Research Article

Combined EEMD with a Novel Flexible Wavelet Threshold Function for Weighing Signal Denoising Approach

Dan Liu,1,2 Xiang Liao,1,2 Shuo Ouyang,3 and Chaoshun Li4

1Hubei Engineering Research Center for Safety Monitoring of New Energy and Power Grid Equipment, School of Electrical & Electronic Engineering, Hubei University of Technology, Wuhan 430068, China
2Xiangyang Industrial Institute of Hubei University of Technology, Xiangyang 441100, China
3Hydrology Bureau of Yangtze River Water Resources Commission, Wuhan 430000, China
4School of Civil & Hydraulic Engineering, Huazhong University of Science & Technology, Wuhan 430000, China

Correspondence should be addressed to Xiang Liao; liaoxiang@hbut.edu.cn

Received 11 May 2022; Revised 11 August 2022; Accepted 12 August 2022; Published 24 August 2022

Academic Editor: Carlos Marques

Copyright © 2022 Dan Liu et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper proposes a method of combining ensemble empirical mode decomposition (EEMD) with a novel flexible wavelet threshold function to reduce the random error in the weighing result that caused by the nonstationary and nonlinear noise in quality characteristic parameter measurement equipment. The original signal is first processed by EEMD, and then, all intrinsic mode functions (IMFs) are processed by a novel flexible threshold function proposed by this paper. Finally, the denoised signal is obtained by adding reconstructed IMFs and residual. Through theoretical analysis, the proposed threshold function can retain more useful information and have continuity at the segmentation point. Moreover, the addition of adjustable parameters makes it more adaptable. Its advantages are verified by comparing the denoised results with other threshold functions in the simulation model. In weighing experiment, the validity of the novel flexible threshold function in weighing signal denoising is verified, and the effectiveness of the proposed method is also quantitatively confirmed by comparing the random errors of the original signal and the denoised signal.

1. Introduction

The mass characteristic parameters of an object—mass, mass center, and moment of inertia—play an important role in many industries [1]. Due to the errors in the manufacturing industry of each component, the quality characteristic parameters of the measured object cannot be directly calculated from the theory, so the quality characteristic parameter measurement equipment was born. It uses sensors to convert the gravity of measured object into electrical signals and then uses three-point method, four-point method, and other measurement methods to obtain related parameters. At present, high precision has been achieved at home and abroad in terms of the mechanical structure and measurement methods of these devices. For example, Gopinath et al. [2] used expensive air-bearing and accurate sensors for accurate measurement. The authors of [3] eliminated side force which may exist on the load cell by adding a ball groove and ball to each load cell. The authors of [4] proposed a flexible measurement technique by fusing data from two systems. However, when the device is working, the electronic circuit and the surrounding environment make the output signal of the sensor contains a lot of noise, which leads to random errors in the quality measurement results. Furthermore, it would increase the error of inertia and other parameters of measured object. If the noise in the weighing signal can be stripped as much as possible, the measurement accuracy of the equipment will be further improved. This is easier than making improvements in equipment structure and measurement methods.

With the development of society, various sensors are subject to more and more interference when measuring. In order to improve the accuracy, many industries have made improvements, such as improving the internal structure of
sensors [5–8] and improving the structure of probes [9, 10] and materials [11, 12] in biological testing and water quality testing. In the use of weighing sensors, the greater interference is the nonlinear, nonsmooth signal generated by the surrounding environment and the circuit itself; the traditional weighing signal processing method can no longer remove the noise in the signal well. In recent years, commonly used processing methods for this type of signals include empirical mode decomposition (EMD) [13, 14], wavelet transform (WT) [15–17], and deep learning-related methods [18]. Some scholars even change the measurement method; for example, the measurement method of [19] has less noise in the original signal when the wiring faults are located. But deep learning needs theoretical standard values to train parameters. Due to the assembly of the device itself, it is difficult to obtain a pure signal corresponding to the standard object. EMD and WT belong to time-frequency analysis methods, which are able to decompose signals according to their own characteristics. Time domain analysis methods have both time and frequency domain information and can obtain better performance; for example, Lee et al. [20] used the time-frequency reflection method for cable fault detection to obtain higher accuracy and a wider range of adaptation scenarios. Many scholars have also proposed improved methods related to EMD and WT to adapt to the complexity of signal and the diversity of applications, such as ensemble empirical mode decomposition (EEMD) [21] and variational mode decomposition (VMD) [16, 22]. There are many ways to improve wavelet denoising methods, such as improving the threshold function to solve the problem that reconstruction signal may oscillate or distort that caused by traditional threshold functions [23–31], setting adaptive threshold [32, 33], finding the optimal wavelet basis [34–36], and setting self-adaptive wavelet decomposition level [15]; some scholars combine EMD and WT [37].

Selecting an appropriate threshold function is the key of wavelet denoising. However, the existence of discontinuities in the traditional hard threshold function may lead to excessive oscillation of the reconstructed signal; the inherent bias of traditional soft threshold function would lead to irritating question such as distortion of the reconstructed signal and low accuracy. Many scholars have proposed that improved threshold functions use different methods. For example, the authors of [23, 24] introduced exponential function into threshold function, which makes it between soft and hard threshold function when coefficient is greater than threshold; on this basis, to increase its adaptability, Wang et al. [25] divided the area between the soft and hard threshold function into five parts by control $\alpha$, but when $\alpha$ takes a certain value, the threshold function is not continuous at threshold. The authors of [26] combined this idea with other improvement methods to improve the threshold function. The authors of [27] used $\sqrt{|d_{pq}|^2 - \lambda^2}$ to replace $|d_{pq} - \lambda|$ to improve threshold function; but they completely set to zero when $d_{pq} < \lambda$, so that a part of useful signal is lost. The authors of [31] introduced a smaller adjustable parameter when coefficient is less than threshold to keep more useful information, but there is still a large deviation between $d_{pq}$ and $d_{pq}$. This paper proposes a novel flexible threshold function to make up for the shortcomings of the above threshold function improvement methods. It can not only ensure that the signal is not distorted to a large extent, and would not cause excessive oscillation of denoised signal, but also can adjust the parameters to achieve the desired denoising effect.

