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Research Article

A Bayesian Adaptive Unscented Kalman Filter for Aircraft Parameter and Noise Estimation

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This paper proposes a new algorithm for the aerodynamic parameter and noise estimation for aircraft dynamical systems. The Bayesian inference method is combined with an unscented Kalman filter to estimate the augmented states and the unknown noise covariance parameters jointly. A Gauss-Newton method is utilized to sequentially maximize the posterior likelihood function for the noise unknown parameter estimation. The performance of the proposed algorithm is evaluated and compared with two other UKFs via a flight scenario of a given aircraft. The results indicate that the proposed algorithm has equivalent performance to the simplified UKF with prior noise information and slightly outperforms the parallel UKF on precision and efficiency in this flight scenario assessment. Then, the consistency and accuracy of the algorithm are further validated by a Monte Carlo simulation with random process noise covariance. This adaptive algorithm provides another feasible and effective way for estimating aerodynamic parameters from the aircraft real flight data with unknown noise characteristics.

1. Introduction

The environment disturbance and sensor noise are the primary contamination sources to aircraft flight data. The disturbance and noise during the flight are usually unknown and unpredictable, and it brings errors and uncertainty in system models and measurements. For accurately estimating aerodynamic parameters from the flight data, the uncertainty and noise must be modelled appropriately and handled in the estimation methods. Thus, the research on parameter estimation from the contaminated data is the heightened spot in the aircraft aerodynamic characteristics analysis, especially for the aerodynamic deviation study between the flight and ground tests.

Kalman filters are the standard solutions to estimate aerodynamic parameters. The classical two-step Bayesian approach, such as extended and unscented Kalman filter (EKF and UKF), had already been utilized for aircraft parameter estimation and system identification in many previous researches [1, 2]. These studies show that UKF outperforms EKF regarding the robustness and accuracy of the performance [1, 3, 4]. So UKF and its improved versions were widely applied in the state and parameter estimation domain. For estimating the parameter from the contaminated flight data, augmentation and parallelization are the two feasible ways commonly adopted in the adaptive Kalman filters. While the state-parameter-noise augmentation technology is less preferable because it may degrade the accuracy of the parameters, the parallelized estimation technology performs pretty well according to the research of Majeed and Narayan Kar [5]. Majeed introduced an adaptive UKF scheme that used a master UKF to estimate the state and aerodynamic parameters and a slave UKF to estimate the noise covariance parallelly. Jategaonkar and Plaetschke [6] compared four algorithms for aircraft parameter estimation accounting for both process and measurement noise. The augmented EKF approach and the three Gauss-Newton based maximum likelihood (ML) approaches were presented and compared in the reference. Finally, the authors concluded that the Gauss-Newton-based ML

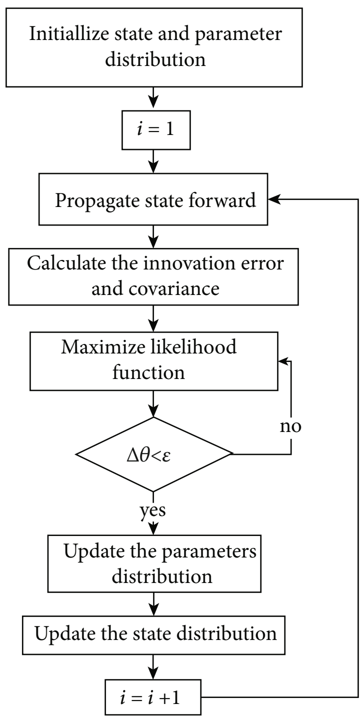

Figure 1: The framework of the Bayesian adaptive estimation algorithm. The flow chart of the algorithm depicts the prediction-inference-update cycle for the aerodynamic parameters and noise covariance estimation.

Table 1: The physical parameters of aircraft.

Aircraft parameters	Values
Mass M	72000 kg
Wing reference area s	124 m^2
Wing span l	34.1 m
Mean aerodynamic chord b_A	4.15 m
Moment of inertia I_{xx}	1658755 $kg \times m^2$
Moment of inertia I_{yy}	2392630 $kg \times m^2$
Moment of inertia I_{zz}	3846326 $kg \times m^2$

method combined with a steady-state filter is generally preferable.

The multiple model adaptive estimation (MMAE) approach provides another way to handle the uncertainty and noise in parameter estimation. MMAE uses several filters running in parallel, each representing a hypothesis of the actual system, to generate enhanced state and parameter estimates. The number of models needed for approximating the actual system is related to the variable dimension. If the uncertainty and noise in the system are a high dimensional variable, then the number of hypothesis models is usually large, and the computational burden will increase significantly. There are some applications of MMAE in aerospace engineering. Cristofaro et al. [7, 8] used an adaptive multiple model approach for aircraft icing detection and identification. Marschke et al. [9, 10] presented a generalized multiple model adaptive estimator, which uses a window of previous data via the autocorrelation matrix to perform the adaptive update, to estimate the state without rate gyros information or process noise covariance.

The ensemble Kalman filter (EnKF) [11] is a recently developed state-of-the-art technology for data assimilation in high-dimensional dynamical systems and has been widely used in the fields of ocean and atmospheric science. The Bayesian-based parameter inference and noise estimation methods are concerned and discussed in the EnKF applications. Delsole and Yang [12] derived generalized maximum likelihood estimates in the EnKF framework by using a Newton-Raphson method to estimate states and parameters. Stroud and Bengtsson [13] adopted a Bayesian approach in EnKF to estimate the state and invariant scalar observation variance; then, they proposed a new Bayesian-based EnKF methodology for sequential state and parameter estimation and introduced three representations of the marginal posterior distribution of the parameters [14]. Dreano et al. [15, 16] proposed an iterative expectation-maximization (EM) algorithm in EnKF to estimate the state and model error covariances; then, Tandeo et al. [17] reviewed the innovation-based methods for jointly estimating model and observation error covariance matrices in ensemble data assimilation. Ueno and Nakamura presented an online EM algorithm [18] and a Bayesian-based iterative algorithm [19] to estimate the noise covariance matrix parameters. Frei and Kunsch [20] combined the particle filter with EnKF for the joint state and noise covariance matrix estimation. Brankart et al. [21] utilized a transformed algorithm and EnKF to reduce the control space of the error covariance parameter to decrease the computational cost of parameter optimization.

