

Research Article

An Efficient Revocable Identity-Based Encryption with Equality Test Scheme for the Wireless Body Area Network

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With the rapid development and popularization of cloud computing, people are willing to upload their own data to the cloud to enjoy the services. However, some personal and private data are not suitable for uploading directly to the cloud. Therefore, these data must be encrypted before uploading to the cloud to ensure the confidentiality. To achieve the confidentiality of data and enjoy cloud services, a notion of identity-based encryption with equality test (IBEET) was proposed. Using IBEET, two ciphertexts encrypted under different public keys can be tested to confirm whether they contain the same plaintext. The equality test can be applied to the wireless body area network system in which the cloud can utilize ciphertexts from patients and medical institutions to perform equality tests to determine whether which patient's status is abnormal. Indeed, revoking illegal or expired users on any cryptosystem is an important issue. To the best of our knowledge, there is little research on the design mechanism of user revocation in the IBEET. In this paper, we propose a novel notion of revocable identity-based encryption with an equality test, called RIBEET. Based on the notion, we present the first RIBEET scheme. Meanwhile, the proposed scheme will be proven to be secure under the bilinear Diffie-Hellman (BDH) assumption.

1. Introduction

With the rapid development and popularization of cloud computing, people are willing to upload their own data to the cloud to enjoy the services. However, some personal and private data are not suitable for uploading directly to the cloud. To ensure the confidentiality of data, several encryption mechanisms [1–4] have been applied to cloud computing. Identity-based encryption (IBE) [5] is one of the encryption mechanisms of public key systems. The system of an IBE contains two roles: the private key generator (PKG) and users (including senders and receivers). Each user utilizes his own identity (e.g., e-mail address, name, or social security number) to register with the PKG to obtain a private key. Senders can regard the identity of the receiver as a public key to encrypt private data. After receiving the encrypted message (ciphertext), the receiver can decrypt it with her/his own private key.

To achieve the confidentiality of data and enjoy cloud services, the first identity-based encryption with equality test (IBEET) was proposed by Ma [6]. Using IBEET, two

ciphertexts encrypted under different public keys can be tested to confirm whether they contain the same plaintext. Ma [6] also gave an application of IBEET used to classify encrypted e-mails. Each encrypted e-mail can be attached with a tag for classification, while the tag can be encrypted under different public keys in the IBEET system. An e-mail server in the cloud can test the equality of any two encrypted tags to classify encrypted e-mails. Subsequently, many studies on IBEET have been published in the literature [7–11].

The equality test can be applied to the wireless body area network (WBAN) system [12–17] in which the cloud can utilize ciphertexts from patients and medical institutions to perform equality tests to determine whether the patient's status is abnormal. Figure 1 shows the architecture of WBANs. A patient is equipped with wearable sensors to collect her/his health record data from sensors of electroencephalogram (EEG), electrocardiogram (ECG), blood pressure, pulse oximeter, insulin pump, electromyogram (EMG), and motion. These health record data are encrypted through the mobile device and uploaded to the cloud server. On the other hand, the medical institution also uploads the

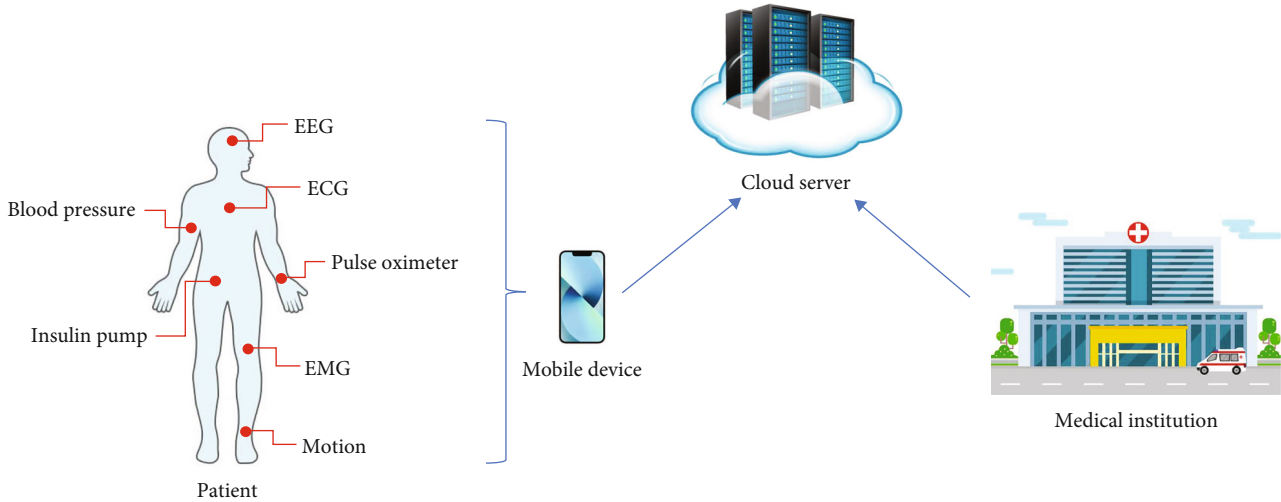


FIGURE 1: The architecture of WBAN.

patient's encrypted health data to the cloud server. The ciphertexts can be tested for equality without knowing the health data of the patient by the cloud server. If the patient's health data are different from the medical institution's health data, it means that the patient's health data are abnormal.

Indeed, revoking illegal or expired users on any cryptosystem is an important issue. In the traditional public key cryptosystem (PKC), public key infrastructures (PKI) must be established to manage each user's certificate which links the user's identity and public key. In addition, the certificate revocation list [18] is also included in the PKI to revoke illegal or expired users. In identity-based public key cryptosystems (ID-PKC), the first IBE was presented by Boneh and Franklin [5] in which a user can be revoked by the PKG, who sends new private keys for all nonrevoked users at each period, if the user did not receive the new private key. So far, many literatures related to revocable IBE [19–26] have been published. To the best of our knowledge, there is little research on design mechanism of user revocation in the IBEET. In this paper, we propose a novel notion of revocable identity-based encryption with equality test, called RIBEET. Based on the notion, we present the first RIBEET scheme. Meanwhile, the scheme will be proven to be secure under the bilinear Diffie-Hellman (BDH) assumption.

1.1. Related Work. In the era of advanced network communication, cloud computing is an indispensable part. The terminal devices on the user side usually do not have high-performance computing power. However, users can entrust large computing tasks to the cloud. Then, the cloud will return the corresponding results to users after finishing the tasks. Indeed, the cloud can assist each user in performing tasks that require a lot of computation, but it also means that the cloud can know each user's data if the data is not encrypted. Typically, users will encrypt data to the cloud if the data is sensitive or private. In addition, encrypted data also needs to be quickly retrieved from the cloud. To achieve

this function, several schemes [3, 4, 27, 28] related to public key encryption with a keyword search were proposed. Although these schemes can retrieve encrypted data, only data encrypted under the same public key can be retrieved.

To support searchable encrypted data under different public keys, Yang et al. [29] proposed a comparison mechanism of two ciphertexts encrypted under different public keys in the traditional public key cryptosystem, called public key encryption with equality test (PKEET). However, the traditional public key cryptosystem must rely on the public key infrastructure to manage each user's certificate which links the user's identity and her/his public key. To avoid the use of public key infrastructure and certificates, Shamir [30] introduced a new concept of ID-PKC in which a user's public key is her/his identity such as name, e-mail, or telephone number. In this way, certificates will no longer be needed in the ID-PKC since the public key is meaningful and can represent the user's identity. Combining the concepts of PKEET and ID-PKC, Ma [6] proposed the first identity-based encryption with equality test, called IBEET. To consider more types of authorizations, Li et al. [31] proposed the IBEET scheme with four types of authorizations. Unfortunately, the proposed scheme of Li et al. [31] is not suitable for the IoT environment because the performance of the scheme is not good. Immediately, Elhabob et al. [10] proposed another IBEET scheme with four types of authorizations which has higher performance.

For the issue of user revocation in the ID-PKC, Boneh and Franklin [5] suggested that the new private keys should be resent to users who have not been revoked at different periods. As a result, secure channels will be established to send these private keys, and the PKG's workload will also increase. To reduce the PKG's workload, Boldyreva et al. [19] hired a binary tree to propose an IBE scheme with user revocation, named revocable IBE (RIBE). However, Boldyreva et al.'s scheme [19] only satisfied the selective-ID security. Later, Libert and Vergnaud [20] proposed another

TABLE 1: Comparisons between the existing schemes and our RIBEET scheme.

Schemes	Public key setting	Avoiding the use of certificates	Supporting equality test of ciphertexts	Providing user revocation
PKEET [29]	PKI-based	No	Yes	Yes
IBE [5]	ID-based	Yes	No	No
RIBE [21]	ID-based	Yes	No	Yes
IBEET [6]	ID-based	Yes	Yes	No
Our RIBEET	ID-based	Yes	Yes	Yes

RIBE scheme which meets the adaptive-ID security. A mechanism for revoking users through public channels was proposed by Tseng and Tsai [21], in which each user's full private key is divided into two parts: a fixed key and a time updated key. The fixed key is delivered to the user through secure channels only once, while the time updated key is delivered to the user through public channels at different periods. Users can be revoked if they do not receive the new time updated keys. For the security of decryption key exposure, Seo and Emura [22] proposed a new RIBE scheme to enhance the security. To reduce the length of public parameters and meet the security of decryption key exposure resistance, Watanabe et al. [23] presented another RIBE scheme. In addition, several lattice-based RIBE schemes [24–26] were proposed to resist quantum attacks.

1.2. Motivation. As mentioned earlier, revoking illegal or expired users on any cryptosystem is still an important issue. In the traditional PKC, the PKEET [29] can hire the certificate revocation list [18] to revoke illegal or expired users. However, the IBE [5] cannot effectively revoke illegal or expired users in the ID-PKC, so the RIBE [21] was proposed. To the best of our knowledge, there is little research on the design mechanism of user revocation in the IBEET [6]. Table 1 shows the comparisons between the PKEET [29], the IBE [5], the RIBE [21], the IBEET [6], and our RIBEET in terms of public key setting, avoiding the use of certificates, supporting the equality test of ciphertexts, and providing user revocation. Hence, we attempt to propose the first revocable identity-based encryption with equality test, called RIBEET.

1.3. Contribution and Organization. Although the existing RIBE schemes [21–26] provide a mechanism to revoke users, they do not extend to support the equality test for ciphertexts. On the other hand, the existing IBEET schemes [6, 10, 31] do not support to revoke users. To the best of our knowledge, there is little research on the design mechanism of user revocation in the IBEET. In this paper, we propose a novel notion of revocable identity-based encryption with

equality test, called RIBEET. In the following, we list specific contributions.

- (i) Based on the existing syntax and security notions of IBEET, we consider the property of user revocation to define a new syntax and security notions of RIBEET
- (ii) Following the syntax of RIBEET, a concrete RIBEET scheme is proposed
- (iii) In the security notions of RIBEET, the proposed scheme is proven to be secure under the bilinear Diffie-Hellman (BDH) assumption
- (iv) We compare the proposed scheme with the previous RIBE scheme and IBEET scheme. We demonstrate that the proposed scheme not only provides user revocation but also supports the equality test for ciphertexts

The rest of the article includes six sections. Preliminaries are given in Section 2. Section 3 presents the syntax and security notions of RIBEET. A concrete RIBEET scheme is proposed in Section 4. The security analysis of the RIBEET scheme is shown in Section 5. We compare the RIBEET scheme with other existing schemes in Section 6. The last section gives the conclusion.

2. Preliminaries

In this section, we introduce two definitions related to a mathematical tool and security assumption. We hire the bilinear pairings [5] as a mathematical tool to construct our RIBEET scheme. To prove the security of the proposed scheme, we consider the bilinear Diffie-Hellman (BDH) problem and then give a BDH assumption [6]. The definition of the bilinear pairings is given as follows.

Definition 1. Let G_1 , G_2 , and G_T be three multiplicative cyclic groups of a prime order q . Assume that a mapping $\hat{e} : G_1 \times G_2 \rightarrow G_T$ is an asymmetric bilinear map. Then, the map \hat{e} satisfies the following properties.

