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Design of Robust Controller for Linear Systems with Markovian Jumping Parameters

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This paper deals with the robustness of the class of uncertain linear systems with Markovian jumping parameters (ULSMJP). The uncertainty is taken to be time-varying norm bounded. Under the assumptions of the boundedness of the uncertainties and the complete access to the system's state and its modes, a sufficient condition for stochastic stabilizability of this class of systems is established. An example is provided to demonstrate the usefulness of the proposed theoretical results.

Keywords: Linear systems with Markovian jumping parameters; Stochastic stability; Markov process; Norm bounded uncertainties

1 INTRODUCTION

This paper deals with the class of systems with Markovian jumping parameters. In the last two decades this class of systems has been

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extensively studied. Theoretical and practical achievements have been reported in the literature. Without any intention of being exhaustive here, we mention [2–4,7–11,15–18,24,26–29]. Other references dealing with this class of systems can be found in the quoted references.

Most of the quoted references, except [5,6,12], have considered certain systems with Markovian jumping parameters. Nowadays, it is well known that these nominal systems may not describe the real systems appropriately and the uncertainties have to be considered if at least the robustness stability is required. The readers are referred to [14] for the different possibilities to model uncertainties.

In [5,6], the uncertain linear and nonlinear piecewise deterministic systems have been investigated. Under some matching conditions, with a state feedback control law, sufficient conditions, which guarantee the stochastic stabilizability robustness of this class of systems have been established. In the work of De souza and Fragoso [12], the disturbance rejection problem has been studied for finite and infinite horizons.

For the deterministic class of linear systems, different approaches to design robust linear controllers have been reported in the literature. Most of these approaches are based on the Lyapunov equation or Riccati equation (see [20,21]). These approaches which are proposed for the design of robust controller for the deterministic class of systems, however, cannot be applied to the class of systems under consideration in this paper. The goal of this paper is to study the stabilizability robustness of the class of uncertain linear systems with Markovian jumping parameters. A feedback control which will assure a good tracking of a given reference and attenuate the effect of the time-varying norm bounded uncertainties is designed. Mainly, we establish a sufficient condition which guarantees the stochastic stability robustness of the class of systems with Markovian jumping parameters under norm bounded uncertainties and we will give an iterative algorithm to determine the parameters of the proposed controller. The paper is organized as follows: in Section 2, we give a brief description of the class of linear systems with Markovian jumping parameters (LSMJP) and recall the definition of the stochastic stabilizability of this class of systems. In Section 3, we establish the sufficient condition for the robust stabilizability of the linear systems with Markovian jumping parameters under some time-varying norm bounded uncertainties.

Section 4 presents a numerical example and the paper is concluded in Section 5 with some final comments.

2 PROBLEM STATEMENT

Before presenting the class of linear systems with Markovian jumping parameters let us consider some examples of this class of systems. Consider an industrial application where we control a dc servomotor driving a given mechanism. The objective is to assure the control of the mechanism's position (linear or angular). Let us assume that the sense of rotation of the actuator is random, which corresponds to a random position of the mechanism, and it is described by a stochastic continuous Markov process with discrete finite state space (two states). Consider also that the complete system (dc servomotor and the mechanism) has two different models, one for each sense of rotation. Let us also assume that in each sense of rotation the system can be described by a linear model with some bounded uncertainties. The problem now is how we can control the system to assure the required performances and eventually assure the robustness of the stability of the closed-loop.

As a second example, let us consider a dynamical system subject to random failures and repairs, so that at each moment it can be in one of the following states: *good functioning*, *intermediate functioning*, *bad functioning* and *failure* for many reasons. The failure mode is not interesting to us since the system cannot be operated. For the other modes, the performances of the system will be different in each mode and the ones obtained in the good functioning mode are certainly better than the ones of the intermediate mode etc. The mode of the system will evolve in a random way between these states. The question which arises is what we can do to assure that the performances of the system will remain the same or at least how we can assure the robustness of the stability of the closed-loop.

These examples fit in the formalism of the class of systems with Markovian jumping parameters which represents a rich class of systems. Other examples of this class of systems can be found in different areas such as manufacturing systems, economics, etc.; see, for example [25].

2.1 Linear Systems with Markovian Jumping Parameters

To describe the class of linear systems with Markovian jumping parameters, let us fix a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. This class of systems owns a hybrid state vector. The first component vector is continuous, the second one is discrete and is always referred to as the mode. Let $\mathbf{x}(t) \in \mathbb{R}^{n_1}$ be the continuous component state vector at time t, $\mathbf{u}(t) \in \mathbb{R}^m$ be the control vector at time t, $\mathbf{y}(t) \in \mathbb{R}^p$ be the system's controlled output, and $\{\mathbf{r}(t),\ t \in [0, T]\}$ be the homogeneous Markov process with right continuous trajectories and taking values on the finite set $\mathcal{B} = \{0, 1, \ldots, N\}$. We assume also that $\mathbf{P}_t := (P_t^1, P_t^2, \ldots, P_t^N)$, with $P_t^i := Prob(\mathbf{r}(t) = i)$; $i = 1, 2, \ldots, N$ satisfies the forward Kolmogorov equation, i.e.

$$\frac{\mathrm{d}\mathbf{P}_t}{\mathrm{d}t} = \Lambda \mathbf{P}_t, \quad 0 \leq t \leq T, \tag{1}$$

$$\mathbf{P}_0 = \bar{\mathbf{P}}, \tag{2}$$

where $\bar{\mathbf{P}}$ is the initial vector probability of the process $\{\mathbf{r}(t), t \in [0, T]\}$ and $\Lambda := [q_{ij}]$ is the stationary transition rate matrix of the process $\{\mathbf{r}(t), t \in [0, T]\}$ with $q_{ij} \geq 0$, and $q_{ii} = -\sum_{j=1,\, j \neq i} q_{ij}$ (see [13]).

The class of linear systems with Markovian jumping parameters is described by the following dynamics:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{r}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{r}(t))\mathbf{u}(t), \quad \mathbf{x}(0) = \mathbf{x}_0, \tag{3}$$

with controlled output

$$\mathbf{y}(t) = \mathbf{C}(\mathbf{r}(t))\mathbf{x}(t) + \mathbf{D}(\mathbf{r}(t))\mathbf{u}(t), \tag{4}$$

where for each $\mathbf{r}(t) = i$, $\mathbf{A}(i)$, $\mathbf{B}(i)$, $\mathbf{C}(i)$ and $\mathbf{D}(i)$ are constant matrices with appropriate dimensions.