The dynamical inverse problem issued in this paper focuses on the estimation of system unknown parameters from contaminated observations. Although several approaches are available for the aircraft parameter and noise estimation, there still exists improvement space in the estimation precision, real-time, and stability performance of algorithms. Thus, we propose a new parameter estimation approach for the aircraft aerodynamic parameter online estimation using the real flight test data. First, we adopt UKF as the state and aerodynamic parameter estimation method by considering its higher performance over other KF methods. Then, the Bayesian statistic inference theorem, which has been validated and applied in numerous data assimilation problems, is introduced into the new algorithm to inference the unknown disturbance and noise. A Gauss-Newton method is adopted for parameter optimization by considering its high computational efficiency and the real-time requirement of the new algorithm. Finally, an algorithm framework of combining UKF with the Bayesian inference and Gauss-Newton method for online aerodynamic parameter estimation from the contaminated flight test data is derived in this paper. The new algorithm's performance is assessed and compared with other UKFs via the simulation data of an aircraft flight scenario. The remainder of this paper is organized as follows. The second section introduces

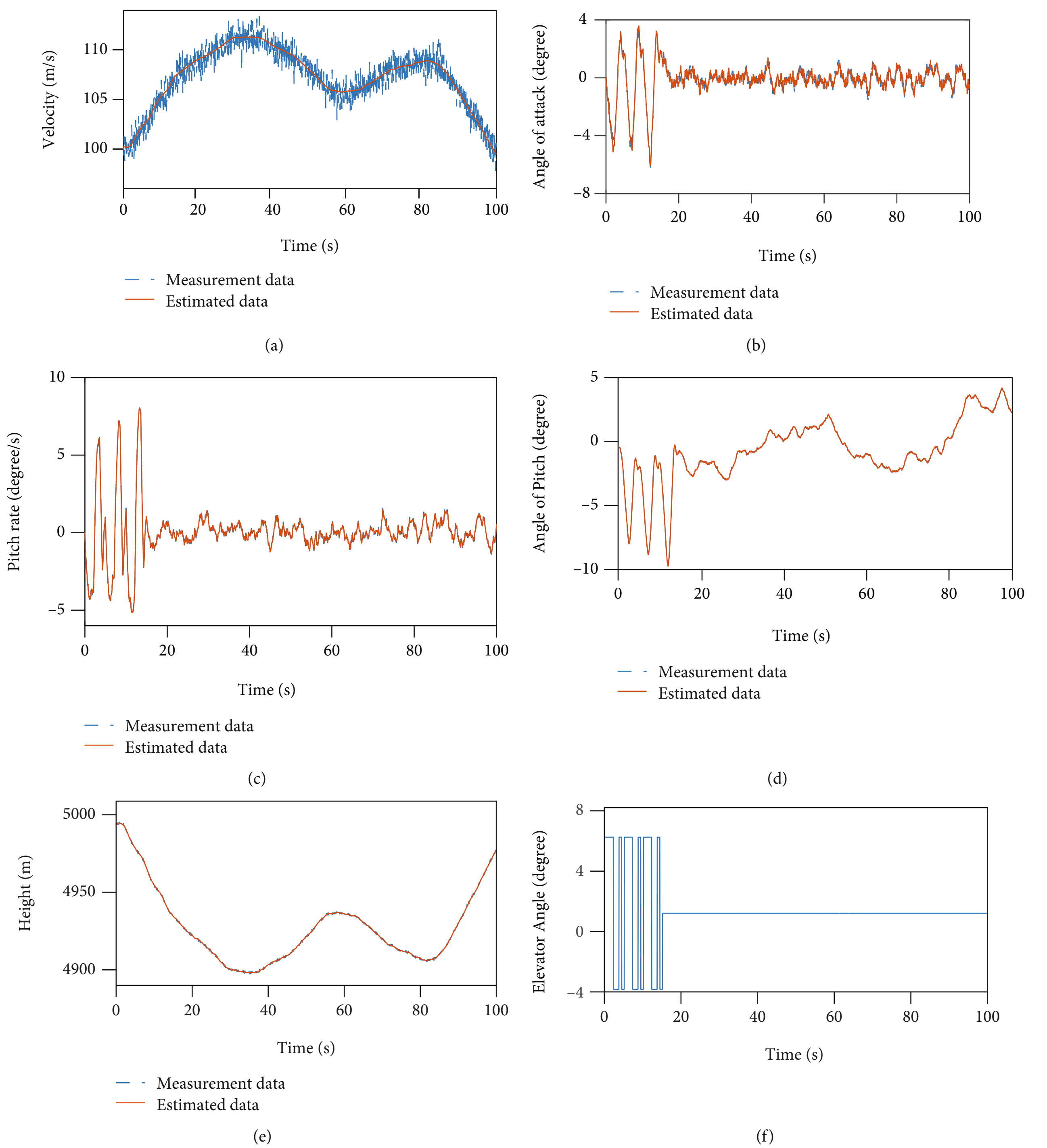

FIGURE 2: Comparison of the system measurement data with the estimated states. The data is estimated by the Bayesian UKF algorithm, and the system inputs are given in the last figure: (a) velocity comparison; (b) angle of attack comparison; (c) pitch rate comparison; (d) angle of pitch comparison; (e) height comparison; (f) system input.

the UKF, Bayesian inference, and the Gauss-Newton method for a nonlinear dynamical system and presents the algorithm framework for the Bayesian adaptive unscented Kalman filter. In the third section, the algorithm's performance is evaluated and compared with other algorithms via a given aircraft flight scenario. In the last section, some conclusions are given.

