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ASYMPTOTIC PERFORMANCE CHARACTERISTICS OF CSMA AND CSMA←CD→CSMA/CD NETWORKS

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This paper presents analytical formulae for performance evaluation of CSMA and CSMA/CD networks with a finite number of buffered users. The asymptotic theory developed in [1] is extended to local area networks so as to investigate the resulting steady state throughput, time delay, probability of blocking, and local stability of each equilibrium. In particular, the stabilization of CSMA and CSMA/CD networks is considered, *i.e.*, how to choose the transmission probability so that a unique steady state with relatively high performance characteristics is achieved.

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1. INTRODUCTION

Local area networks provide the communication links among users within a limited geographical area. Since the maximum distance between users is normally not more than a few kilometers, the propagation delay is short compared to the packet transmission time. This short delay allows users to listen for a carrier and act accordingly. A class of random access protocols, called CSMA (carrier sense multiple access) protocols, uses hardware at each user to sense the state of the channel before transmitting. By making use of the sensed information, a user is able to defer its transmission if the channel is busy. Thus, CSMA protocols for local area networks reduce the level of collision and achieve the throughput higher than e^{-1} achieved by ALOHA protocols whereby users attempt transmissions whenever they have packets ready. A further refinement of the random access protocols, called CSMA/CD (carrier sense multiple access with collision detection) protocols, results from the use of hardware to sense the channel during transmission of a packet as well as before transmission.

CSMA protocols have been studied in [2]–[19]. However, most previous studies were limited to CSMA and CSMA/CD with unbuffered users. Throughput-delay characteristics of CSMA and CSMA/CD with unbuffered users have been analyzed in [2]–[9]. In [10]–[11], the stability of CSMA and CSMA/CD with unbuffered users has been considered. The effects of various packet sizes on CSMA/CD with unbuffered users have been addressed in [12]–[14]. In [15]–[17], the queuing analysis of CSMA and CSMA/CD with buffered users has been addressed. In [18]–[19], CSMA/CD with buffered users has been investigated by equilibrium point analysis. However, the results from [15]–[19] were numerical in nature and required recursive procedures for performance evaluation.

The purpose of this paper is to extend the result of [1] to local area networks and develop a unified analytical method for performance evalution of CSMA and CSMA/CD with a finite number of buffered users. The structure of this paper is as follows: In Section 2, the model under consideration is introduced. In Section 3, the asymptotic approximation is described. In Section 4, performance analysis is presented. The conclusions are formulated in Section 5.

2. THE MODEL

2.1. The Channel

Consider a channel consisting of a noiseless collision feedforward channel and a noiseless, collisionless feedback channel. In addition, it is assumed that

(a) the propagation delay is nonzero,
(b) time is slotted with a slot duration equal to the propagation delay and transmission of a packet across a feedforward channel requires τ unit intervals of time,
(c) feedback signal at $n + \tau + 1$ is received by all users that have been active during $[n, n + \tau)$.

2.2. The Network

It is assumed that

(a_1) the network consists of $1 << M < \infty$ users communicating through the collision channel,
(b_1) every user has a buffer capable of storing $1 \leq \mathrm{N} < \infty$ packets,
(c_1) the input traffic is a Bernoulli sequence with a parameter p_a, *i.e.*, during each time slot $[n, n + 1)$, every user generates a packet with probability p_a. Since at most one packet can be successfully transmitted across the feedforward channel during τ time slots, to avoid triviality, it is assumed that $M\tau p_a \leq 1$,
(d_1) If a packet is generated by a user having its buffer full, the newly generated packet is rejected; otherwise, it is stored in the first empty cell of the buffer,
(e_1) if a positive acknowledgement is received by a user at time $n + \tau + 1$, the packet transmitted during $[n, n + \tau)$ is eliminated from the first cell of its buffer; otherwise, it is stored and attempted for transmission at a later time slot,
(f_1) if a packet is eliminated from the first cell of a buffer, a packet stored in its mth cell, $m = 2,\ldots, N$, instantaneouly moves down to be stored in cell $m - 1$,
(g_1) all busy users sense the feedforward channel. If the channel is idle, every busy user attempts a transmission at the beginning of time slot with a probabilty p. If a collision occurrs, the time required for all active users to abort their transmissions is k time slots where $1 \leq k \leq \tau$.

