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*Research Article*

# Sampled Characteristic Modeling and Forgetting Gradient Learning Algorithm for Robot Servo Systems

**Hongbo Bi, Dong Chen, Yanjuan Li, and Ting You**

*College of Electrical and Information Engineering, Quzhou University, Quzhou, Zhejiang 324000, China*

Correspondence should be addressed to Ting You; 8991548@qq.com

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Servo systems of robotic exhibit nonlinear coupling with multidimensional characteristics, which poses a challenge to existing modeling and identification techniques. According to a kind of robot servo system which runs repetitively operations over a prespecified finite time interval, a low-order sampling characteristic modeling method is derived in this work. Characteristic parameters are allowed to vary from both time axis and iteration one; the forgetting gradient learning algorithm is utilized to estimate characteristic parameters. Furthermore, the effectiveness of the proposed algorithms is proved via theoretical analyses and numerical simulations.

## 1. Introduction

As an important part of the industrial Internet system, robots are attracting more and more attention. Existing robot control methods mostly depend on models. Because of the dynamic characteristics of controlled objects, the complexity of control tasks and operating environment, it is often difficult to establish accurate mathematical models. Nonnegligible unmodeled dynamics have higher requirements on system models and the robust performance of closed-loop systems needs to be improved. In order to reduce the impact of unmodeled dynamics on the performance of control systems, most existing methods build high-order models of systems. Modeling and control technology for high-order systems is extremely complex and difficult to implement. The feature modeling method provides a predictable way to model complex systems. It does not need to establish a precise dynamic model of the system nor does it use a system reduction or linearization method. Instead, it expresses the model as a low-order time-varying difference equation, taking into account factors, such as system state, external environment, and control variables, and compresses the dynamic information into feature parameters. The Euler approximation is often used to discretize the continuous system, and the equivalent characteristic model [1–6] is given. The feature model can also be performed with exact discretization to give a sampling feature model of the system. Without limiting the sampling interval and control tasks, a feature model with rapidly changing (even abrupt) characteristic parameters may be obtained [7, 8].

Stochastic gradient algorithm and its derivative algorithm are popular for training network weights and avoid the matrix operation compared with the recurrence least squares algorithm and the small online computation, which has attracted much attention [9–17]. The consistent convergence results of the parameters are still available for the stochastic gradient algorithm under weaker incentives. The martingale theory and stochastic process theory are powerful tools to analyze the convergence of recurrence algorithms. From the published results, most methods are targeted for stationary systems, where the estimated parameters are stationary.

When dealing with time-varying systems, it is found that the stochastic gradient algorithms do not have the ability to track the time-varying parameters. In the field of time-varying system identification, more consideration is given to how to correct the recursive algorithms, that is, how to construct the correction algorithm of the recursive algorithms, so as to achieve effective tracking of the time-varying parameters and improve the convergence rate of the parameters. It is well known that if the time-varying parameter change law is not known, there is no consistent parameter estimation convergence [18–21]. Based on this conclusion, one abandons the attempt to achieve complete tracking of time-varying parameters and instead works on a correction algorithm of how to construct and analyze recursive algorithms, expecting to give lower upper bounds on the estimation error of time-varying parameters. Common correction algorithms such as weighted recursive algorithms include rectangular windows, exponential windows (forgetting factors), etc. For irreducible nonstationary processes, these results can more accurately evaluate the parameter estimation accuracy, which has important applications in practical engineering applications.

When the system parameters are repetitive and independent, the instant change system runs repeatedly on a finite interval, the system parameters change over time, but the next time, it repeats this change, the instant change parameter to a repetition invariant. "Recursive" algorithms are constructed along the repetition axis, and the learning algorithm gives its complete estimates regardless of parameter slow change, fast change, and even mutation [22, 23]. However, in practical situations, there are often system parameters that change not only along the time domain but also with the number of iterations, that is, iteration dependence. If the time-varying parameter changes with the number of iterations are unknown, then similar to the recurrence algorithm, the iterative learning identification algorithm based on the principle of repeated invariant cannot track the system parameters effectively. We consider introducing the forgetting factor in the iterative learning algorithm and propose the forgetting gradient learning algorithm to estimate the iteration-dependent time-dependent system parameters. Then, under the premise of satisfying the repeated continuous incentive conditions, the performance analysis of the proposed algorithm is used and the simulation examples are completed to illustrate the effectiveness of the forgetting gradient learning algorithm.

## 2. Description of the Problem

Actual robot servo systems are mostly continuous nonlinear time-varying coupling processes, considering the following systems:

$$y^{(n)}(t) = f\Big(y(t), \dot{y}(t), \cdots, y^{(n-1)}(t), u(t), \dot{u}(t), \cdots, u^{(m)}(t), t\Big) + \xi(t), \tag{1}$$

where $u$ stands for input and $y$ for output, and $\xi(t)$ represents external disturbances.

With the development of computer control technology, the analysis and synthesis process of the robot servo system needs a sampling model. The following discrete nonlinear time-varying coupling processes are also considered:

$$y(k+1) = f\big(y(k), y(k-1), \cdots, y(k-n), u(k), u(k-1), \cdots, u(k-m), k\big) + \xi(k). \tag{2}$$

Here, we consider relaxing the initial condition that $f(0,0,\cdots,0,t) \wedge f(0,0,\cdots,0,k)$ can be arbitrary values.

The goal of this paper is to build a sampling feature model for a robot servo system. For iteration-dependent and time-varying feature parameters in the model, the amnestic gradient learning algorithm is used to estimate them. To further validate the effectiveness of the proposed learning algorithm, the random process theory is used to analyze the model and a numerical example is given to verify the effectiveness of the proposed algorithm.

The rest of this paper is arranged as follows: the third part establishes the sampling characteristic model of the servo system, the fourth part puts forward the forgotten gradient learning algorithm, the fifth part gives the convergence analysis process of the learning algorithm, the sixth part verifies by numerical examples, and the seventh part gives the conclusion of this paper.