- (1) Bilinearity: $\hat{e}(g_1^a, g_2^b) = \hat{e}(g_1, g_2)^{ab}$ for $g_1 \in G_1$, $g_2 \in G_2$, and $a, b \in \mathbb{Z}_q^*$
- (2) Nondegeneracy: $\hat{e}(g_1, g_2) \neq 1$ for some $g_1 \in G_1$ and $g_2 \in G_2$
- (3) Computability: $\hat{e}(g_1, g_2)$ can be efficiently computed for $g_1 \in G_1$, $g_2 \in G_2$

For the asymmetric bilinear map, the BDH problem is to compute $\hat{e}(g_1, g_2)^{abc}$ by given a tuple $\langle q, G_1, G_2, G_T, \hat{e}, g_1, g_1^a, g_1^c, g_2, g_2^a, g_2^b \rangle$. We define the BDH assumption as follows.

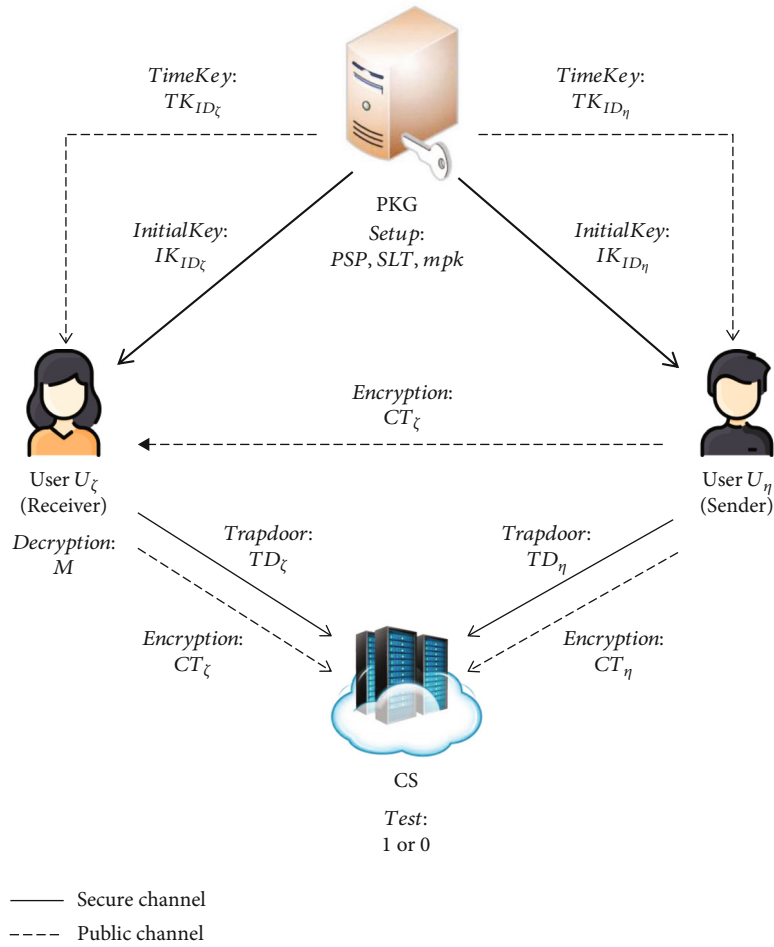


FIGURE 2: The syntax of RIBEET.

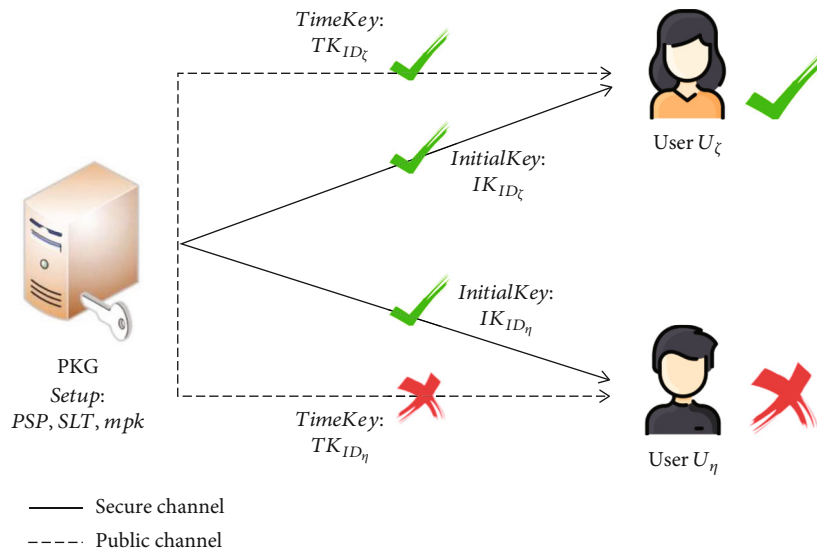


FIGURE 3: The procedure for user revocation.

TABLE 2: Notations.

Notations	Meaning
PSP	The public system parameters
SLT	The system life time
mpk	The master private key
ID	The user's identity
IK _{ID}	The user initial key
TK _{ID}	The user time key
M	The message
CT	The ciphertext
TD _{ID}	The trapdoor
(CT_ζ, TD_ζ)	The ciphertext-trapdoor pair of the user U_ζ
(CT_η, TD_η)	The ciphertext-trapdoor pair of the user U_η

Definition 2. On inputting a tuple $(q, G_1, G_2, G_T, \hat{e}, g_1, g_1^a, g_1^c, g_2, g_2^a, g_2^b)$, we say that the BDH problem holds if no algorithm \mathcal{A} has nonnegligible advantage in computing $\hat{e}(g_1, g_2)^{abc}$. The advantage can be denoted as $\Pr[\mathcal{A}(g_1, g_1^a, g_1^c, g_2, g_2^a, g_2^b) = \hat{e}(g_1, g_2)^{abc}] < \epsilon$.

3. Syntax and Security Notions

3.1. Syntax of RIBEET. Based on the syntax of IBEET schemes [6], we employ the revocation technique [21] to present a new syntax of RIBEET depicted in Figure 2 which consists of three roles and seven algorithms, namely Setup, InitialKey, TimeKey, Encryption, Decryption, Trapdoor, and Test. The first role is the private key generator (PKG) who is responsible for executing the first three algorithms, and the second role is the users who can, respectively, utilize Encryption, Decryption, and Trapdoor algorithms for encryption, decryption, and authorization. The last role is the cloud server (CS) who runs the Test algorithm to compare the two ciphertexts. For the user revocation, we use Figure 3 to illustrate how users are revoked by the PKG. If the PKG stops sending the time key to a user, it means that the user has been revoked since both initial key and time key are required to execute Decryption and Trapdoor algorithms. Here, we arrange some notations used in these algorithms in Table 2. The algorithms of RIBEET are described in detail as follows.

- (i) Setup: this algorithm is performed by the PKG who takes a security parameter k and a time period t as input to produce the public system parameters PSP, the system life time SLT, and the master private key mpk
- (ii) InitialKey: this algorithm is performed by the PKG who takes the public system parameter PSP, the master private key mpk, and a user's identity $ID \in \{0, 1\}^*$ as input to produce user initial key IK_{ID}

- (iii) TimeKey: this algorithm is performed by the PKG who takes the public system parameter PSP, the master private key mpk, a user's identity $ID \in \{0, 1\}^*$, and a period $T \in SLT$ as input to produce user time key TK_{ID}
- (iv) Encryption: this algorithm is performed by a user (sender) who takes the public system parameter PSP, a user's identity $ID \in \{0, 1\}^*$, a period $T \in SLT$, and a message $M \in \{0, 1\}^\lambda$ as input to produce a ciphertext CT
- (v) Decryption: this algorithm is performed by a user (receiver) who takes the public system parameter PSP, the receiver's initial key IK_{ID}, the receiver's time key TK_{ID}, and the ciphertext CT as input to produce the message M
- (vi) Trapdoor: this algorithm is performed by a user who takes her/his initial key IK_{ID} and time key TK_{ID} as input to produce the trapdoor TD_{ID}
- (vii) Test: this algorithm is performed by the CS who takes the public system parameters PSP and two ciphertext-trapdoor pairs (CT_ζ, TD_ζ) and (CT_η, TD_η) from any two users U_ζ and U_η as input to produce 1 or 0

3.2. Security Notions of RIBEET. In this section, we define the security notions of RIBEET which includes four types of adversaries. Two of these types are the same as the security notions of IBEET [6]. Considering the revoked users from RIBEET, we need to add two types of adversaries in the security notions. These four types of adversaries are presented as follows.

- (1) Type I adversary: such an adversary can obtain all information (including time key TK_{ID}) transmitted through public channels. The adversary can be regarded as an outside attacker
- (2) Type II adversary: such an adversary owns her/his initial key IK_{ID}, but he does not have the current time key TK_{ID}. The adversary can be regarded as a revoked user
- (3) Type III adversary: this adversary is identical to the type I adversary, except that she/he possesses the trapdoor TD
- (4) Type IV adversary: this adversary is identical to the type II adversary, except that she/he possesses the trapdoor TD

Following the security notions of IBEET [6], we consider revoked users to define the new security notions of RIBEET. Definitions 3 and 4, respectively, are given to state IND-ID-CCA and OW-ID-CCA security of an RIBEET scheme.

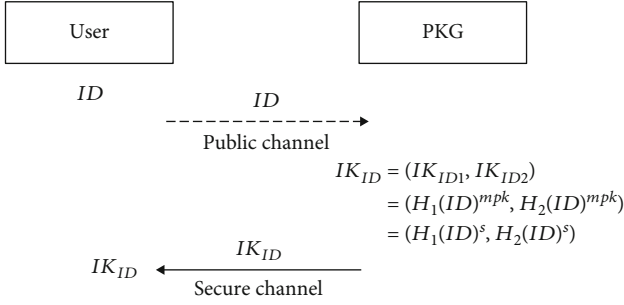


FIGURE 4: InitialKey procedure.

Definition 3 (IND-ID-CCA). Let \mathcal{A} be a type I or type II adversary for an RIBEET scheme and \mathcal{B} be a challenger in the following game. The scheme is IND-ID-CCA secure if the advantage that \mathcal{A} wins the game is negligible.

- (1) Setup. The challenger \mathcal{B} takes a security parameter k and a time period t as input to produce the public system parameters PSP, the system life time SLT, and the master private key mpk. The public system parameters PSP and the system life time SLT are sent to the adversary \mathcal{A}
- (2) Phase 1. Several queries below can be issued by the adversary \mathcal{A}
 - (a) InitialKey query(ID): given an identity ID, the challenger \mathcal{B} generates an initial key IK_{ID} as the response by running the InitialKey algorithm of the RIBEET scheme
 - (b) TimeKey query(ID, T): given an identity ID and a period T , the challenger \mathcal{B} generates a time key $T K_{ID}$ as the response by running the TimeKey algorithm of the RIBEET scheme
 - (c) Decryption query(ID, T , CT): given an identity ID, a period T , and a ciphertext CT, the challenger \mathcal{B} generates the resulting message M as the response by running the Decryption of the RIBEET scheme
 - (d) Trapdoor query(ID, T): given an identity ID and a period T , the challenger \mathcal{B} generates a trapdoor $T D_{ID}$ as the response by running the Trapdoor of the RIBEET scheme
- (3) Challenge. Two messages M_0^* , M_1^* , an identity ID^* , and a period T^* are submitted by the adversary \mathcal{A} . The challenger \mathcal{B} chooses M_b^* from these two messages, where $b \in \{0, 1\}$ is a random coin. The challenger \mathcal{B} then generates a ciphertext CT^* as the challenge one by running the Encryption of the RIBEET scheme with (ID^*, T^*, M_b^*) . Here, the following restrictions must be satisfied

- (i) The adversary \mathcal{A} cannot issue the Trapdoor query with ID^*
 - (ii) The adversary \mathcal{A} cannot issue the InitialKey query with ID^* if it is the type I adversary
 - (iii) The adversary \mathcal{A} cannot issue the TimeKey query with (ID^*, T^*) if it is the type II adversary
- (4) Phase 2. Under the above restrictions, \mathcal{A} can execute the same tasks as in phase 1
 - (5) Guess. The adversary \mathcal{A} outputs a guess $b' \in \{0, 1\}$ and wins the game if $b' = b$. The advantage that \mathcal{A} wins the game can be denoted as $\text{Adv}_{\mathcal{A}}(k) = |\Pr[b' = b] - 1/2|$

Definition 4 (OW-ID-CCA). Let \mathcal{A} be a type III or type IV adversary for an RIBEET scheme and \mathcal{B} be a challenger in the following game. The scheme is OW-ID-CCA secure if the advantage that \mathcal{A} wins the game is negligible.

