t) + \boldsymbol{D}_{\alpha(t)}\boldsymbol{u}_k(t) \\ \boldsymbol{z}_k(t) = \mathrm{sat}_{y_0}(\boldsymbol{y}_k(t)), \end{cases} \tag{44}$$

where $\alpha(t)$ is random switched sequence chosen as 1 and 2 (as shown in Figure 1); system parameter matrices are chosen as

$$\begin{gathered} \boldsymbol{A}_1 = \begin{bmatrix} -3 & -1.5 \\ -1 & -2 \end{bmatrix}, \\ \boldsymbol{B}_1 = \begin{bmatrix} 1.5 & 2 \\ 2 & 1.5 \end{bmatrix}, \\ \boldsymbol{C}_1 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \\ \boldsymbol{D}_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \\ \boldsymbol{A}_2 = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}, \\ \boldsymbol{B}_2 = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}, \\ \boldsymbol{C}_2 = \begin{bmatrix} 1 & 2 \\ 0.6 & 1.5 \end{bmatrix}, \\ \boldsymbol{D}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.6 \end{bmatrix} \end{gathered} \tag{45}$$

Threshold of output saturation is $\boldsymbol{y}_0 = [2\ 1.5]^{\mathrm{T}}$. When algorithm (5) is used to control the switched system (44), and let us suppose that run time of system is given by $0 \le t \le 5$s, the initial state is given by $\boldsymbol{x}_k(0) = [2, 0.5]^{\mathrm{T}}$, the initial control input is given by $\boldsymbol{u}_0(t) = [0, 0]^{\mathrm{T}}$, and sampling time is 0.01 s, such that the desired trajectory is

$$\begin{gathered} y_{d1}(t) = 1.5e^{0.05t} \sin(\pi t), \\ y_{d2}(t) = 1.1e^{0.05t} \sin\left(\pi t + \frac{\pi}{6}\right). \end{gathered} \tag{46}$$

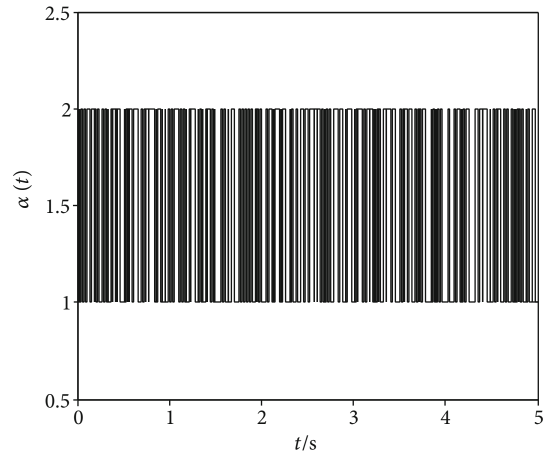

Figure 1: The random switched sequence of $\alpha(t)$.

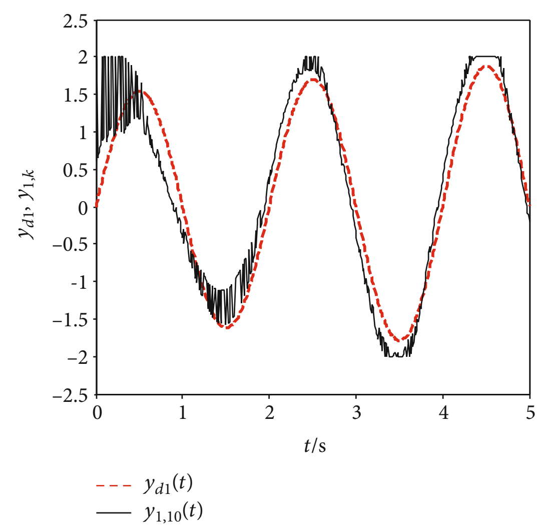

Figure 2: System output y_1 at 10th iteration.