In order to reduce the random error of the measured object mass obtained by the sensor of the mass characteristic measuring device, we propose a method combining EEMD with a novel flexible wavelet threshold function for processing weighing signal with noise. First, this paper briefly introduces the principles of EEMD and WT and makes a theoretical analysis of the flexible threshold function, then the validity of the improved threshold function is verified through the simulation in MATLAB, and finally, an experimental platform is built to verify the authenticity and feasibility of the denoising method.

2. The Theoretical Analysis of Denoising Method

This section gives a theoretical analysis of the proposed weighing signal denoising method. Firstly, the noise reduction process of EEMD and WT methods is briefly introduced, and then, the novel flexible threshold function is proposed, and its characteristics are analyzed. Finally, the methods of other parameters in the process of denoising, such as wavelet basis, number of decomposition layers, and threshold, are introduced.

2.1. EEMD. EEMD adds white noise with uniform distribution and zero average to the original signal to solve the modal aliasing problem in EMD, so it is more effective than EMD in removing noise from original signal. The specific decomposition process is as follows:

First, add $n$ groups of white noise with the same signal-to-noise ratio to the original signal ($x$) to get noise-added signals ($s_i$, $i = 1 \cdots n$).

Second, $s_i$ is decomposed by EMD to obtain the corresponding intrinsic mode functions ($\text{IMF}_{i,j}$ ($j = 1 \cdots m$) and residual components ($\text{rec}_j$).

Finally, calculate the mean of $\text{IMF}_{i,j}$ and $\text{rec}_j$, which are $\text{IMF}_i$ and $\text{rec}_i$ of $x$.

2.2. Improved Threshold Function. The wavelet transform is based on Fourier transform. WT decomposes a signal into superposition of multiple signals based on the scale and wavelet function. Among them, the coefficients based on scale function are the low-frequency components which show the approximate contour information of original signal. We also call them approximate coefficients. The coefficients based on wavelet function are the high-frequency components called detail coefficients. They show the detail part and the degree of complexity. Various noises mixed in the signal are contained in the high-frequency components. The number of expansion coefficients of the useful signal is
smaller and the amplitude is larger, while noise signal is opposite. Therefore, we can set a threshold for all detail components, discard the coefficients that smaller than threshold, and retain the ones that larger than threshold; finally with the coefficients of the approximate components, obtain the processed signal. To sum up, we can use three steps to finish the process of wavelet threshold denoising.

1. Wavelet Decomposition. Select wavelet base; determine the number of decomposition layer; set an appropriate threshold.
2. Detail Coefficient Quantization. Select a threshold function to handle high-frequency components.
3. Wavelet Reconstruction. It is the inverse of wavelet decomposition.

From above steps, we can know selecting an appropriate threshold function is vital for wavelet denoising.

The denoised signal obtained by the traditional soft threshold function is odd function and has a large SNR, but the discontinuity at the segmented point may cause excessive oscillation of the denoised signal. Therefore, an improved threshold function is proposed to make up for the shortcomings of both.

The express of novel flexible threshold function proposed by this paper is as follows:

\[ d_{pq} = g(d_{pq}) \]

where \( d_{pq} \) is the \( q \)-th wavelet coefficient of the \( p \)-th layer of the high-frequency components; \( d_{pq} \) is the wavelet coefficient of denoised signal, \( \lambda \) is the threshold; \( k \) is a positive number, and its value controls the speed of tending towards hard threshold function, usually \( k = 1 \); and \( \alpha \) and \( m \) are positive integers, which are adjustable parameters.

From the expression, we can know that the flexible threshold function has three characteristics:

1. The flexible threshold function is odd function

2. The flexible threshold function is continuous at the segment point. It can be inferred from:

\[
\lim_{d_{pq} \to -\lambda} \hat{d}_{pq} = \lim_{d_{pq} \to \lambda} \hat{d}_{pq} = \hat{d}_{pq} \bigg|_{d_{pq}=\lambda} = 0, \tag{2}
\]

\[
\lim_{d_{pq} \to -\lambda} \hat{d}_{pq} = \lim_{d_{pq} \to \lambda} \hat{d}_{pq} = \hat{d}_{pq} \bigg|_{d_{pq}=-\lambda} = 0 \tag{3}
\]

The flexible threshold function is continuous on \( R \), which can avoid unnecessary oscillations in the reconstructed signal. It overcomes shortcomings of hard threshold function.

When \( d_{pq} > \lambda \), the flexible threshold function greatly decreases the deviation between \( \hat{d}_{pq} \) and \( d_{pq} \).

It can be inferred from

\[
\lim_{d_{pq} \to \infty} \hat{d}_{pq} = \infty = d_{pq}. \tag{4}
\]

This solves the problem of denoised signal distortion caused by a constant deviation of soft threshold function.

To observe the advantages of flexible threshold function more clearly, compare it with the traditional and reference improved threshold functions. Set the threshold to 0.3. For observing more clearly, only draw the comparison chart of first quadrant, as shown in Figure 1.

Observing Figure 1, the flexible threshold function has the following characteristics:

1. This threshold function is not directly set to zero when \( d_{pq} < \lambda \). Part of useful information can be retained.
2. This threshold function has the smallest deviation between \( \hat{d}_{pq} \) and \( d_{pq} \) compared with selected references. As \( d_{pq} \) increases, the flexible threshold function gradually approaches hard threshold function, and at the fastest speed, it tends to hard threshold function. Increasing \( k \) can accelerate the speed at which the flexible threshold function approaches the hard threshold.

Through theoretical analysis, these verify that the flexible threshold function has certain advantages.