- (1) Setup. The challenger \mathcal{B} takes a security parameter k and a time period t as input to produce the public system parameters PSP, the system life time SLT, and the master private key mpk. The public system parameters PSP and the system life time SLT are sent to the adversary \mathcal{A}
- (2) Phase 1. Several queries below can be issued by the adversary \mathcal{A}
 - (a) InitialKey query(ID): given an identity ID, the challenger \mathcal{B} generates an initial key IK_{ID} as the response by running the InitialKey algorithm of the RIBEET scheme
 - (b) TimeKey query(ID, T): given an identity ID and a period T , the challenger \mathcal{B} generates a time key $T K_{ID}$ as the response by running the TimeKey algorithm of the RIBEET scheme
 - (c) Decryption query(ID, T , CT): given an identity ID, a period T , and a ciphertext CT, the challenger \mathcal{B} generates the resulting message M as the response by running the Decryption of the RIBEET scheme
 - (d) Trapdoor query(ID, T): given an identity ID and a period T , the challenger \mathcal{B} generates a trapdoor $T D_{ID}$ as the response by running the Trapdoor of the RIBEET scheme
- (3) Challenge. An identity ID^* and a period T^* are submitted by the adversary \mathcal{A} . The challenger \mathcal{B} randomly chooses M^* and then generates a ciphertext CT^* as the challenge one by running the Encryption

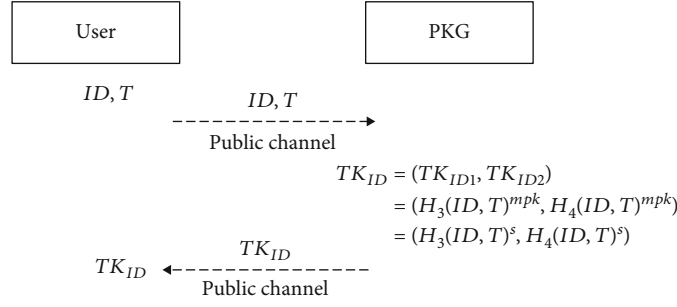


FIGURE 5: TimeKey procedure.

TABLE 3: Comparisons of the proposed RIBEET with the existing RIBE and several IBEET.

Schemes	The cost of performing encryption	The cost of performing decryption	The cost of performing equality test	Supporting equality test of ciphertexts	Providing user revocation
RIBE [21]	1 · Pair + 2 · Exp (8.7843 ms)	1 · Pair (7.8351 ms)	—	No	Yes
IBEET [6]	2 · Pair + 6 · Exp (18.5178 ms)	2 · Pair + 2 · Exp (16.6194 ms)	4 · Pair (31.3404 ms)	Yes	No
IBEET [10]	2 · Pair + 4 · Exp (17.5686 ms)	2 · Pair + 1 · Exp (16.1448 ms)	2 · Pair + 2 · Exp (16.6194 ms)	Yes	No
IBEET [11]	2 · Pair + 9 · Exp (19.9416 ms)	2 · Pair + 2 · Exp (16.6194 ms)	2 · Pair + 2 · Exp (16.6194 ms)	Yes	No
Our RIBEET	2 · Pair + 5 · Exp (18.0432 ms)	2 · Pair + 2 · Exp (16.6194 ms)	4 · Pair (31.3404 ms)	Yes	Yes

TABLE 4: Comparison of communication cost.

Schemes	PK	CT	TD
RIBE [21]	$2 G_1 $	$ G_1 + Z_q $	—
IBEET [6]	$ G_1 $	$4 G_1 + Z_q $	$ G_1 $
IBEET [10]	$ G_1 $	$2 G_1 + 2 Z_q $	$ G_1 $
IBEET [11]	$ G_1 + Z_q $	$3 G_1 + Z_q $	$4 G_1 $
Our RIBEET	$2 G_2 $	$3 G_1 + Z_q $	$ G_2 $

of the RIBEET scheme with (ID^*, T^*, M^*) . Here, the following restrictions must be satisfied

- (a) The adversary \mathcal{A} cannot issue the InitialKey query with ID^* if it is the type III adversary
- (b) The adversary \mathcal{A} cannot issue the TimeKey query with (ID^*, T^*) if it is the type IV adversary
- (4) Phase 2. Under the above restrictions, \mathcal{A} can execute the same tasks as in phase 1

- (5) Guess. The adversary \mathcal{A} outputs a guess M' and wins the game if $M^* = M'$. The advantage that \mathcal{A} wins the game can be denoted as $\text{Adv}_{\mathcal{A}}(k) = \Pr[M^* = M']$.

4. Concrete RIBEET Scheme

A revocable identity-based encryption with equality test scheme, which we denote by RIBEET, consists of algorithms Setup, InitialKey, TimeKey, Encryption, Decryption, Trapdoor, and Test. Each of the algorithms is described as follows.

- (1) Setup: this algorithm is performed by the PKG who takes a security parameter k and a time period t as input to produce an asymmetric bilinear map $\hat{e} : G_1 \times G_2 \rightarrow G_T$ and a system life time $\text{SLT} = \{T_0, T_1, \dots, T_t\}$, where G_1, G_2 , and G_T are multiplicative cyclic groups of prime order q . The PKG first chooses two arbitrary generators $g_1 \in G_1$ and $g_2 \in G_2$ and picks eight cryptographic one-way hash functions $H_1 : \{0, 1\}^* \rightarrow G_2$, $H_2 : \{0, 1\}^* \rightarrow G_2$, $H_3 : \{0, 1\}^* \rightarrow G_2$, $H_4 : \{0, 1\}^* \rightarrow G_2$, $H_5 : G_T \times G_1 \times G_1 \rightarrow \{0, 1\}^{\lambda+l}$, $H_6 : \{0, 1\}^\lambda \rightarrow G_2$, $H_7 : \{0, 1\}^{\lambda+l} \rightarrow Z_q^*$, and $H_8 : G_T \rightarrow G_2$, where λ and l are fixed lengths. Then, a random value $s \in Z_q^*$ is chosen, and $P_{\text{pub}} = g_1^s$ is computed. The public system parameters are $\text{PSP} = \{q, G_1$

, $G_2, G_T, \hat{e}, g_1, g_2, P_{\text{pub}}, H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8$ }, the system life time is $SLT = \{T_0, T_1, \dots, T_t\}$, and the master private key is $\text{mpk} = s$

- (2) InitialKey: this algorithm is performed by the PKG who takes the public system parameter PSP, the master private key mpk , and a user's identity $ID \in \{0, 1\}^*$ as input to produce user initial key

$$\begin{aligned} \text{IK}_{\text{ID}} &= (\text{IK}_{\text{ID1}}, \text{IK}_{\text{ID2}}) \\ &= \left(H_1(\text{ID})^{\text{mpk}}, H_2(\text{ID})^{\text{mpk}} \right) \\ &= (H_1(\text{ID})^s, H_2(\text{ID})^s). \end{aligned} \quad (1)$$

Here, the procedure of this algorithm is depicted in Figure 4.

- (3) TimeKey: this algorithm is performed by the PKG who takes the public system parameter PSP, the master private key mpk , a user's identity $ID \in \{0, 1\}^*$, and a period $T \in SLT$ as input to produce user time key

$$\begin{aligned} \text{TK}_{\text{ID}} &= (\text{TK}_{\text{ID1}}, \text{TK}_{\text{ID2}}) \\ &= \left(H_3(\text{ID}, T)^{\text{mpk}}, H_4(\text{ID}, T)^{\text{mpk}} \right) \\ &= (H_3(\text{ID}, T)^s, H_4(\text{ID}, T)^s). \end{aligned} \quad (2)$$

Here, the procedure of this algorithm is depicted in Figure 5.

- (4) Encryption: this algorithm is performed by a sender who takes the public system parameter PSP, a user's identity $ID \in \{0, 1\}^*$, a period $T \in SLT$, and a message

$M \in \{0, 1\}^\lambda$ as input to produce ciphertexts $\text{CT} = (\text{CT}_1, \text{CT}_2, \text{CT}_3, \text{CT}_4)$ which are shown as follows

- (a) $\text{CT}_1 = g_1^r$
- (b) $\text{CT}_2 = g_1^u$
- (c) $\text{CT}_3 = H_5(\hat{e}(P_{\text{pub}}, H_1(\text{ID}) \cdot H_3(\text{ID}, T))^u, \text{CT}_1, \text{CT}_2) \oplus (M || V)$
- (d) $\text{CT}_4 = H_6(M)^r \cdot H_8(\hat{e}(P_{\text{pub}}, H_2(\text{ID}) \cdot H_4(\text{ID}, T))^u)$

Here, $r = H_7(M, V)$ and the two values $V \in \{0, 1\}^l$ and $u \in Z_q^*$ are chosen in random.

- (5) Decryption: this algorithm is performed by a receiver who takes the public system parameter PSP, the receiver's initial key $\text{IK}_{\text{ID}} = (\text{IK}_{\text{ID1}}, \text{IK}_{\text{ID2}})$, the receiver's time key $\text{TK}_{\text{ID}} = (\text{TK}_{\text{ID1}}, \text{TK}_{\text{ID2}})$, and the ciphertext $\text{CT} = (\text{CT}_1, \text{CT}_2, \text{CT}_3, \text{CT}_4)$ as input to produce the message M . The detailed process is shown as follows:

- (a) Compute $\text{CT}_3 \oplus H_5(\hat{e}(\text{CT}_2, \text{IK}_{\text{ID1}} \cdot \text{TK}_{\text{ID1}}), \text{CT}_1, \text{CT}_2)$ to obtain $M' || V'$
- (b) Compute $r' = H_7(M', V')$
- (c) Produce the message M' as M if $\text{CT}_1 = g_1^{r'}$ and $\text{CT}_4 = H_6(M)^{r'} \cdot H_8(\hat{e}(\text{CT}_2, \text{IK}_{\text{ID2}} \cdot \text{TK}_{\text{ID2}}))$ both hold

The correctness of obtaining $M' || V'$ can be demonstrated as follows.

$$\begin{aligned} \text{CT}_3 \oplus H_5(\hat{e}(\text{CT}_2, \text{IK}_{\text{ID1}} \cdot \text{TK}_{\text{ID1}}), \text{CT}_1, \text{CT}_2) &= H_5(\hat{e}(P_{\text{pub}}, H_1(\text{ID}) \cdot H_3(\text{ID}, T))^u, \text{CT}_1, \text{CT}_2) \oplus (M' || V') \oplus H_5(\hat{e}(\text{CT}_2, \text{IK}_{\text{ID1}} \cdot \text{TK}_{\text{ID1}}), \text{CT}_1, \text{CT}_2) \\ &= H_5(\hat{e}(g_1^s, H_1(\text{ID}) \cdot H_3(\text{ID}, T))^u, \text{CT}_1, \text{CT}_2) \oplus (M' || V') \oplus H_5(\hat{e}(g_1^u, H_1(\text{ID})^s \cdot H_3(\text{ID}, T)^s), \text{CT}_1, \text{CT}_2) \\ &= H_5(\hat{e}(g_1, H_1(\text{ID}) \cdot H_3(\text{ID}, T))^{su}, \text{CT}_1, \text{CT}_2) \oplus (M' || V') \oplus H_5(\hat{e}(g_1, H_1(\text{ID}) \cdot H_3(\text{ID}, T))^{su}, \text{CT}_1, \text{CT}_2) = M' || V'. \end{aligned} \quad (3)$$

$\text{TK}_{\eta 2}, \text{CT}_{\eta 3}, \text{CT}_{\eta 4}$), from any two users U_ζ and U_η as input to produce 1 or 0 according to the following steps

- (6) Trapdoor: this algorithm is performed by a user who takes her/his initial key $\text{IK}_{\text{ID}} = (\text{IK}_{\text{ID1}}, \text{IK}_{\text{ID2}})$ and time key $\text{TK}_{\text{ID}} = (\text{TK}_{\text{ID1}}, \text{TK}_{\text{ID2}})$ as input to produce the trapdoor $\text{TD}_{\text{ID}} = \text{IK}_{\text{ID2}} \cdot \text{TK}_{\text{ID2}} = H_2(\text{ID})^s \cdot H_4(\text{ID}, T)^s$

- (7) Test: this algorithm is performed by the CS who takes the public system parameters PSP and two ciphertext-trapdoor pairs $(\text{CT}_\zeta, \text{TD}_\zeta)$ and $(\text{CT}_\eta, \text{TD}_\eta)$, where $\text{C}_{\text{T}\zeta} = (\text{CT}_{\zeta 1}, \text{CT}_{\zeta 2}, \text{CT}_{\zeta 3}, \text{CT}_{\zeta 4})$ and $\text{C}_{\text{T}\eta} = (\text{CT}_{\eta 1}, \text{C}$

- (a) Compute R_ζ and R_η as follows:

- (i) $R_\zeta = \text{CT}_{\zeta 4} / H_8(\hat{e}(\text{CT}_{\zeta 2}, \text{TD}_\zeta)) = H_6(M_\zeta)^{H_7(M_\zeta, V_\zeta)}$
- (ii) $R_\eta = \text{CT}_{\eta 4} / H_8(\hat{e}(\text{CT}_{\eta 2}, \text{TD}_\eta)) = H_6(M_\eta)^{H_7(M_\eta, V_\eta)}$

TABLE 5: Comparison of energy cost.