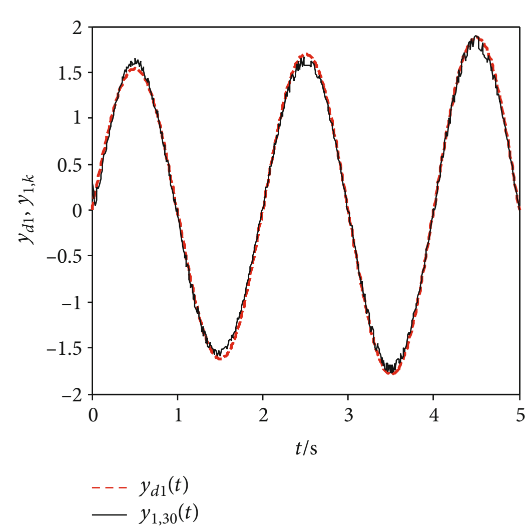

Figure 3: System output y_1 at 30th iteration.

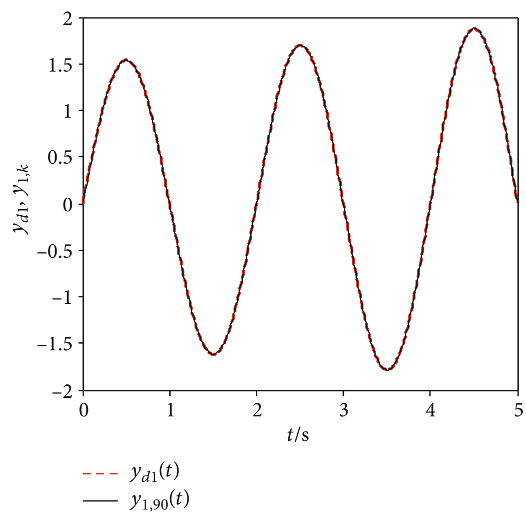

Figure 4: System output y_1 at 90th iteration.

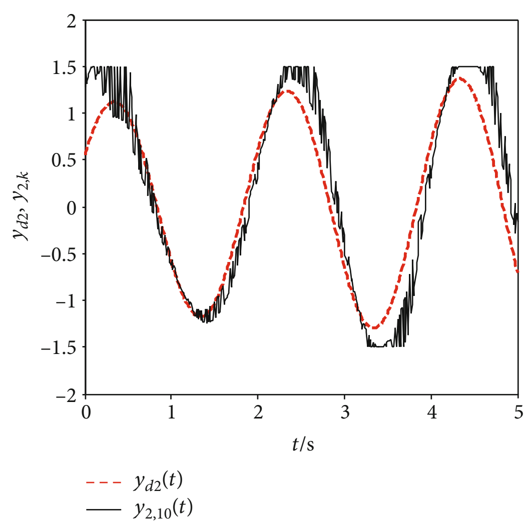

Figure 5: System output y_2 at 10th iteration.

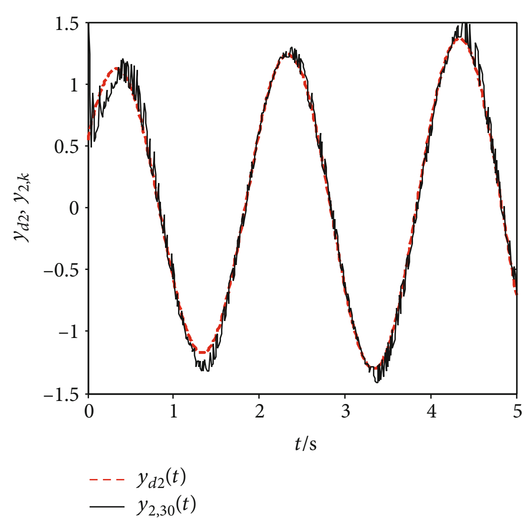

Figure 6: System output y_2 at 30th iteration.