2.3. Evolution Equations

Let $h_i(n)$, $i = 1,\dots, N$, be the occupancy of the i-th layer of buffers at time n, and $v(n)$ be the indicator of the feedforward channel being idle at time n. Due to the assumptions formulated above, the evolution of h_i and v can be characterized as follows:

$$\begin{aligned} h_i(n+1) &= h_i(n) + \zeta_i(n, n+1) - \psi_{i,i-1}(n+1) + \psi_{i+1,i}(n+1) \\ v(n+1) &= v(n) + \eta(n,n+1) - \mu(n, n+1), \end{aligned} \tag{1}$$

$$i = 1,\dots, N,\ \psi_{N+1,N}(n) = 0,\ \forall n,$$

$$M \geq h_1 \geq \dots \geq h_N,\ v(n) \in \{0, 1\},\ \forall n.$$

The sequence $\zeta_i(n, n+1)$, $n = 0, 1,\dots$, represents the process of packet arrivals into the ith layer during $[n, n+1)$. The event $\zeta_i(n, n+1) = l$, $l = 0, 1,\dots, M$ means that l packets have arrived into the ith layer during $[n, n+1)$. Due to assumptions (c_1) and (d_1),

$$\begin{aligned} &P[\zeta_i(n+1) = l | h_1(n),\dots, h_N(n)] \\ &\qquad = \binom{h_{i-1}(n) - h_i(n)}{l} p_a^l (1-p_a)^{h_{i-1}(n)-h_i(n)-l}, \end{aligned} \tag{2}$$

$l = 0, 1,\dots, h_{i-1}(n) - h_i(n)$, $i = 1,\dots, N$, $h_0(n) = M$, $\forall n$.

The sequence of $\psi_{1,0}(n+1)$, $n = 0, 1,\dots$, represents the process of successful transmission. The event $\psi_{1,0}(n+1) = 1$ means that a successful transmission has occurred during $[n - \tau, n)$. Due to assumptions (b), (e_1), and (g_1), the conditional probability distribution of $\psi_{1,0}(n+1)$ can be represented as follows:

$$\begin{aligned} &P[\psi_{1,0}(n+1) = 1 | h_i(n-\tau),\dots, h_i(n), \forall i \in \{1,\dots, N\}, v(n-\tau),\dots, v(n)] \\ &\qquad = v(n-\tau) F_1(h_1(n-\tau)), \end{aligned} \tag{3}$$

where $F_1(h_1(n - \tau)) = h_1(n - \tau)p(1-p)^{h_1(m-\tau)-1}$

The sequence of $\psi_{i,i-1}(n+1)$, $i = 2,\dots, N$, $n = 0, 1,\dots$, represents the transition of a packet from buffer layer i to buffer layer $i - 1$. The event $\psi_{i,i-1}(n+1) = 1$ means that a packet has moved at the end of time slot $[n, n+1)$ from buffer layer i to buffer layer $i - 1$. Due to assumption (f_1),

$$\begin{aligned} &P[\psi_{i,i-1}(n+1) = 1 | h_i(n-\tau),\dots, h_i(n), \forall i \in \{1,\dots, N\}, v(n-\tau),\dots, v(n)] \\ &= \frac{h_i(n) + E[\zeta_i(n, n+1) | h_1(n),\dots, h_N(n)]}{h_1(n)} v(n-\tau) F_1(h_1(n-\tau)),\ i = 2,\dots, N. \end{aligned} \tag{4}$$

The sequence of $\eta(n,n+1)$, $n = 0, 1, 2,\dots$, represents the process of channel being idle during $[n, n+1)$. The event $\eta(n, n+1) = 0$ means that the channel becomes busy during

$[n, n+1)$. The event $\eta(n, n+1) = 1$ means that the channel becomes idle during $[n, n+1)$. Due to assumptions (e_1) and (g_1),

$$P[\eta(n, n+1) = 1|\ldots] = 1 - \sum_{i=k}^{\tau-1} v(n-i)F_1(h_1(n-i)) - \sum_{i=0}^{k-1} v(n-i)(1 - F_0(h_1(n-i))), \tag{5}$$

where $F_0(h_1(n-\tau)) = (1-p)^{h_1(m-\tau)}$. If $\tau = k$, then CSMA/CD turns out to be CSMA.

