Research Article

The Time-Dependent Reliability Analysis of Brake Piston Special-Shaped Seal of the Caliper Disc Brake

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1. Introduction

With the progress of vehicle technology and the improvement of vehicle speed, higher requirements are put forward for the quality of vehicle braking system. The caliper disc brake is widely used in automobile braking systems because of its excellent characteristics such as stable braking, good heat dissipation, high safety, and convenient installation and maintenance [1, 2]. The sensitivity and reliability of the caliper disc brakes have a great relationship with the rationality of the sealing structure of the brake piston. The function of the sealing structure is to ensure that the brake fluid cannot leak through the sealing ring during the braking process and provide sufficient reset force for the brake piston at the end of the braking process. At present, the brake piston seal structure adopts the form of O-shaped or rectangular section seal ring. Although the traditional standard O-shaped or rectangular section seal form is widely used because of its simple structure, low cost, compact design structure, and easy installation, the brake piston seal is under time-dependent and complex working conditions such as high temperature, variable pressure and vibration, leakage, wear, insufficient elasticity, and other phenomena are easy to occur, to reduce the braking efficiency and even brake failure, affecting the driving safety. On the premise of ensuring the braking efficiency of caliper disc brake, research and explore new structural forms which can meet the special-shaped seal of high-efficiency brake piston, to improve the reliability of the sealing performance of caliper disc brake piston.

Based on the caliper disc brake piston special-shaped seal structure plum flower ring as the research object, the brake piston stress of the plum flower ring is seen as a random process, to have the maximum stress as equivalent stress, equivalent stress analysis how the probability distribution of changes over by stress; according to the stress-strength interference theory model, the whole working time was divided into \( n \) periods to calculate the reliability, and the time-varying failure reliability model of the quincunx seal ring of the brake piston was established. At the same time, the failure of the brake piston seal ring was divided into shear stress failure and leakage failure by using the total
probability formula and stress-strength interference model, and the time-varying failure reliability of the brake piston quin-shaped seal ring and O-shaped seal ring under single failure mode and multiple failure modes was analyzed and calculated.

2. Time-Dependent Reliability Calculation of Brake Piston Seal of Caliper Disc Brake

Reliability design is regarded as a leap of safety design concept. In the process of reliability analysis, load, strength, and other design parameters are treated as random variables, so it can describe load properties and material properties objectively and scientifically. At the same time, the goal of reliability design is to ensure that the strength is greater than the load, and the safety degree of the design can be expressed quantitatively. As shown in Figure 1, when the brake piston works under random load, the contact force of the seal ring changes with the passage of time. If reliability and failure rate are taken as its performance indicators, the reliability model of the seal ring can be expressed as a function of reliability over time [3]. Real-time variable reliability analysis is to analyze the probability distribution of the performance function of the brake piston seal ring when the input is random variable.

2.1. The Calculation Method of Time-Dependent Reliability. Structural reliability can be expressed as the ability of a system to accomplish the specified task under a certain times and conditions or the probability that the product can work normally at the specified time in the normal service environment. Generally, the limit state function for the reliability of time-invariant structures is a random variable model.

\[ Z = g(X) = g(X_1, X_2, \ldots, X_n), \]  

(1)

When analyzing the reliability, variables \( X_1, X_2, \ldots, X_n \) are generally used to replace the factors affecting the structure. It can be seen from the above formula that the traditional study of reliability does not take into account the change of pressure over time and the influence of other factors. During the working period, the brake piston of clamp disc brake will be affected by time-varying factors such as abrasion of sealing structure materials and variable pressure, and its sealing performance will be reduced, resulting in the decrease of contact stress with the increase of working time. The ability of the piston seal ring to complete the sealing function under the given conditions will be reduced, that is, the working reliability of the piston seal structure will be reduced [4, 5].