## 3. Characteristic Modeling for the Servo System

We consider the following discrete nonlinear system:

$$\begin{aligned} y(k+1) &= f\big(y(k), y(k-1), \cdots, y(k-n), u(k), u(k-1), \cdots, u(k-m), k\big) \\ &\quad - f\big(0, y(k-1), \cdots, y(k-n), u(k), u(k-1), \cdots, u(k-m), k\big) \\ &\quad + f\big(0, y(k-1), \cdots, y(k-n), u(k), u(k-1), \cdots, u(k-m), k\big) \\ &\quad - f(0,0,\cdots,0,k) + f(0,0,\cdots,0,k) + \xi(k) \\ &= \frac{\partial f(\cdot)}{\partial y(k)} y(k) + \frac{\partial f(\cdot)}{\partial y(k-1)} y(k-1) + \cdots \frac{\partial f(\cdot)}{\partial u(k-m)} u(k-m) \\ &= \varphi^{\mathrm{T}}(k)\theta(k) + v(k), \end{aligned} \tag{3}$$

where $\varphi(k) = [y(k), y(k-1), \cdots, u(k-m)], \theta(k) = [(\partial f(\cdot)/\partial y(k)), (\partial f(\cdot)/\partial y(k-1)), \cdots, (\partial f(\cdot)/\partial u(k-m))], v(k) = f(0, 0, \cdots, 0, k) + \xi(k)$; in the same way, we can obtain the following:

$$y^{(n)}(t) = \frac{\partial f(\cdot)}{\partial y(t)} y(t) + \frac{\partial f(\cdot)}{\partial \dot{y}(t)} \dot{y}(t) + \cdots \frac{\partial f(\cdot)}{\partial u^{(m)}(t)} u^{(m)}(t) + f(0, 0, \cdots, 0, t) + \xi(t), \tag{4}$$

which denotes $a_i(t) = (\partial f(\cdot)/\partial y^{(i)}(t)), b_i(t) = (\partial f(\cdot)/\partial y^{(j)}(t))$, where $i = 0, 1, \cdots, n$; $j = 0, 1, \cdots, m$, then

$$y^{(n)}(t) = \sum_{i=0}^{n} a_i(t) y^{(i)}(t) + \sum_{j=0}^{m} b_j(t) u^{(j)}(t) + f(0, 0, \cdots, 0, t) + \xi(t). \tag{5}$$

The characteristic modeling method establishes a characteristic model for a nonlinear higher-order system by compressing the characteristic information to the characteristic parameters. Obviously, the lower the order of the characteristic model, the faster the change of the feature parameters. We consider building a first-order sampling characteristic model for the controlled system [7], and we can see that

$$y(k+1) = a(k) y(k) + b(k) u(k) + v(k). \tag{6}$$

In general, we can also set up second-order and third-order sampling feature models of the controlled system, that is, formula (5) can be written into formula (3).

*3.1. Forgetting Gradient Learning Algorithm.* We consider the following single-input single-output (SISO) discrete time-varying system repetitively operates over a prespecified finite time interval:

$$A\left(q^{-1}, k, t\right) y_k(t) = B\left(q^{-1}, k, t\right) u_k(t) + v_k(t), \tag{7}$$

where $t = (0, 1, \cdots, N)$ denotes time domain, and $k = (1, 2, \cdots)$ denotes iteration domain. $u_k(t)$ and $y_k(t)$ represent the input and output of the system, respectively. $v_k(t)$ is the interference variable. $A(q^{-1}, k, t)$ and $B(q^{-1}, k, t)$ are time-varying polynomials of shift operators of SISO discrete systems, where $A(q^{-1}, k, t) = 1 + a_{1,k}(t) + a_{2,k}(t) + \cdots + a_{n_a,k}(t)$ and $B(q^{-1}, k, t) = b_{1,k}(t) + b_{2,k}(t) + \cdots + b_{n_b,k}(t)$. $a_{1,k}(t), a_{2,k}(t), \cdots, a_{n_a,k}(t)$ and $b_{1,k}(t), b_{2,k}(t), \cdots, b_{n_b,k}(t)$ are the unknown parameters. We denote $\varphi_k(t) = [-y_k(t-1), -y_k(t-2), \cdots, -y_k(t-n_a), u_k(t-1), u_k(t-2), \cdots, u_k(t-n_b)]^{\mathrm{T}}, \theta_k(t) = [a_{1,k}(t), a_{2,k}(t), \cdots, a_{n_a,k}(t), b_{1,k}(t), b_{2,k}(t), \cdots, b_{n_b,k}(t)]^T$, and $n = n_a + n_b$.

Equation (6) can be rewritten into the following regression model:

$$y_k(t) = \varphi_k^T(t) \theta_k(t) + v_k(t). \tag{8}$$

Similar to the stochastic gradient algorithm, a forgetting gradient learning algorithm for identifying iteratively dependent time-varying systems is presented:

$$\widehat{\theta}_k(t) = \widehat{\theta}_{k-1}(t) + \frac{\varphi_k(t)}{r_k(t)} \left[ y_k(t) - \varphi_k^T(t) \widehat{\theta}_{k-1}(t) \right], \tag{9}$$

$$r_k(t) = \lambda r_{k-1}(t) + \left\| \varphi_k(t) \right\|^2. \tag{10}$$

*3.2. Convergence Analysis of Forgetting Gradient Algorithm.* The learning algorithm obtains parameter estimation based on the input and output data obtained when the system runs repeatedly in the operating interval. Because the operating interval is limited, we cannot obtain the convergence analysis results in the conventional sense. Only repetitive convergence results can be obtained. That is, for $t \in \{0, 1, \cdots, N-1\}$, the corresponding convergence classification is as follows

Repetitive consistency:

$$\lim_{k \longrightarrow \infty} \widehat{\theta}_k(t) = \theta(t), a.s. \tag{11}$$

Repetitive boundedness:

$$\lim_{k \longrightarrow \infty} \mathrm{E} \left\| \widehat{\theta}_k(t) - \theta_k(t) \right\|^2 \leq \varepsilon < \infty, a.s. \tag{12}$$

When the system parameters are iteratively independent, using the learning algorithm, we can obtain the repeated consistency convergence results of the parameters. When the system parameters are iteratively dependent, we analyze the convergence performance of the forgetting gradient learning algorithm represented by equations (9) and (10).

For fixed time $t \in \{0, 1, \cdots, N-1\}$, $F_k(t)$ is denoted as a $\sigma$ algebra consisting of the input and output data obtained by $k$ repeated operations. In order to analyze the convergence of the proposed learning algorithm, the following assumptions are derived.