For the stability of this class of systems, there exist many definitions in the literature (see [15,25] and references therein). In the rest of this paper, we will use the following one.

Let $\mathbf{x}(t, \mathbf{x}_0, i)$ represent the corresponding solution of system (3) at time t when the control $\mathbf{u}(\cdot) = 0$ is used and the initial conditions are respectively $\mathbf{x}_0$ and i.

DEFINITION 2.1 [17] For system (3)–(4), the equilibrium point 0 is stochastically stable, if for every initial state $(\mathbf{x}_0, \mathbf{r}_0)$ the following holds:

$$\mathbf{E}\left\{\int_0^{\infty} \|\mathbf{x}(t, \mathbf{x}_0, \mathbf{r}_0)\|^2 \mathrm{d}t \mid \mathbf{x}_0, \mathbf{r}_0\right\} < \infty. \tag{5}$$

Let $\mathbf{x}(t, \mathbf{x}_0, i, \mathbf{u})$ represent the corresponding solution of system (3) at time t when the control $\mathbf{u}(\cdot)$ is used and the initial conditions are respectively $\mathbf{x}_0$ and i.

DEFINITION 2.2 System (3)–(4) is said to be stochastically stabilizable if, for all finite $\mathbf{x}_0 \in \mathbb{R}^{n_1}$ and $i \in \mathcal{B}$, there exists a state feedback control, $\mathbf{u}(\cdot)$, such that

$$\lim_{T\to\infty} E_{\mathbf{u}(\cdot)}\left\{\int_0^{T} \mathbf{x}'(t, \mathbf{x}_0, i, \mathbf{u})\mathbf{x}(t, \mathbf{x}_0, i, \mathbf{u})\, \mathrm{d}t \mid \mathbf{x}_0,\ \mathbf{r}(0) = \mathbf{r}_0\right\} \le \mathbf{x}_0'\tilde{\mathbf{P}}\mathbf{x}_0, \tag{6}$$

where $\tilde{\mathbf{P}}$ is a symmetric positive-definite matrix.

Remark 2.1 Notice that the upper bound in this definition depends on the initial conditions of the system.

The following theorem was established by Ji and Chizeck [17]. It states the necessary and sufficient conditions for stochastic stabilizability of this class of systems. For the proof, the reader is referred to this reference.

THEOREM 2.1 [17] *The system above is stochastically stabilizable if and only if there exists a control law* $\mathbf{u}(t) = -\mathbf{K}(\mathbf{r}(t))\mathbf{x}(t)$ *such that, for any given positive-definite and symmetric matrix* $\mathbf{Q}(i)$, *the unique set of solutions*, $\mathbf{P}(i)$, *of the N coupled equations*

$$\begin{aligned}&[\mathbf{A}(i) - \mathbf{B}(i)\mathbf{K}(i)]'\mathbf{P}(i) + \mathbf{P}(i)[\mathbf{A}(i) - \mathbf{B}(i)\mathbf{K}(i)] + \sum_{j\in\mathcal{B}} q_{ij}\mathbf{P}(j)\\ &\quad = -\mathbf{Q}(i), \quad \forall i \in \mathcal{B},\end{aligned} \tag{7}$$

are positive-definite symmetric.

Proof See [17].

In the standard formulation of the control problem, we often use a nominal model of the system for the design procedure. But in real life, the matrices $\mathbf{A}(i)$, $\mathbf{B}(i)$, $\mathbf{C}(i)$ and $\mathbf{D}(i)$ for each $\mathbf{r}(t)=i$ with value in $\mathcal{B}$ are not precisely known for many reasons, well known by the control community, and we always retrieve a discrepancy between the used nominal model and the real process. These uncertainties, which can be divided into two categories, i.e., matched and mismatched uncertainties, can make the feedback controller inefficient and in worst case, the real system can become unstable. In order to avoid these problems, when designing the controller, we need to take into account the system's uncertainties. From the practical point of view, the purpose of each used controller consists in assuring the asymptotic tracking and the disturbance rejection. In the next section, we will propose a control law which guarantees simultaneously the asymptotic tracking and the disturbance rejection for the class of systems under study.

Remark 2.2 [17] The autonomous linear system with Markovian jumping parameters is said to be stable if and only if for any given positive-definite and symmetric matrix $\mathbf{Q}(i)$ the unique set of solutions, $\mathbf{P}(i)$, of the N coupled equations:

$$\mathbf{A}'(i)\mathbf{P}(i)+\mathbf{P}(i)\mathbf{A}(i)+\sum_{j\in\mathcal{B}} q_{ij}\mathbf{P}(j)=-\mathbf{Q}(i),\quad \forall i\in\mathcal{B} \tag{8}$$

are positive-definite and symmetric.

Remark 2.3 If for some mode i (for any $i\in\mathcal{B}$) the jump rate q_{ij} is 0, the system will remain at this mode for ever. The mode is called *absorbing mode* and the robustness condition is similar to the one used in the deterministic case.

In this paper, the adopted Euclidean norm of vector $\mathbf{x}$, denoted by $\|\mathbf{x}\|$ is $\|\mathbf{x}\|=(x_1^2+x_2^2+\cdots+x_n^2)^{1/2}$ where x_i, for $i=1,\ldots,n$, denotes the ith element of the vector $\mathbf{x}$. The induced Euclidean norm of matrix $\|\mathbf{M}\|$ is given by $\|\mathbf{M}\|=[\lambda_{\max}(\mathbf{M}'\mathbf{M})]^{1/2}$, where $\mathbf{M}'$ denotes the transpose of matrix $\mathbf{M}$ and $\lambda_{\min}(\mathbf{M})$ and $\lambda_{\max}(\mathbf{M})$ denote respectively the minimum eigenvalue and the maximum eigenvalue of matrix $(\mathbf{M})$.

In the remainder of this paper, we assume that the system has the same dimension at each mode, and that the mode $\mathbf{r}(t)$ and the continuous state $\mathbf{x}(t)$ are available for controller at each time t.