In order to accurately calculate the brake piston seal reliability, based on the definition of reliability, considering the time factor, the two scholars of domestic Li Yi and Yan Yun through research suggests that for most of the system model structure, its load effect and structural resistance are changes over time, and changes in the reliability of the stochastic process is a dynamic process. At the same time, the two scholars obtained the relationship expression of the time-varying reliability function of the system structure:

\[ Z(t) = g[R(t), S(t)] = R(t) - S(t). \]  

(2)

In Equation (2), \( S(t) \) is the loading effect, namely, the random application process of the medium pressure load, \( R(t) \) is the random process of the system structure resistance, that is, the contact stress of the sealing ring, and \( Z(t) \) is the function of the sealing ring in a certain limit state. The random process is a linear or nonlinear function. Then, the reliability probability of the piston sealing ring in the period \( T \) can be expressed as:

\[ P(T) = P\{Z(t) > 0, t \in [0, T]\} = P\{R(t) - S(t) > 0, t \in [0, T]\}. \]  

(3)

The contact stress of the brake piston quin-shaped seal ring is greater than the pressure of the brake fluid at every moment \( t \), during the working period. In this case, the brake piston seal ring can be in a reliable normal working state. According to the complementary theorem of probability, the failure probability of the piston seal ring within the service period \( T \) can be expressed as:

\[ P_f = 1 - P_s(T) = P\{R(t_i) < S(t_i), t_i \in [0, T]\}. \]  

(4)

Formula (4) shows that in the working process of the brake piston quin-shaped seal ring, as long as there is a moment when the contact stress of \( t_i \) sealing structure is less than the medium pressure, the sealing failure phenomenon will occur. The reliability probability and failure probability expressed by Equation (3) and Equation (4) are both related to the factor of time \( T \), which is called time-dependent reliability analysis.

2.2. Stress-Strength Interference Model. The stress-strength interference model is the basic model for calculating the reliability of components. In the process of calculating the reliability, the basic idea of interference analysis with random variables can be expressed as follows: as shown in Figure 2, the probability density function curves of stress and strength are drawn in the same coordinate system. Generally, an interference phenomenon happens when the two curves intersect. It will appear failure where the stress is larger than the strength [6]. The stress and strength in the definition are both generalized, and the stress can be the external factors leading to the failure of the sealing ring or sealing system, such as the pressure, friction, and wear of the brake fluid. Strength is the resistance of the sealing ring or sealing system corresponding to various stresses, such as static strength, fatigue static strength, and contact stress. When the probability density function of strength \( S \) is \( f_S(s) \) and the probability density function of stress \( s \) is \( f_s(s) \), the probability density function of the reliability \( R \) can be expressed as
load in the working process, the probability density function
and the greater the failure possibility of the structure.
the probability that the stress is greater than the strength,
seal rings under a single load or under a single failure mode.
The larger the area of the stress strength interference area,
that is, the larger the shadow part in the
Figure 2 is the
stress strength interference phenomenon occurs. The shaded
part in Figure 2 is the
stress-strength interference area
distribution due to some external or internal reason, the
ring can be expressed as follows:
Equation (7) is suitable for calculating the reliability of
the seal ring produce a certain contact stress, which is
the friction force produced by the contact surface between
contact stress is greater than the
As is shown in Figure 3, the reliability calculation of
the brake piston seal ring can be equivalent to the reliability of
the series system constituting its failure mode [7, 8].
When the working load of the sealing ring is a de
value \( L \), there is an independent relationship among the fail-
ure modes. The reliability of the sealing ring can be equiva-
ent to the reliability of the series system under independent
failure. The reliability \( R_L \) can be expressed as follows:
In formula (8), when the load is determined value \( L \), if
the applied load obeys the random variable of the density
function \( f_L(L) \), the relationship expression of the reliability
R of the sealing ring can be obtained according to the stress
strength interference theory and the calculation formula of
full probability, as shown in
\[
R = \int_{-\infty}^{+\infty} P(S > s) \int_{s}^{+\infty} f_{i}(S) dS dS = \int_{-\infty}^{+\infty} f_{i}(S) \int_{s}^{+\infty} f_{j}(S) dS dS. 
\]
(5)
Then, the failure probability \( F \) is expressed as follows:
\[
F = 1 - R = \int_{-\infty}^{+\infty} f_{i}(s) \int_{s}^{+\infty} f_{j}(S) dS dS = \int_{s}^{+\infty} f_{i}(S) \int_{-\infty}^{+\infty} f_{j}(S) dS dS. 
\]
(6)
When the stress distribution overlaps with the strength
distribution due to some external or internal reason, the
stress strength interference phenomenon occurs. The shaded
part in Figure 2 is the “stress-strength interference area”.
The larger the area of the stress strength interference area,
that is, the larger the shadow part in the figure, the greater
the probability that the stress is greater than the strength,
and the greater the failure possibility of the structure.
When the brake piston seal ring is subjected to a single
load in the working process, the probability density function
of strength \( S \) is expressed as \( f_S(S) \), and the probability
density function of load \( L \) is expressed as \( f_L(L) \). As the stress-
strength interference theory shows, the reliability of the seal
ring can be expressed as follows:
\[
R = \int_{-\infty}^{+\infty} f_L(L) \int_{s}^{+\infty} f_S(S) dS dL. 
\]
(7)
Equation (7) is suitable for calculating the reliability of
seal rings under a single load or under a single failure mode.
In practice, seal rings are often affected by different failure
modes which are independent of each other. When there
are multiple failure modes in the working process of the seal-
ing ring, every failure mode will cause the sealing ring to fail.