2. Materials and Methods

2.1. Nonlinear Dynamical System Model. The algorithm is proposed for a nonlinear dynamical system. First, the generic nonlinear system can be represented by a model with additive Gaussian noise:

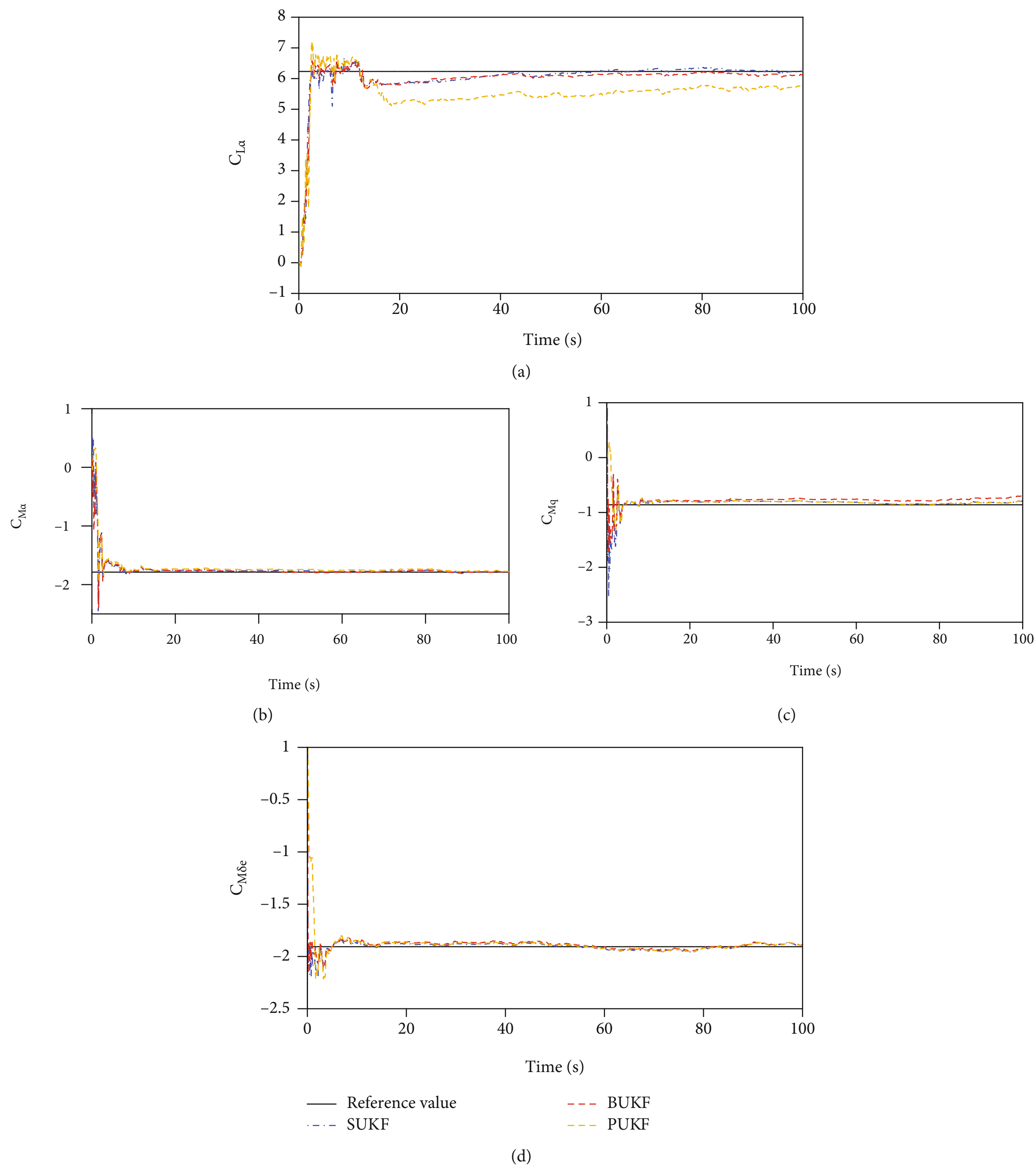

Figure 3: Comparison of the parameter estimation performance of the three algorithms. The algorithms are the Bayesian UKF (BUKF) which is proposed in this paper, the simplified UKF (SUKF) with prior noise knowledge, and the parallel UKF (PUKF) proposed in reference [5]: (a) estimation results of $C_{L\alpha}$; (b) estimation results of $C_{M\alpha}$; (c) estimation results of C_{Mq}; (d) estimation results of $C_{M\delta e}$.

$$
\begin{gathered}
\dot{x}(t) = f[x(t), u(t), \Theta] + w(t),\ w(t) \sim \mathcal{N}[0, Q(t,\theta)], \\
y(t) = g[x(t), u(t), \Theta], \\
z(k) = y(k) + v(t), \quad k = 1, 2, \cdots N, \quad v(t) \sim \mathcal{N}[0, R(t,\theta)],
\end{gathered} \tag{1}
$$

where x is the system state variable, u is the input variable, y is the output variable, z is the discrete measurement signal, N is the length of the sampling data, t denotes the current time, Θ is the system unknown parameters, w and v represent the additive Gaussian process and measurement noise, the covariance matrices Q and R are assumed to be time-variant and depend on unknown parameter θ, and $f(\bullet)$ and $g(\bullet)$ represent general nonlinear functions.

The system parameter Θ and noise covariance parameter θ are commonly unknown. The system parameter can be estimated through the augmented system in Equation (2), where the variables x_a and w_a denote the augmented state and process noise, respectively. The noise parameter estimation is difficult and has a strong correlation with the system unknown parameter estimation. Here, we attempt to utilize the Bayesian inference with the optimization method for the noise parameter estimation.

$$
\dot{x}_a = \begin{bmatrix} \dot{x} \\ \dot{\Theta} \end{bmatrix} = \begin{bmatrix} f[x(t), u(t), \Theta] \\ 0 \end{bmatrix} + w_a(t) \triangleq f_a[x(t), u(t), \Theta] + w_a(t). \tag{2}
$$

2.2. Unscented Kalman Filter for Augmented State Estimation. The augmentation approach estimates the system state and unknown parameters jointly. Many previous research confirmed the effectiveness and efficiency of UKF for augmented state estimation with a priori noise information. A standard UKF algorithm can estimate the augmented system in Equation (2) as follows [1, 5]:

(1) Initialization

$$
\begin{gathered}
\widehat{x}_0^a = E\{x_0^a\} = \begin{bmatrix} \widehat{x}_0 \\ \Theta_0 \end{bmatrix}, \\
\widehat{P}_0^a = E\left\{ (x_0^a - \widehat{x}_0^a)(x_0^a - x\wedge_0^a)^T \right\} = \begin{bmatrix} P_{x0} & 0 \\ 0 & P_{\Theta 0} \end{bmatrix}.
\end{gathered} \tag{3}
$$

(2) Sigma points calculation and prediction

$$
\begin{gathered}
\tilde{\boldsymbol{\chi}}_k^a = \left[\widehat{\mathbf{x}}_k^a \quad \widehat{\mathbf{x}}_k^a - \gamma\sqrt{\widehat{\mathbf{P}}_k^a} \quad \widehat{\mathbf{x}}_k^a + \gamma\sqrt{\widehat{\mathbf{P}}_k^a} \right], \\
\tilde{\boldsymbol{\chi}}_{k+1}^a = \widehat{x}_k^a + \int_{t_k}^{t_{k+1}} f_a\left(\widehat{\boldsymbol{\chi}}_k^x, \mathbf{u}_k, \widehat{\boldsymbol{\chi}}_k^\Theta \right) dt, \\
\tilde{\mathbf{x}}_{k+1}^a = \sum_{i=0}^{2L} \varpi_i^{(m)} \tilde{\boldsymbol{\chi}}_{i,k+1}^a, \\
\tilde{\mathbf{P}}_{k+1} = \sum_{i=0}^{2L} \varpi_i^{(c)} \left[\tilde{\boldsymbol{\chi}}_{i,k+1}^a - \tilde{\mathbf{x}}_{k+1}^a \right] \left[\tilde{\boldsymbol{\chi}}_{i,k+1}^a - \tilde{\mathbf{x}}_{k+1}^a \right]^T + Q,
\end{gathered} \tag{4}
$$

where L is the dimension of the augmented state, superscript "a" represents the augmented variable, $\chi^a = \left[\chi^x \quad \chi^\Theta \right]^T$ denotes the sigma points matrix, χ^x represents the variable corresponding to the system state, and χ^Θ represents the variable corresponding to the system unknown parameters. The coefficients and weights in Equation (4) can be calculated by Equation (5), where the superscripts "m" and "c" of the weight ϖ are for the computation of mean and covariance, respectively. According to the reference [1], the values of a, b, and κ are chosen as 0.009, 2, and 0, respectively.

$$
\begin{cases}
\lambda = a^2(L+\kappa) - L, \\
\varpi_0^{(m)} = \dfrac{\lambda}{L+\lambda}, i = 0, \\
\varpi_0^{(c)} = \dfrac{\lambda}{L+\lambda} + \left(1 - a^2 + b\right), i = 0, \\
\varpi_i^{(m)} = \varpi_i^{(c)} = \dfrac{1}{2(L+\lambda)}, i = 1, \cdots, 2L, \\
\gamma = \sqrt{L+\lambda}.
\end{cases} \tag{5}
$$

(3) Measurement update

$$
\begin{gathered}
\mathbf{Y}_{k+1} = g\left(\tilde{\boldsymbol{\chi}}_{k+1}^x, \mathbf{u}_{k+1}, \tilde{\boldsymbol{\chi}}_{k+1}^\Theta \right), \\
\tilde{\mathbf{y}}_{k+1} = \sum_{i=0}^{2L} \varpi_i^{(m)} \mathbf{Y}_{i,k+1}, \\
\mathbf{P}_{\tilde{y}\tilde{y}_{k+1}} = \sum_{i=0}^{2L} \varpi_i^{(c)} [\mathbf{Y}_{i,k+1} - \tilde{\mathbf{y}}_{k+1}][\mathbf{Y}_{i,k+1} - \tilde{\mathbf{y}}_{k+1}]^T + R, \\
\mathbf{P}_{\tilde{x}\tilde{y}_{k+1}} = \sum_{i=0}^{2L} \varpi_i^{(c)} \left[\tilde{\boldsymbol{\chi}}_{i,k+1}^x - \tilde{\mathbf{x}}_{k+1} \right] [\mathbf{Y}_{i,k+1} - \tilde{\mathbf{y}}_{k+1}]^T, \\
\mathbf{K}_{k+1} = \mathbf{P}_{\tilde{x}\tilde{y}_{k+1}} \mathbf{P}_{\tilde{y}\tilde{y}_{k+1}}^{-1}, \\
\mu_{k+1} = \mathbf{z}_{k+1} - \tilde{\mathbf{y}}_{k+1}, \\
\widehat{\mathbf{x}}_{k+1} = \tilde{\mathbf{x}}_{k+1} + \mathbf{K}_{k+1}\mu_{k+1}, \\
\widehat{\mathbf{P}}_{k+1} = \tilde{\mathbf{P}}_{k+1} - \mathbf{K}_{k+1} \mathbf{P}_{\tilde{y}\tilde{y}_{k+1}} \mathbf{K}_{k+1}^T,
\end{gathered} \tag{6}
$$

where "~" denotes the prediction values, "^" denotes the updated values, and μ is the innovation error between the observations and the projection of the forecasts onto the observation space. The diagonal elements in the covariance matrices Q and R are unknown and required to be inferenced by the Bayesian method.