Energy cost	RIBE [21]	IBE _{ET} [6]	IBEET [10]	IBEET [11]	Our RIBEET
Performing encryption	358.256 μ J	755.224 μ J	716.508 μ J	813.292 μ J	735.868 μ J

(b) Compute $\widehat{e}(\text{CT}_{\zeta_1}, R_\eta)$ and $\widehat{e}(\text{CT}_{\eta_1}, R_\zeta)$

$$(i) \widehat{e}(\text{CT}_{\zeta_1}, R_\eta) = \widehat{e}(\mathcal{g}_1^{H_7(M_\zeta, V_\zeta)}, H_6(M_\eta)^{H_7(M_\eta, V_\eta)}) = \widehat{e}(\mathcal{g}_1, H_6(M_\eta))^{H_7(M_\zeta, V_\zeta) \cdot H_7(M_\eta, V_\eta)}$$

$$(ii) \widehat{e}(\text{CT}_{\eta_1}, R_\zeta) = \widehat{e}(\mathcal{g}_1^{H_7(M_\eta, V_\eta)}, H_6(M_\zeta)^{H_7(M_\zeta, V_\zeta)}) = \widehat{e}(\mathcal{g}_1, H_6(M_\zeta))^{H_7(M_\zeta, V_\zeta) \cdot H_7(M_\eta, V_\eta)}$$

(c) Return 1 if $\widehat{e}(\text{CT}_{\zeta_1}, R_\eta) = \widehat{e}(\text{CT}_{\eta_1}, R_\zeta)$. Otherwise, return 0

In the following, we present the details of [leftmargin = 0em]

$$(i) R_\zeta = \text{CT}_{\zeta_4}/H_8(\widehat{e}(\text{CT}_{\zeta_2}, \text{TD}_\zeta)) = H_6(M_\zeta)^{r_\zeta} \cdot H_8(\widehat{e}(P_{\text{pub}}, H_2(\text{ID}_\zeta) \cdot H_4(\text{ID}_\zeta, T))^{u_\zeta})/H_8(\widehat{e}(\mathcal{g}_1^{u_\zeta}, H_2(\text{ID}_\zeta)^s \cdot H_4(\text{ID}_\zeta, T)^s)) = H_6(M_\zeta)^{r_\zeta} \cdot H_8(\widehat{e}(\mathcal{g}_1^s, H_2(\text{ID}_\zeta) \cdot H_4(\text{ID}_\zeta, T))^{u_\zeta})/H_8(\widehat{e}(\mathcal{g}_1^{u_\zeta}, H_2(\text{ID}_\zeta)^s \cdot H_4(\text{ID}_\zeta, T)^s)) = H_6(M_\zeta)^{r_\zeta} \cdot H_8(\widehat{e}(\mathcal{g}_1, H_2(\text{ID}_\zeta) \cdot H_4(\text{ID}_\zeta, T))^{su_\zeta})/H_8(\widehat{e}(\mathcal{g}_1, H_2(\text{ID}_\zeta) \cdot H_4(\text{ID}_\zeta, T))^{su_\zeta}) = H_6(M_\zeta)^{H_7(M_\zeta, V_\zeta)}$$

$$(ii) R_\eta = \text{CT}_{\eta_4}/H_8(\widehat{e}(\text{CT}_{\eta_2}, \text{TD}_\eta)) = H_6(M_\eta)^{r_\eta} \cdot H_8(\widehat{e}(P_{\text{pub}}, H_2(\text{ID}_\eta) \cdot H_4(\text{ID}_\eta, T))^{u_\eta})/H_8(\widehat{e}(\mathcal{g}_1^{u_\eta}, H_2(\text{ID}_\eta)^s \cdot H_4(\text{ID}_\eta, T)^s)) = H_6(M_\eta)^{r_\eta} \cdot H_8(\widehat{e}(\mathcal{g}_1^s, H_2(\text{ID}_\eta) \cdot H_4(\text{ID}_\eta, T))^{u_\eta})/H_8(\widehat{e}(\mathcal{g}_1^{u_\eta}, H_2(\text{ID}_\eta)^s \cdot H_4(\text{ID}_\eta, T)^s)) = H_6(M_\eta)^{r_\eta} \cdot H_8(\widehat{e}(\mathcal{g}_1, H_2(\text{ID}_\eta) \cdot H_4(\text{ID}_\eta, T))^{su_\eta})/H_8(\widehat{e}(\mathcal{g}_1, H_2(\text{ID}_\eta) \cdot H_4(\text{ID}_\eta, T))^{su_\eta}) = H_6(M_\eta)^{H_7(M_\eta, V_\eta)}$$

5. Security Analysis

In this section, we give four theorems to show that the proposed scheme has the IND-ID-CCA security for type I and II adversaries and the OW-ID-CCA security for type III and IV adversaries.

Theorem 5. *If the BDH assumption holds, the proposed RIBEET scheme satisfies the IND-ID-CCA security in the security game. More precisely, suppose that \mathcal{A}_1 is a PPT type I adversary who has at least ε advantage to break the RIBEET scheme. Then, there exists an algorithm \mathcal{B} to solve the BDH*

problem with the advantage

$$\varepsilon' \geq \left(\frac{1}{q_{H_5}}\right) \left[\frac{\varepsilon}{e(q_{IK} + q_T + 1)} - \frac{q_D}{q} - \frac{q_{H_8}}{q} \right], \quad (4)$$

where q_{H_5} , q_{H_8} , q_{IK} , q_T , q_D , and e , respectively, are the number of queries to random oracle H_5 , random oracle H_8 , Initialkey query, Trapdoor query, Decryption query, and Euler's number.

Proof. An algorithm \mathcal{B} is constructed to solve the BDH problem. The algorithm \mathcal{B} is given a BDH tuple $\langle q, G_1, G_2, G_T, \widehat{e}, \mathcal{g}_1, \mathcal{g}_1^a, \mathcal{g}_1^c, \mathcal{g}_2, \mathcal{g}_2^a, \mathcal{g}_2^b \rangle$ which is defined in Section 2. The algorithm \mathcal{B} can be seen as a challenger to find the answer of the BDH problem. The answer $A = \widehat{e}(\mathcal{g}_1, \mathcal{g}_2)^{abc}$ can be found by interacting with the PPT type I adversary \mathcal{A}_1 in the following security game.

(1) *Setup:* the challenger \mathcal{B} utilizes the BDH tuple to set $P_{\text{pub}} = \mathcal{g}_1^a$ and then generates the public system parameters $\text{PSP} = \{q, G_1, G_2, G_T, \widehat{e}, \mathcal{g}_1, \mathcal{g}_2, P_{\text{pub}}, H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8\}$, where H_i is a random oracle for $i = 1, 2, \dots, 8$. In addition, the system life time $\text{SLT} = \{T_0, T_1, \dots, T_t\}$ can be generated by the challenger \mathcal{B} . Then, \mathcal{B} gives \mathcal{A}_1 the public system parameters PSP and system life time SLT. Here, the adversary \mathcal{A}_1 can issue queries to each random oracle as follows

(a) H_1 query(ID): \mathcal{A}_1 can utilize ID to obtain a response to the random oracle H_1 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called List H_1 which is composed of tuples, and the format of the tuple is $\langle \text{ID}, U_{\text{ID}}, u, rb \rangle$. The response is acquired from the List H_1 which is initially empty and can be updated by the following steps

(i) \mathcal{B} returns U_{ID} as the response if ID exists in a tuple $\langle \text{ID}, U_{\text{ID}}, u, rb \rangle$ from the List H_1

(ii) Otherwise, \mathcal{B} picks a random value $u \in Z_q^*$ and a random bit $rb \in \{0, 1\}$ to compute

$$U_{\text{ID}} = \begin{cases} \mathcal{g}_2^u, & \text{if } rb = 0, \\ \mathcal{g}_2^{bu}, & \text{if } rb = 1, \end{cases} \quad (5)$$

where $\Pr[rb = 0] = \delta$ and $\Pr[rb = 1] = 1 - \delta$ (which will be

discussed later). Then, \mathcal{B} adds the tuple $\langle \text{ID}, U_{\text{ID}}, u, rb \rangle$ to the ListH_1 and returns U_{ID} to \mathcal{A}_1

(b) $H_2\text{query}(\text{ID})$: \mathcal{A}_1 can utilize ID to obtain a response to the random oracle H_2 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called ListH_2 which is composed of tuples, and the format of the tuple is $\langle \text{ID}, V_{\text{ID}}, v, rb \rangle$. The response is acquired from the ListH_2 which is initially empty and can be updated by the following steps:

- (a) \mathcal{B} returns V_{ID} as the response if ID exists in a tuple $\langle \text{ID}, V_{\text{ID}}, v, rb \rangle$ from the ListH_2
- (b) Otherwise, \mathcal{B} picks a random value $v \in Z_q^*$ and utilizes ID to find rb in the ListH_2 . Then, \mathcal{B} computes

$$V_{\text{ID}} = \begin{cases} g_2^v, & \text{if } rb = 0, \\ g_2^{bv}, & \text{if } rb = 1, \end{cases} \quad (6)$$

and adds the tuple $\langle \text{ID}, V_{\text{ID}}, v, rb \rangle$ to ListH_2 . \mathcal{B} returns V_{ID} to \mathcal{A}_1

(c) $H_3\text{query}(\text{ID}, T)$: \mathcal{A}_1 can utilize (ID, T) to obtain a response to the random oracle H_3 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called ListH_3 which is composed of tuples, and the format of the tuple is $\langle \text{ID}, T, U_{\text{IDT}}, \eta \rangle$. The response is acquired from the ListH_3 which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns U_{IDT} as the response if (ID, T) exists in a tuple $\langle \text{ID}, T, U_{\text{IDT}}, \eta \rangle$ from the ListH_3
- (ii) Otherwise, \mathcal{B} picks a random value $\eta \in Z_q^*$ to compute $U_{\text{IDT}} = g_2^\eta$. Then, \mathcal{B} adds the tuple $\langle \text{ID}, T, U_{\text{IDT}}, \eta \rangle$ to the ListH_3 and returns U_{IDT} to \mathcal{A}_1

(d) $H_4\text{query}(\text{ID}, T)$: \mathcal{A}_1 can utilize (ID, T) to obtain a response to the random oracle H_4 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called ListH_4 which is composed of tuples, and the format of the tuple is $\langle \text{ID}, T, V_{\text{IDT}}, \zeta \rangle$. The response is acquired from the ListH_4 which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns V_{IDT} as the response if (ID, T) exists in a tuple $\langle \text{ID}, T, V_{\text{IDT}}, \zeta \rangle$ from the ListH_4
- (ii) Otherwise, \mathcal{B} picks a random value $\zeta \in Z_q^*$ to compute $V_{\text{IDT}} = g_2^\zeta$. Then, \mathcal{B} adds the tuple $\langle \text{ID}, T, V_{\text{IDT}}, \zeta \rangle$ to the ListH_4 and returns V_{IDT} to \mathcal{A}_1