3. Force Model of Brake Piston of Caliper
Disc Brake
The friction force produced by the contact surface between
the brake piston seal ring and the brake piston can make the
seal ring produce a certain contact stress, which is
used to resist the pressure of the brake fluid acting on
its surface. When the contact stress is greater than the
pressure of brake fluid, Von-Mises stress will make the seal
ring leak in the direction of pressure. As a result, it will
Failure mode 2

Displacing the original contact state and reduce the sealing performance.

Assuming that the seal structure of the brake piston is a rectangular sealing ring, the deformation diagram of the rectangular sealing ring in the working process is shown in Figure 4. When the brake fluid pressure is \( p_0 \), the sealing ring and the piston will only have a relatively sliding contact without force deformation, and what remains unchanged is the width of the contact, that is, the pressure of the deformed rectangular sealing ring on the piston is distributed in a rectangular shape, which can be expressed by a rectangular \( abce \). At the same time, to keep the rectangular sealing ring well sealed in the contact width, in the pressure direction of the brake fluid, the pressure drops from \( p_0 \) to \( p = 0 \), so that the pressure distribution generated by the oil pressure can be represented by a triangle \( \epsilon_{\text{min}} \). The sealing condition of the rectangular seal is that the contact stress is greater than the brake fluid pressure. The contact pressure can be expressed by the area \( A_1 \) of rectangular \( abce \), and the medium pressure can be expressed by the area \( A_2 \) of triangle \( \epsilon_{\text{min}} \). Then, the condition of sealing structure is that the area \( A_1 \) of rectangular.

\( abce \) is larger than the area \( A_2 \) of triangle \( \epsilon_{\text{min}} \). The average stress produced by sealing ring to piston is \( p_0/2 \) [9].

The hold force \( F \) of the sealing ring to the piston is:

\[
F = \pi \cdot d \cdot b_0 \cdot \frac{p_0}{2},
\]  

(10)

where \( d \) is the inner diameter of the piston sleeve of brake caliper in mm, \( b_0 \) is the diameter of the sealing ring in mm, and \( p_0 \) is the braking pressure of hydraulic oil in MPa.

Assuming that the friction coefficient between the seal ring and the piston sleeve of the brake caliper is expressed as \( \mu \), the friction resistance \( F_0 \) required to ensure good sealing performance can be expressed as follows:

\[
F_0 = \mu N = \mu \pi \cdot d \cdot b_0 \cdot \frac{p_0}{2}.
\]  

(11)

The friction provided by the seal ring is expressed as \( F_\epsilon \). The radial deformation \( \epsilon \) of the seal ring is the main cause of friction \( F_\epsilon \):

\[
\epsilon = \frac{(h_0 - h)}{h_0} \times 100\%,
\]  

(12)

\[
F_\epsilon = \mu \cdot \sigma \cdot S = \mu \cdot \sigma \cdot \pi \cdot d \cdot b_0,
\]  

(13)

where \( \mu \) is the friction coefficient of rubber material, \( h \) is the thickness of the compressed seal ring in mm, \( h_0 \) is the thickness of the seal ring without pressure in mm, \( S \) is the contact area between the seal ring and the piston sleeve of brake pliers in mm\(^2\), and \( \sigma \) is the compressive stress in MPa. The number of \( \sigma \) is determined by the function of the deformation of the seal ring and the parameters of the rubber material. The material parameters will change with the change of temperature because the rubber is a hyperelastic body. Therefore, the relationship between stress and strain is not linear throughout the working period.