2.3. Bayesian Inference Method. Since the unknown noise characteristics will degrade the performance of UKF badly, the sequential Bayesian inference is adopted to enhance the adaptivity of UKF. The Bayesian inference method maximizes the posterior probability density function to calculate the unknown parameters:

$$\theta = \arg\max_{\theta} p(\theta|z_k), \tag{7}$$

where z_k is the k_{th} measurements. The above posterior distribution function can be written recursively via Bayesian theorem:

$$p(\theta|z_k) \propto p(\theta|z_{k-1})p(z_k|\theta, z_{k-1}). \tag{8}$$

The formula in Equation (8) defines a recursive form of the parameter distribution over time. The first term on the right side is the posterior distribution of θ via the previous measurements; the second term is the likelihood at the current step. The innovation information is used to approximate the likelihood [14]:

$$p(z_k|\theta, z_{k-1}) \propto \left|\widehat{S}_k(\theta)\right|^{-1/2} \exp\left[-\frac{1}{2}\widehat{e}_k^T(\theta)\widehat{S}_k(\theta)^{-1}\widehat{e}_k(\theta)\right], \tag{9}$$

where $\widehat{e}_k$ is the innovation error, $\widehat{S}_k$ is the innovation covariance matrix, the μ_{k+1} and $\mathbf{P}_{\tilde{y}\tilde{y}_{k+1}}$ in Equation (6) can, respectively, represent the innovations $\widehat{e}_k$ and $\widehat{S}_k$.

Then, a representation of the parameter posterior distribution is needed to update the parameter distribution recursively. Stroud presented three representations of the parameter distribution in reference [14], including a grid-based distribution, a Gaussian approximation, and a particle approximation with kernel resampling. Here, we choose the Gaussian approximation method for the new algorithm.

The parameter posterior distribution at each step k is approximated by a normal distribution with mean m_k and covariance matrix C_k; then, the probability density function can be written as follows:

$$p(\theta|z_k) \propto \exp\left[-\frac{1}{2}(\theta - m_k)^T C_k^{-1}(\theta - m_k)\right]. \tag{10}$$

Equations (8)–(10) define the update expression of the parameter posterior distribution. An optimization algorithm can be used to calculate the parameter's statistic characteristics recursively. The log-likelihood of the parameter posterior probability function is given by

$$\begin{aligned} l(\theta) &= \ln p(\theta|z_{k-1}) + \ln p(z_k|\theta, z_{k-1}) \\ &= -\frac{1}{2}(\theta - m_k)^T C_k^{-1}(\theta - m_k) - \frac{1}{2}\widehat{e}_k^T(\theta)\widehat{S}_k(\theta)^{-1}\widehat{e}_k(\theta) - \frac{1}{2}\ln\left|\widehat{S}_k(\theta)\right|. \end{aligned} \tag{11}$$

Then, the parameter's mean and covariance matrix at $k+1$ step can be computed by maximizing the log-likelihood function:

$$\begin{aligned} m_{k+1} &= \arg\max_{\theta} l(\theta), \\ C_{k+1} &= -\left[\frac{\partial^2 l(\theta)}{\partial\theta\partial\theta^T}\right]^{-1}_{\theta=m_{k+1}}. \end{aligned} \tag{12}$$

Table 2: The RMSE and efficiency of the three UKF algorithms.

Algorithms	RMSE $C_{L\alpha}$	$C_{M\alpha}$	C_{Mq}	$C_{M\delta e}$	Computational time (s)
SUKF	0.1167	0.1133	0.1558	0.0385	0.0312
BUKF	0.1191	0.0999	0.1466	0.0376	0.0319
PUKF	0.1592	0.1335	0.1396	0.0686	0.0322

2.4. Gauss-Newton Method for Parameter Optimization. For the parameter optimization problem in Equation (12), the conventional optimization scheme like genetic algorithm is not suitable by considering the computational efficiency. Here, we derive a Gauss-Newton recursive formula for the parameter optimization problem and to satisfy the requirement of real-time performance. The Newton direction is utilized as the parameter descent direction to guarantee the steepest descent.

The parameter is maximized by taking the partial derivatives of the log-likelihood function in Equation (11) with respect to every unknown parameter to zero. The log-likelihood function is simplified before calculating the partial derivatives. First, the UKF deterministic sampling process decides that the innovation error e is independent of the unknown parameter θ, and the innovation covariance S is a summation of the parameter-dependent noise covariance and the deterministic forecast state covariance, which is independent of θ. The innovation covariance matrix determinant term is assumed to be constant by ignoring the parameter θ influence on the matrix's determinant. Finally, the log-likelihood function can be written as follows:

$$l_s(\theta) = -\frac{1}{2}(\theta - m_k)^T C_k^{-1}(\theta - m_k) - \frac{1}{2}\widehat{e}_k^T\widehat{S}_k(\theta)^{-1}\widehat{e}_k + c, \tag{13}$$

where c is constant and independent of θ. The simplified function is directly derived by θ, and the recursive formula of parameter mean and covariance is expressed as follows:

$$\begin{cases} m_{k+1} = m_k - G_k^{-1} g_k, \\ g_k = C_k^{-1}(\theta - m_k) - \dfrac{1}{2}\widehat{e}_k^T\widehat{S}_k^{-1}(\theta)\dfrac{\partial\widehat{S}_k(\theta)}{\partial\theta}\widehat{S}_k^{-1}(\theta)\widehat{e}_k. \\ G_k = C_k^{-1} - \dfrac{1}{2}G_{\text{innov}}(\theta), \end{cases} \tag{14}$$

$$C_{k+1} = \left[C_k^{-1} - \frac{1}{2}G_{\text{innov}}(m_{k+1})\right]^{-1}, \tag{15}$$