2.3. Selection of Threshold, Decomposition Layer, and Wavelet Base. In the processing of using wavelet transform to denoise, the reasonable setting of threshold will directly affect the quality of signal denoising. Setting threshold too large will cause the loss of the expansion coefficients of some useful signals in the high-frequency components and cause distortion of the reconstructed signal. Setting threshold too small will cause some expansion coefficients of noise to remain and result in low signal denoising accuracy.

The traditional general threshold setting expression is as follows [38]:

\[
\lambda = \sigma \sqrt{2 \ln N}, \tag{5}
\]

where \( N \) is the number of coefficients and \( \sigma \) is the standard deviation of noise signal.
In addition to general threshold setting method, the commonly used threshold setting methods include “rigsure” based on the unbiased risk principle, “heursure” based on the heuristic principle, and “minimaxi” based on the maximum-min principle. These three methods can all be set directly in the threshold selection rule when using the “wden” function in MATLAB. Although their focus is different, the threshold is not adaptable with the number of decomposition layers. Mallat and Hwang [39] have proved that the coefficient corresponding to noise decreases with the increase of the number of decomposition layers. When using the traditional general threshold denoising, the original information of the signal is often removed, resulting in the loss of the original information of the signal, making the denoising effect poor. Therefore, many scholars put forward improved methods to adapt to different decomposition levels on the basis of equation (5), among which the best method is to multiply a coefficient which is related with the number of layers on the basis of (5) [40]. The setting expression is as follows:

$$\lambda = \sigma \sqrt{2 \ln (N)/\ln (p+1)},$$  \hspace{1cm} (6)

where $p$ is the number of layers of current wavelet decomposition, $N$ is the of wavelet coefficients of current decomposition layer, and $\sigma$ is the standard deviation of noise. If noise is unknown, we usually use (7) to calculate its value.

$$\sigma = \frac{\text{median}(d_p)}{0.6745}.$$  \hspace{1cm} (7)

The number of decomposition layers is also a very important factor when performing wavelet decomposition on a signal. With its increase, the number of detail component coefficients will increase, and detail information of original signal can be better displayed, and the accuracy of signal denoising will be improved. However, when the number of decomposition layers is increased to the optimal, increasing its value will not only reduce the effect of signal denoising but also increase the amount of calculation, which will slow down the signal processing speed. Many scholars use different evaluation indexes of denoised signal to find the optimal decomposition level. For example, Li et al. [41] used the sum of 4 evaluation indexes to determine the optimal level of wavelet transform. However, these methods need to be repeatedly tested and verified based on experimental results. Chen et al. [42] obtained a conclusion that the energy of the wavelet is positively correlated with the complexity of the signal; Yu et al. [43] found the optimal decomposition level by finding the maximum energy increment. The most ideal weighing signal should be a straight line when it is stable, so the optimal decomposition layer can be found by looking for the smallest energy increment.

The energy of the $p$-th level detail coefficients:

$$E_p = \sum_{k=1}^{m} d_{p,q}^2.$$  \hspace{1cm} (8)

The energy increment of the $p$-th level detail coefficients:

$$\Delta E_p = E_{p+1} - E_p.$$  \hspace{1cm} (9)
Calculate the energy increment for each layer and then find the smallest energy increment. This paper analyzes the raw data of [44] at 8-axis and 10-axis low speed to judge whether this method is effective. The energy increment of each layer is shown in Table 1. Observing Table 1, we can see that the energy increment of the fifth layer is the smallest. Therefore, the decomposition level should be selected as 5 which is consistent with the decomposition level selected in [44].

The selection principle of wavelet basis mainly considers five factors: orthogonality, regularity, vanishing moment, tight support interval, and symmetry. The five factors interact and restrict each other. Different wavelet bases have different emphases among various factors [45]. Haar wavelet has the shortest support length, only the first-order vanishing moment, and has poor local analysis ability in frequency domain, but it is the simplest. Db-N series are mostly compactly supported, continuous, and orthogonal wavelets, but their smoothness is poor. Coif-N has better symmetry than db series and has 2N order vanishing moment, which can reduce the phase shift during reconstruction but have poor tight support. Sym-N is also an improvement of Db-N series. They are approximately symmetric, which can reduce the phase shift during reconstruction [46]. So in this paper, we only consider four widely used compactly supported orthogonal wavelets: “haar,” “db,” “sym,” and “coif” series. The selection of wavelet bases can be selected by the evaluation indexes of noise reduction signals in subsequent experiments.

Based on the above analysis, the specific flow chart of the method combining EEMD with improved wavelet threshold function for processing weighing signal with noise is shown in Figure 2. The original signal is preprocessed by EEMD to get several IMFs and rec. Then, an improved threshold function is used to complete the wavelet denoising of all IMFs to get reconstructed IMFs, finally adding the rec to obtain the denoised signal.

### 3. Evaluation Index

The more mature evaluation methods in the field of signal analysis include signal-to-noise ratio (SNR) and mean square error (MSE). Among them, SNR is the ratio of the energy of original signal to the energy of noise signal; MSE is the square root of variance between original signal and denoised signal. The higher the SNR, the better the denoising effect; the smaller the MSE, the better the denoised signal. So this paper select SNR and MSE as the main evaluation indicators. Besides these two indicators, many papers choose smoothness (R) as an evaluation indicator. R is the ratio of the variance of denoised signal to the variance of original signal. The smaller the value of R, the smoother the denoised signal. However, when R alone is used as the evaluation index, mistakes are often made. Therefore, this paper uses smoothness (R) as an auxiliary evaluation index.