In order to ensure good sealing performance, the sealing rings need to be satisfied: \( \sigma_{\text{xy}} \). From Equation (11) and Equation (13), we can come to the conclusion that \( \sigma_{\text{min}} = p_0/2 \).

\[
\frac{2}{3} E \left(1 + 2 \mu \cdot \frac{b_0}{2h_0} \right) \left(1 - \frac{1}{\sqrt{1 - \epsilon_{\text{min}}}} + \epsilon_{\text{min}} - 1 \right) = \frac{p_0}{2}.
\]  

(14)

In Equation (14), \( \epsilon_{\text{min}} \) is the minimum deformation of the sealing ring in the sealing process.

When the brake pressure disappears, the brake piston returns to the original position which is the starting position of the brake under the reset force of the seal ring. The reset force and the internal pressure are equal.

\[
F_{j\omega} = F_0 = \pi \cdot \mu \cdot \pi \cdot d \cdot b_0 \cdot \frac{p_0}{2}.
\]  

(15)

The relationship between the maximum contact stress of the sealing ring and the hydraulic pressure is approximately linear, which conforms to the self-tight sealing mechanism [10].

\[
P_\epsilon = P_0 + kP,
\]  

(16)

where \( P_\epsilon \) is the maximum contact stress on the sealing ring, \( P_0 \) is the maximum contact pressure on the sealing ring during installation, the \( P \) on behalf of the pressure of brake fluid, and \( k \) is the fluid pressure transfer coefficient and can be
taken as 1 for O-ring. As for other types of sealing ring, it depends on the type of sealing ring [11–13].

4. Time-Dependent Reliability Analysis of Special-Shaped Seal Ring of Brake Piston

As a key component of caliper disc brakes, the brake piston sealing ring will not only reduce the braking performance but also greatly reduce the safety of driving. Therefore, the reliability of the sealing ring can affect the efficient braking performance of the vehicle. In this paper, time-dependent reliability analysis of special-shaped seal ring of the brake piston takes plum-shaped seal ring as an example (as shown in Figure 5). There are four sealing contact planes designed at the contact between the quincunx seal ring and the groove of the brake piston and the brake caliper piston, to achieve good self-sealing function; at the same time, grooves are designed on the four seal contact planes to store lubricating oil, so as to improve the lubrication capacity of the special-shaped seal ring and reduce the wear in the reciprocating process and effectively reduce the leakage. Because the cross section of the special-shaped seal ring is similar to plum blossom, it is called plum blossom seal ring.

The failure modes of the seal ring can be divided into shear failure and leakage failure. The maximum shear stress of the plum ring is \(\sigma_{xy}\), and the shear resistant strength is \(\tau_0\). Plum blossom seal ring will fail due to shear failure when the maximum shear stress of the sealing ring is greater than its shear resistant strength. The shear failure function of plum blossom seal ring is assumed to be as follows:

\[
g(x) = [\tau_b] - \sigma_{xy}, \quad (17)
\]

where \(\sigma_{xy}\) is the maximum shear stress of the plum-shaped seal ring which can be calculated in the finite element software ANSYS Workbench, \([\tau_b]\) is the allowable shear strength of rubber, \(p\) is the pressure of brake fluid, and \((\sigma_{xy})_{\text{max}}\) is expressed by the maximum contact stress of plum-blossom seal ring. Therefore, the leakage failure function can be described in

\[
g(x) = (\sigma_{xy})_{\text{max}} - p. \quad (18)
\]

4.1. Determination of Maximum Shear Stress Distribution.

The maximum shear stress of the plum-shaped seal ring of the brake piston is related to the parameters of compression, medium pressure, and friction coefficient. Because each parameter has the characteristics of randomness, the stress \(\sigma_{xy}\) also belongs to random variables and has certain distribution characteristics. The uncertainty of the load means that the load applied to the brake piston is a random quantity subject to a certain distribution. The load value for calculating the reliability is not fixed. Its load value fluctuates around the average load, and the fluctuation range is expressed by the coefficient of variation [14, 15].

Friction coefficient, medium pressure, and compression are selected as the main factors affecting the two failure modes of quincunx seal ring. Set the above three factors as random variables and assume that the three random variables obey a normal distribution. The mean value and variation coefficient of each random variable are shown in Table 1.