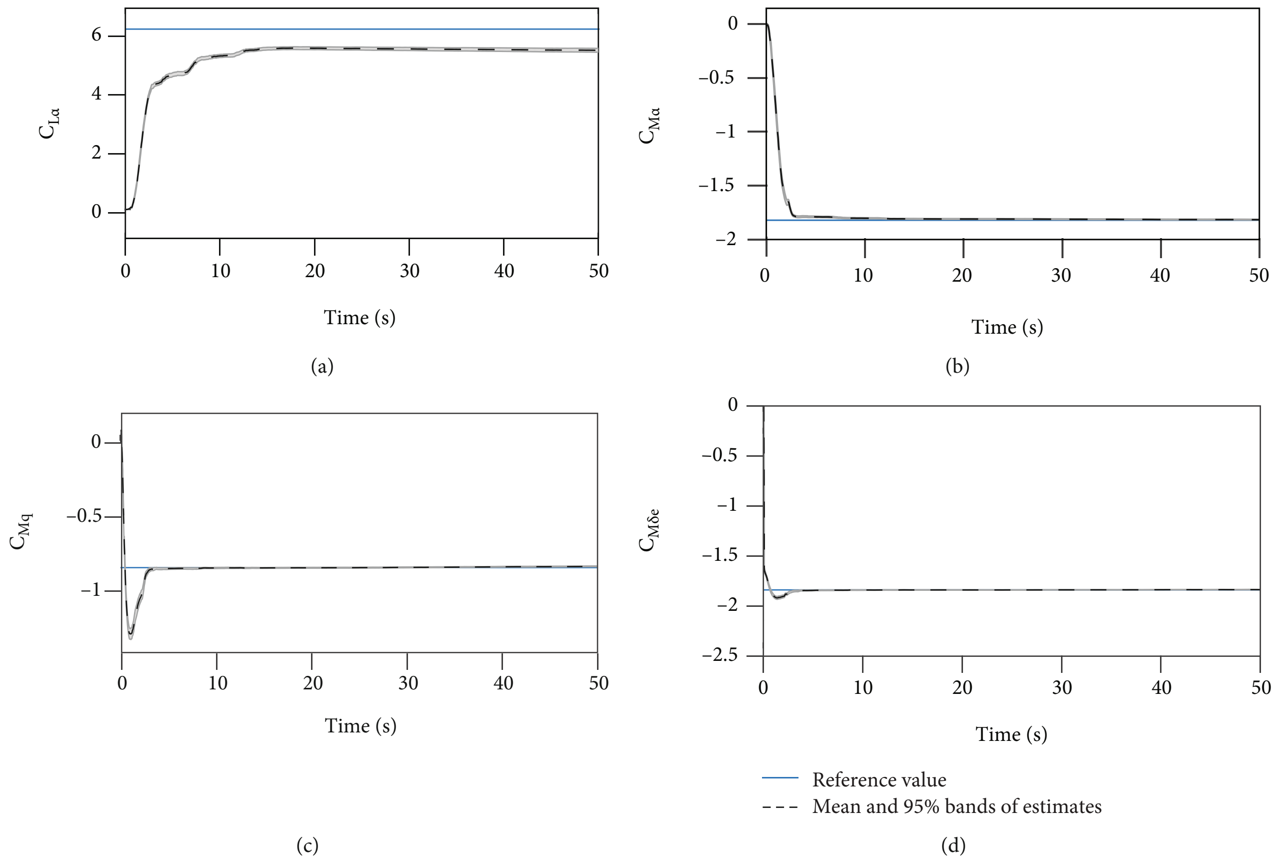

Figure 4: Histories of mean values of the four estimated aerodynamic derivatives in Monte Carlo simulation. The means with 95% bands for every step are fitted by the normal distribution and compared with the reference values: (a) statistics of $C_{L\alpha}$; (b) statistics of $C_{M\alpha}$; (c) statistics of C_{Mq}; (d) statistics of $C_{M\delta e}$.

where g_k and G_k are the gradient and Hessian matrix of the log-likelihood function, G_{innov} is an $n_p \times n_p$ matrix which represents the Hessian matrix of the innovation-related term, n_p is the dimension of the unknown parameters, $\{g_{ij}\}$ denotes the matrix element at the i_{th} row and j_{th} column and can be given by

$$g_{ij} = -\hat{e}_k^T \hat{S}_k^{-1} \left(\frac{\partial \hat{S}_k}{\partial \theta_i} \hat{S}_k^{-1} \frac{\partial \hat{S}_k}{\partial \theta_j} + \frac{\partial \hat{S}_k}{\partial \theta_j} \hat{S}_k^{-1} \frac{\partial \hat{S}_k}{\partial \theta_i} \right) \hat{S}_k^{-1} \hat{e}_k, \quad i, j = 1, 2, \cdots, n_p. \tag{16}$$

The partial derivative of the innovation covariance matrix S is involved in Equations (14)–(16); if only the diagonal elements of the noise covariance are concerned, then the partial derivative is also a diagonal matrix, and its diagonal elements can be given by

$$\frac{\partial \hat{S}_k(\theta)}{\partial \theta_i} = \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_j \end{bmatrix}, \quad d_j = \begin{cases} 0, j \neq i \\ 1, j = i \end{cases}, \quad i, j = 1, 2, \cdots, n_p. \tag{17}$$

Then, the updates for the unknown parameters' posterior distribution can be calculated by Equations (14)–(17). First, the mean updates are iterated several times by Equation (14) until the error converges. Then, the covariance updates are calculated by Equation (15) using the mean updates. This Gauss-Newton-based optimization algorithm can be utilized in online parameter optimization but also requires a very close initialization and is vulnerable to disturbances.

2.5. Bayesian Adaptive Unscented Kalman Filter. By combining the above methods, we present a Bayesian adaptive unscented Kalman filter to handle the aircraft aerodynamic parameter estimation problem with the unknown noise characteristics. The inference and optimization are utilized in every filtering step to give the noise characteristic estimates. The framework of this approach is shown in Figure 1. The parameter inference procedure is added between the prediction and update steps of UKF to estimate the noise characteristics. Thus, the proposed algorithm consists of prediction, parameter inference and optimization, and update three steps.

The Bayesian adaptive unscented Kalman filter approach for aerodynamic parameters and noise covariance estimation is summarized below:

(1) Initialize the augmented states with $x_a \sim \mathcal{N}(\hat{x}_0^a, \hat{P}_0^a)$ and the unknown noise covariance parameters with $\theta \sim \mathcal{N}(m_0, C_0)$

(2) Propagate the augmented states forward by Equation (4) for $i = 1, 2, \cdots, N-1$

(3) Calculate the i_{th} step innovation error and covariance using the parameter's mean value m_i by Equation (6); using the innovation information in Equation (14) to iteratively calculate the mean update m_{i+1} of the noise parameters until the iterative error is less than a small tolerance; using m_{i+1} in Equation (15) to obtain the noise parameter covariance updates C_{i+1}

(4) Update the system augmented state by Equation (6) using the parameter's mean value of $(i+1)_{\text{th}}$ step and the state forecast of i_{th} step, then repeat steps (2)-(4) until the algorithm stops