*Hypothesis 1.* $v_k(t)$ satisfies

$$E\left[ v_k(t) \,|\, F_{k-1}(t) \right] = 0, a.s. \tag{13}$$

*Hypothesis 2.* There exists uniformly bounded $\sigma_v(t)$ with respect to $t$ such that

$$E\left[ v_k^2(t) \,|\, F_{k-1}(t) \right] \leq \sigma_v^2, a.s. \tag{14}$$

*Hypothesis 3.* The following repetitive persistent excitation conditions are established:

$$\alpha(t) I \leq \frac{1}{N} \sum_{i=1}^{N} \varphi_{k+i} \varphi_{k+i}^T \leq \beta(t) I, a.s, \tag{15}$$

where both $\alpha(t)$ and $\beta(t)>0$, $N >$ n.

From formula (10), we obtain as follows:

$$r_k(t) = \lambda r_{k-1}(t) + \left\| \varphi_k(t) \right\|^2 = \sum_{i=1}^{k} \lambda^{k-i} \left\| \varphi_i(t) \right\|^2 + \lambda^k r_0(t). \tag{16}$$

Then,

$$
\begin{aligned}
Nr_k(t) &= N\sum_{i=1}^{k}\lambda^{k-i}\left\|\varphi_i(t)\right\|^2 + N\lambda^k r_0(t) \\
&\le \sum_{i=1}^{k}\lambda^{k-i}\left\|\varphi_i(t)\right\|^2 + \sum_{i=0}^{k}\lambda^{k-i}\left\|\varphi_i(t)\right\|^2 + \cdots \\
&+ \sum_{i=2-N}^{k}\lambda^{k-i}\left\|\varphi_i(t)\right\|^2 + N\lambda^k r_0(t) \\
&\le \sum_{i=1}^{k}\lambda^{k-i}\left[\sum_{l=0}^{N-1}\left\|\varphi_{i-l}(t)\right\|^2\right] + \left\|\varphi_k(t)\right\|^2 + \sum_{i=0}^{1}\lambda^{i}\left\|\varphi_{k-i}(t)\right\|^2 \\
&+ \cdots + \sum_{i=0}^{N-2}\lambda^{i}\left\|\varphi_{k-i}(t)\right\|^2 + N\lambda^k r_0(t),
\end{aligned}
\tag{17}
$$

where Hypothesis 3 is satisfied, $\sum_{l=0}^{N-1}\lambda^i\|\varphi_{i-l}(t)\|^2 \le nN\beta$, then

$$
\begin{aligned}
Nr_k(t) &\le nN\beta\sum_{i=1}^{k}\lambda^{k-i} + \left\|\varphi_k(t)\right\|^2 + \sum_{i=0}^{1}\left\|\varphi_{k-i}(t)\right\|^2 \\
&+ \cdots \sum_{i=0}^{N-2}\left\|\varphi_{k-i}(t)\right\|^2 + N\lambda^k r_0(t)
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&\le nN\beta\sum_{i=1}^{k}\lambda^{k-i} + (N-1)nN\beta + N\lambda^k r_0(t) \\
&= \frac{1-\lambda^k}{1-\lambda}nN\beta + (N-1)nN\beta + N\lambda^k r_0(t),
\end{aligned}
$$

i.e.,

$$
r_k(t) \le \frac{1-\lambda^k}{1-\lambda}n\beta + (N-1)n\beta + \lambda^k r_0(t). \tag{19}
$$

Take limit of both sides of this inequality

$$
\begin{aligned}
\lim_{k\longrightarrow\infty} r_k(t) &\le \frac{1}{1-\lambda}n\beta + (N-1)n\beta \\
&= \frac{nN\beta - \lambda(N-1)n\beta}{1-\lambda}.
\end{aligned}
\tag{20}
$$

For system (6), we define the transition matrix as follows:

$$
L_{k+1,k}(t) = \left[I - \frac{\varphi_k(t)\varphi_k^T(t)}{r_k(t)}\right]. \tag{21}
$$

The upper bound of $\lambda_{\max}[L_{k+N,k}^{\mathrm{T}}(t)L_{k+N,k}(t)]$ is then solved, and this bound is denoted as $A(t)$. Let $x_k(t)$ be the unit eigenvector corresponding to the maximum eigenvalue $\lambda_{\max}(t)$ of the matrix $L_{k+N,k}^{\mathrm{T}}(t)L_{k+N,k}(t)$, and we construct the difference equation

$$
x_{k+1}(t) = \left[I - \frac{\varphi_k(t)\varphi_k^{\mathrm{T}}(t)}{r_k(t)}\right]x_k(t), \tag{22}
$$

then $x_{k+N}(t) = L_{k+N,k}(t)x_k(t)$

Taking norm on both sides of the abovementioned equation, and it follows that $\|x_{k+N}(t)\|^2 = x_k^{\mathrm{T}}(t)L_{k+N,k}^{\mathrm{T}}(t)$ $L_{k+N,k}(t)x_k(t) \le A(t)$, according to (22) and $r_k(t) > \|\varphi_k(t)\|^2$,

$$
\begin{aligned}
x_{k+1}^{\mathrm{T}}(t)x_{k+1}(t) &= x_k^{\mathrm{T}}(t)\left[I - 2\frac{\varphi_k(t)\varphi_k^{\mathrm{T}}(t)}{r_k(t)} + \left[\frac{\varphi_k(t)\varphi_k^{\mathrm{T}}(t)}{r_k(t)}\right]^2\right]x_k(t) \\
&\le x_k^{\mathrm{T}}(t)\left[I - 2\frac{\varphi_k(t)\varphi_k^{\mathrm{T}}(t)}{r_k(t)} + \frac{\varphi_k(t)\varphi_k^{\mathrm{T}}(t)}{r_k(t)}\right]x_k(t) \\
&\le x_k^{\mathrm{T}}(t)x_k(t) - \frac{\left\|\varphi_k^{\mathrm{T}}(t)x_k(t)\right\|^2}{r_k(t)}.
\end{aligned}
\tag{23}
$$