Sometimes, different single evaluation index will get different results, when using them to evaluate the effect of signal denoising. Therefore, if a comprehensive index that reasonably combines multiple single evaluation indicators from different perspectives can be put forward, the effect of signal denoising can be comprehensively evaluated. SNR, MSE, R, and cross-correlation coefficient (ρ) are the most commonly used single evaluation indexes. Among them, SNR reflects the effect of noise on the original signal; MSE shows the overall deviation information of signal; R reflects the local characteristics of the signal; the information reflected by ρ is basically same [47]. Similar to literature [48], a multimechanism model was established. In consequence, we select SNR, MSE, and R to form a comprehensive index (H). The coefficient of variation is a dimensionless

<table>
<thead>
<tr>
<th>Axis</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>8X</td>
<td>0.3159</td>
<td>2.0812</td>
<td>0.0628</td>
<td>6.6245</td>
<td>-6.5094</td>
<td>-0.6848</td>
<td>-0.7281</td>
<td>2.0635</td>
<td>1.9878</td>
</tr>
<tr>
<td>10X</td>
<td>0.2462</td>
<td>1.1112</td>
<td>-0.1537</td>
<td>6.1573</td>
<td>-3.6586</td>
<td>-2.6136</td>
<td>-0.4973</td>
<td>1.4405</td>
<td>3.4303</td>
</tr>
</tbody>
</table>

Table 1: The results of calculating the energy increment of 1-9 layers.

![Figure 2: EEMD combined with improved wavelet threshold function.](image_url)
quantity, which is obtained by dividing the standard deviation of an index value by its mean value. It can objectively reflect the degree of variation of an index value [49]. So, coefficient of variation is selected to calculate the weight of different single evaluation index. The specific calculation steps of the comprehensive index \((H)\) are as follows:

First, normalize the SNR, MSE, and \(R\) with different value ranges to obtain the dimensionless values PSNR, PMSE, and PR. Their value ranges are \([0, 1]\), where 1 is the best and 0 is the worst.

The calculation formula of PSNR:

\[
PSNR = \frac{\text{SNR}-\min (\text{SNR})}{\max (\text{SNR})-\min (\text{SNR})}. \tag{10}
\]

The calculation formula of PMSE and PR:

\[
\text{PI}_n = \frac{\max (I_n) - I_n}{\max (I_n) - \min (I_n)}. \tag{11}
\]

Then, calculate the coefficients of variation of each indicator:

\[
\text{CVPI}_n = \frac{\sigma_{\text{PI}_n}}{\mu_{\text{PI}_n}}, \tag{12}
\]

where \(\sigma_{\text{PI}_n}\) is the standard deviation of the \(n\)-th indicator and \(\mu_{\text{PI}_n}\) is the mean of the \(n\)-th indicator.

Calculate the weight of each indicator:

\[
W_{\text{PX}_n} = \frac{\text{CVPI}_n}{\sum_{n=1}^{3} \text{CVPI}_n}. \tag{13}
\]

Finally, three indicators are linearly combined to get the comprehensive indicator:

\[
H = \sum_{n=1}^{3} W_{\text{PX}_n} \times \text{PX}_n. \tag{14}
\]

From the above steps, we can know that the higher the \(H\), the better the denoising effect.

For verifying the validity of the comprehensive index \((H)\), a signal composed of three sinusoidal signals with different frequencies and a low-frequency trend signal is now constructed.

\[
S = 5 \sin \left(\frac{2\pi t}{1000}\right) + 4 \sin \left(\frac{2\pi t}{600}\right) + 3 \sin \left(\frac{2\pi t}{75}\right) + 0.002t. \tag{15}
\]

Add SNR = 10 noise to \(S\) to obtain noisy signal, and perform wavelet denoising on the noisy signal. We choose the number of decomposition levels to be 5, and in the hard threshold function, select (6) as the threshold setting method. Use different evaluation indicators and comprehensive index \((H)\) in “sym” series to select the optimal wavelet base. The evaluation indexes of denoised signals obtained under different wavelet bases are shown in Table 2.

From Table 2, we can know that the SNR is the largest, the MSE is the smallest, and \(R\) is smaller when wavelet base is sym9. That is, the traditional evaluation indexes consider sym9 to be the optimal wavelet base in “sym” series. When the wavelet base is sym9, \(H\) is the largest; that is, sym9 is the optimal wavelet base under the comprehensive evaluation index \((H)\). We are able to know that the result obtained by the comprehensive index \((H)\) is consistent with the result obtained by the traditional evaluation indicators, so the comprehensive index \((H)\) proposed in this paper is effective.

### 4. Simulation Analysis

The most ideal weighing signal should be constant. Set its value to 300 and use “awgn” function of MATLAB; add noise of SNR = 5, 10, 15 to it to simulate noisy signals obtained from the load cell. The specific waveforms are shown in Figure 3.

The specific processing of the three analog noise signals is as follows:

Firstly, the EEMD decomposition of the three noise signals is performed to obtain the respective IMFs. Where the ensemble number is 200, the ratio of the standard deviation of the added noise and that of noisy signal is 0.2.
Secondly, select the optimal wavelet bases. The evaluation indexes of denoised signals when using harr, db-N, coif-N, and sym-N wavelet bases are obtained under the conditions of hard threshold function, threshold, and decomposition layer set according to part 2.3, respectively, and the corresponding $H$-values are calculated. The wavelet basis with the largest $H$ value is optimal.

Finally, the evaluation indexes of denoised waveforms of the flexible threshold function with different parameters and other comparison methods are obtained under the conditions of optimal wavelet, threshold value set in Section 2.3, and decomposition layers. Then, parameter values are determined to obtain the final denoised signal.

Figure 4 shows the waveforms of MF1-IMF7 and residual component obtained by performing EEMD decomposition on the normalized noisy signals. Observing Figure 4, the noise mainly exists in the first three high-
Then, calculate residual component to obtain the denoised waveforms reconstructed IMF1-IMF7 components; finally, process the IMF1-IMF7 components to obtain the minimum energy method. Then, use and the number of decomposition layers determined by (6) for threshold setting; choose the hard threshold function evaluation indexes when using wavelet denoising. See formula (1), we can know that there are 3 frequency components. According to the EEMD denoised principle, the first three components should be removed, and the remaining components form the reconstructed signal. However, this could lose part of useful information. Therefore, all IMFs are denoised by wavelet, and they all participate in the formation of the recombined signal.