(e) $H_5\text{query}(W, \text{CT}_1, \text{CT}_2)$: \mathcal{A}_1 can utilize $(W, \text{CT}_1, \text{CT}_2)$ to obtain a response to the random oracle H_5 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called ListH_5 which is composed of tuples, and the format of the tuple is $\langle W, \text{CT}_1, \text{CT}_2, \omega \rangle$. The response is acquired from the ListH_5 which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns ω as the response if $(W, \text{CT}_1, \text{CT}_2)$ exists in a tuple $\langle W, \text{CT}_1, \text{CT}_2, \omega \rangle$ from the ListH_5
- (ii) Otherwise, \mathcal{B} picks a random value $\omega \in \{0, 1\}^{\lambda+l}$ and adds the tuple $\langle W, \text{CT}_1, \text{CT}_2, \omega \rangle$ to the ListH_5 . Then, \mathcal{B} returns ω to \mathcal{A}_1

(f) $H_6\text{query}(M)$: \mathcal{A}_1 can utilize M to obtain a response to the random oracle H_6 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called ListH_6 which is composed of tuples, and the format of the tuple is $\langle M, Q \rangle$. The response is acquired from the ListH_6 which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns Q as the response if M exists in a tuple $\langle M, Q \rangle$ from the ListH_6
- (ii) Otherwise, \mathcal{B} picks a random point $Q \in G_2$ and adds the tuple $\langle M, Q \rangle$ to the ListH_6 . Then, \mathcal{B} returns Q to \mathcal{A}_1

(g) $H_7\text{query}(M, V)$: \mathcal{A}_1 can utilize (M, V) to obtain a response to the random oracle H_7 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called ListH_7 which is composed of tuples, and the format of the tuple is $\langle M, V, \gamma \rangle$. The response is acquired from the ListH_7 which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns γ as the response if (M, V) exists in a tuple $\langle M, V, \gamma \rangle$ from the ListH_7
- (ii) Otherwise, \mathcal{B} picks a random value $\gamma \in Z_q^*$ and adds the tuple $\langle M, V, \gamma \rangle$ to the ListH_7 . Then, \mathcal{B} returns γ to \mathcal{A}_1

(h) $H_8\text{query}(N)$: \mathcal{A}_1 can utilize N to obtain a response to the random oracle H_8 from the challenger \mathcal{B} .

To obtain the response, \mathcal{B} maintains a list, called $ListH_8$ which is composed of tuples, and the format of the tuple is $\langle N, S \rangle$. The response is acquired from the $ListH_8$ which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns S as the response if N exists in a tuple $\langle N, S \rangle$ from the $ListH_8$
 - (ii) Otherwise, \mathcal{B} picks a random point $S \in G_2$ and adds the tuple $\langle N, S \rangle$ to the $ListH_8$. Then, \mathcal{B} returns S to \mathcal{A}_1
- (2) Phase 1: the adversary \mathcal{A}_1 can, respectively, utilize ID , (ID, T) , (ID, T, CT) and (ID, T) to issue the InitialKey query, Timekey query, Decryption query, and Trapdoor query. The response to each query can be obtained as follows

- (a) InitialKey query(ID): \mathcal{A}_1 utilizes ID to issue the query, while \mathcal{B} , respectively, finds the corresponding tuples $\langle ID, U_{ID}, u, rb \rangle$ and $\langle ID, V_{ID}, v, rb \rangle$ from the $ListH_1$ and the $ListH_2$ according to ID . If $rb = 1$, \mathcal{B} interrupts this game. If $rb = 0$, \mathcal{B} use u and v to define $IK_{ID} = (IK_{ID1}, IK_{ID2}) = ((g_2^a)^u, (g_2^a)^v)$. Then \mathcal{B} returns IK_{ID} as the user initial key to \mathcal{A}_1
- (b) Timekey query(ID, T): \mathcal{A}_1 utilizes (ID, T) to issue the query, while \mathcal{B} , respectively, finds the corresponding tuples $\langle ID, T, U_{IDT}, \eta \rangle$ and $\langle ID, T, V_{IDT}, \zeta \rangle$ from the $ListH_3$ and the $ListH_4$ according to (ID, T) . \mathcal{B} use η and ζ to define $TK_{ID} = (TK_{ID1}, TK_{ID2}) = ((g_2^a)^\eta, (g_2^a)^\zeta)$. Then, \mathcal{B} returns TK_{ID} as the user time key to \mathcal{A}_1
- (c) Decryption query(ID, T, CT): \mathcal{A}_1 utilizes (ID, T, CT) to issue the query, while \mathcal{B} , respectively, finds the corresponding tuples $\langle ID, U_{ID}, u, rb \rangle$, $\langle ID, V_{ID}, v, rb \rangle$, $\langle ID, T, U_{IDT}, \eta \rangle$, and $\langle ID, T, V_{IDT}, \zeta \rangle$ from the $ListH_1$, $ListH_2$, $ListH_3$, and the $ListH_4$ according to ID and T . The response of this query is acquired from these lists by performing the following tasks

- (i) If $rb = 0$, \mathcal{B} , respectively, uses ID and (ID, T) to run InitialKeyquery and Timekey query to obtain IK_{ID} and TK_{ID} . Then, \mathcal{B} utilizes IK_{ID} , TK_{ID} , and CT to run the Decryption algorithm to produce the message M which is sent to \mathcal{A}_1
- (ii) If $rb = 1$, \mathcal{B} utilizes CT_1 and CT_2 , which are from $CT = (CT_1, CT_2, CT_3, CT_4)$, to find the corresponding tuple $\langle W, CT_1, CT_2, \omega \rangle$ from the $ListH_5$. Then, $M' || V' = CT_3 \oplus \omega$ can be computed by using CT_3

and ω . Further, \mathcal{B} utilizes M' and V' to find the corresponding tuples $\langle M, V, \gamma \rangle$ from the $ListH_7$ and $\langle M, Q \rangle$ from the $ListH_6$. Obviously, γ and Q can be obtained. If S can be found in the corresponding tuple $\langle N, S \rangle$ from the $ListH_8$ such that $CT_4 = Q^\gamma \cdot S$ holds, \mathcal{B} will confirm whether $CT_1 = g_1^\gamma$ holds. If $CT_1 = g_1^\gamma$, the message M' is sent to \mathcal{A}_1

- (d) Trapdoor query(ID, T): \mathcal{A}_1 utilizes (ID, T) to issue the query, while \mathcal{B} , respectively, uses ID and (ID, T) to run InitialKey query and Timekey query to obtain $IK_{ID} = (IK_{ID1}, IK_{ID2})$ and $TK_{ID} = (TK_{ID1}, TK_{ID2})$. Then, \mathcal{B} utilizes IK_{ID2} and TK_{ID2} to produce the trapdoor $TD_{ID} = IK_{ID2} \cdot TK_{ID2}$ which is sent to \mathcal{A}_1
- (3) Challenge: when the phase 1 is over, \mathcal{A}_1 outputs a tuple $\langle ID^*, T^*, M_0^*, M_1^* \rangle$ as the target of the challenge. \mathcal{B} utilizes ID^* to find the corresponding tuples $\langle ID, U_{ID}, u, rb \rangle$ from the $ListH_1$. If $rb = 0$, \mathcal{B} interrupts this game. If $rb = 1$, \mathcal{B} randomly selects $\mathfrak{b} \in \{0, 1\}$ and $V \in \{0, 1\}^l$ to run H_7 query with $M_{\mathfrak{b}}^*$ and V . Then, γ can be obtained. \mathcal{B} utilizes γ to set $CT_1^* = g_1^\gamma$. In addition, \mathcal{B} sets $CT_2^* = g_1^c$, while a random value $CT_3 \in \{0, 1\}^{\lambda+l}$ and a random point $CT_4 \in G_2$ are chosen. Finally, the challenge ciphertext $C T^* = (CT_1^*, CT_2^*, CT_3^*, CT_4^*)$ is sent to \mathcal{A}_1
- (4) Phase 2: \mathcal{A}_1 can issue the same query as phase 1, but it must be under the condition of $ID \neq ID^*$ and $CT \neq CT^*$
- (5) Guess: \mathcal{A}_1 responds to \mathcal{B} with a guess $\mathfrak{b}' \in \{0, 1\}$. If $\mathfrak{b}' \neq \mathfrak{b}$, \mathcal{B} responds with failure and terminates. Otherwise, \mathcal{A}_1 wins the game. Then, \mathcal{B} randomly selects a tuple $\langle W^*, CT_1^*, CT_2^*, \omega^* \rangle$ from the $ListH_5$ and calculates $H_5(\widehat{e}(g_1, g_2)^{abcu^*} \cdot \widehat{e}(g_1^{ac}, g_2)^{\eta^*}, CT_1^*, CT_2^*) = (M_{\rho}^* || V) \oplus CT_3^*$, where $\widehat{e}(g_1, g_2)^{abcu^*} \cdot \widehat{e}(g_1^{ac}, g_2)^{\eta^*} = W^*$. Hence, \mathcal{B} can output the BDH solution $A = (W^* / \widehat{e}(g_1^{ac}, g_2)^{\eta^*})^{u^{*-1}}$ due to $\widehat{e}(g_1, g_2)^{abc} = (W^* / \widehat{e}(g_1^{ac}, g_2)^{\eta^*})^{u^{*-1}}$

□

Analysis. Let us start with two cases, namely, the simulation of H_i query for $i = 1, 2, \dots, 8$ and the simulation of decryption query. For the H_1, H_2, H_3, H_4, H_6 , and H_7 queries, it is obvious that the simulations are perfect because there exists no relationship between the constructions of these queries and the solution of the BDH problem. For the H_5 and H_8 queries, we consider two events $E_{H_5}^*$ and $E_{H_8}^*$ which, respectively, issues the H_5 query with $(\widehat{e}$

$(g_1, g_2)^{abcu^*} \cdot \widehat{e}(g_1^{ac}, g_2)^{\eta^*}, CT_1^*, CT_2^*$) and the H_8 query with $\widehat{e}(g_1, g_2)^{abcv^*} \cdot \widehat{e}(g_1^a, g_2)^{\zeta^*}$. We say that the simulations of H_5 and H_8 queries are perfect if $E_{H_5}^*$ and $E_{H_8}^*$ do not happen. For the decryption query, we consider an event E_{DecErr} where the challenger \mathcal{B} cannot decrypt the ciphertext. Assume that q_D is the number of decryption query. Then, we obtain $\Pr[E_{\text{DecErr}}] \leq q_D/q$

$$\Pr[\mathbf{b} = \mathbf{b}'] = \Pr[\mathbf{b} = \mathbf{b}' | E] \Pr[E] + \Pr[\mathbf{b} = \mathbf{b}' | \neg E] \Pr[\neg E] \leq \Pr[E] + \left(\frac{1}{2}\right) \cdot \Pr[\neg E] = \Pr[E] + \frac{1}{2} \cdot (1 - \Pr[E]) = \frac{1}{2} \cdot \Pr[E] + \frac{1}{2}. \quad (7)$$

According to above inequality, $\varepsilon = \Pr[\mathbf{b} = \mathbf{b}'] - 1/2$ and $E = (E_{H_5}^* \vee E_{H_8}^* \vee E_{\text{DecErr}}) | \neg E_{\text{Abort}}$, we have

$$\varepsilon = \Pr[\mathbf{b} = \mathbf{b}'] - \frac{1}{2} \leq \Pr[E] \leq \frac{\Pr[E_{H_5}^*] + \Pr[E_{H_8}^*] + \Pr[E_{\text{DecErr}}]}{\Pr[\neg E_{\text{Abort}}]}. \quad (8)$$

Moreover, we obtain

$$\Pr[E_{H_5}^*] \geq \varepsilon \cdot \Pr[\neg E_{\text{Abort}}] - \Pr[E_{\text{DecErr}}] - \Pr[E_{H_8}^*]. \quad (9)$$