2.2 Tracking Control Law

In this subsection, we describe the control law which will be used in the rest of this paper to assure asymptotic tracking and disturbance rejection. The control law, we use here, is based on a given dynamics that we have to follow precisely. The control law requires the choice of four parameters $\mathbf{A}_r(i)$, $\mathbf{B}_r(i)$, $\mathbf{K}_1(i)$ and $\mathbf{K}_2(i)$, for all $i \in \mathcal{B}$. This robust control law is described by the following dynamics:

$$\dot{\mathbf{x}}_r(t) = \mathbf{A}_r(\mathbf{r}(t))\mathbf{x}_r(t) + \mathbf{B}_r(\mathbf{r}(t))\mathbf{y}(t), \tag{9}$$

$$\mathbf{u}(t) = \mathbf{K}_2(\mathbf{r}(t))\mathbf{x}(t) + \mathbf{K}_1(\mathbf{r}(t))\mathbf{x}_r(t), \tag{10}$$

where $\mathbf{x}_r(t) \in \mathbb{R}^{n_2}$; $\mathbf{A}_r(\mathbf{r}(t))$, $\mathbf{B}_r(\mathbf{r}(t))$, $\mathbf{K}_1(\mathbf{r}(t))$ and $\mathbf{K}_2(\mathbf{r}(t))$ are constant matrices with appropriate dimensions for each value of $\mathbf{r}(t)$. Their values are chosen to assure the required performances. Note that the eigenvalue assignment design technique can be used to determine these matrices for each value of $\mathbf{r}(t) \in \mathcal{B}$.

Equation (9) describes the dynamics that we have to track and Eq. (10) is the expression of the control law. It is assumed in the rest of this paper that the system is stochastically controllable and observable.

3 ROBUST STABILIZABILITY OF ULSMJP

The aim of this section is to establish, under some appropriate assumptions on the system's uncertainties, the sufficient condition which guarantees the stochastic stabilizability of the class of ULSMJP.

Let us consider the following uncertain dynamical systems:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}(\mathbf{r}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{r}(t))\mathbf{u}(t)\\ &\quad + \Delta\mathbf{A}(\mathbf{r}(t), t)\mathbf{x}(t) + \Delta\mathbf{B}(\mathbf{r}(t), t)\mathbf{u}(t), \end{aligned}\tag{11}$$

$$\begin{aligned}\mathbf{y}(t) &= \mathbf{C}(\mathbf{r}(t))\mathbf{x}(t) + \mathbf{D}(\mathbf{r}(t))\mathbf{u}(t) + \Delta\mathbf{C}(\mathbf{r}(t), t)\mathbf{x}(t)\\ &\quad + \Delta\mathbf{D}(\mathbf{r}(t), t)\mathbf{u}(t), \end{aligned}\tag{12}$$

$$\mathbf{x}(0) = \mathbf{x}_0,$$

where the matrices $\Delta\mathbf{A}(\cdot)$, $\Delta\mathbf{B}(\cdot)$, $\Delta\mathbf{C}(\cdot)$ and $\Delta\mathbf{D}(\cdot)$ are real-valued functions representing time-varying norm bounded uncertainties. Note that, when the Markov process is at time t at state i, these matrices are time-varying with bounded entries.

For each mode $i \in \mathcal{B}$, the admissible uncertainties in Eqs. (11)–(12) are assumed to be of the following forms:

$$\Delta\mathbf{A}(i,t) = \mathbf{H}_1(i)\mathbf{F}(i,t)\mathbf{E}_1(i), \tag{13}$$

$$\Delta\mathbf{B}(i,t) = \mathbf{H}_1(i)\mathbf{F}(i,t)\mathbf{E}_2(i), \tag{14}$$

$$\Delta\mathbf{C}(i,t) = \mathbf{H}_2(i)\mathbf{F}(i,t)\mathbf{E}_1(i), \tag{15}$$

$$\Delta\mathbf{D}(i,t) = \mathbf{H}_2(i)\mathbf{F}(i,t)\mathbf{E}_2(i), \tag{16}$$

where $\mathbf{H}_1(i)$, $\mathbf{H}_2(i)$, $\mathbf{E}_1(i)$ and $\mathbf{E}_2(i)$ are known constant matrices of appropriate dimensions and $\mathbf{F}(i,t)$ is an unknown matrix function satisfying the following condition:

$$\mathbf{F}'(i,t)\mathbf{F}(i,t) \leq \mathbf{I}, \quad \forall i \in \mathcal{B}, \quad t \geq 0, \tag{17}$$

with $\mathbf{I}$ the matrix identity of appropriate dimension and $\mathbf{F}(i,t)$ being Lebesgue measurable.

Remark 3.1 The parameter uncertainty structure as in (13)–(16), when $\mathbf{r}_t = i$, $i \in \mathcal{B}$, has been widely used in the problems of robust control and robust filtering of uncertain systems (see, e.g. [22,23] and the references therein) and many practical systems possess parameter uncertainties which can be either exactly modeled, or overbounded by (17). Observe that the unknown matrix $\mathbf{F}(i,t)$ in (13)–(16) can even be allowed to be state-dependent, i.e. $\mathbf{F}(i,t) = \mathbf{F}(i,\mathbf{x},t)$ as long as (17) is satisfied.

Combining the controller dynamics and the system dynamics, we obtain the following closed-loop systems dynamics:

$$\begin{aligned}
\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{x}}_r(t) \end{bmatrix}
&= \begin{bmatrix} \mathbf{A}(\mathbf{r}(t)) + \mathbf{B}(\mathbf{r}(t))\mathbf{K}_2(\mathbf{r}(t)) & \mathbf{B}(\mathbf{r}(t))\mathbf{K}_1(\mathbf{r}(t)) \\ \mathbf{B}_r(\mathbf{r}(t))\mathbf{C}(\mathbf{r}(t)) + \mathbf{B}_r(\mathbf{r}(t))\mathbf{D}(\mathbf{r}(t))\mathbf{K}_2(\mathbf{r}(t)) & \mathbf{A}_r(\mathbf{r}(t)) + \mathbf{B}_r(\mathbf{r}(t))\mathbf{D}(\mathbf{r}(t))\mathbf{K}_1(\mathbf{r}(t)) \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}_r(t) \end{bmatrix} \\
&\quad + \begin{bmatrix} \Delta\mathbf{A}(\mathbf{r}(t),t) + \Delta\mathbf{B}(\mathbf{r}(t),t)\mathbf{K}_2(\mathbf{r}(t)) & \Delta\mathbf{B}(\mathbf{r}(t),t)\mathbf{K}_1(\mathbf{r}(t)) \\ \mathbf{B}_r(\mathbf{r}(t))[\Delta\mathbf{C}(\mathbf{r}(t),t) + \Delta\mathbf{D}(\mathbf{r}(t),t)\mathbf{K}_2(\mathbf{r}(t))] & \mathbf{B}_r(\mathbf{r}(t))\Delta\mathbf{D}(\mathbf{r}(t),t)\mathbf{K}_1(\mathbf{r}(t)) \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}_r(t) \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\mathbf{y}(t) &= [\mathbf{C}(\mathbf{r}(t)) + \mathbf{D}(\mathbf{r}(t))\mathbf{K}_2(\mathbf{r}(t)) \;\; \mathbf{D}(\mathbf{r}(t))\mathbf{K}_1(\mathbf{r}(t))] \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}_r(t) \end{bmatrix} \\
&\quad + [\Delta\mathbf{C}(\mathbf{r}(t),t) + \Delta\mathbf{D}(\mathbf{r}(t),t)\mathbf{K}_2(\mathbf{r}(t)) \;\; \Delta\mathbf{D}(\mathbf{r}(t),t)\mathbf{K}_1(\mathbf{r}(t))] \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}_r(t) \end{bmatrix}.
\end{aligned}$$