The wavelet base needs to be determined through evaluation indexes when using wavelet denoising. See formula (6) for threshold setting; choose hard threshold function and the number of decomposition layers determined by the minimum energy method. Then, use “haar” wavelet, “db” series, “coif” series, and “sym” series wavelet bases to process the IMF1-IMF7 components to obtain different reconstructed IMF1-IMF7 components; finally, add the residual component to obtain the denoised waveforms under different wavelet bases. Calculate SNR, MSE, and R of denoised signals obtained on different wavelet bases. Then, calculate H of denoised signals obtained on different wavelet bases. When H is the largest, the denoising effect is considered to be the best, and the wavelet base is the optimal wavelet base. The H for the best wavelet bases denoising waveforms obtained by the best wavelet base in each series are shown in Table 3. So, select sym5 when SNR = 5; select db5 when SNR = 10; select sym9 when SNR = 15.

Observing equation (1), we can know that there are 3 adjustable parameters in the flexible threshold function. The convergence speed would further increase by increasing the value of k. The appropriate values of α and m can save some useful information when the coefficient is less than the threshold. It can be seen from equation (1) that when the value of m is large, α must be large to achieve the desired effect, so we take m = 1, 2, 3. Obtain the relationship between evaluation indexes and parameter α when m = 1, 2, and 3, respectively. Then, compare with the results obtained from other threshold functions; the specific information is shown in Figures 5 and 6.

Observing Figure 5, we can see that when m is constant, within a certain range, SNR increases with the increase of value of α and MSE decreases with the increase of α, which means that the denoised signal is getting better and better. Meantime, the rate of change of SNR and MSE gradually slows down. When α increases to a certain value, SNR reaches the maximum and MSE reaches the minimum. The indexes of SNR and MSE think that the denoised signal at this time is the best. The auxiliary index R increases continuously with the increase of α in the full range. This thinks that the denoised signal is getting worse. Comparing the horizontal and vertical coordinates of (a), (b), and (c) and the graphs trend, it can be seen that m = 2 and 3 are magnifications to the details of m = 1. Therefore, the desired denoising effect can be achieved by changing variable parameters of flexible threshold function; that is, the flexible threshold function has better adaptability. R is relatively low when α is small, but R is high when α is large.

From Figure 6, it can be found that the H of the flexible threshold function is basically larger than others obtained by other threshold functions. Therefore, the comprehensive index (H) thinks that the flexible threshold function has better denoising performance.

We require the processed signal to ensure a certain degree of smoothness. Therefore, the R obtained by the threshold function when the parameters are determined must be less than the R under the hard threshold function. So, we use variations of SNR, MSE, and R rather than to further determine the value of α. The specific process is as follows:
First, calculate variations of SNR, MSE, and $R$:

$$b_{X_{ik}} = |X_{i,k+1} - X_k|.$$  \hspace{1cm} (16)

Second, variations are normalized:

$$bb_{X_i} = \frac{b_{X_{ik}} - \min (b_{X_i})}{\max (b_{X_i}) - \min (b_{X_i})}.$$  \hspace{1cm} (17)

Finally, linear combination obtains parameter determination index ($L$):

$$L = bb_{\text{SNR}} + bb_{\text{MSE}} - bb_{R}.$$  \hspace{1cm} (18)

We select $m = 2$ and calculate the corresponding $L$. First, take the value of $\alpha$ when $L$ is about 1. If the SNR and MSE of the denoised signal do not meet the requirements at this time, we can reduce $L$ at a rate of 0.1 until the requirements are met. If the overall $L$ value is very small, just take the largest $L$ value. Denormalize the processed data in different threshold function and calculate the corresponding indicators. The specific information is shown in Table 4.

![Figure 6: The $H$ corresponding to different $\alpha$. (a) $m = 1$, (b) $m = 2$, and (c) $m = 3$.](image)

<table>
<thead>
<tr>
<th>Noise signal</th>
<th>Threshold function</th>
<th>SNR</th>
<th>MSE</th>
<th>$R$</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR = 5</td>
<td>Soft</td>
<td>56.3176</td>
<td>0.4584</td>
<td>0.0843</td>
<td>0.3142</td>
</tr>
<tr>
<td></td>
<td>Hard</td>
<td>56.8883</td>
<td>0.4292</td>
<td>0.1057</td>
<td>0.5910</td>
</tr>
<tr>
<td></td>
<td>Reference [23]</td>
<td>56.4272</td>
<td>0.4526</td>
<td>0.0852</td>
<td>0.4164</td>
</tr>
<tr>
<td></td>
<td>Reference [26]</td>
<td>56.7896</td>
<td>0.4341</td>
<td>0.0936</td>
<td>0.6680</td>
</tr>
<tr>
<td></td>
<td>Reference [27]</td>
<td>56.8047</td>
<td>0.4334</td>
<td>0.0940</td>
<td>0.6770</td>
</tr>
<tr>
<td>$m = 2, \alpha = 2$</td>
<td></td>
<td>56.9815</td>
<td>0.4246</td>
<td>0.0966</td>
<td>0.8194</td>
</tr>
<tr>
<td>SNR = 10</td>
<td>Soft</td>
<td>60.9984</td>
<td>0.2674</td>
<td>0.0906</td>
<td>0.2854</td>
</tr>
<tr>
<td></td>
<td>Hard</td>
<td>61.2953</td>
<td>0.2584</td>
<td>0.1116</td>
<td>0.2560</td>
</tr>
<tr>
<td></td>
<td>Reference [23]</td>
<td>61.0600</td>
<td>0.2655</td>
<td>0.0920</td>
<td>0.3200</td>
</tr>
<tr>
<td></td>
<td>Reference [26]</td>
<td>61.2349</td>
<td>0.2602</td>
<td>0.0999</td>
<td>0.3634</td>
</tr>
<tr>
<td></td>
<td>Reference [27]</td>
<td>61.2462</td>
<td>0.2599</td>
<td>0.1007</td>
<td>0.3617</td>
</tr>
<tr>
<td>$m = 2, \alpha = 7$</td>
<td></td>
<td>61.8412</td>
<td>0.2427</td>
<td>0.1109</td>
<td>0.7241</td>
</tr>
<tr>
<td>SNR = 15</td>
<td>Soft</td>
<td>66.4437</td>
<td>0.1429</td>
<td>0.0990</td>
<td>0.3962</td>
</tr>
<tr>
<td></td>
<td>Hard</td>
<td>66.9319</td>
<td>0.1351</td>
<td>0.1313</td>
<td>0.5748</td>
</tr>
<tr>
<td></td>
<td>Reference [23]</td>
<td>66.5576</td>
<td>0.1410</td>
<td>0.1022</td>
<td>0.4940</td>
</tr>
<tr>
<td></td>
<td>Reference [26]</td>
<td>66.8472</td>
<td>0.1364</td>
<td>0.1210</td>
<td>0.6034</td>
</tr>
<tr>
<td></td>
<td>Reference [27]</td>
<td>66.8583</td>
<td>0.1362</td>
<td>0.1207</td>
<td>0.6209</td>
</tr>
<tr>
<td>$m = 2, \alpha = 2$</td>
<td></td>
<td>66.9561</td>
<td>0.1347</td>
<td>0.1242</td>
<td>0.6909</td>
</tr>
</tbody>
</table>