The sequence of $\mu(n, n+1)$, $n = 0, 1, 2, \ldots$, represents the process of channel being idle at time n. The event $\mu(n, n+1) = 0$ means that the channel is busy at time n. The event $\mu(n, n+1) = 1$ means that the channel is idle at time n. Due to assumptions (e_1) and (g_1),

$$P[\mu(n, n+1) = 1|\ldots] = v(n). \tag{6}$$

3. ASYMPTOTIC APPROXIMATION

In terms of the normalized variables $x_i = h_i/M$, $u = v/M$, equation (1) can be rewritten as

$$\begin{aligned} x_i(n+1) &= x_i(n) + \frac{1}{M}[\zeta_i(n, n+1) - \psi_{i,i-1}(n+1) + \psi_{i+1,i}(n+1)] \\ u(n+1) &= u(n) + \frac{1}{M}[\eta(n, n+1) - \mu(n, n+1)], \end{aligned} \tag{7}$$

$$i = 1, \ldots, N, \ \psi_{N+1,N}(n+1) = 0, \forall n,$$

$$1 \geq x_1 \geq x_2 \geq \cdots \geq x_N, \ u \in \{0, 1/M\}.$$

In order to show that the conditions of Theorem 1 of [1] are met, it is sufficient to show that $\eta(n, n+1)$, $\mu(n, n+1)$ satisfy (12) of [1]. Since the random variables $\eta(n, n+1)$, $\mu(n, n+1)$ take values 0, 1, the conditions of Theorem 1 of [1] are satisfied.

Thus, the asymptotic approximation of (7) can be shown as follows:

$$\begin{aligned} y_i(n+1) &= y_i(n) + \epsilon[(y_{i-1}(n) - y_i(n))Mp_a \\ &\quad - \frac{y_i(n) - y_{i+1}(n)}{y_1(n)} Mw(n-\tau)F_1(y_1(n-\tau))] \\ w(n+1) &= w(n) + \epsilon[1 - \sum_{i=k}^{\tau-1} Mw(n-i)F_1(y_1(n-i)) \\ &\quad - \sum_{i=0}^{k-1} Mw(n-i)(1 - F_0(y_1(n-i))) - Mw(n)] \end{aligned} \tag{8}$$

$$i = 1, \ldots, N, \epsilon = 1/M, y_0(n) = 1, y_{N+1}(n) = 0, \forall n,$$

$$y_i \in [0, 1], 1 \geq y_1 \geq y_2 \geq \cdots \geq y_N, w \in [0, 1/M],$$

where $F_1(y_1(n - \tau)) = My_1(n - \tau)p(1 - p)^{My_1(n-\tau)-1}$, $F_0(y_1(n - \tau)) = (1 - p)^{My_1(n-\tau)}$.

4. PERFORMANCE ANALYSIS

Steady State. Let y_{is} denote the steady state value of the averaged normalized occupancy of the i-th layer of buffers, $i = 1, \ldots, N$. Then from (8),

$$y_{is} = y_{1s} \cdot \frac{y_{(i-1)s} - y_{Ns}}{1 - y_{Ns}}, \; i = 1, \ldots, N,$$

$$w_s = 1/[M(1 + (\tau - k)F_1(y_{1s}) + k(1 - F_0(y_{1s})))] \tag{9}$$

where y_{1s} and y_{Ns} are related by

$$(1 - y_{Ns})Mp_a = \frac{F_1(y_{1s})}{1 + (\tau - k)F_1(y_{1s}) + k(1 - F_0(y_{1s}))}. \tag{10}$$

The left-hand side of (10) is referred to as a load line and the right-hand side of (10) is referred to as a transmission line. Thus, every intersection of the load line and the transmission line defines a steady state of the network. Figure 1 shows the load and transmission lines for CSMA/CD.

Local Stability of the Equilibria. The slope of the load line and the transmission line at the intersection defines the stability properties, i.e., Theorem B of [1] holds. The proof is obtained using the Jury stability test.

Steady-State Performance Characteristics. If equation (10) has a unique solution then, as it follows from Theorem A of [1], there exists M_0, such that for $M \geq M_0$, the network has the following performance characteristics:

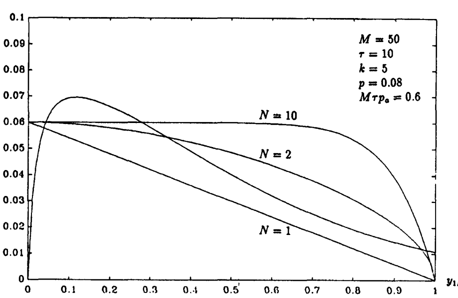

Figure 1 *Load and transmission lines for CSMA/CD*

(i) The steady-state throughput is

$$TP = \frac{F_1(y_{1s})}{1 + (\tau - k)F_1(y_{1s}) + k(1 - F_0(y_{1s}))}$$

where y_{1s} is the solution of (10).