Using the form of uncertainties given by Eqs. (13)–(16), we obtain the following dynamics:

$$\dot{\bar{\mathbf{x}}}(t) = [\bar{\mathbf{A}}(\mathbf{r}(t)) + \bar{\mathbf{H}}(\mathbf{r}(t))\mathbf{F}(\mathbf{r}(t), t)\bar{\mathbf{E}}(\mathbf{r}(t))]\bar{\mathbf{x}}, \tag{18}$$

$$\mathbf{y}(t) = [\bar{\mathbf{C}}(\mathbf{r}(t)) + \mathbf{H}_2(\mathbf{r}(t))\mathbf{F}(\mathbf{r}(t),t)\bar{\mathbf{E}}(\mathbf{r}(t))]\bar{\mathbf{x}}, \tag{19}$$

where

$$\bar{\mathbf{x}}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}_r(t) \end{bmatrix},$$

$$\bar{\mathbf{A}}(\mathbf{r}(t)) =$$
$$\begin{bmatrix} \mathbf{A}(\mathbf{r}(t)) + \mathbf{B}(\mathbf{r}(t))\mathbf{K}_2(\mathbf{r}(t)) & \mathbf{B}(\mathbf{r}(t))\mathbf{K}_1(\mathbf{r}(t)) \\ \mathbf{B}_r(\mathbf{r}(t))\mathbf{C}(\mathbf{r}(t)) + \mathbf{B}_r(\mathbf{r}(t))\mathbf{D}(\mathbf{r}(t))\mathbf{K}_2(\mathbf{r}(t)) & \mathbf{A}_r(\mathbf{r}(t)) + \mathbf{B}_r(\mathbf{r}(t))\mathbf{D}(\mathbf{r}(t))\mathbf{K}_1(\mathbf{r}(t)) \end{bmatrix},$$

$$\bar{\mathbf{H}}(\mathbf{r}(t)) = \begin{bmatrix} \mathbf{H}_1(\mathbf{r}(t)) \\ \mathbf{B}_r(\mathbf{r}(t))\mathbf{H}_2(\mathbf{r}(t)) \end{bmatrix},$$

$$\bar{\mathbf{E}}(\mathbf{r}(t)) = [\mathbf{E}_1(\mathbf{r}(t)) + \mathbf{E}_2(\mathbf{r}(t))\mathbf{K}_2(\mathbf{r}(t)) \quad \mathbf{E}_2(\mathbf{r}(t))\mathbf{K}_1(\mathbf{r}(t))],$$

$$\bar{\mathbf{C}}(\mathbf{r}(t)) = [(\mathbf{C}(\mathbf{r}(t)) + \mathbf{D}(\mathbf{r}(t))\mathbf{K}_2(\mathbf{r}(t)) \quad \mathbf{D}(\mathbf{r}(t))\mathbf{K}_1(\mathbf{r}(t))].$$

Before establishing the sufficient condition which guarantees the stochastic stabilizability robustness of ULSMJP, let us introduce the following proposition which will be used in the proof of the result of this section.

PROPOSITION 3.1 [23] *Given any constant* $\rho > 0$ *and matrices* $\mathbf{H}, \mathbf{E}, \mathbf{F}$ *with compatible dimensions such that* $\mathbf{F}'\mathbf{F} \leq \mathbf{I}$, *then for any* $\mathbf{x}$

$$2(\mathbf{x}'\mathbf{PHFEx}) \leq \rho(\mathbf{x}'\mathbf{PHH}'\mathbf{Px}) + \frac{1}{\rho}(\mathbf{x}'\mathbf{E}'\mathbf{Ex}). \tag{20}$$

THEOREM 3.1 *The system* (11)–(12) *is robust stochastically stabilizable for all admissible uncertainties if there exists a control law given by Eqs.* (9)–(10), *such that for a given selection of scalars* $\rho(i) > 0$, $i \in \mathcal{B}$, *the following set of N coupled algebraic Riccati equations*:

$$\bar{\mathbf{A}}'(i)\mathbf{P}(i) + \mathbf{P}(i)\bar{\mathbf{A}}(i) + \rho(i)\mathbf{P}(i)\bar{\mathbf{H}}(i)\bar{\mathbf{H}}'(i)\mathbf{P}(i) + \sum_{j\in\mathcal{B}} q_{ij}\mathbf{P}(j) + \frac{1}{\rho(i)}\bar{\mathbf{E}}'(i)\bar{\mathbf{E}}(i) + \mathbf{Q}(i) = 0 \tag{21}$$

has symmetric positive-definite solution, $\mathbf{P}(i)$ *for any given symmetric positive-definite matrix* $\mathbf{Q}(i)$.

Proof Suppose that the condition (21) is satisfied and let the candidate stochastic Lyapunov function $v(i, \bar{\mathbf{x}})$ be defined by:

$$v(i, \bar{\mathbf{x}}) = \bar{\mathbf{x}}'\mathbf{P}(i)\bar{\mathbf{x}}. \tag{22}$$

Consider now the weak infinitesimal operator $\tilde{\mathcal{A}}$ of the process $\{(\mathbf{r}(t), \bar{\mathbf{x}}(t)),\ t \in [0, T]\}$, which is the natural stochastic analog of the deterministic derivative (see [19]). This weak infinitesimal operator $\tilde{\mathcal{A}}$ is defined by

$$\tilde{\mathcal{A}}v(i, \bar{\mathbf{x}}) = \lim_{h \to 0} \frac{1}{h}[E\{v(\mathbf{r}(t+h), \bar{\mathbf{x}}(t+h)) \mid \bar{\mathbf{x}}(t),\ \mathbf{r}(t) = i\} - v(\mathbf{r}(t) = i,\ \bar{\mathbf{x}}(t))]$$

which is given by

$$\tilde{\mathcal{A}}v(i, \bar{\mathbf{x}}) = \bar{\mathbf{x}}'[\bar{\mathbf{A}}'(i)\mathbf{P}(i) + \mathbf{P}(i)\bar{\mathbf{A}}(i) + 2\mathbf{P}(i)\bar{\mathbf{H}}(i)\mathbf{F}(i,t)\bar{\mathbf{E}}(i) + \sum_{j \in \mathcal{B}} q_{ij}\mathbf{P}(j)]\bar{\mathbf{x}}. \tag{23}$$