We transpose both sides of this inequality and that

$$
\frac{\left\|\varphi_k^{\mathrm{T}}(t)x_k(t)\right\|^2}{r_k(t)} \le \left\|x_k(t)\right\|^2 - \left\|x_{k+1}(t)\right\|^2, \tag{24}
$$

then

$$
\sum_{i=0}^{N-1}\frac{\left\|\varphi_{k+i}^{\mathrm{T}}(t)x_{k+i}(t)\right\|^2}{r_{k+i}(t)} \le \left\|x_k(t)\right\|^2 - \left\|x_{k+N}(t)\right\|^2 \le 1 - A(t), \tag{25}
$$

for any $i \in [0, N-1]$,

$$
\begin{aligned}
\left\|x_{k+i}(t) - x_k(t)\right\| &= \left\|\sum_{j=0}^{i-1}\frac{\varphi_{k+i}(t)\varphi_{k+j}^{\mathrm{T}}(t)}{r_{k+j}(t)}x_{k+j}(t)\right\| \\
&\le \sum_{j=0}^{i-1}\left\|\frac{\varphi_{k+i}(t)}{\sqrt{r_{k+j}(t)}}\right\|\left\|\frac{\varphi_{k+j}^{\mathrm{T}}(t)x_{k+j}(t)}{\sqrt{r_{k+j}(t)}}\right\| \\
&\le \sqrt{i(1-A(t))}.
\end{aligned}
\tag{26}
$$

Taking trace to repetitive excitation condition A3), we obtain as follows:

$$
\sum_{i=0}^{N-1}\left\|\varphi_{k+i}(t)\right\|^2 \le nN\beta, a.s. \tag{27}
$$

In the condition (A3), we multiply $x_k^T(t)$ to the left and $x_k(t)$ to the right and use the formulas (15), (19)–(22) to obtain as follows:

$$
\begin{aligned}
\alpha N &\leq x_k^{\mathrm{T}}(t)\sum_{i=0}^{N-1}\varphi_{k+i}(t)\varphi_{k+j}^{\mathrm{T}}(t)x_k(t) \\
&\leq \sqrt{\frac{nN\beta-\lambda(N-1)n\beta}{1-\lambda}}\left[x_k^{\mathrm{T}}(t)\sum_{i=0}^{N-1}\frac{\varphi_{k+i}(t)\varphi_{k+j}^{\mathrm{T}}(t)}{\sqrt{r_{k+i}(t)}}x_k(t)-x_k^{\mathrm{T}}(t)\sum_{i=0}^{N-1}\frac{\varphi_{k+i}(t)\varphi_{k+j}^{\mathrm{T}}(t)}{\sqrt{r_{k+i}(t)}}x_{k+i}(t)+x_k^{\mathrm{T}}(t)\sum_{i=0}^{N-1}\frac{\varphi_{k+i}(t)\varphi_{k+j}^{\mathrm{T}}(t)}{\sqrt{r_{k+i}(t)}}x_{k+i}(t)\right] \\
&\leq \sqrt{\frac{nN\beta-\lambda(N-1)n\beta}{1-\lambda}}\left[\left\|x_k^{\mathrm{T}}(t)\sum_{i=0}^{N-1}\frac{\varphi_{k+i}(t)\varphi_{k+j}^{\mathrm{T}}(t)}{\sqrt{r_{k+i}(t)}}\left(x_k(t)-x_{k+i}(t)\right)\right\|+\left\|x_k^{\mathrm{T}}(t)\sum_{i=0}^{N-1}\frac{\varphi_{k+i}(t)\varphi_{k+j}^{\mathrm{T}}(t)}{\sqrt{r_{k+i}(t)}}\right\|x_{k+i}(t)\right] \\
&\leq \sqrt{\frac{nN\beta-\lambda(N-1)n\beta}{1-\lambda}}\left[\left\|x_k^{\mathrm{T}}(t)\right\|\sum_{i=0}^{N-1}\left\|\frac{\varphi_{k+i}(t)\varphi_{k+j}^{\mathrm{T}}(t)}{\sqrt{r_{k+i}(t)}}\right\|\left\|x_k(t)-x_{k+i}(t)\right\|+\left\|x_k^{\mathrm{T}}(t)\right\|\sum_{i=0}^{N-1}\left\|\varphi_{k+i}(t)\frac{\varphi_{k+j}^{\mathrm{T}}(t)}{\sqrt{r_{k+i}(t)}}x_{k+i}(t)\right\|\right] \\
&\leq \sqrt{\frac{nN\beta-\lambda(N-1)n\beta}{1-\lambda}}\left[\sqrt{nN\beta}\sqrt{\frac{N(N-1)}{2}}\sqrt{1-A(t)}+\sqrt{nN\beta}\sqrt{1-A(t)}\right].
\end{aligned}
\tag{28}
$$

It is derived as follows:

$$
A(t)\leq 1-\frac{(1-\lambda)\alpha^2 N}{((1/N)+2(\sqrt{(N-1/2N)})+(N-1/2))\left(n^2N\beta^2-\lambda n^2N\beta^2+\lambda n^2\beta^2\right)}. \tag{29}
$$

**Lemma 1.** *Let the nonnegative sequence $x_k(t), a_k(t), b_k(t)$ satisfy the following relation:*

$$
x_{k+1}(t)\leq\left(1-a_k(t)\right)x_k(t)+b_k(t), \tag{30}
$$

*where $a_k(t)\in([0,1]), \sum_{k=1}^{\infty}a_k(t)=\infty, x_k(0)<\infty$, then*

$$
\lim_{k\longrightarrow\infty}x_k(t)\leq\frac{b_k(t)}{a_k(t)}, \tag{31}
$$

*where the right limit is assumed to exist.*

Suppose the observed noise $v_k(t)$ and the parameter iteration-dependent system parameter change rate $\omega_k(t)=\theta_k(t)-\theta_{k-1}(t)$ are zero-mean random noise sequences unrelated to the input $u_k(t)$ and the following relationships are satisfied:

$$
\begin{aligned}
&\text{(A2) } \mathrm{E}\left[v_k(t)\right]=0, \mathrm{E}\left[\omega_k(t)\right]=0, \mathrm{E}\left[v_k(t)\omega_k(i)\right]=0, \\
&\text{(A3) } \mathrm{E}\left[v_k(t)v_k(i)\right] \\
&\qquad \mathrm{E}\left[v_k^2(t)\right]=\sigma_v^2(t)\leq\sigma_v^2<\infty, \\
&\text{(A4)} \\
&\qquad \mathrm{E}\left[\left\|\omega_k(t)\right\|^2\right]=\sigma_\omega^2(t)\leq\sigma_\omega^2<\infty.
\end{aligned}
\tag{32}
$$

If PE condition (A1) is satisfied, the parameter estimation error $\widehat{\theta}_k(t)-\theta_k(t)$ given by the forgotten gradient learning algorithm is repetitively bounded, which is solved below.