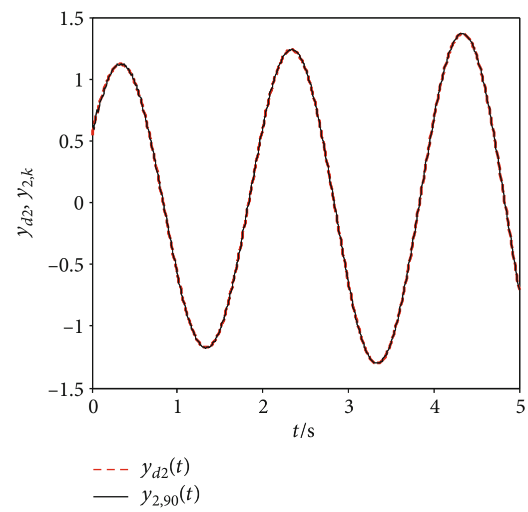

Figure 7: System output y_2 at 90th iteration.

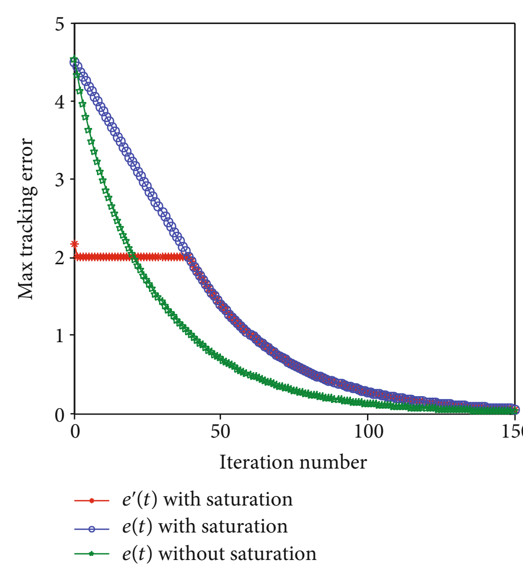

Figure 8: Tracking error convergence curve of y_1.

Learning gain matrix is chosen as

$$\boldsymbol{L} = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.12 \end{bmatrix}, \tag{47}$$

which satisfies the convergence condition of $\rho_1 = 0.9680 < 1$, $\rho_2 = 0.9280 < 1$.

The simulation results are shown in Figures 2–9. From Figures 2–7, we can see that when designed algorithm (5) is used to control the switched system with output saturation, system output can realize the complete tracking of desired trajectory in finite time interval under random switched sequences as iterations increase. At the same time, from Figures 8 and 9, we can see that the

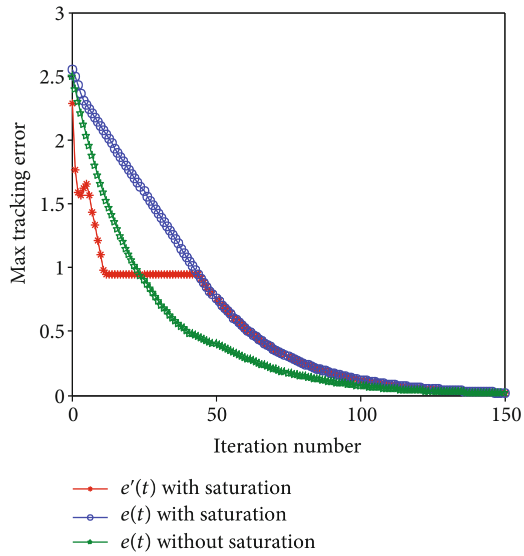

FIGURE 9: Tracking error convergence curve of y_2.