(ii) The steady-state time delay, TD, is

$$TD = \frac{My_s(1 + (\tau - k)F_1(y_{1s}) + k(1 - F_0(y_{1s})))}{F_1(y_{1s})}$$

where $y_s = \sum_{i=1}^{N} y_{is}$.

(iii) The steady-state probability of packet rejection (blocking), P_R, is

$$P_R = y_{Ns}(y_{1s}).$$

Remark 1. The normalized throughput, TP_N, with respect to the transmission time, τ, is

$$TP_N = \frac{\tau F_1(y_{1s})}{1 + (\tau - k)F_1(y_{1s}) + k(1 - F_0(y_{1s}))} \tag{11}$$

If $k << \tau$, then from (11), $\lim_{\tau \to \infty} TP_N = 1$. This relationship coincides with the one obtained in [4], [5], and [10].

Remark 2. If equation (10) has multiple solutions, global analysis of TP, TD, and P_R can be conducted using the large deviations approach as in [1].

The transmission lines of CSMA/CD with various transmission probabilities are shown in Figure 2. As it follows from Figure 2, the transmission probability, p, may lead to elimination of the saturation phenomena and to stabilization of the network in a steady state with relatively high performance characteristics. This phenomenon can be characterized as follows:

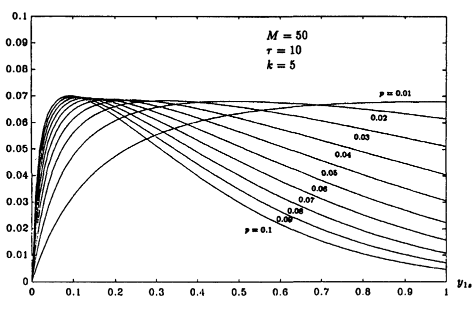

Figure 2 . *Transmission lines for CSMA/CD*

THEOREM.—*There exists M_0 and p such that a network defined by (a_1)–(g_1) in a channel defined by (a)–(c) has a unique globally asymptotically stable steady state $y_{1s}, y_{2s}, \ldots, y_{Ns}$, w_s provided that $M \geq M_0$ and p satisfies the condition*

$$(k + 1)(1 + Mln(1 - p)) - k(1 - p)^M \geq 0. \tag{12}$$

Proof. From the local stability property of equilibria, the network has a unique steady state if *TP* is a monotonically increasing function with respect to y_{1s}. Then

$$\frac{\partial TP}{\partial y_{1s}} = (k + 1)\frac{\partial F_1}{\partial y_{1s}} + k(F_1\frac{\partial F_0}{\partial y_{1s}} - F_0\frac{\partial F_1}{\partial y_{1s}}) \geq 0, \forall y_{1s} \in [0, 1].$$

By simple algebraic manipulations, this is satisfied if

$$(k + 1)(1 + M\ln(1 - p)) - k(1 - p)^M \geq 0.$$

From Theorem B of [1], the unique steady state is asymptotically stable.

Remark 3. Since $M >> 1$, without losing much generality,

$$1 + M\ln(1 - p) \cong 1 - pM, (1 - p)^M \cong e^{-pM}.$$

If $k >> 1$, then from (12), the required p becomes $p \leq \frac{\sqrt{2k}}{kM}$.

5. CONCLUSIONS

In this paper, an analytical method for performance analysis and design of CSMA/CD with a finite number of buffered users is developed. The method developed here can be utilized as follows:

a. Find the intersections of the load and transmission lines (10). These intersections define the number and the position of the steady states as well as their stability properties.
b. If the intersection is unique, the steady state performance characteristics are calculated according to (i)–(iii).
c. If the intersection is not unique, estimate the residence time, as in [1], and evaluate the global (average) performance characteristics as in Corollary 2 of [1].
d. If the transmission probability is used for stabilization of the network, choose the maximum value of the transmission probability from (12) that stabilizes the network.

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