In order to obtain the required upper bound for $\tilde{\mathcal{A}}v(i, \bar{\mathbf{x}})$, we will use the inequality of Proposition 3.1. By applying inequality (20) to various terms in the expression (23), we have

$$2[\bar{\mathbf{x}}'\mathbf{P}(i)\bar{\mathbf{H}}(i)\mathbf{F}(i,t)\bar{\mathbf{E}}(i)\bar{\mathbf{x}}] \le \rho(i)\bar{\mathbf{x}}'\mathbf{P}(i)\bar{\mathbf{H}}(i)\bar{\mathbf{H}}'(i)\mathbf{P}(i)\bar{\mathbf{x}} + \frac{1}{\rho(i)}\bar{\mathbf{x}}'\bar{\mathbf{E}}'(i)\bar{\mathbf{E}}(i)\bar{\mathbf{x}}.$$

Given any admissible uncertainties and using the condition (21), then the expression (23) becomes

$$\begin{aligned}\tilde{\mathcal{A}}v(i, \bar{\mathbf{x}}) &\le \bar{\mathbf{x}}'[\bar{\mathbf{A}}'(i)\mathbf{P}(i) + \mathbf{P}(i)\bar{\mathbf{A}}(i) + \rho(i)\mathbf{P}(i)\bar{\mathbf{H}}(i)\bar{\mathbf{H}}'(i)\mathbf{P}(i) \\ &\quad + \sum_{j \in \mathcal{B}} q_{ij}\mathbf{P}(j) + \frac{1}{\rho(i)}\bar{\mathbf{E}}'(i)\bar{\mathbf{E}}(i)]\bar{\mathbf{x}} \\ &\le -\bar{\mathbf{x}}'\mathbf{Q}(i)\bar{\mathbf{x}}.\end{aligned}$$

Dividing both sides by $v(i, \bar{\mathbf{x}})$, for $\bar{\mathbf{x}} \neq 0$, we obtain

$$\begin{aligned}\frac{\tilde{\mathcal{A}}v(i, \bar{\mathbf{x}})}{v(i, \bar{\mathbf{x}})} &\le -\frac{\bar{\mathbf{x}}'\mathbf{Q}(i)\bar{\mathbf{x}}}{\bar{\mathbf{x}}'\mathbf{P}(i)\bar{\mathbf{x}}} \\ &\le \frac{-\lambda_{\min}(\mathbf{Q}(i))}{\lambda_{\max}(\mathbf{P}(i))}.\end{aligned}$$

If we define a real number α as

$$\alpha = \min_{i \in \mathcal{B}} \left\{ \frac{\lambda_{\min}(\mathbf{Q}(i))}{\lambda_{\max}(\mathbf{P}(i))} \right\}$$

which is positive, then we obtain

$$\tilde{\mathcal{A}}v(i, \mathbf{x}) \leq -\alpha v(i, \bar{\mathbf{x}}).$$

Using now Dynkin's formula and Gronwall–Bellman lemma and the proof used by Ji and Chizeck [17], we obtain

$$\lim_{T \to \infty} E\left\{ \int_0^T \bar{\mathbf{x}}'(t)\bar{\mathbf{x}}(t)\,\mathrm{d}t \mid \bar{\mathbf{x}}_0, \mathbf{r}_0 \right\} \leq \bar{\mathbf{x}}'_0\tilde{\mathbf{P}}\bar{\mathbf{x}}_0,$$

where

$$\tilde{\mathbf{P}} = \max_{i \in \mathcal{B}} \frac{\mathbf{P}_i}{\alpha \|\mathbf{P}_i\|},$$

which proves the sufficient condition.

For the output, based on Eq. (19), we get

$$\begin{aligned}
\|\mathbf{y}(t)\|^2 =& \mathbf{x}'(t)(\mathbf{C}(\mathbf{r}(t)) + \mathbf{H}_2(\mathbf{r}(t))\mathbf{F}(\mathbf{r}(t), t)\bar{\mathbf{E}}(\mathbf{r}(t)))'(\mathbf{C}(\mathbf{r}(t)) \\
&+ \mathbf{H}_2(\mathbf{r}(t))\mathbf{F}(\mathbf{r}(t), t)\bar{\mathbf{E}}(\mathbf{r}(t)))\mathbf{x}(t) \\
=& \mathbf{x}'(t)(\mathbf{C}'(\mathbf{r}(t))\mathbf{C}(\mathbf{r}(t)) + 2\mathbf{C}'(\mathbf{r}(t))\mathbf{H}_2(\mathbf{r}(t))\mathbf{F}(\mathbf{r}(t), t)\bar{\mathbf{E}}(\mathbf{r}(t)) \\
&+ (\mathbf{H}_2(\mathbf{r}(t))\mathbf{F}(\mathbf{r}(t), t)\bar{\mathbf{E}}(\mathbf{r}(t)))'(\mathbf{H}_2(\mathbf{r}(t))\mathbf{F}(\mathbf{r}(t), t)\bar{\mathbf{E}}(\mathbf{r}(t)))\mathbf{x}(t).
\end{aligned} \tag{24}$$