Solution: Define the parameter estimation error vector

$$
\widetilde{\theta}_k(t)=\widehat{\theta}_k(t)-\theta_k(t). \tag{33}
$$

We assume $\widetilde{\theta}_0(t)$ is independent of $v_k(t)$, and $E[\|\widetilde{\theta}_0(t)\|^2]<\infty$, it can be obtained by using the formulas (7) and (9).

$$
\begin{aligned}
\widetilde{\theta}_k(t) &= \widehat{\theta}_{k-1}(t)-\theta_k(t)+\theta_{k-1}(t)-\theta_{k-1}(t) \\
&\quad+\frac{\varphi_k(t)}{r_k(t)}\left[\varphi_k^{\mathrm{T}}(t)\theta_k(t)+v_k(t)-\varphi_k^{\mathrm{T}}(t)\widehat{\theta}_{k-1}(t)\right], \\
&= \widetilde{\theta}_{k-1}(t)-\omega_k(t)-\frac{\varphi_k(t)\varphi_k^{\mathrm{T}}(t)}{r_k(t)}\widetilde{\theta}_{k-1}(t) \\
&\quad+\frac{\varphi_k(t)\varphi_k^{\mathrm{T}}(t)}{r_k(t)}\omega_k(t)+\frac{\varphi_k(t)}{r_k(t)}v_k(t), \\
&= \left[I-\frac{\varphi_k(t)\varphi_k^{\mathrm{T}}(t)}{r_k(t)}\right]\widetilde{\theta}_{k-1}(t)-\left[I-\frac{\varphi_k(t)\varphi_k^{\mathrm{T}}(t)}{r_k(t)}\right]\omega_k(t) \\
&\quad+\frac{\varphi_k(t)}{r_k(t)}v_k(t), \\
&= L_{k+1,k-N+1}(t)\widetilde{\theta}_{k-N}(t)-\sum_{i=0}^{N}L_{k+1,k-i+1}(t)\omega_{k-i}(t) \\
&\quad+\sum_{i=0}^{N-1}L_{k+1,k-i+1}(t)\frac{\varphi_{k-i}(t)}{r_{k-i}(t)}v_{k-i}(t).
\end{aligned}
\tag{34}
$$

Taking norms on both sides of formula (34)

$$\begin{aligned}\left\|\tilde{\theta}_k(t)\right\|^2 \le{}& \tilde{\theta}_{k-N}^{\mathrm{T}}(t)L_{k+1,k-N+1}^{\mathrm{T}}(t)L_{k+1,k-N+1}(t)\tilde{\theta}_{k-N}(t)\\ &+ 2\tilde{\theta}_{k-N}^{\mathrm{T}}(t)L_{k+1,k-N+1}^{\mathrm{T}}(t)\left[\sum_{i=0}^{N-1}L_{k+1,k-i+1}(t)\frac{\varphi_{k-i}(t)}{r_{k-i}(t)}v_{k-i}(t)\right.\\ &\left.-\sum_{i=0}^{N}L_{k+1,k-i+1}(t)\omega_{k-i}(t)\right]\\ &+\left\|\sum_{i=0}^{N-1}L_{k+1,k-i+1}(t)\frac{\varphi_{k-i}(t)}{r_{k-i}(t)}v_{k-i}(t)-\sum_{i=0}^{N}L_{k+1,k-i+1}(t)\omega_{k-i}(t)\right\|^2.\end{aligned} \tag{35}$$

Taking expectation on both sides of formula (35),

$$\begin{aligned}\mathrm{E}\left(\left\|\tilde{\theta}_k(t)\right\|^2\right) \le{}& A(t)\mathrm{E}\left\|\tilde{\theta}_{k-N}(t)\right\|^2 + N\sum_{i=0}^{N-1}A(t)\frac{\left\|\varphi_{k-i}(t)\right\|^2}{\left\|r_{k-i}(t)\right\|^2}\left\|v_{k-i}(t)\right\|^2\\ &+(N+1)\sum_{i=0}^{N}A(t)\left\|\omega_{k-i}(t)\right\|^2\\ \le{}& A(t)\mathrm{E}\left\|\tilde{\theta}_{k-N}(t)\right\|^2 + N\sum_{i=0}^{N-1}A(t)\frac{1}{r_{k-i}(t)}\sigma_v^2\\ &+(N+1)\sum_{i=0}^{N}A(t)\sigma_\omega^2\\ \le{}& A(t)\mathrm{E}\left\|\tilde{\theta}_{k-N}(t)\right\|^2 + N^2A(t)\frac{1-\lambda}{\lambda^{N-1}n\alpha}\sigma_v^2\\ &+N(N+1)A(t)\sigma_\omega^2.\end{aligned} \tag{36}$$

From Lemma 1 and $A(t)<1$, it is derived as follows:

$$\begin{aligned}\mathrm{E}\left(\left\|\tilde{\theta}_k(t)\right\|^2\right) \le{}& \frac{N^2\left(1-\lambda/\lambda^{N-1}n\alpha\right)\sigma_v^2+N(N+1)\sigma_\omega^2}{1-A(t)}\\ \le{}& \frac{1+2N\sqrt{(N-1/2N)}+\left(N^2-N/2\right)}{(1-\lambda)\alpha^2}\\ &\left(n^2N\beta^2-\lambda n^2N\beta^2+\lambda n^2\beta^2\right)\left(\frac{1-\lambda}{\lambda^{N-1}n\alpha}\sigma_v^2+N(N+1)\sigma_\omega^2\right).\end{aligned} \tag{37}$$

Here, we give the convergence analysis results of the forget gradient learning algorithm in the case of parameter iteration dependence. According to formula (37), we can obtain the bounded convergence effect of the algorithm, that is, the parameter bounds converge to the true values and we give the bounds of the convergence bounds. From formula (37), when $\sigma_\omega^2=0$, which is equal to the system parameters iterate independently, we take $\lambda = 1$. A random gradient learning algorithm is obtained.