In order to ensure that the quincunx seal ring meets the strength requirements during operation, the maximum stress is generally taken as its maximum value during operation. In the process of time-dependent reliability analysis of plum-blossom seal failure, the whole working time is divided into \(n\) time periods to calculate the reliability. Therefore, the maximum shear force is selected from the simulated data of each time period to represent the shear force of that period. From the knowledge of probability and mathematical statistics, we can know that [16] a random variable \(X\) obeys a normal distribution, and the \(n\) samples \(X_1, X_2 \cdots X_n\) of \(X\), the maximum values taken from these samples form the sequence \(X_m = \max (X_1, X_2, \cdots, X_n)\), the distribution function of \(X_m\) is represented by \(F_m(x)\), and the density function is represented by \(f_m(x)\), then its expression is:

\[
F_m(x) = [F(x)]^n, \quad (19)
\]

\[
f_m(x) = n[F(x)]^{n-1}f(x), \quad (20)
\]

where \(F(x)\) represents the distribution function of a single sample and \(f(x)\) represents the density function of a single sample. At a certain time, the probability distribution \(F(x)\) of shear stress \(X\) obeys the normal distribution with a mean value of \(\mu\) and standard deviation of \(\sigma\). When the sample size increases infinitely, the limit distribution of statistics is similar to that of sampling. Therefore, the maximum order statistics \(X_m\) of random variables \((X_1, X_2, \cdots, X_n)\) obey the extreme value I distribution. The distribution function is:

\[
F_m(x) = \exp \{-\exp [-a_n(x - u_n)]\}. \quad (21)
\]
Table 1: The mean values and variation coefficients of the random variables.

<table>
<thead>
<tr>
<th>Program</th>
<th>Compression Mean Values</th>
<th>Medium Pressure Variance Coefficient of Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean values</td>
<td>0.55 mm</td>
<td>5 MPa</td>
</tr>
<tr>
<td>Variation coefficients</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The mean and variance are expressed as:

\[
E(X_n) = \frac{0.5772}{a_w} + u_w, \tag{22}
\]

\[
\text{Var}(X_n) = \frac{\pi^2}{6a_n}, \tag{23}
\]

where

\[
u_n = \sigma \sqrt{\frac{2 \ln n - \sigma \frac{\ln (\ln n)}{2} + \ln 4\pi}{2\sqrt{2 \ln n}}} + \mu, \tag{23a}
\]

\[
a_n = \frac{\sqrt{2 \ln n}}{\sigma}. \tag{23b}
\]

The distribution function of the equivalent shear stress of the maximum shear stress \( X(n) \) in each period is obtained by Equation (21). Taking the plum-blossom seal ring as an example, the test shows that at the significant level (\( \alpha = 0.05 \)), the maximum shear stress obeys the normal distribution \((\mu_s = 2.8069 \times 10^6, \sigma_s = 2.5095 \times 10^5)\), the cumulative distribution function of the equivalent shear stress in \( n \) time periods is expressed as follows:

\[
F_{m}(m) = \left\{ \int_{-\infty}^{x} \frac{1}{0.2509 \sqrt{2\pi}} e^{-1/2(X-2.8069/0.2509)^2} dX \right\}^{n}. \tag{24}
\]

By deriving the cumulative distribution function Equation (24), the expression of the probability density function \( f_{m}(x) \) of the equivalent shear stress can be obtained.

\[
f_{m}(m) = n \left\{ \int_{-\infty}^{x} \frac{1}{0.2509 \sqrt{2\pi}} e^{-1/2(X-2.8069/0.2509)^2} dX \right\}^{n-1}
\]

\[
\cdot \frac{1}{0.2509 \sqrt{2\pi}} e^{-1/2(X-2.8069/0.2509)^2}. \tag{25}
\]

4.2. Determination of Maximum Shear Resistance Stress Distribution. According to the actual situation of the shear stress on the plum-shaped seal ring, the time-dependent effect of the stress should be taken into account when calculating the time-dependent reliability of the failure of the plum-shaped seal ring. Therefore, on the basis of the shear stress and shear strength of the plum-shaped seal ring, the influence of the random process of stress is considered, and the function can be expressed as follows:

\[
g(X, t) = \delta(t) - s Y', t, \tag{26}
\]

where \( \delta(t) \) is a random variable of shear strength of rubber sealing ring, \( s Y', t, \) is a random process of shear stress, and \( Y' \) is a related random variable. The time-dependent reliability index corresponding to the function can be expressed as follows:

\[
\beta(t) = \frac{\mu_g(t)}{\sigma_g(t)} = \frac{E[g(X, t)]}{\text{Var}[g(X, t)]}, \tag{27}
\]

where \( \mu_g(t) \) is the mean of the function and \( \sigma_g(t) \) is the standard deviation of the function. The expression of time-dependent reliability corresponding to the function can be expressed as

\[
R = \varphi(\beta(t)) = P\left\{ \delta(t) > s (Y', t) \right\}, \tag{28}
\]

where \( t \in [0, T] \) and \( R \) represents the working reliability of the sealing ring. To ensure the sealing performance of the sealing ring, the shear stress of the sealing ring must be less than its shear strength within each time \( t \) in the working process.