Table 4: Denoising indexes of different threshold functions.
Observing Table 4, we can find the denoised signal that using the flexible threshold function has the largest SNR, the smallest MSE, the moderate R, and the largest H when adding different levels of noise. This verifies that the flexible threshold function is better than others.

5. Weighing Experiment Verification

In order to verify whether the proposed method has a good denoising effect on the actual weighing signal and whether it can reduce the random error in the weighing result, this section first designs the hardware and software structure of the three-point weighing system and obtains the corresponding actual weighing signals. Then, they are processed for noise reduction. This paper describes one noisy actual weighing signal processing in detail. In order to better verify the superiority of the method, it is compared with other methods in this process. Finally, the correlation statistics of the quality values obtained before and after noise removal are calculated and compared.

5.1. Design of the Experiment. In order to verify the effectiveness of the noise reduction method proposed in this paper, a mass measurement system is built to obtain the mass signal
from the weighing sensor output. The system structure diagram is shown in Figure 7(a). The weighing sensor selected for the system is the CZL-A series sensor that fully meets the requirements of Class C3 of R60 International and Class C3 of GB/T7551-2008. It converts the mass of the object to be measured into an electrical signal for transmission. The weighing platform for the object to be measured is mounted on three load cells located on the same circumference and distributed 120 degrees apart. When the sensor is under pressure, the resistance value of the Wheatstone bridge inside the sensor changes and outputs the corresponding mv-level voltage signal, which is then amplified, filtered, and converted to digital-to-analog by the HX711 chip-based circuit and then passed to ATmega64, the master of lower computer (LC) which will judge whether the data is valid. If the data is valid, it will be sent to the upper computer (UC) through serial communication to store and display the data and vice versa to reacquire the signal. The specific workflow is shown in Figure 7(b). When LC completes the initialization and receives the command from UC, it will perform signal acquisition weight.

5.2. The Processing of a Group of Actual Weighing Signal. The original digital signal of weighing sensor was randomly acquired and imported into MATLAB. Firstly, EEMD decomposition is performed on the normalized signal data, and waveform of IMFs1-7 and rec is obtained as shown in Figure 8. Then, all IMF components are denoised by WT using different threshold functions. Finally, the reconstructed IMFs plus rec to get different de-noised signals. Calculate and compare the SNR, MSE, and $R$ of different denoised waveforms, and judge whether the flexible threshold function is better than others.

The $H$ values of denoising signals obtained at the same threshold, decomposition level, and threshold function with different wavelet bases are calculated. The specific process is similar to Section 4. Then, select the wavelet base with maximum $H$. Table 5 shows the $H$ for the best wavelet bases denoising waveforms obtained by the best wavelet base in each series. Therefore, the wavelet base is selected as sym6.

For the IMF1-IMF7 components in Figure 8, different threshold functions of the traditional and references are used to perform wavelet denoising; then, the SNR, MSE, and $R$ of

<table>
<thead>
<tr>
<th>Threshold function</th>
<th>SNR</th>
<th>MSE</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>13.1423</td>
<td>0.1020</td>
<td>0.1612</td>
</tr>
<tr>
<td>Hard</td>
<td>13.4079</td>
<td>0.0990</td>
<td>0.1699</td>
</tr>
<tr>
<td>Reference [23]</td>
<td>13.1924</td>
<td>0.1014</td>
<td>0.1620</td>
</tr>
<tr>
<td>Reference [26]</td>
<td>13.3398</td>
<td>0.0997</td>
<td>0.1662</td>
</tr>
<tr>
<td>Reference [27]</td>
<td>13.3552</td>
<td>0.0996</td>
<td>0.1658</td>
</tr>
</tbody>
</table>

Figure 9: The SNR, MSE, and $R$ corresponding to different $\alpha$. (a) $m = 1$, (b) $m = 2$, and (c) $m = 3$. 

Table 6: The SNR, MSE, and $R$ of denoised signals that use different threshold functions of actual weighing signal.
the respective denoised signals are calculated. The specific values are shown in Table 6.