$$\begin{aligned}\hat{\theta}_k(t) &= \hat{\theta}_{k-1}(t) + \frac{\varphi_k(t)}{r_k(t)}\left[y_k(t)-\varphi_k^{\mathrm{T}}(t)\hat{\theta}_{k-1}(t)\right],\\ r_k(t) &= r_{k-1}(t)+\left\|\varphi_k(t)\right\|^2.\end{aligned} \tag{38}$$

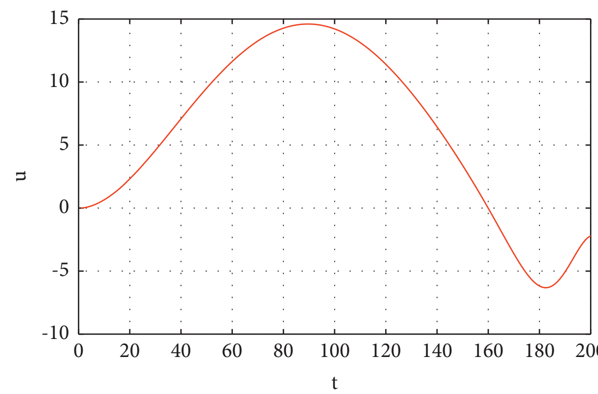


Figure 1: Input values.

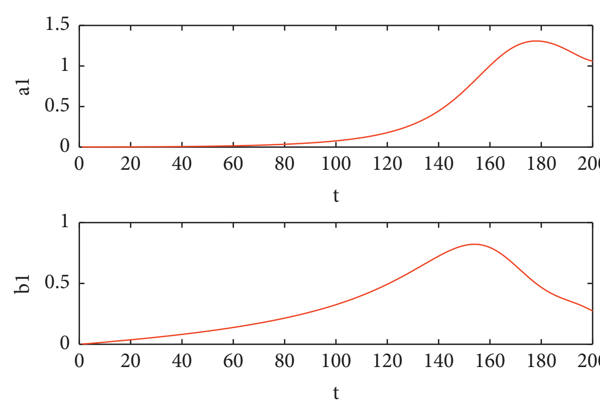


Figure 2: Parameters estimation.

In this case, the consistent convergence result of parameters can be obtained according to formula (37), that is, the parameter estimation converges completely to the parameter truth value.

## 4. Numerical Results

This section completes numerical examples to demonstrate that the learning identification algorithm can be used to estimate time-varying parameters in dynamic systems as shown in Figures 1 and 2.

*Example 1.* Consider the following nonlinear system:

$$\begin{cases}\dot{x}_1 = x_2,\\ \dot{x}_2 = \dfrac{1-e^{-x_1}}{1+e^{-x_1}} + \cos t\sin\left(x_2u\right)+u+\dot{u},\\ y = x_1.\end{cases} \tag{39}$$

The expected trajectory is $y_d(t) = 10(10t^3 - 15t^4 + 6t^5)$. Using the sampling feature modeling method provided in this paper and the adaptive learning control method provided in reference [7], a first-order sampling feature model is established. Where the sampling time $T = 0.005$, the initial

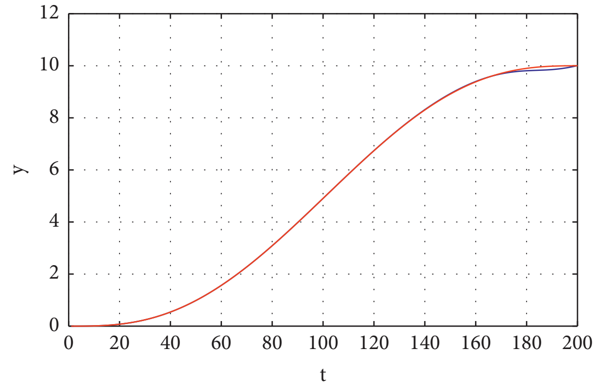


Figure 3: Tracking performance.

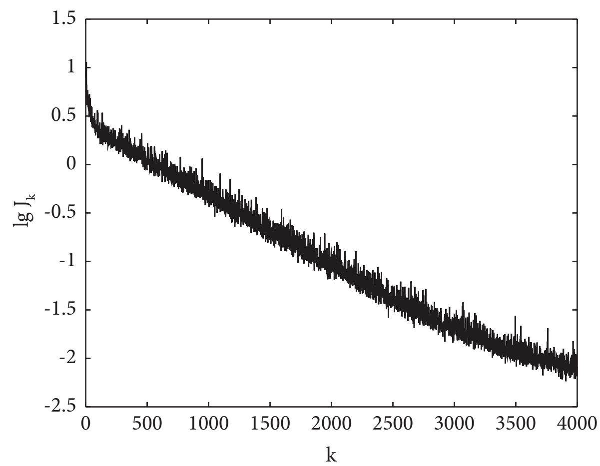


Figure 4: The error with respect to repetition.

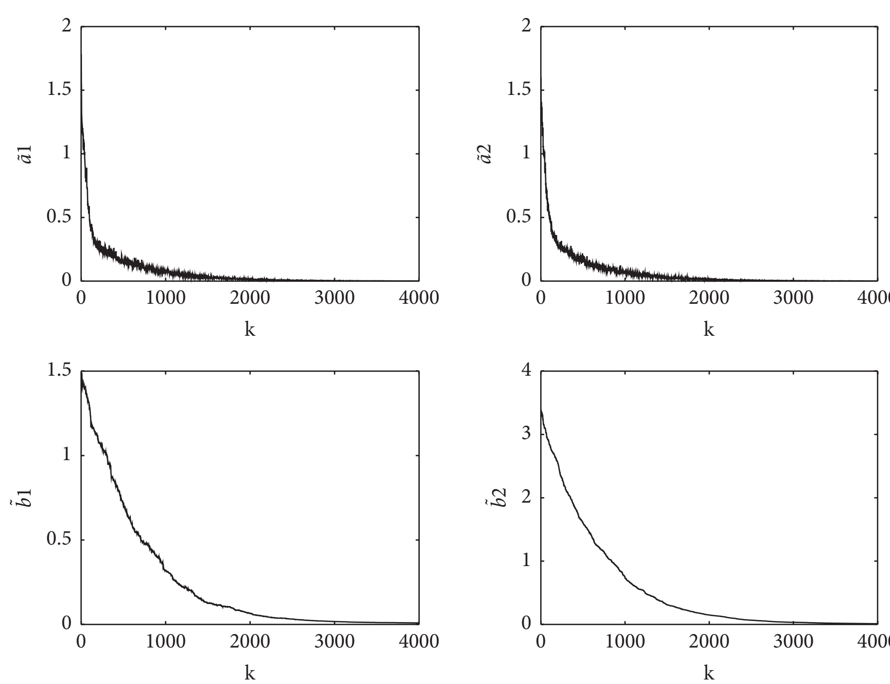


Figure 5: Parameter estimation errors after learning.