3. Results and Discussion

3.1. Aircraft Longitudinal Dynamical Model. For assessing the performance of the Bayesian adaptive UKF algorithm, an aircraft longitudinal motion is modelled to generate the simulation data. The motion in the wind axes can be expressed as follows:

$$\begin{aligned}
\dot{V} &= -\frac{q_\infty s}{M} C_D + \frac{P_x}{M} + g_x, \\
\dot{\alpha} &= -\frac{q_\infty s}{MV \cos\beta} C_L + \frac{P_z}{MV\cos\beta} + \frac{g_z}{V\cos\beta} - \\
&\quad \frac{1}{\cos\beta}(p\cos\alpha\sin\beta - q\cos\beta + r\sin\alpha\sin\beta), \\
\dot{q} &= \frac{q_\infty s b_A}{I_{yy}} C_M + \frac{M_y}{I_{yy}} + \frac{I_{zz} - I_{xx}}{I_{yy}} pr - \frac{I_{xz}}{I_{yy}} (p^2 - r^2), \\
\dot{\varphi} &= q\cos\phi - r\sin\phi, \\
\dot{h} &= V\cos\alpha\cos\beta\sin\varphi - V\sin\beta\cos\varphi\sin\phi - \\
&\quad V\sin\alpha\cos\beta\cos\varphi\cos\phi,
\end{aligned} \tag{18}$$

where V is the aircraft velocity; α and β denote the angle of attack and sideslip; p, q, and r represent roll, pitch, and yaw rates, respectively; φ is the pitch angle; ϕ is the roll angle; h is the aircraft height; q_∞ is dynamic pressure; P_x and P_z represent engine thrust; M_y is the thrust moment; g_x and g_z are the gravitational acceleration components; s is wing reference area; b_A is the mean aerodynamic chord; and M and I represent the aircraft mass and moment of inertial, respectively.

The longitudinal aerodynamic coefficients C_D, C_L, and C_M are the drag, lift, and pitch moment coefficients, respectively, and can be modelled by a polynomial formula by Equation (19), where δ_e denotes the elevator angle.

$$\begin{aligned}
C_D &= C_{D0} + C_{D\alpha}\alpha, \\
C_L &= C_{L0} + C_{L\alpha}\alpha + C_{L\delta e}\delta_e, \\
C_M &= C_{M0} + C_{M\alpha}\alpha + C_{M\dot{\alpha}}\dot{\alpha} + C_{Mq} q + C_{M\delta e}\delta_e.
\end{aligned} \tag{19}$$

The aerodynamic derivatives in Equation (19) are usually unknown and need to be estimated from the flight data. Then, the unknown system intrinsic parameters generally include the following:

$$\Theta = \begin{bmatrix} C_{D0} & C_{D\alpha} & C_{L0} & C_{L\alpha} & C_{L\delta_e} & C_{M0} & C_{M\alpha} & C_{M\dot{\alpha}} & C_{Mq} & C_{M\delta_e} \end{bmatrix}^T. \tag{20}$$

3.2. Algorithm Performance Assessment. A steady-level cruise scenario of an aircraft with similar aerodynamic and flight dynamical characteristics as Boeing 737 (B737) is utilized to generate the flight simulation data. The aircraft's aerodynamic parameters are calculated by a computational fluid dynamics tool. The physical parameters of the aircraft are listed in Table 1. The aircraft cruises at the height of 5 km with a velocity of 100 m/s. The aircraft's initial angular rate is assumed to be zero, and its flight attitude is trimmed to zero by the elevator angle of 1.06 degrees.

The process and measurement noises are assumed to be white Gaussian noise with zero mean and a given covariance. The pseudo-random number generated via the given noise characteristics is added to the system state and measurement models during every simulation step. The process and measurement noise covariances for this scenario are given by

$$\begin{aligned}
Q &= \operatorname{diag}\begin{bmatrix} 0.01 & 2.5\times10^{-3} & 2.5\times10^{-3} & 1\times10^{-4} & 0 \end{bmatrix}, \\
R &= \operatorname{diag}\begin{bmatrix} 0.7164 & 1.0\times10^{-8} & 1.5\times10^{-8} & 2.5\times10^{-7} & 0.25 \end{bmatrix}.
\end{aligned} \tag{21}$$

In order to improve the identifiability of the aerodynamic stability and control derivatives, three "3211" disturbance signals with the amplitude of 5 degrees are consecutively added to the trimmed input signal. The simulation generates 100 s of data with a sampling time of 0.05 s to evaluate the algorithm.

The Bayesian UKF estimation algorithm proposed in this paper is compared with the simplified UKF and the parallelized UKF [5] to evaluate the performance. The accurate noise characteristics are given to the simplified UKF while the other two UKFs run with unknown noise characteristics. The process and measurement noise covariance matrices Q and R are unknown and need to be estimated. Since the influence of Q can be projected to the measurement noise influence according to UKF, so only the measurement noise covariance matrix R is estimated, and the process noise covariance matrix Q is simply regarded as a diagonal matrix with a small amount of $1e-6$ to avoid the morbidity of the state error covariance matrix. Assuming the five state variables are all observed, and considering the height is not vulnerable to the disturbance and noise, then the covariance of height is not estimated in R and regarded as a constant value of 0.01 m. Finally, the four noise covariance parameters θ and four noise insensitive aerodynamic derivatives Θ in Equation (22) are assumed to be unknown and need to be estimated.

$$\begin{aligned}\Theta &= \left[C_{L\alpha} \quad C_{M\alpha} \quad C_{Mq} \quad C_{M\delta e} \right]^T, \\ \theta &= \left[R_V \quad R_\alpha \quad R_q \quad R_\varphi \right]^T.\end{aligned} \tag{22}$$

For this flight scenario, the system states are initialized by the measurement data at $t = 0$ s. The unknown parameters are, respectively, initialized by $\Theta_0 = [0, 0, 0, 0]^T$, $\theta_0 = [10^{-4}, 10^{-4}, 10^{-4}, 10^{-4}]^T$. The augmented state error covariance matrix is initialized as an identity matrix, and the initial noise parameter error covariance is $P_{\theta 0} = \mathrm{diag}\,(1, 8 \times 10^{-4}, 10^{-4}, 10^{-4})$. Specifically, the noise parameter initialization is only needed in the running of the Bayesian and parallel UKFs, and the same parameter initialization is adopted in all three UKFs.