Generally speaking, the sampling data of shear strength of sealing rings can be obtained mainly through experiments, and then, the correlation distribution and digitally characteristics can be obtained by statistical calculation. Assuming that the shear strength of plum-shaped rubber seal obeys the normal distribution \((\mu_s = 4.46, \sigma_s = 0.29)\), the shear strength of O-shaped rubber seal obeys the normal distribution \((\mu_s = 4.6, \sigma_s = 0.32) \) \[17, 18\]. The same method can be used to calculate the failure time-depending reliability of plum-shaped seal ring and O-shaped seal ring under leakage and the failure time-depended reliability of standard seal structure.

4.3. Time-Dependent Reliability Distribution of Seal Ring under Single Failure Mode. The action time history of random loads is not considered in the calculation of static reliability model. The calculation of static reliability model directly uses the stress strength interference theory to solve the problem, which can express the reliability of the corresponding load within the action time. Based on the actual working condition of plum-shaped seal ring of brake piston of forceps disc brake, and taking the action time of the shear force as the measurement index of the working life, this paper establishes the time-varying reliability model of the failure of the plum blossom seal ring.

Assuming that the probability density function of the maximum shear resistant stress \( \delta_{sH} \) of the plum blossom seal ring is expressed by \( f_{\delta_{sH}}(\delta_{sH}) \), the cumulative distribution function of contact stress \( s_{H} \) is expressed by \( F_{s_{H}}(s_{H}) \), and the probability density function is expressed by \( f_{s_{H}}(s_{H}) \). When calculating the reliability of the plum-blossom seal
ring, according to the stress-strength interference theory, the reliability of the seal ring in a period of time can be expressed as follows:

\[
R = \int_{-\infty}^{\infty} f_{\delta_H}(\delta) \int_{-\infty}^{\infty} f_{n_1}(s_H) ds_H d\delta_H = \int_{-\infty}^{\infty} f_{\delta_H}(\delta_H) F_{n_1}(\delta_H) d\delta_H.
\]  

(29)

In \( n \) time periods, as Equation (19) and Equation (20) show, when the shear strength of plum blossom seal rings does not change or deteriorate, the cumulative distribution function \( F_m(s) \) and probability density function \( f_m(s) \) of the equivalent shear stress can be expressed as follows:

\[
F_m(s_H) = \left[ F_{n_1}(s_H) \right]^n,
\]

(30)

\[
f_m(s_H) = n \left[ F_{n_1}(s_H) \right]^{n-1} f_{n_1}(s_H).
\]

(31)

By substituting Equation (30) and Equation (31) into Equation (29), the time-dependent reliability of plum-blossom seal rings in \( n \) time periods can be calculated as follows:

\[
R(n) = \int_{-\infty}^{\infty} f_{\delta_H}(\delta) \int_{-\infty}^{\infty} n \left[ F_{n_1}(s_H) \right]^{n-1} f_{n_1}(s_H) ds_H d\delta_H
\]

\[
= \int_{-\infty}^{\infty} f_{\delta_H}(\delta_H) \left[ F_{n_1}(\delta_H) \right]^n d\delta_H
\]

(32)

\[
= \int_{-\infty}^{\infty} n \left[ f_{n_1}(s_H) \right]^{n-1} f_{n_1}(s_H) \left[ 1 - F_{n_1}(s_H) \right] ds_H.
\]