By Proposition 3.1, we can get

$$\begin{aligned}
&2\mathbf{x}'\mathbf{C}'(\mathbf{r}(t))\mathbf{H}_2(\mathbf{r}(t))\mathbf{F}(\mathbf{r}(t), t)\bar{\mathbf{E}}(\mathbf{r}(t)) \\
&\leq \mathbf{x}'\mathbf{C}'(\mathbf{r}(t))\mathbf{H}_2(i)\mathbf{H}_2'(\mathbf{r}(t))\mathbf{C}(\mathbf{r}(t))\mathbf{x} + \mathbf{x}'\bar{\mathbf{E}}'(\mathbf{r}(t))\bar{\mathbf{E}}(\mathbf{r}(t))\mathbf{x} \\
&\leq (\|\mathbf{C}(\mathbf{r}(t))\|^2\|\mathbf{H}_2(\mathbf{r}(t))\|^2 + \|\bar{\mathbf{E}}(\mathbf{r}(t))\|^2)\|\mathbf{x}\|^2
\end{aligned}$$

and

$$\begin{aligned}
&\mathbf{x}'(\mathbf{H}_2(\mathbf{r}(t))\mathbf{F}(\mathbf{r}(t),t)\bar{\mathbf{E}}(\mathbf{r}(t)))'(\mathbf{H}_2(\mathbf{r}(t))\mathbf{F}(\mathbf{r}(t),t)\bar{\mathbf{E}}(\mathbf{r}(t)))\mathbf{x} \\
&\quad \leq \|\mathbf{H}_2(\mathbf{r}(t))\|^2\|\bar{\mathbf{E}}(\mathbf{r}(t))\|^2\|\mathbf{x}\|^2.
\end{aligned}$$

Then Eq. (24) can be rewritten as

$$\begin{aligned}
\|\mathbf{y}(t))\|^2 &\leq (\|\mathbf{C}(i)\|^2 + \|\bar{\mathbf{E}}(i)\|^2)(1 + \|\mathbf{H}_2(i)\|^2)\|\mathbf{x}\|^2 \\
&\leq \beta\|\mathbf{x}\|^2
\end{aligned}$$

with

$$\beta = \max_{i\in\mathcal{B}}\left\{(\|\mathbf{C}(i)\|^2 + \|\bar{\mathbf{E}}(i)\|^2)(1 + \|\mathbf{H}_2(i)\|^2)\right\}.$$

Then

$$\begin{aligned}
\lim_{T\to\infty} E\left\{\int_0^T \|\mathbf{y}(t)\|^2 \mathrm{d}t \,|\, \mathbf{y}_0, \mathbf{r}(0)\right\} &\leq \beta \lim_{T\to\infty}\left\{\int_0^T \|\mathbf{x}(t)\|^2 \mathrm{d}t \,|\, \mathbf{x}_0, \mathbf{r}(0)\right\} \\
&\leq \tilde{c}\|\mathbf{y}_0\|^2,
\end{aligned}$$

where $\tilde{c}$ is a positive constant.

Remark 3.2 The special case where $\mathbf{A}_r(i) = \mathbf{B}_r(i) = \mathbf{K}_1(i) = 0$ for all $i \in \mathcal{B}$, was considered by Boukas [5]. It represents the robust state feedback control under matching conditions. Under additional assumptions Boukas [5] proposes a procedure for the design of the robust controller which assures the robust stabilizability of the system under study.

Remark 3.3 The solution of the coupled Riccati equations type is given by algorithm of Abou-Kandil *et al.* [1]. The choice of the parameters $\rho(i), i = 1, \ldots, N$ is discussed in [22].

Based on the previous theorem, we can propose a design procedure for the robust controller. For instance, a way to design the robust controller can be obtained by following the procedure where the steps are as follows.

Step 1: Choose $\bar{\mathbf{A}}(i)$ such that the nominal system (18) is stochastically stable. In order, we propose the following method.

Choose adequately the desired n eigenvalues λ_k^i, $k = 1, 2, \ldots, n$ for each mode i, $i = 1, 2, \ldots, N$ to get the desired specifications; and construct $\bar{\mathbf{A}}(i)$ by using

$$\begin{aligned}\bar{\mathbf{A}}(i) &:= \mathbf{T}^{-1}(i)\mathrm{diag}\,[\lambda_1^i, \cdots, \lambda_n^i]\mathbf{T}(i)\\ &= \begin{bmatrix} \bar{\mathbf{A}}_1(i) & \bar{\mathbf{A}}_2(i) \\ \bar{\mathbf{A}}_3(i) & \bar{\mathbf{A}}_4(i) \end{bmatrix},\end{aligned}$$

where $\mathbf{T}(i)$ is any adequately invertible chosen matrix and diag( ) is a diagonal matrix.

Step 2: Given a positive-definite and symmetric matrix $\mathbf{Q}(i)$, initialize $\rho(i)$, $i = 1, 2, \ldots, N$, to some starting value; e.g. set $\rho(i) = 1$, $i = 1, 2, \ldots, N$. Determine the symmetric positive-definite matrix $\mathbf{P}(i)$, $i = 1, 2, \ldots, N$ by using the condition (21) and the algorithm [1]. If the solution is not positive-definite symmetric, then replace $\rho(i) = \rho(i)/2$ and repeat step 2.

Step 3: Determine the matrices $\mathbf{A}_r(i)$, $\mathbf{B}_r(i)$, $\mathbf{K}_1(i)$ and $\mathbf{K}_2(i)$ by using the matrix $\bar{\mathbf{A}}(i)$ such that

$$\begin{aligned}\mathbf{K}_1(i) &= \mathbf{B}^+(i)\bar{\mathbf{A}}_2(i),\\ \mathbf{K}_2(i) &= \mathbf{B}^+(i)(\bar{\mathbf{A}}_1(i) - \mathbf{A}(i)),\\ \mathbf{B}_r(i) &= \bar{\mathbf{A}}_3(i)(\mathbf{C}(i) + \mathbf{D}(i)\mathbf{K}_2(i))^+,\\ \mathbf{A}_r(i) &= \mathbf{A}_4(i) - \mathbf{B}_r(i)\mathbf{D}(i)\mathbf{K}_1(i),\end{aligned}$$

where $\mathbf{M}^+$ denotes the pseudo-inverse of matrix $\mathbf{M}$.

Step 4: Obtain the robust controller from Eqs. (9)–(10).