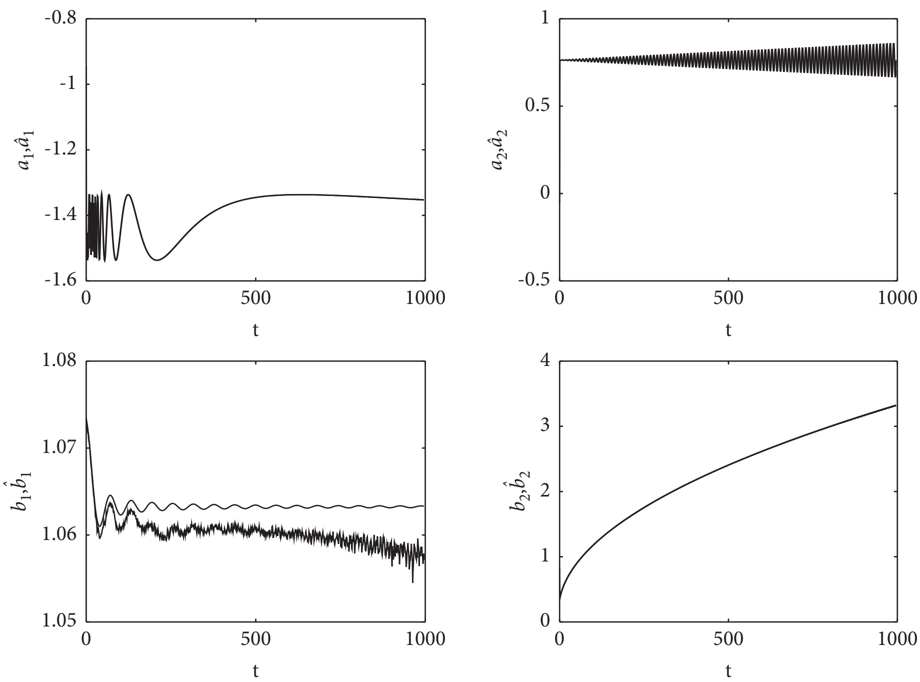


Figure 6: Parameter estimations after learning.

value $r = 1$; the forgetting factor $\lambda = 0.65$, we can obtain as follows:

Tracking performance is shown in Figure 3.

From the simulation results, using the sampled characteristic modeling method provided in this paper, although fast time-varying characteristic parameters are obtained, the output of the system enables efficient tracking of desired trajectories.

*Example 2.* Consider the following finite interval time-varying system.

$$\begin{aligned} y_k(t+1) + a_{1,k}(t)y_k(t) + a_{2,k}(t)y_k(t-1) \\ = b_{1,k}(t)u_k(t) + b_{2,k}(t)u_k(t-1) + v_k(t+1), \end{aligned} \tag{40}$$

where

$$\begin{aligned} a_{1,k}(t) &= -1.5 + 0.1\sin\left(\frac{50}{t}\right) + 0.001\sqrt{k}, \\ a_{2,k}(t) &= 0.7 + \frac{0.1t}{1000} * \sin\left(\frac{\pi t}{5}\right) + 0.001\sqrt{k}, \\ b_{1,k}(t) &= 1 + 0.1\frac{\sin(2\pi t/61)}{t} + 0.001\sqrt{k}, \\ b_{2,k}(t) &= 0.1 + 0.1\sqrt{t} + 0.001\sqrt{k}. \end{aligned} \tag{41}$$

In the simulation, we set finite interval length $N = 1000$. For $t = 1, 2, \cdots, N, k = 1, 2, \cdots, 4000, u_k(t)$ as uniformly distributed random variables on [-0.5, 0.5], forgetting factor $\lambda = 0.7$, $v_k(t) = 0.01$ *randn*. Here, *randn* is the production function of random variables that obey (0, 1) normal distribution. In the random dependence learning algorithms

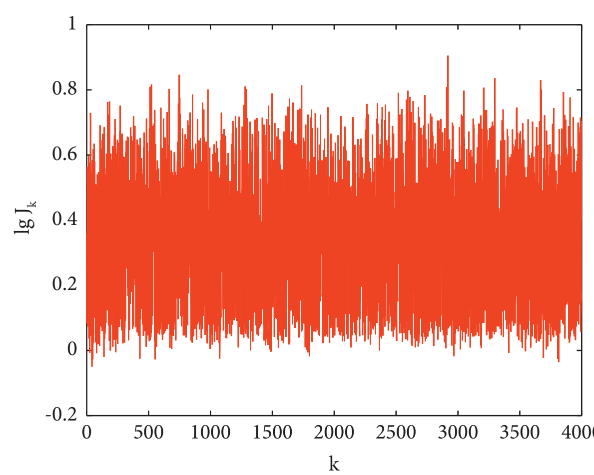


Figure 7: The error with respect to every interval.

(8) and (9), we set the initial value $r_0(t) = 1$, $\widehat{\theta}_0(t) = 0$. To examine the convergence performance, we define $J_k = \max_{1 \le t \le N} \lg|e_k(t)|, e_k(t) = y_k(t) - \varphi_k^T(t)\widehat{\theta}_{k-1}(t)$.

The simulation results are shown in Figure 4–6. The prediction error is shown in Figure 4, and the parameter estimation error is shown in Figure 5, and the parameters estimate values in Figure 6. It can be seen from Figure 4 that the prediction error decreases rapidly with the increase of the number of iterations. In Figure 5, the parameter estimation error asymptotically converges to zero in a small field, and the simulation results show the uniform convergence of the parameter estimation, which can almost converge to real values.