Observing Table 6. The SNR and $R$ are the smallest and the MSE is the largest when using soft threshold function. That is, the denoised waveform obtained by soft threshold function is smoothest but has distortion. The SNR and $R$ are the largest and the MSE is the smallest when using hard threshold function. That is, the accuracy of the denoised waveform obtained by the hard threshold is higher, but there is oscillation. The SNR, MSE, and $R$ of denoised signal using improved threshold functions in the references literature are all in the middle position. That is, improved threshold functions improve the distortion of soft threshold and the oscillation of hard threshold. Different improvement methods have different focuses. The smoothness of the denoised signal is greatly improved when use the threshold function in [23]; the SNR and MSE are improved greatly in [27], but these three improved threshold functions are not adjustable and have poor adaptability.

Calculate evaluating indexes of the flexible threshold function to obtain the relationship between their and parameter $\alpha$ when $m = 1, 2,$ and $3,$ respectively, and compare with the results obtained in Table 6; the specific information is shown in Figures 9 and 10.

**Figure 10:** The $H$ corresponding to different $\alpha$. (a) $m = 1$, (b) $m = 2$, and (c) $m = 3$.

**Table 7:** The partial denoising information used in the flexible threshold function.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$\alpha$</th>
<th>SNR</th>
<th>MSE</th>
<th>$R$</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>13.4338</td>
<td>0.0687</td>
<td>0.1667</td>
<td>0.3992</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>13.6391</td>
<td>0.0964</td>
<td>0.1692</td>
<td>0.4335</td>
</tr>
<tr>
<td></td>
<td>77</td>
<td>17.3387</td>
<td>0.0629</td>
<td>0.7889</td>
<td>0.8529</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>13.3831</td>
<td>0.0992</td>
<td>0.1663</td>
<td>0.3786</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>13.3669</td>
<td>0.0957</td>
<td>0.1692</td>
<td>0.4275</td>
</tr>
<tr>
<td></td>
<td>318</td>
<td>17.7333</td>
<td>0.0601</td>
<td>0.8049</td>
<td>0.7821</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>17.3706</td>
<td>0.0994</td>
<td>0.1663</td>
<td>0.3645</td>
</tr>
<tr>
<td>3</td>
<td>72</td>
<td>17.1781</td>
<td>0.0955</td>
<td>0.1698</td>
<td>0.4194</td>
</tr>
<tr>
<td></td>
<td>1133</td>
<td>17.7204</td>
<td>0.0602</td>
<td>0.8033</td>
<td>0.7057</td>
</tr>
</tbody>
</table>

**Table 8:** The denoising indexes of different threshold functions of actual weighing signal.

<table>
<thead>
<tr>
<th>Threshold function</th>
<th>SNR</th>
<th>MSE</th>
<th>$R$</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>105.1390</td>
<td>46.9345</td>
<td>0.1612</td>
<td>0.3332</td>
</tr>
<tr>
<td>Hard</td>
<td>105.4045</td>
<td>45.5213</td>
<td>0.1699</td>
<td>0.6045</td>
</tr>
<tr>
<td>Reference [23]</td>
<td>105.1891</td>
<td>46.6646</td>
<td>0.1620</td>
<td>0.4175</td>
</tr>
<tr>
<td>Reference [26]</td>
<td>105.3364</td>
<td>45.8798</td>
<td>0.1662</td>
<td>0.5927</td>
</tr>
<tr>
<td>Reference [27]</td>
<td>105.3518</td>
<td>45.7982</td>
<td>0.1658</td>
<td>0.6430</td>
</tr>
<tr>
<td>$m = 2, \alpha = 4$</td>
<td>105.4317</td>
<td>45.3793</td>
<td>0.1666</td>
<td>0.7932</td>
</tr>
</tbody>
</table>
For understanding more specific information, we extract some information of some key points in Figures 9 and 10. The specific values are shown in Table 7.

Figures 9 and 10 and Table 7 draw the same conclusions as in part 4. The flexible threshold function is more effective than others in denoising process of actual weighing signal.

Use L as the indicator to select the parameters of the novel threshold function, and denormalize denoised data under different threshold functions; the various indicators are shown in Table 8.

From Table 8, we can know that, compared to other threshold functions, the flexible threshold function has the largest SNR and H, the smallest MSE, and the appropriate R. Compared with other improved methods, the proposed method has obvious advantages in dealing with noisy weighing signals. In addition, the parameters can also be adjusted to achieve the desired denoising effect, which has good adaptability. Figure 11 shows the specific signal waveforms after denoising that use different threshold functions.

5.3 Weighing Experiment. Collect the signals of the three sensors when platform in no-load, measuring standard object (1.8 kg), and measuring the measured object. Use the EEMD combined with the flexible threshold function method in this paper to process the 9 original signals. For verifying the validity of the flexible threshold function again, this paper changes threshold function to traditional threshold functions, and the above operation is repeated. The SNR, MSE, and R of the denoised signals under different conditions are shown in Tables 9–11.

![Figure 11: The waveform graphs of denoised signals.](image)

**Table 9: The SNR, MSE, and R of denoised signals when platform in no-load.**

<table>
<thead>
<tr>
<th>Threshold function</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SNR</td>
<td>MSE</td>
<td>R</td>
</tr>
<tr>
<td>Soft</td>
<td>12.7807</td>
<td>0.1284</td>
<td>0.0147</td>
</tr>
<tr>
<td>Hard</td>
<td>13.8761</td>
<td>0.1132</td>
<td>0.1074</td>
</tr>
<tr>
<td>Ours</td>
<td>14.0033</td>
<td>0.1115</td>
<td>0.0810</td>
</tr>
</tbody>
</table>

**Table 10: The SNR, MSE, and R of denoised signals when platform in weighing standard object.**

<table>
<thead>
<tr>
<th>Threshold function</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SNR</td>
<td>MSE</td>
<td>R</td>
</tr>
<tr>
<td>Soft</td>
<td>11.0699</td>
<td>0.1328</td>
<td>0.0934</td>
</tr>
<tr>
<td>Hard</td>
<td>11.4096</td>
<td>0.1277</td>
<td>0.1053</td>
</tr>
<tr>
<td>Ours</td>
<td>11.5347</td>
<td>0.1259</td>
<td>0.1020</td>
</tr>
</tbody>
</table>