When the plum blossom seal ring passes through working \( n \) period, the relationship between its failure rate and reliability is shown in Equation (33) [18]. Therefore, the expression of the failure rate of plum blossom seal ring is:

\[
h(n) = \frac{R(n) - R(n-1)}{R(n)} = \frac{\int_{-\infty}^{\infty} f_{\delta_H}(\delta_H) \left[ 1 - F_{n_1}(\delta_H) \right]^n d\delta_H}{\int_{-\infty}^{\infty} f_{\delta_H}(\delta_H) \left[ F_{n_1}(\delta_H) \right]^n d\delta_H}
\]

\[
= \frac{\int_{-\infty}^{\infty} [n - (n + 1) F_{n_1}(s_H)] \left[ F_{n_1}(s_H) \right]^{n-1} f_{n_1}(s_H) \left[ 1 - F_{n_1}(s_H) \right] ds_H}{\int_{-\infty}^{\infty} [n F_{n_1}(s_H)]^{n-1} f_{n_1}(s_H) \left[ 1 - F_{n_1}(s_H) \right] ds_H}.
\]

(33)

According to the relationship between the reliability of the plum blossom seal ring of the brake piston under the shear stress failure mode and the stress action time and the shear stress and antishear stress distribution and parameters introduced earlier, the reliability of plum blossom seal ring and O seal ring in shear stress failure mode can be obtained, as shown in Figure 6. As can be seen from the figure, under the same conditions, the reliability of the shaped seal ring is greater than that of the traditional O seal ring.

4.4. Time-Dependent Reliability Distribution of Seal Ring under Multiple Failure Mode. Based on the two failure modes of leakage failure and shear failure of quincunx seal ring, the time-varying failure reliability model of seal ring is established in this paper. It can be seen from Equation (9) that under the assumption that each failure mode is not related to each other, the reliability of the sealing ring can be obtained according to the logical relationship of the series system.

\[
R = \prod_{i=1}^{m} R_i = \prod_{i=1}^{m} \int_{-\infty}^{\infty} f_{\delta_i}(\delta_i) d\delta_i d\delta_i.
\]

(34)

The above formula is based on the assumption that the failure modes are independent of each other, but in practice, most of the failure modes have a certain failure correlation. In this case, Equation (34) needs to be revised, and the reliability results [19–21] can be revised according to the experimental data or practical experience. However, the reliability
obtained by this method is not very accurate, and there is a big error between the calculated reliability and the practical reliability.

In the actual work of the seal ring, due to the influence of various uncertain factors, few failure modes are completely independent of each other. Therefore, it is of great engineering significance to establish a multimode reliability model considering common cause failure. It has been said that in the working process of the sealing ring, due to the influence of external environmental factors and their own factors, two failure forms of shear failure and leakage failure will occur. According to the stress strength interference theory and considering various failure modes of the sealing ring, a time-dependent failure reliability model is established.

When the shear force on the seal ring is equivalent to force in \( n \) time periods, the failure between the two failure modes is independent. The reliability of the seal ring under multiple failure modes can be expressed by Equation (8) and Equation (9):

\[
R(n) = \int_{-\infty}^{+\infty} \left[ \prod_{i=1}^{m} \left( 1 - F_i(L) \right) \right] n[F_i(L)]^{m-1}f_i(L) dL. \tag{35}
\]

According to the load distribution function obtained above, the time-dependent failure reliability of seal rings under various failure modes can be obtained by substituting Equation (35) as shown in Figure 7. It can be seen from the figure that the reliability of the sealing ring decreases with the increase of time, and the reliability under the multi-effect failure mode is smaller than that under the failure mode, respectively.

5. Conclusions

(1) Based on time-varying reliability of the basic concepts and theoretical basis of stress-strength interference model, based on the service life of brake piston plum blossom seal ring measures, established the brake piston reciprocating motion situation many times the load equivalent model of sealing ring, and the brake piston seal in the shear failure and leakage failure of reliability model under two kinds of failure modes

(2) Based on the force model of the brake piston of the caliper disc brake, the caliper force and friction resistance of the seal ring before and after the braking power disappeared are analyzed

(3) For sealing ring shear failure and leakage failure of the existence of the two failure modes, respectively, set up special seal structure of the plum blossom seal ring under single load and failure mode of the stress-strength interference model, calculate the reliability under two kinds of failure modes; under the same conditions, the reliability of the plum blossom seal ring is greater than the reliability of the O ring

Data Availability

The (data type) data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Xin Bai and Huiting Ni are responsible for relevant investigation; Liqun Lu for conceptualization; Leilei Shi for computing method; Meng Sun, Xin Bai, and Leilei Shi for model establishment; Liqun Lu, Meng Sun, and Leilei Shi for reliability study; Liqun Lu and Huiting Ni for verification; Liqun Lu for resources; and Liqun Lu and Meng Sun for writing the manuscript.
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