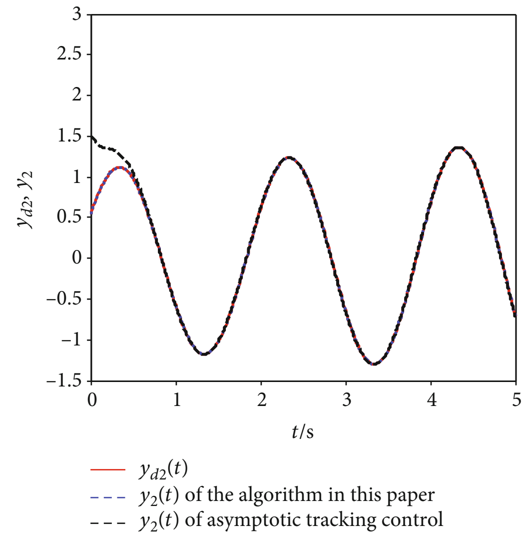

FIGURE 11: y_2 comparison of asymptotic tracking control and the algorithm in this paper.

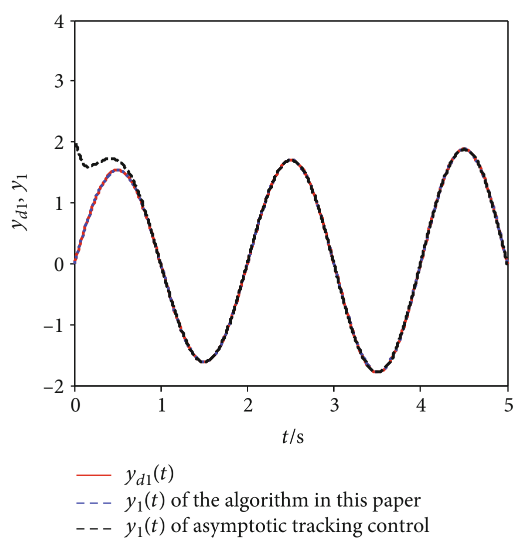

FIGURE 10: y_1 comparison of asymptotic tracking control and the algorithm in this paper.

convergence speed of system with output saturation is slower than system without output saturation. The reason is in that when the system has sensor saturation, output error cannot all be used as feedback, but eventually, the error still converges to zero.

In order to better reflect the difference between the proposed algorithm (5) and the traditional asymptotic tracking control, the proposed algorithm and the asymptotic tracking control method are simulated and compared. The simulation results are shown in Figures 10 and 11. Figure 10 shows the tracking result of the system output $y_1(t)$ to the desired trajectory $\boldsymbol{y}_{\mathrm{d}1}(t)$. Figure 11 shows the tracking result of the system output $y_2(t)$ to the desired trajectory $\boldsymbol{y}_{\mathrm{d}2}(t)$. As can be seen from Figures 10 and 11, the proposed algorithm can completely track the desired trajectory in the entire time interval $t \in [0, 5]$ s. However, the traditional asymptotic tracking control can only make the output asymptotically track the desired trajectory with time but cannot achieve complete tracking of the desired trajectory.

5. Conclusions

For the random switched system with saturation constraints, an iterative learning control algorithm is proposed to correct the control input by using the actual measurement error. And the convergence of each subsystem is strictly proved in the sense of λ-norm, and the convergence conditions of the algorithm are given. The research results show that under any fixed initial state and random switched rules, the algorithm can achieve the complete tracking of the system output to the desired trajectory with the increase of the number of iterations in a finite time interval.

In this paper, only the sensor saturation constraint problem is considered. In practical engineering, the actuator saturation constraint phenomenon is also common. Therefore, in the future work, the iterative learning control problem of switched systems with actuator saturation constraints can be further studied on the basis of this paper.

The algorithm in this paper requires that the initial state of the system is fixed at each iteration, that is, it satisfies the strict repetition of the initial positioning, which has some limitations in practical application. If the mathematical model of the system is known, we can use the idea of [35] to further relax the initial state of each iteration to the random initial state.

Data Availability

The data used to support the findings of this study are included within the article. Readers only need to obtain the research results according to the data provided by the numerical examples in this paper.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this article.

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