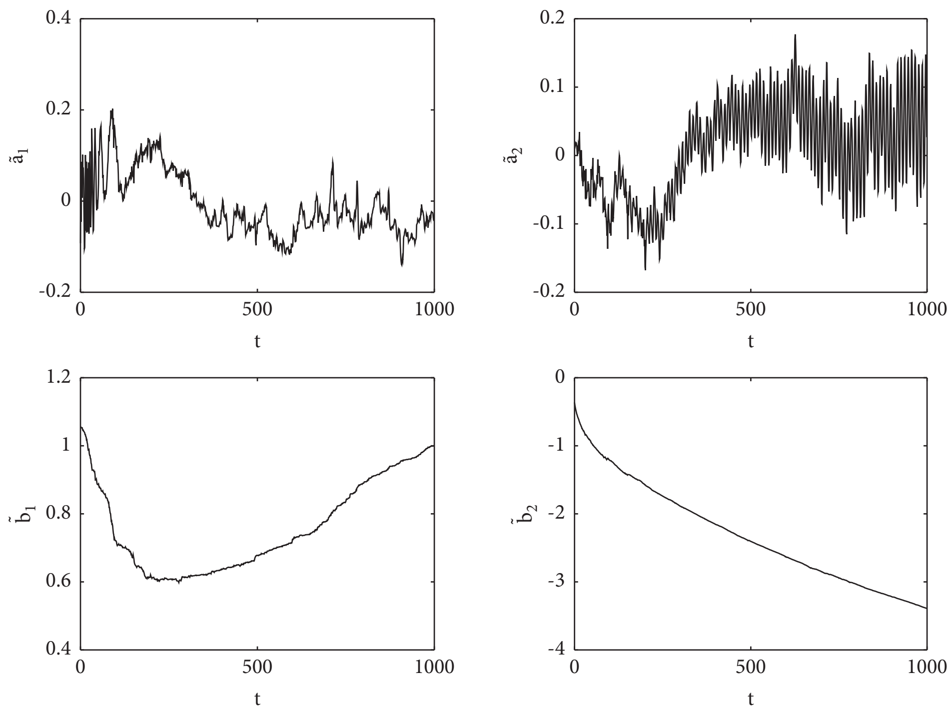


Figure 8: Parameter estimation errors after the last recursive process.

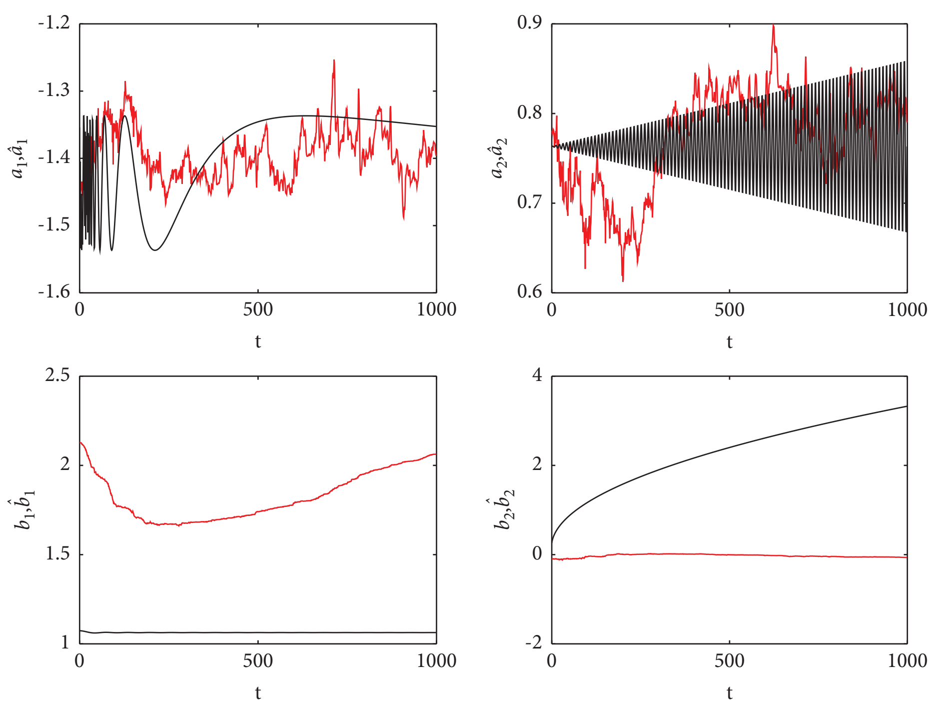


Figure 9: Parameter estimations after the last recursive process.

To demonstrate the effectiveness of the proposed algorithm, the simulation results are compared with stochastic gradient method appeared in reference [17], where the finite interval length $N = 1000 * 4000$ and the other conditions are same to which of the abovementioned simulation. The errors with respect to every interval are shown in Figure 7. Parameter estimation errors after last recursive process and parameter estimations after last recursive process are shown in Figures 8 and 9.

From the simulation results, the bounded convergence result is guaranteed, but the identification result is weaker than the result of the method presented in this paper.

## 5. Conclusions

As modeling technology is of great importance to robots for industrial Internet systems, the proposed sampled characteristic model and forgetting gradient learning identification method can be used to solve the parameter estimation problem of time-varying systems running round-trip over finite intervals. The forgetting gradient learning algorithm is derived for time-varying systems under finite-interval repeat operations. We prove the repetition boundedness of the learning algorithm under repeated continuous excitation conditions and give the estimation error bounds given by the proposed algorithm. The completed simulations also verify the effectiveness of the learning algorithm. Further, we present the convergence analysis results of the stochastic gradient learning algorithm, which can obtain consistent convergence results for the parameters when parameter iterations are independent. The main purpose of this paper is to propose this learning identification method and to clarify the connection and difference between the learning identification and the existing recursive identification algorithms. For the completeness of the theory and the expression simplicity, for the consistency analysis of the learning algorithm, we learn from the mature results of the learning algorithm. However, there are still differences between the two algorithms, such as the recurrence algorithm requires the PE condition along the time domain, while the learning algorithm requires the repeated PE condition; the assumption of the convergence consistency of the learning algorithm and the estimation of the obtained convergence rate are allowed to depend on time. Systematic results for recursive identification are presented in literature [17], from which we can learn for follow-up studies, including the case of system interference such as colored noise, continuous excitation, improvements of SPR conditions, and convergence rate estimation.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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