4 ILLUSTRATIVE EXAMPLE

Let us assume in this example a system with three modes, i.e. $\mathcal{B} = \{1, 2, 3\}$ and the transition rate matrix Λ between these modes is given by

$$\Lambda = \begin{bmatrix} -3 & 0.5 & 2.5 \\ 1 & -2 & 1 \\ 1.7 & 0.3 & -2 \end{bmatrix}.$$

The dynamics of the system in each mode is assumed to be described by the following differential equations:

mode 1

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -2.5 & 0.3 & 0.8 \\ 1 & -3 & 0.2 \\ 0 & 0.5 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0.707 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0.12\sin(\frac{1}{4}\pi t) & 0.15(\sin(\frac{1}{4}\pi t) + \cos(\frac{1}{4}\pi t)) & 0 \\ 0 & 0.125\cos(\frac{1}{4}\pi t) & 0.075\sin(\frac{1}{4}\pi t) \\ 0.12\sin(\frac{1}{4}\pi t) & 0.15\sin(\frac{1}{4}\pi t) & 0.075\sin(\frac{1}{4}\pi t) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0.09\sin(\frac{1}{4}\pi t) & 0.15\cos(\frac{1}{4}\pi t) & 0.12\sin(\frac{1}{4}\pi t) \\ 0.105\sin(\frac{1}{4}\pi t) & 0.125\cos(\frac{1}{4}\pi t) & 0.105\sin(\frac{1}{4}\pi t) \\ 0.195\sin(\frac{1}{4}\pi t) & 0 & 0.225\sin(\frac{1}{4}\pi t) \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix},$$

$$\mathbf{y}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix};$$

mode 2

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -2.5 & 1.2 & 0.3 \\ -0.5 & 5 & -1 \\ 0.25 & 1.2 & 5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0.707 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.707 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0.15\sin(\frac{1}{4}\pi t) & 0.135\sin(\frac{1}{4}\pi t) \\ 0.09\cos(\frac{1}{4}\pi t) + 0.0625\sin(\frac{1}{4}\pi t) & 0.15\cos(\frac{1}{4}\pi t) & 0.0625\sin(\frac{1}{4}\pi t) \\ 0.075\sin(\frac{1}{4}\pi t) & 0 & 0.075\sin(\frac{1}{4}\pi t) \end{bmatrix}$$

$$\times \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0.075\sin(\frac{1}{4}\pi t) & 0.105\sin(\frac{1}{4}\pi t) & 0 \\ 0.135\cos(\frac{1}{4}\pi t) & 0.135\sin(\frac{1}{4}\pi t) & 0.075\sin(\frac{1}{4}\pi t) \\ 0 & 0.15\sin(\frac{1}{4}\pi t) & 0.09\sin(\frac{1}{4}\pi t) \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix},$$

$$\mathbf{y}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix};$$

mode 3

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 2 & 1.5 & -0.4 \\ 2.2 & 3 & 0.7 \\ 1.1 & 0.9 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0.707 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0.125\sin(\frac{1}{4}\pi t) & 0.1\sin(\frac{1}{4}\pi t) & 0.125\sin(\frac{1}{4}\pi t) \\ 0.0875\cos(\frac{1}{4}\pi t) & 0.1\cos(\frac{1}{4}\pi t) & 0.075\cos(\frac{1}{4}\pi t) \\ 0.0875\cos(\frac{1}{4}\pi t) & 0.1\cos(\frac{1}{4}\pi t) & 0.075\cos(\frac{1}{4}\pi t) + 0.15\sin(\frac{1}{4}\pi t) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0.125\sin(\frac{1}{4}\pi t) & 0.125\sin(\frac{1}{4}\pi t) & 0.1625\sin(\frac{1}{4}\pi t) \\ 0.075\cos(\frac{1}{4}\pi t) & 0.0625\cos(\frac{1}{4}\pi t) & 0.0875\cos(\frac{1}{4}\pi t) \\ 0.075\cos(\frac{1}{4}\pi t) & 0.0625\cos(\frac{1}{4}\pi t) + 0.15\sin(\frac{1}{4}\pi t) & 0.0875\cos(\frac{1}{4}\pi t) + 0.125\sin(\frac{1}{4}\pi t) \end{bmatrix}$$

$$\times \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix},$$

$$\mathbf{y}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}.$$

We assume also that the initial conditions at time $t = 0$ are given by

$$\mathbf{x}(0) = [0\ 0\ 0]', \qquad \mathbf{x}_r(0) = [0\ 0\ 0]', \qquad \mathbf{P}_0 = [\tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}]'.$$

In this example, the norm bounded uncertainty form is used, that is, for each mode $i \in \mathcal{S}$, these uncertainties are given by

$$\mathbf{F}(1,t) = \mathbf{F}(2,t) = \mathbf{F}(3,t) = \begin{bmatrix} \sin(\frac{1}{4}\pi t) & 0 & 0 \\ 0 & \cos(\frac{1}{4}\pi t) & 0 \\ 0 & 0 & \sin(\frac{1}{4}\pi t) \end{bmatrix}$$

which satisfy the inequality $\mathbf{F}(i,t)'\mathbf{F}(i,t) \le \mathbf{I}, \quad \forall t \ge 0,\ i = 1, 2, 3.$

Let us also assume that the required matrices $\mathbf{H}_1(i)$, $\mathbf{E}_1(i)$ and $\mathbf{E}_2(i)$, $i \in \mathcal{B}$, are given as follows:

mode 1

$$\mathbf{H}_1(1) = \begin{bmatrix} 0.3 & 0.3 & 0 \\ 0 & 0.25 & 0.3 \\ 0.3 & 0 & 0.3 \end{bmatrix}, \quad \mathbf{E}_1(1) = \begin{bmatrix} 0.4 & 0.5 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}, \quad \mathbf{E}_2(1) = \begin{bmatrix} 0.3 & 0 & 0.4 \\ 0 & 0.5 & 0 \\ 0.35 & 0 & 0.35 \end{bmatrix};$$

mode 2

$$\mathbf{H}_1(2) = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.3 & 0.25 \\ 0 & 0 & 0.3 \end{bmatrix}, \ \mathbf{E}_1(2) = \begin{bmatrix} 0 & 0.5 & 0.45 \\ 0.3 & 0.25 & 0 \\ 0.25 & 0 & 0.25 \end{bmatrix}, \ \mathbf{E}_2(2) = \begin{bmatrix} 0.25 & 0.35 & 0 \\ 0.45 & 0 & 0 \\ 0 & 0.5 & 0.3 \end{bmatrix};$$

mode 3

$$\mathbf{H}_1(3) = \begin{bmatrix} 0.25 & 0 & 0.25 \\ 0 & 0.25 & 0 \\ 0 & 0.25 & 0.3 \end{bmatrix}, \ \mathbf{E}_1(3) = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.35 & 0.4 & 0.3 \\ 0 & 0.4 & 0.5 \end{bmatrix}, \ \mathbf{E}_2(3) = \begin{bmatrix} 0.5 & 0 & 0.25 \\ 0.3 & 0.25 & 0.35 \\ 0 & 0.5 & 0.4 \end{bmatrix}.$$