**Table 11: The SNR, MSE, and R of denoised signals when platform in weighing measured object.**

<table>
<thead>
<tr>
<th>Threshold function</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SNR</td>
<td>MSE</td>
<td>R</td>
</tr>
<tr>
<td>Soft</td>
<td>14.1197</td>
<td>0.0985</td>
<td>0.1169</td>
</tr>
<tr>
<td>Hard</td>
<td>14.3827</td>
<td>0.0955</td>
<td>0.1564</td>
</tr>
<tr>
<td>Ours</td>
<td>14.4844</td>
<td>0.0944</td>
<td>0.1357</td>
</tr>
</tbody>
</table>
Figure 12: The waveform graphs of weighing signals after denoising: (a) measured object, (b) standard object (1.8 kg), and (c) no-load.

Figure 13: The relative errors before and after signal denoising.
From Tables 9–11, we can know that all denoised weighing signals obtained by the flexible threshold function have the largest SNR, the smallest MSE, and the moderate R. The weighing signals processing by the method proposed in this paper have good denoised accuracy.

From the principle of three-point weighing [50], we can get

\[ m = m_1 + m_2 + m_3. \]  \hspace{1cm} (19)

So, the corresponding weight signal waveforms can be obtained by adding the three sensor signals under different conditions. Figure 12 is the comparison waveforms before and after signal denoising.

Calculate the mass of the measured object by analyzing original weighing signals and denoised weighing signals. First is to analyze original data. The data value of the standard object minus the data value of unloaded and then divided by the mass of the standard object to obtain the slope of the sensor. The data value of the measured object minus the data value when the weighing platform is empty and then divided by the slope of the sensor; then, 345 corresponding measured object mass values can be obtained. Take its average value as the benchmark quality of the measured object, and draw a graph of the relative error of each data point. Then, perform the above operations on the signals after denoising to get the relative error map of denoised signals. Finally, Figure 13 shows the relative errors in the two cases.

Some specific values are shown in Table 12.

<table>
<thead>
<tr>
<th></th>
<th>Original signal</th>
<th>Denoised signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark mass of measured object (kg)</td>
<td>3.4313</td>
<td>3.4313</td>
</tr>
<tr>
<td>Standard deviation of quality</td>
<td>0.0025</td>
<td>0.0015</td>
</tr>
<tr>
<td>Maximum relative error (%)</td>
<td>0.21070</td>
<td>0.1703</td>
</tr>
<tr>
<td>Average relative error (%)</td>
<td>0.0570</td>
<td>0.0344</td>
</tr>
<tr>
<td>Standard deviation of relative error</td>
<td>0.0453</td>
<td>0.0283</td>
</tr>
</tbody>
</table>

6. Conclusion

Aiming at the digital weighing signal with noise in quality characteristic parameter measurement equipment, this paper proposes a signal denoising method combining EEMD and a novel flexible threshold function. In order to overcome the discontinuity of hard threshold and reduce the deviation of soft threshold in the process of wavelet threshold denoising, this paper proposed a novel flexible threshold function. It is not only continuous on R but also the quantized coefficients \( \tilde{d}_{pq} \) can quickly converge to the actual values \( d_{pq} \) when coefficients are greater than threshold. The nonzero operation can effectively retain the useful signal when coefficients are less than threshold. The inclusion of adjustable parameters makes it adaptable. The denoised weighing signal of using the novel flexible threshold function has the largest SNR and \( H \), the smallest MSE, and appropriate \( R \) by comparing with others in simulation experiment. This validates its effectiveness. In the actual weighing experiment, the standard deviation of quality values is reduced by 40%; the maximum relative error of denoised signal was reduced by 21.52%; the average relative error of denoised signal was reduced by 39.65%, and the standard deviation of the relative error was reduced by 37.53%. This proved that the weighing signal denoising method effectively reduces the random error in result. In the future, this paper will conduct further research on the influence of the signal noise reduction method on the indirect mass characteristic parameters such as the center of mass and the moment of inertia.

**Abbreviations**

- **EEMD**: Ensemble empirical mode decomposition
- **IMFs**: Intrinsic mode functions
- **rec**: Residual components
- **SNR**: Signal-to-noise ratio
- **MSE**: Mean square error
- **R**: Smoothness
- **H**: The comprehensive indicator
- **EMD**: Empirical mode decomposition
- **WT**: Wavelet transform
- **VMD**: Variational mode decomposition
- \( \tilde{d}_{pq} \): The \( q \)-th wavelet coefficient of the \( p \)-th layer of high-frequency components of signal
- \( \tilde{q}_{pq} \): The wavelet coefficient of reconstructed signal
- \( \lambda \): Threshold
- \( N \): The number of coefficients
- \( \sigma \): The standard deviation
- \( E_p \): The energy of the \( p \)-th level detail coefficients
- \( \Delta E_p \): The energy increment of the \( p \)-th level detail coefficients
- \( \rho \): Cross-correlation coefficient
- **PX**: The dimensionless values
$$\mu: \text{ The mean}$$
$$W: \text{ The weight}$$
$$b_X: \text{ The variations of } X$$
$$bb_X: \text{ Normalize the variations of } X$$
$$L: \text{ The parameter determination index}$$
$$UC: \text{ Upper computer}$$
$$LC: \text{ Lower computer}.$$

Data Availability

The data that support the findings of this study are available on request from the corresponding author.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Acknowledgments

This research was supported by the Open Foundation of Hubei Engineering Research Center for Safety Monitoring of New Energy and Power Grid Equipment (grant number HBSKF202125) and National Natural Science Foundation of China (grant number 51809097).

References