Since $\Pr[\neg E_{\text{Abort}}] = \delta^{q_{\text{IK}} + q_T} (1 - \delta)$, we can gain $\Pr[\neg E_{\text{Abort}}] \geq 1/e^{(q_{\text{IK}} + q_T + 1)}$ when $\delta = 1 - (1/(q_{\text{IK}} + q_T + 1))$. Then, we have

$$\Pr[E_{H_5}^*] \geq \frac{\varepsilon}{e^{(q_{\text{IK}} + q_T + 1)}} - \frac{q_D}{q} - \frac{q_{H_8}}{q}. \quad (10)$$

Here, the adversary \mathcal{A}_1 can distinguish the target ciphertext CT^* is the real one when $E_{H_5}^*$ occurs. In addition, the tuple $\langle \widehat{e}(g_1, g_2)^{abcu^*} \cdot \widehat{e}(g_1^{ac}, g_2)^{\eta^*}, CT_1^*, CT_2^* \rangle$ has been added in the List_{H_5} . If the challenger \mathcal{B} picks the correct tuple from the List_{H_5} , \mathcal{B} wins this security game. Meanwhile, the advantage of solving the BDH problem is

$$\varepsilon' \geq \frac{1}{q_{H_5}} \Pr[E_{H_5}^*] \geq \frac{1}{q_{H_5}} \left[\frac{\varepsilon}{e^{(q_{\text{IK}} + q_T + 1)}} - \frac{q_D}{q} - \frac{q_{H_8}}{q} \right]. \quad (11)$$

Theorem 6. *If the BDH assumption holds, the proposed RIBEET scheme satisfies the IND-ID-CCA security in the security game. More precisely, suppose that \mathcal{A}_2 is a PPT type 2 adversary who has at least ε advantage to break the RIBEET scheme. Then, there exists an algorithm \mathcal{B} to solve the BDH problem with the advantage*

$$\varepsilon' \geq \left(\frac{1}{q_{H_5}} \right) \left[\frac{\varepsilon}{e^{(q_{\text{TK}} + q_T + 1)}} - \frac{q_D}{q} - \frac{q_{H_8}}{q} \right], \quad (12)$$

Next, we discuss an event E which states that the simulation of this security game will not be interrupted. Here, we can obtain $E = (E_{H_5}^* \vee E_{H_8}^* \vee E_{\text{DecErr}}) | \neg E_{\text{Abort}}$, where E_{Abort} is defined as the event that the challenger \mathcal{B} interrupts this security game. Since \mathcal{B} guesses \mathbf{b}' with the advantage $\leq 1/2$, the $\Pr[\mathbf{b} = \mathbf{b}' | \neg E] \leq 1/2$ can be obtained if E does not occur. Further, we have

where q_{H_5} , q_{H_8} , q_{TK} , q_T , q_D , and e , respectively, are the number of queries to random oracle H_5 , random oracle H_8 , Time-key query, Trapdoor query, Decryption query, and Euler's number.

Proof. An algorithm \mathcal{B} is constructed to solve the BDH problem. The algorithm \mathcal{B} is given a BDH tuple $\langle q, G_1, G_2, G_T, \widehat{e}, g_1, g_2, g_1^a, g_1^c, g_2^a, g_2^b \rangle$ which is defined in Section 2. The algorithm \mathcal{B} can be seen as a challenger to find the answer of the BDH problem. The answer $A = \widehat{e}(g_1, g_2)^{abc}$ can be found by interacting with the PPT type II adversary \mathcal{A}_2 in the following security game.

- (1) Setup: the challenger \mathcal{B} utilizes the BDH tuple to set $P_{\text{pub}} = g_1^a$ and then generates the public system parameters $\text{PSP} = \{q, G_1, G_2, G_T, \widehat{e}, g_1, g_2, P_{\text{pub}}, H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8\}$, where H_i is a random oracle for $i = 1, 2, \dots, 8$. In addition, the system life time $\text{SLT} = \{T_0, T_1, \dots, T_t\}$ can be generated by the challenger \mathcal{B} . Then, \mathcal{B} gives \mathcal{A}_2 the public system parameters PSP and system life time SLT . Here, the adversary \mathcal{A}_2 can issue queries to each random oracle as follows
 - (a) H_1 query(ID): \mathcal{A}_2 can utilize ID to obtain a response to the random oracle H_1 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called List_{H_1} which is composed of tuples, and the format of the tuple is $\langle \text{ID}, U_{\text{ID}}, u \rangle$. The response is acquired from the List_{H_1} which is initially empty and can be updated by the following steps
 - (i) \mathcal{B} returns U_{ID} as the response if ID exists in a tuple $\langle \text{ID}, U_{\text{ID}}, u \rangle$ from the List_{H_1}
 - (ii) Otherwise, \mathcal{B} picks a random value $u \in Z_q^*$ to compute $U_{\text{ID}} = g_2^u$. Then, \mathcal{B} adds the tuple $\langle \text{ID}, U_{\text{ID}}, u \rangle$ to the List_{H_1} and returns U_{ID} to \mathcal{A}_2

(b) H_2 query(ID): \mathcal{A}_2 can utilize ID to obtain a response to the random oracle H_2 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called List H_2 which is composed of tuples, and the format of the tuple is $\langle \text{ID}, V_{\text{ID}}, v \rangle$. The response is acquired from the List H_2 which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns V_{ID} as the response if ID exists in a tuple $\langle \text{ID}, V_{\text{ID}}, v \rangle$ from the List H_2
- (ii) Otherwise, \mathcal{B} picks a random value $v \in Z_q^*$ to compute $V_{\text{ID}} = g_2^v$. Then, \mathcal{B} adds the tuple $\langle \text{ID}, V_{\text{ID}}, v \rangle$ to the List H_2 and returns V_{ID} to \mathcal{A}_2

(c) H_3 query(ID, T): \mathcal{A}_2 can utilize (ID, T) to obtain a response to the random oracle H_3 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called List H_3 which is composed of tuples, and the format of the tuple is $\langle \text{ID}, T, U_{\text{IDT}}, \eta, rb \rangle$. The response is acquired from the List H_3 which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns U_{IDT} as the response if (ID, T) exists in a tuple $\langle \text{ID}, T, U_{\text{IDT}}, \eta, rb \rangle$ from the List H_3
- (ii) Otherwise, \mathcal{B} picks a random value $\eta \in Z_q^*$ and a random bit $rb \in \{0, 1\}$ to compute

$$U_{\text{IDT}} = \begin{cases} g_2^\eta, & \text{if } rb = 0, \\ g_2^{b\eta}, & \text{if } rb = 1, \end{cases} \quad (13)$$

where $\Pr[rb = 0] = \delta$ and $\Pr[rb = 1] = 1 - \delta$ (which will be discussed later). Then, \mathcal{B} adds the tuple $\langle \text{ID}, T, U_{\text{IDT}}, \eta, rb \rangle$ to the List H_3 and returns U_{IDT} to \mathcal{A}_2

(d) H_4 query(ID, T): \mathcal{A}_2 can utilize (ID, T) to obtain a response to the random oracle H_4 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called List H_4 which is composed of tuples, and the format of the tuple is $\langle \text{ID}, T, V_{\text{IDT}}, \zeta, rb \rangle$. The response is acquired from the List H_4 which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns V_{IDT} as the response if (ID, T) exists in a tuple $\langle \text{ID}, T, V_{\text{IDT}}, \zeta, rb \rangle$ from the List H_4
- (ii) Otherwise, \mathcal{B} picks a random value $\zeta \in Z_q^*$ and utilizes (ID, T) to find rb in the List H_4 . Then, \mathcal{B} com-

putes

$$V_{\text{IDT}} = \begin{cases} g_2^\zeta, & \text{if } rb = 0, \\ g_2^{b\zeta}, & \text{if } rb = 1, \end{cases} \quad (14)$$

and add the tuple $\langle \text{ID}, T, V_{\text{IDT}}, \zeta, rb \rangle$ to List H_4 . \mathcal{B} returns V_{IDT} to \mathcal{A}_2

(e) H_5 query(W, CT $_1$, CT $_2$): \mathcal{A}_2 can utilize (W, CT $_1$, CT $_2$) to obtain a response to the random oracle H_5 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called List H_5 which is composed of tuples, and the format of the tuple is $\langle W, \text{CT}_1, \text{CT}_2, \omega \rangle$. The response is acquired from the List H_5 which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns ω as the response if $(W, \text{CT}_1, \text{CT}_2)$ exists in a tuple $\langle W, \text{CT}_1, \text{CT}_2, \omega \rangle$ from the List H_5
- (ii) Otherwise, \mathcal{B} picks a random value $\omega \in \{0, 1\}^{\lambda+l}$ and adds the tuple $\langle W, \text{CT}_1, \text{CT}_2, \omega \rangle$ to the List H_5 . Then, \mathcal{B} returns ω to \mathcal{A}_2

(f) H_6 query(M): \mathcal{A}_2 can utilize M to obtain a response to the random oracle H_6 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called List H_6 which is composed of tuples, and the format of the tuple is $\langle M, Q \rangle$. The response is acquired from the List H_6 which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns Q as the response if M exists in a tuple $\langle M, Q \rangle$ from the List H_6
- (ii) Otherwise, \mathcal{B} picks a random point $Q \in G_2$ and adds the tuple $\langle M, Q \rangle$ to the List H_6 . Then, \mathcal{B} returns Q to \mathcal{A}_2

(g) H_7 query(M, V): \mathcal{A}_2 can utilize (M, V) to obtain a response to the random oracle H_7 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called List H_7 which is composed of tuples, and the format of the tuple is $\langle M, V, \gamma \rangle$. The response is acquired from the List H_7 which is initially empty and can be updated by the following steps

- (i) \mathcal{B} returns γ as the response if (M, V) exists in a tuple $\langle M, V, \gamma \rangle$ from the List H_7