Based on the proposed algorithm if we choose the same eigenvalues for the three modes and the matrices $\mathbf{T}(1) = \mathbf{T}(2) = \mathbf{T}(3) = \mathbf{T}$ with the numerical values

$$\begin{array}{ccccccl} -18.75, & -8.25, & -13.6, & -10.5, & -12.45, & -9.65 & \text{eigenvalues} \end{array}$$

$$\mathbf{T} = \begin{bmatrix} 0.0457 & 0.4023 & -0.1372 & -0.5602 & 0.2583 & -0.0265 \\ -0.7501 & 0.4814 & -0.7165 & 0.4495 & -0.4371 & 0.1074 \\ -0.5860 & -0.3267 & 0.5407 & 0.4965 & 0.1708 & -0.5800 \\ 0.2930 & 0.4092 & -0.2145 & 0.2616 & 0.2265 & 0.6445 \\ -0.0328 & -0.1719 & -0.2388 & -0.3489 & 0.5324 & -0.2399 \\ 0.0703 & 0.5502 & 0.2693 & 0.2180 & -0.6151 & -0.4225 \end{bmatrix},$$

we obtain the following expression for $\bar{\mathbf{A}}(i)$, $i = 1, 2$:

$$\bar{\mathbf{A}}(1) = \bar{\mathbf{A}}(2) =$$

$$\begin{bmatrix} -8.6949 & 0.1744 & 0.6304 & -0.0723 & -1.8088 & 0.0356 \\ -4.1814 & -15.2306 & -1.9633 & 4.8802 & 5.2402 & 6.0079 \\ -9.7455 & -1.8326 & -16.0953 & 1.8841 & 10.4481 & 5.9355 \\ 5.3195 & 1.1901 & 2.4710 & -11.5805 & -6.1133 & -2.8973 \\ -0.3145 & -0.4335 & -0.4160 & -1.1461 & -11.9730 & 0.0513 \\ 1.0874 & 0.7281 & 0.9453 & 1.6556 & 2.3253 & -9.6257 \end{bmatrix},$$

which gives in turn the following matrices for the required controller:

$$\mathbf{A}_r(i) = \begin{bmatrix} -11.5805 & -6.1133 & -2.8973 \\ -1.1461 & -11.9730 & 0.0513 \\ 1.6556 & 2.3253 & -9.6257 \end{bmatrix}, \quad i = 1, 2, 3;$$

$$\mathbf{B}_r(i) = \begin{bmatrix} 5.3195 & 1.1901 & 2.4710 \\ -0.3145 & -0.4335 & -0.4160 \\ 1.0874 & 0.7281 & 0.9453 \end{bmatrix}, \quad i = 1, 2, 3;$$

$$\mathbf{K}_1(i) = \begin{cases} \begin{bmatrix} -0.1023 & -2.5584 & 0.0504 \\ 4.8802 & 5.2402 & 6.0079 \\ 1.8841 & 10.4481 & 5.9355 \end{bmatrix} & \text{if the mode is 1,} \\ \begin{bmatrix} -0.1023 & -2.5584 & 0.0504 \\ 4.8802 & 5.2402 & 6.0079 \\ 2.6649 & 14.7781 & 8.3953 \end{bmatrix} & \text{if the mode is 2,} \\ \begin{bmatrix} -0.1023 & -2.5584 & 0.0504 \\ 4.8802 & 5.2402 & 6.0079 \\ 1.8841 & 10.4481 & 5.9355 \end{bmatrix} & \text{if the mode is 3;} \end{cases}$$

$$\mathbf{K}_2(i) = \begin{cases} \begin{bmatrix} -8.7622 & -0.1777 & -0.2399 \\ -5.1814 & -12.2306 & -2.1633 \\ -9.7455 & -2.3326 & -14.0953 \end{bmatrix} & \text{if the mode is 1,} \\ \begin{bmatrix} -8.7622 & -1.4506 & 0.4673 \\ -3.6814 & -20.2306 & -0.9633 \\ -14.1379 & -4.2894 & -29.8378 \end{bmatrix} & \text{if the mode is 2,} \\ \begin{bmatrix} -15.1272 & -1.8750 & 1.4574 \\ -6.3814 & -18.2306 & -2.6633 \\ -10.8455 & -2.7326 & -14.0953 \end{bmatrix} & \text{if the mode is 3.} \end{cases}$$

The required robustness conditions for stochastic stability (21) are verified for $\rho(i) = 1$, $\mathbf{Q}(i) = \mathbf{I}$ for all $i = 1, 2, 3$ and the simulation results (Fig. 1) show the robust controller trajectories for ULSMJP. We have used Matlab on PC to do the simulation.

5 CONCLUSION

In this paper, we have dealt with the uncertain class of linear systems with Markovian jumping parameters. Under the assumptions of the controllability of the system, the observability of the continuous state

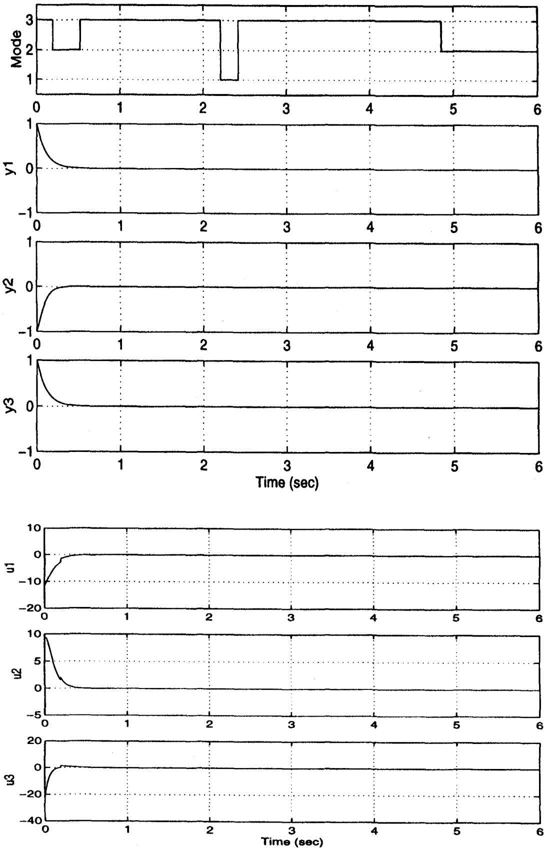

FIGURE 1 Evolution mode, states and controls trajectories versus times.

$\mathbf{x}(t)$ and the mode i and the norm bounded uncertainties, a robust controller design approach in the time domain has been presented for this class of systems. Sufficient condition guaranteeing the stability robustness has been established which is in terms of a set of coupled Ricatti equations.

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