Figures 2(a)–2(e) compare the simulated measurement data with the five estimated states by the Bayesian UKF algorithm. The results show that the estimated states are consistent with the measurement data, and the algorithm can effectively filter the noise. The system input is disturbed by three "3211" signals between 0 and 15 s (see Figure 2(f)).

Figure 3 compares the parameter estimation performance of the three algorithms, which are the Bayesian UKF, the simplified UKF, and the parallel UKF. The three UKFs are denoted as BUKF, SUKF, and PUKF in Figure 3 and Table 2, respectively. The four unknown aerodynamic derivatives are estimated by the three UKFs and compared with the reference value which is presented in the simulation program. The estimated results of the four derivatives all converge to the reference values rapidly from the zero initials. The estimates of aerodynamic derivatives show that the Bayesian UKF has the equivalent precision as the simplified UKF with prior noise characteristics knowledge, while the error of the parallel UKF is more obvious than the other two methods. Especially for the estimates of $C_{L\alpha}$, the results of parallel UKF converge more slowly. The root mean square error (RMSE) of the estimated results to the reference values is compared in Table 2. The calculation of RMSE uses the relative error (see Equation (23)) for reflecting the estimates' overall deviation from the reference values. The results in Table 2 indicate that the estimates of $C_{M\delta e}$ are more accurate than the other three derivatives; the accuracy of Bayesian UKF estimates is equivalent to the simplified UKF estimates; the estimation error of parallel UKF is bigger than Bayesian UKF for the three derivatives of $C_{L\alpha}$, $C_{M\alpha}$, and $C_{M\delta e}$ in this flight scenario. The average computational time of every filtering step in Table 2 shows that the efficiency of Bayesian UKF is slightly lower than simplified UKF and higher than parallel UKF on the same computing platform.

$$\begin{aligned}\mathrm{RMSE} &= \sqrt{\frac{1}{N}\sum_{i=1}^{N} (e_i^r)^T e_i^r}, \\ e^r &= \frac{\widehat{\Theta} - \Theta_{\mathrm{true}}}{\Theta_{\mathrm{true}}}.\end{aligned} \tag{23}$$

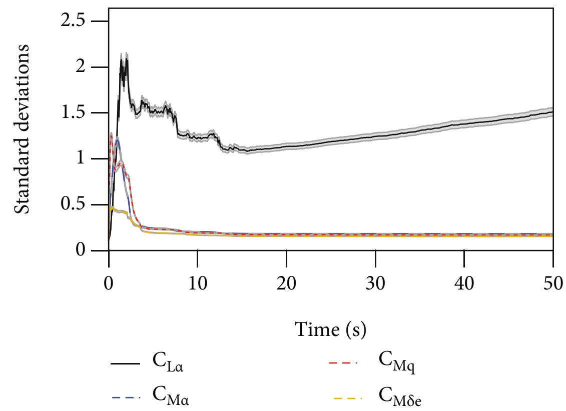

Figure 5: Histories of standard deviations of the four estimated aerodynamic derivatives in Monte Carlo simulation. The four derivatives' standard deviations with 95% bands for every step are fitted by the normal distribution.

The performance of the Bayesian adaptive algorithm is further assessed by a Monte Carlo simulation with 1800 realizations. For every realization, the diagonal elements of the process noise covariance are randomly chosen from the ±10% range of the given values of Q in Equation (21). The Monte Carlo simulation results are sequentially fitted by the normal distribution, and the statistics of the mean and standard deviation with their 95% bands are given in Figures 4 and 5. The histories of the estimated mean values with 95% bands are compared with the reference values in Figures 4(a)–4(d). The Monte Carlo results validate the robustness and effectiveness of the adaptive algorithm for the aerodynamic parameter estimation with unknown noise characteristics. The narrow 95% bands of the mean values indicate the excellent consistency of estimates for the varied process noise covariances. The three derivatives in the pitch direction converge to the reference values rapidly, while the steady-state mean estimates of $C_{L\alpha}$ slightly deviate from the reference value (about 11% less than the reference value). The standard deviations with 95% bands for the derivatives are shown in Figure 5. The banded curves also illustrate the estimation accuracy and consistency of the Bayesian UKF algorithm. The Monte Carlo simulation shows that the estimation accuracy of $C_{L\alpha}$ is lower than other derivatives, which may be related to the slow convergence of the derivatives in the aircraft lift direction under disturbance and noise.

4. Conclusions

This paper issues the problem of system state and intrinsic parameter joint estimation with unknown noise characteristics. A Bayesian adaptive unscented Kalman filter is proposed for this problem. A Gauss-Newton-based noise parameter optimization method is introduced into the UKF filtering procedure for maximizing the posterior probability likelihood function. The proposed algorithm is evaluated and assessed by a steady-level cruise scenario of a B737-like aircraft. A simplified UKF and a parallel UKF are applied to the same flight scenario with the Bayesian adaptive UKF for the performance comparison. The results demonstrate that the Bayesian adaptive UKF algorithm can estimate the system augmented states from the flight data with unknown noise characteristics. The adaptive filter performs very well with relatively high accuracy and efficiency. The comparison with the other two UKFs illustrates that the Bayesian UKF has equivalent performance to the simplified UKF with prior noise information but slightly outperforms the parallel UKF on the aerodynamic derivatives' estimation precision and computational time for this flight scenario. Then, the effectiveness and consistency of the proposed algorithm are further validated by a Monte Carlo simulation.

The algorithm proposed here has the potential application prospect in the aerodynamic parameter estimation from the aircraft real flight test data and provides another way of handling the parameter-noise simultaneously estimating problem. This algorithm possesses the same inherent merits and defects of Newton methods, so the further improvement in precision, stability, and efficiency of the algorithm is required in future researches.

Data Availability

The raw and process data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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