- (ii) Otherwise, \mathcal{B} picks a random value $\gamma \in Z_q^*$ and adds the tuple $\langle M, V, \gamma \rangle$ to the $\text{List}H_7$. Then, \mathcal{B} returns γ to \mathcal{A}_2
- (h) H_8 query(N): \mathcal{A}_2 can utilize N to obtain a response to the random oracle H_8 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called $\text{List}H_8$ which is composed of tuples, and the format of the tuple is $\langle N, S \rangle$. The response is acquired from the $\text{List}H_8$ which is initially empty and can be updated by the following steps
- (i) \mathcal{B} returns S as the response if N exists in a tuple $\langle N, S \rangle$ from the $\text{List}H_8$
 - (ii) Otherwise, \mathcal{B} picks a random point $S \in G_2$ and adds the tuple $\langle N, S \rangle$ to the $\text{List}H_8$. Then, \mathcal{B} returns S to \mathcal{A}_2
- (2) Phase 1: the adversary \mathcal{A}_2 can, respectively, utilize ID , (ID, T) , $(\text{ID}, T, \text{CT})$, and (ID, T) to issue the InitialKey query, Timekey query, Decryption query, and Trapdoor query. The response to each query can be obtained as follows
- (a) InitialKey query(ID): \mathcal{A}_2 utilizes ID to issue the query, while \mathcal{B} , respectively, finds the corresponding tuples $\langle \text{ID}, U_{\text{ID}}, u \rangle$ and $\langle \text{ID}, V_{\text{ID}}, v \rangle$ from the $\text{List}H_1$ and the $\text{List}H_2$ according to ID . \mathcal{B} use u and v to define $\text{IK}_{\text{ID}} = (\text{IK}_{\text{ID}1}, \text{IK}_{\text{ID}2}) = ((g_2^a)^u, (g_2^a)^v)$. Then, \mathcal{B} returns IK_{ID} as the user initial key to \mathcal{A}_1
 - (b) Timekey query(ID, T): \mathcal{A}_2 utilizes (ID, T) to issue the query, while \mathcal{B} , respectively, finds the corresponding tuples $\langle \text{ID}, T, U_{\text{ID}T}, \eta, rb \rangle$ and $\langle \text{ID}, T, V_{\text{ID}T}, \zeta, rb \rangle$ from the $\text{List}H_3$ and the $\text{List}H_4$ according to (ID, T) . If $rb = 1$, \mathcal{B} interrupts this game. If $rb = 0$, \mathcal{B} use η and ζ to define $\text{TK}_{\text{ID}} = (\text{TK}_{\text{ID}1}, \text{TK}_{\text{ID}2}) = ((g_2^a)^\eta, (g_2^a)^\zeta)$. Then, \mathcal{B} returns TK_{ID} as the user time key to \mathcal{A}_2
 - (c) Decryption query(ID, T, CT): \mathcal{A}_2 utilizes $(\text{ID}, T, \text{CT})$ to issue the query, while \mathcal{B} , respectively, finds the corresponding tuples $\langle \text{ID}, U_{\text{ID}}, u \rangle$, $\langle \text{ID}, V_{\text{ID}}, v \rangle$, $\langle \text{ID}, T, U_{\text{ID}T}, \eta, rb \rangle$, and $\langle \text{ID}, T, V_{\text{ID}T}, \zeta, rb \rangle$ from the $\text{List}H_1$, $\text{List}H_2$, $\text{List}H_3$, and $\text{List}H_4$ according to ID and T . The response of this query is acquired from these lists by performing the following tasks
- (i) If $rb = 0$, \mathcal{B} , respectively, uses ID and (ID, T) to run InitialKey query and Timekey query to obtain IK_{ID} and TK_{ID} . Then, \mathcal{B} utilizes IK_{ID} , TK_{ID} , and CT to run the Decryption algorithm to produce the message M which is sent to \mathcal{A}_2
 - (ii) If $rb = 1$, \mathcal{B} utilizes CT_1 and CT_2 , which are from $\text{CT} = (\text{CT}_1, \text{CT}_2, \text{CT}_3, \text{CT}_4)$, to find the corresponding tuple $\langle W, \text{CT}_1, \text{CT}_2, \omega \rangle$ from the $\text{List}H_5$. Then, $M' || V' = \text{CT}_3 \oplus \omega$ can be computed by using CT_3 and ω . Further, \mathcal{B} utilizes M' and V' to find the corresponding tuples $\langle M, V, \gamma \rangle$ from the $\text{List}H_7$ and $\langle M, Q \rangle$ from the $\text{List}H_6$. Obviously, γ and Q can be obtained. If S can be found in the corresponding tuple $\langle N, S \rangle$ from the $\text{List}H_8$ such that $\text{CT}_4 = Q^\gamma \cdot S$ holds, \mathcal{B} will confirm whether $\text{CT}_1 = g_1^\gamma$ holds. If $\text{CT}_1 = g_1^\gamma$, the message M' is sent to \mathcal{A}_2
- (1) Trapdoor query(ID, T): \mathcal{A}_2 utilizes (ID, T) to issue the query, while \mathcal{B} , respectively, uses ID and (ID, T) to run InitialKey query and Timekey query to obtain $\text{IK}_{\text{ID}} = (\text{IK}_{\text{ID}1}, \text{IK}_{\text{ID}2})$ and $\text{TK}_{\text{ID}} = (\text{TK}_{\text{ID}1}, \text{TK}_{\text{ID}2})$. Then, \mathcal{B} utilizes $\text{IK}_{\text{ID}2}$ and $\text{TK}_{\text{ID}2}$ to produce the trapdoor $\text{TD}_{\text{ID}} = \text{IK}_{\text{ID}2} \cdot \text{TK}_{\text{ID}2}$ which is sent to \mathcal{A}_2
- (3) Challenge: when phase 1 is over, \mathcal{A}_2 outputs a tuple $\langle \text{ID}^*, T^*, M_0^*, M_1^* \rangle$ as the target of the challenge. \mathcal{B} utilizes (ID^*, T^*) to find the corresponding tuples $\langle \text{ID}, T, U_{\text{ID}T}, \eta, rb \rangle$ from the $\text{List}H_3$. If $rb = 0$, \mathcal{B} interrupts this game. If $rb = 1$, \mathcal{B} randomly selects $\mathfrak{b} \in \{0, 1\}$ and $V \in \{0, 1\}^l$ to run H_7 query with M_0^* and V . Then, γ can be obtained. \mathcal{B} utilizes γ to set $\text{CT}_1^* = g_1^\gamma$. In addition, \mathcal{B} sets $\text{CT}_2^* = g_1^c$, while a random value $\text{CT}_3 \in \{0, 1\}^{\lambda+l}$ and a random point $\text{CT}_4^* \in G_2$ are chosen. Finally, the challenge ciphertext $\text{CT}^* = (\text{CT}_1^*, \text{CT}_2^*, \text{CT}_3^*, \text{CT}_4^*)$ is sent to \mathcal{A}_2
- (2) Phase 2: \mathcal{A}_2 can issue the same query as phase 1, but it must be under the condition of $\text{ID} \neq \text{ID}^*$ and $\text{CT} \neq \text{CT}^*$
- (3) Guess: \mathcal{A}_2 responds to \mathcal{B} with a guess $\mathfrak{b}' \in \{0, 1\}$. If $\mathfrak{b}' \neq \mathfrak{b}$, \mathcal{B} responds with failure and terminates. Otherwise, \mathcal{A}_2 wins the game. Then, \mathcal{B} randomly selects a tuple $\langle W^*, \text{CT}_1^*, \text{CT}_2^*, w^* \rangle$ from the $\text{List}H_5$ and outputs the BDH solution $A = (W^* / \tilde{e}(g_1^{ac}, g_2)^{u^*})^{\eta^{*-1}}$ due to $\tilde{e}(g_1, g_2)^{abc} = (W^* / \tilde{e}(g_1^{ac}, g_2)^{u^*})^{\eta^{*-1}}$

The security analysis is similar to Theorem 5. We obtain that \mathcal{B} 's advantage to solve the BDH problem is $\epsilon' \geq (1/$

$$q_{H_5})\Pr[E_{H_5}^*] \geq (1/q_{H_5})[(\epsilon/e(q_{TK} + q_T + 1)) - (q_D/q) - (q_{H_8}/q)]. \quad \square$$

Theorem 7. *If the BDH assumption holds, the proposed RIBEET scheme satisfies the OW-ID-CCA security in the security game. More precisely, suppose that \mathcal{A}_3 is a PPT type 3 adversary who has at least ϵ advantage to break the RIBEET scheme. Then, there exists an algorithm \mathcal{B} to solve the BDH problem with the advantage*

$$\epsilon' \geq \left(\frac{1}{q_{H_5}}\right) \left[\frac{\epsilon - (1/2^\lambda)}{e(q_{IK} + 1)}\right] - \frac{q_D}{q}, \quad (15)$$

where q_{H_5} , q_{IK} , q_D , and e , respectively, are the number of queries to random oracle H_5 , Initialkey query, Decryption query, and Euler's number.

Proof. An algorithm \mathcal{B} is constructed to solve the BDH problem. The algorithm \mathcal{B} is given a BDH tuple $\langle q, G_1, G_2, G_T, \hat{e}, g_1, g_1^a, g_1^b, g_2, g_2^a, g_2^b \rangle$ which is defined in Section 2. The algorithm \mathcal{B} can be seen as a challenger to find the answer of the BDH problem. The answer $A = \hat{e}(g_1, g_2)^{abc}$ can be found by interacting with the PPT type III adversary \mathcal{A}_3 in the following security game.

(i) Setup: The challenger \mathcal{B} utilizes the BDH tuple to set $P_{pub} = g_1^a$, and then generates the public system parameters $PSP = \{q, G_1, G_2, G_T, \hat{e}, g_1, g_2, P_{pub}, H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8\}$, where H_i is a random oracle for $i = 1, 2, \dots, 8$. In addition, the system life time $SLT = \{T_0, T_1, \dots, T_t\}$ can be generated by the challenger \mathcal{B} . Then \mathcal{B} gives \mathcal{A}_3 the public system parameters PSP and system life time SLT . Here, the adversary \mathcal{A}_3 can issue queries to each random oracle as below

(a) H_1 query(ID): \mathcal{B} answers \mathcal{A}_3 in the same form as the proof of Theorem 5

(b) H_2 query(ID): \mathcal{A}_3 can utilize ID to obtain a response to the random oracle H_2 from the challenger \mathcal{B} . To obtain the response, \mathcal{B} maintains a list, called $ListH_2$ which is composed of tuples, and the format of the tuple is $\langle ID, V_{ID}, v, rb \rangle$. The response is acquired from the $ListH_2$ which is initially empty and can be updated by the following steps

(i) \mathcal{B} returns V_{ID} as the response if ID exists in a tuple $\langle ID, V_{ID}, v, rb \rangle$ from the $ListH_2$

(ii) Otherwise, \mathcal{B} picks a random value $v \in Z_q^*$ and utilizes ID to find rb in the $ListH_2$. Then \mathcal{B} computes $V_{ID} = g_2^v$ and adds the tuple $\langle ID, V_{ID}, v, rb \rangle$ to $ListH_2$. \mathcal{B} returns V_{ID} to \mathcal{A}_3

(c) $H_3 - H_8$ queries: \mathcal{B} answers \mathcal{A}_3 in the same form as the proof of Theorem 5

(ii) Phase 1: The adversary \mathcal{A}_3 can, respectively, utilize ID , (ID, T) , (ID, T, CT) and (ID, T) to issue the *InitialKey query*, *Timekey query*, *Decryption query* and *Trapdoor query*. The response to each query can be obtained as follows

(1) *InitialKey query* (ID): \mathcal{B} answers \mathcal{A}_3 in the same form as the proof of Theorem 5

(2) *Timekey query* (ID, T): \mathcal{B} answers \mathcal{A}_3 in the same form as the proof of Theorem 5

(3) *Decryption query* (ID, T, CT): \mathcal{A}_3 utilizes (ID, T, CT) to issue the query, while \mathcal{B} , respectively, finds the corresponding tuples $\langle ID, U_{ID}, u, rb \rangle$, $\langle ID, V_{ID}, v, rb \rangle$, $\langle ID, T, U_{IDT}, \eta \rangle$ and $\langle ID, T, V_{IDT}, \zeta \rangle$ from the $ListH_1$, $ListH_2$, $ListH_3$ and the $ListH_4$ according to ID and T . The response of this query is acquired from these lists by performing the following tasks

(i) If $rb = 0$, \mathcal{B} , respectively, uses ID and (ID, T) to run *InitialKeyquery* and *Timekey query* to obtain IK_{ID} and TK_{ID} . Then \mathcal{B} utilizes IK_{ID} , TK_{ID} and CT to run *Decryption* algorithm to produce the message M which is sent to \mathcal{A}_3

(ii) If $rb = 1$, \mathcal{B} utilizes CT_1 and CT_2 , which are from $CT = (CT_1, CT_2, CT_3, CT_4)$, to find the corresponding tuple $\langle W, CT_1, CT_2, \omega \rangle$ from the $ListH_5$. Then $M' || V' = CT_3 \oplus \omega$ can be computed by using CT_3 and ω . Further, \mathcal{B} utilizes M' and V' to find the corresponding tuples $\langle M, V, \gamma \rangle$ from the $ListH_7$ and $\langle M, Q \rangle$ from the $ListH_6$. Obviously, γ and Q can be obtained. After that, \mathcal{B} utilizes ID and (ID, T) to run *InitialKey query* and *Timekey query* to obtain IK_{ID2} and TK_{ID2} and computes $IK_{ID2} \cdot TK_{ID2} = g_2^{a(v+\zeta)}$. If S can be found in the corresponding tuple $\langle \hat{e}(CT_2, g_2^{a(v+\zeta)}), S \rangle$ from the $ListH_8$ such that $CT_4 = Q^v \cdot S$ holds, \mathcal{B} will confirm whether $CT_1 = g_1^\gamma$ holds. If $CT_1 = g_1^\gamma$, the message M' is sent to \mathcal{A}_3

(d) *Trapdoor query* (ID, T): \mathcal{A}_3 utilizes (ID, T) to issue the query, while \mathcal{B} , respectively, uses ID and (ID, T) to run *InitialKey query* and *Timekey query* to obtain $IK_{ID} = (IK_{ID1}, IK_{ID2})$ and $TK_{ID} = (TK_{ID1}, TK_{ID2})$. Then \mathcal{B} utilizes IK_{ID2} and TK_{ID2} to produce

the trapdoor $TD_{ID} = IK_{ID2} \cdot TK_{ID2}$ which is sent to \mathcal{A}_3

- (e) *Challenge*: When the phase 1 is over, \mathcal{A}_3 outputs a tuple $\langle ID^*, T^*, M^* \rangle$ as the target of the challenge. \mathcal{B} utilizes ID^* to find the corresponding tuples $\langle ID, U_{ID}, u, rb \rangle$ from the $ListH_1$. If $rb = 0$, \mathcal{B} interrupts this game. If $rb = 1$, \mathcal{B} randomly selects $V \in \{0, 1\}^l$ to run H_7 query with M^* and V . Then γ can be obtained. \mathcal{B} utilizes γ to set $CT_1^* = g_1^\gamma$ and find H_6 query $(M^*)^\gamma$ and H_8 query $(\tilde{e}(CT_2^*, g_2^{a(v^* + \zeta^*)}))$ to get Q and S such that $CT_4^* = Q^\gamma \cdot S$. In addition, \mathcal{B} sets $CT_2^* = g_1^c$, while a random value $CT_3 \in \{0, 1\}^{\lambda+l}$ is chosen. Finally, the challenge ciphertext $CT^* = (CT_1^*, CT_2^*, CT_3^*, CT_4^*)$ is sent to \mathcal{A}_3 .
- (f) *Phase 2*: \mathcal{A}_3 can issue the same query as the phase 1, but it must be under the condition of $ID \neq ID^*$ and $CT \neq CT^*$.
- (g) *Guess*: \mathcal{A}_3 responds to \mathcal{B} with a guess $M' \in \{0, 1\}^\lambda$. If $M' \neq M$, \mathcal{B} responds with failure and terminates. Otherwise, \mathcal{A}_3 wins the game. Then \mathcal{B} randomly selects a tuple $\langle W^*, CT_1^*, CT_2^*, \omega^* \rangle$ from the $ListH_5$, and outputs the BDH solution $A = (W^* / \tilde{e}(g_1^{ac}, g_2)^{\eta^*})^{u^{*^{-1}}}$ due to $\tilde{e}(g_1, g_2)^{abc} = (W^* / \tilde{e}(g_1^{ac}, g_2)^{\eta^*})^{u^{*^{-1}}}$.

The security analysis is similar to Theorem 5. We obtain that \mathcal{B} 's advantage to solve the BDH problem is $\epsilon' \geq (1/q_{H_5}) \Pr[E_{H_5}^*] \geq (1/q_{H_5})[(\epsilon - (1/2^\lambda)) / e(q_{IK} + 1)] - q_D/q$. \square

Theorem 8. *If the BDH assumption holds, the proposed RIBEET scheme satisfies the OW-ID-CCA security in the security game. More precisely, suppose that \mathcal{A}_4 is a PPT type 4 adversary who has at least ϵ advantage to break the RIBEET scheme. Then, there exists an algorithm \mathcal{B} to solve the BDH problem with the advantage.*

$$\epsilon' \geq \left(\frac{1}{q_{H_5}} \right) \left[\frac{\epsilon - (1/2^\lambda)}{e(q_{TK} + 1)} \right] - \frac{q_D}{q}, \quad (16)$$

where q_{H_5} , q_{TK} , q_D , and e , respectively, are the number of queries to random oracle H_5 , Timekey query, Decryption query, and Euler's number.

Proof. An algorithm \mathcal{B} is constructed to solve the BDH problem. The algorithm \mathcal{B} is given a BDH tuple $\langle q, G_1, G_2, G_T, \tilde{e}, g_1, g_1^a, g_1^c, g_2, g_2^a, g_2^b \rangle$ which is defined in Section 2. The algorithm \mathcal{B} can be seen as a challenger to find the answer of the BDH problem. The answer $A = \tilde{e}(g_1, g_2)^{abc}$ can be found by interacting with the PPT type IV adversary \mathcal{A}_4 in the following security game.

- (i) *Setup*: The challenger \mathcal{B} utilizes the BDH tuple to set $P_{pub} = g_1^a$, and then generates the public system

parameters $PSP = \{q, G_1, G_2, G_T, \tilde{e}, g_1, g_2, P_{pub}, H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8\}$, where H_i is a random oracle for $i = 1, 2, \dots, 8$. In addition, the system life time $SLT = \{T_0, T_1, \dots, T_t\}$ can be generated by the challenger \mathcal{B} . Then \mathcal{B} gives \mathcal{A}_4 the public system parameters PSP and system life time SLT . Here, the adversary \mathcal{A}_4 can issue queries to each random oracle as below

- (a) $H_1 - H_3$ queries: \mathcal{B} answers \mathcal{A}_4 in the same form as the proof of Theorem 6
- (b) $H_4 - H_8$ queries: \mathcal{B} answers \mathcal{A}_4 in the same form as the proof of Theorem 5
- (ii) Phase 1: The adversary \mathcal{A}_4 can, respectively, utilize ID , (ID, T) , (ID, T, CT) and (ID, T) to issue the *InitialKey query*, *Timekey query*, *Decryption query* and *Trapdoor query*. The response to each query can be obtained as follows
- (a) *InitialKey query* (ID): \mathcal{B} answers \mathcal{A}_4 in the same form as the proof of Theorem 6
- (b) *Timekey query* (ID, T): \mathcal{B} answers \mathcal{A}_4 in the same form as the proof of Theorem 6
- (c) *Decryption query* (ID, T, CT): \mathcal{B} answers \mathcal{A}_4 in the same form as the proof of Theorem 7
- (d) *Trapdoor query* (ID, T): \mathcal{B} answers \mathcal{A}_4 in the same form as the proof of Theorem 7
- (1) *Challenge*: When the phase 1 is over, \mathcal{A}_4 outputs a tuple $\langle ID^*, T^*, M^* \rangle$ as the target of the challenge. \mathcal{B} utilizes (ID^*, T^*) to find the corresponding tuples $\langle ID, T, U_{IDT}, \eta, rb \rangle$ from the $ListH_3$. If $rb = 0$, \mathcal{B} interrupts this game. If $rb = 1$, \mathcal{B} randomly selects $V \in \{0, 1\}^l$ to run H_7 query with M^* and V . Then γ can be obtained. \mathcal{B} utilizes γ to set $CT_1^* = g_1^\gamma$ and find H_6 query $(M^*)^\gamma$ and H_8 query $(\tilde{e}(CT_2^*, g_2^{a(v^* + \zeta^*)}))$ to get Q and S such that $CT_4^* = Q^\gamma \cdot S$. In addition, \mathcal{B} sets $CT_2^* = g_1^c$, while a random value $CT_3 \in \{0, 1\}^{\lambda+l}$ is chosen. Finally, the challenge ciphertext $CT^* = (CT_1^*, CT_2^*, CT_3^*, CT_4^*)$ is sent to \mathcal{A}_4 .
- (2) *Phase 2*: \mathcal{A}_4 can issue the same query as the phase 1, but it must be under the condition of $ID \neq ID^*$ and $CT \neq CT^*$.
- (3) *Guess*: \mathcal{A}_4 responds to \mathcal{B} with a guess $M' \in \{0, 1\}^\lambda$. If $M' \neq M$, \mathcal{B} responds with failure and terminates. Otherwise, \mathcal{A}_4 wins the game. Then \mathcal{B} randomly

selects a tuple $\langle W^*, CT_1^*, CT_2^*, \omega^* \rangle$ from the $List_{H_5}$, and outputs the BDH solution $A = (W^* \tilde{e}(g_1^{ac}, g_2)^{u^*})^{\eta^{s-1}}$ due to $\tilde{e}(g_1, g_2)^{abc} = (W^* \tilde{e}(g_1^{ac}, g_2)^{u^*})^{\eta^{s-1}}$

The security analysis is similar to Theorem 5. We obtain that \mathcal{B} 's advantage to solve the BDH problem is $\epsilon' \geq (1/q_{H_5}) \Pr[E_{H_5}^* \geq (1/q_{H_5})][(\epsilon - (1/2^\lambda))/e(q_{TK} + 1)] - (q_D/q)$. \square

Theorem 9. *The proposed RIBEET scheme is secure for brute force attacks if the discrete logarithm problem is hard.*

Proof. As mentioned in the concrete RIBEET scheme, the public system parameters are $PSP = \{q, G_1, G_2, G_T, \tilde{e}, g_1, g_2, P_{pub}, H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8\}$, the system life time is $SLT = \{T_0, T_1, \dots, T_t\}$ and the master private key is $mpk = s$. Based on the discrete logarithm problem, we ensure that the adversary cannot recover the master private key m $pk = s$ form $P_{pub} = g_1^s$. In addition, the security of the user initial key IK_{ID} and user time key TK_{ID} is also based on the discrete logarithm problem due to $IK_{ID} = (IK_{ID1}, IK_{ID2}) = (H_1(ID)^{mpk}, H_2(ID)^{mpk}) = (H_1(ID)^s, H_2(ID)^s)$ and $TK_{ID} = (TK_{ID1}, TK_{ID2}) = (H_3(ID, T)^{mpk}, H_4(ID, T)^{mpk}) = (H_3(ID, T)^s, H_4(ID, T)^s)$. Hence, the proposed RIBEET scheme can resist brute force attacks. \square

6. Comparison

In this section, we compare the proposed RIBEET scheme with the previous RIBE scheme [21] and IBEET scheme [6]. In order to analyze the cost of performing encryption, decryption and equality test, we first define two notations as follows.

- (1) Pair: time to perform a bilinear pairing $\tilde{e} : G_1 \times G_2 \rightarrow G_T$
- (2) Exp: time to perform an exponentiation in G_1, G_2 or G_T

We gain Pair = 7.8351 ms and Exp = 0.4746 ms from the literature [32]. These two execution times are obtained under the hardware device with Intel Core i7-8550U 1.80 GHz processor. Meanwhile, the prime number q selected in the cryptosystem setting phase is 256-bit. In addition, three multiplicative cyclic groups G_1, G_2 , and G_T of the prime order q are chosen in the simulation.

In Table 3, we list the comparisons of our proposed RIBEET scheme with the RIBE scheme [21] and several IBEET schemes [6, 10, 11] in terms of the cost of performing encryption, decryption and equality test, and two properties related to user revocation and equality test of ciphertexts. For the cost of performing encryption and decryption, Tseng and Tsai's RIBE scheme [21] has better performance than the other two schemes. However, Tseng and Tsai's RIBE scheme does not support equality test of ciphertexts. Although the existing IBEET schemes [6, 10, 11] and our

proposed RIBEET scheme support equality test of ciphertexts, the IBEET schemes does not have a mechanism to revoke users. Conversely, our proposed RIBEET scheme not only provides user revocation, but also retains the cost of encryption, decryption and equality test with the existing IBEET schemes. Additionally, Table 4 compares our RIBEET scheme with the RIBE scheme [21] and several IBEET schemes [6, 10, 11] in terms of $|PK|$, $|CT|$, and $|TD|$ which are, respectively, denoted as the bit length of user public key, ciphertext and trapdoor. We observed that the communication cost of our RIBEET scheme is similar to that of other schemes.

As mentioned in Section 1, the data collected from sensors on the patients is finally encrypted by the mobile device and then transmitted to the cloud. For the analysis of energy cost, we employ the "ampere" app to measure the voltage and current on the mobile device. After running this app, we obtain 14.28 V and 2856 mA on the mobile device. Table 5 lists the energy cost of performing encryption on the mobile device by using the formula $W = U \cdot I \cdot t$, where W, U, I , and t , respectively, are watt, voltage, current, and time.

7. Conclusions

We considered the existing syntax of IBEET and the property of user revocation to present the new syntax of RIBEET. Under the new syntax, we proposed a concrete RIBEET scheme. Meanwhile, we demonstrated that the proposed scheme has the IND-ID-CCA security for type I and II adversaries and the OW-ID-CCA security for type III and IV adversaries. We compared the proposed scheme with the previous RIBE scheme and IBEET scheme. We showed that the proposed scheme not only supports equality test for ciphertexts but also provides user revocation.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

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