Mathematical Problems in Engineering, 2002, Vol. 8(3), pp. 233–248 Taylor & Francis Taylor & Francis Group

Design of a Decentralized Detection Filter for a Large Collection of Interacting LTI Systems*

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(Received 11 March 2002)

In this paper, the problem of designing a decentralized detection filter for a large homogeneous collection of LTI systems is considered. The collection of systems considered here draws inspiration from platoons of vehicles, and the considered interactions amongst systems in the collection are banded and lower triangular, mimicking the typical "look-ahead" nature of interactions in a platoon of vehicles. A fault in a system propagates to other systems in the collection via such interactions.

The decentralized detection filter for the collection is composed of interacting detection filters, one for each system. The feasibility of communicating the state estimates to other systems in the collection is assumed here. An important concern is the propagation of state estimation errors. In order that the state estimation errors not amplify as they propagate, a $\mathcal{H}_\infty$ constraint on the state estimation error propagation dynamics is imposed. A sufficient condition for constructing a decentralized detection filter for the collection is presented. An example is provided to illustrate the design procedure.

Key words: Detection filter, Decentralized estimation, Fault detection, String stability, Interconnected systems

1 INTRODUCTION

In this paper, we consider a large, homogeneous collection of interacting systems. The collection of systems considered here draws inspiration from a platoon of vehicles. The structure of each system, including its interactions with other systems in the collection, is assumed identical and linear. The interactions are assumed to be banded and lower triangular, mimicking the "look-ahead" nature of interactions amongst vehicles in a platoon. This structure for the collection is reasonable, as in vehicle platoons [14].

A malfunction in any system can degrade the performance of the entire collection and can lead to serious consequences. Moreover, fault diagnosis in a large collection of systems is a complex problem because any system in the collection can fail and each system can fail in a number of ways; a fault in any system propagates from one system to another in finite time. The problem may further be compounded by the disparity in information available to each system. The problem of detecting and isolating a fault consists of two tasks: identifying the system in which a fault has occurred and then diagnosing the fault in the failed system. It is conceivable that the level of difficulty of diagnosing a fault in the collection depends on the nature of the fault.

* Presented at the ASME International Mechanical Engineering Congress and Exposition, November 2001; submitted to Mathematical Problems in Engineering, October 2001.

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ISSN 1024-123X print; ISSN 1563-5147 online

A persistent, but a novel theme in this paper is the design of decentralized estimation algorithms that are insensitive to the size of the collection. For example, in the case of vehicle platoons, it is desirable that the diagnosability of a fault in any vehicle in the platoon be independent of the number of the vehicles. Such an insensitivity enables one to equip every vehicle with identical hardware and software for FDI and yet be able to guarantee the detection and isolation of faults in the collection. In this sense, insensitivity of the algorithms to the size of the collection is central to achieving the scalability and robustness of the designed detection filter.

There are two kinds of redundancy – physical and analytical. Building physical redundancy in a system involves employing redundant sensors to measure or infer the same variable. Analytical redundancy involves the usage of the knowledge of the model and sensors to obtain the estimate of states and the outputs. The directionality of the evolution of output residuals (the difference between estimated and actual outputs) provides the required diagnostic information. We focus mainly on the problem of detecting *actuation failures* using analytic redundancy in this paper. For an exposition of failure detection and tolerance, we will refer the readers to [4, 6, 11].

The approach of fault detection and identification using fault detection filters was first developed by Beard [1] and Jones [3]. A geometric approach for the development of a detection filter is presented by Massoumnia [5]; we briefly review the results of this paper. A complete algebraic approach for the development of a detection filter, using frequency domain techniques is presented by Viswanadham *et al.* [7]. An eigensystem assignment problem for the design of the detection filter is considered by White and Speyer [8].

In this paper, we combine the approaches of the decentralized control and detection filter design to arrive at a decentralized detection filter for a large collection of interacting LTI systems. A decentralized detection filter is constructed by designing local detection filters for each system in the collection and allowing the local detection filters to communicate the state estimates, as in Šiljak *et al.* [13]. The design of decentralized detection filters was considered by [2] for a platoon of vehicles; however, this approach does not deal with the propagation of state estimation errors.

This paper is organized as follows: In the second section, we briefly review the basic concepts and algorithms associated with the design of a detection filter; the chief source for the review is [5]. In the third section, we design a detection filter for a large collection of interacting LTI systems. We provide examples in both sections to illustrate the concepts and the procedures involved.

2 DESIGN OF A DETECTION FILTER FOR ACTUATION FAULTS

The design of a Luenberger observer for an LTI system assumes accurate knowledge of the plant and the inputs. This is a strong assumption and it limits its practical usage. In this paper, we will make this assumption nevertheless.

In the absence of any disturbances or failures in the plant, actuators or the sensors, as with any observer, the state estimation error and the output estimation error must decay. In the event of an occurrence of a fault, when modeled as an unknown input, it is desirable that the evolution of the output residual of the observer corresponds in a one-to-one manner with the occurrence of a fault. If such an observer can be designed, then it is called a detection filter. There are many instances in which one may not be able to design a detection filter; for example, if there are more faults (modeled as unknown inputs) than the outputs, it is impossible to distinguish between the occurrence of a fault in correspondence with the evolution of the output residual. In this section, we will briefly review the results relevant to the design of a detection filter.

Model of a Plant with Actuation Faults We consider a LTI system with multiple faults of the form:

$$\dot{x} = Ax + F\mu, \tag{1}$$

$$y = Cx. \tag{2}$$

Here $\mu = [\mu_1, \mu_2, \ldots, \mu_r]^T$ are the faults which affect the system through the fault vectors $F = [f_1, f_2, \ldots, f_r]$. Clearly, if $r > m$, the number of outputs, one cannot distinguish between the effects of all the faults from the output. We will, therefore, assume that $r = m$. In addition, we make the following *assumptions* about this system,

1. The matrix C is epic and of rank m. This assumption implies that all the measurements are independent; the underlying assumption throughout the development of the detection filter is that sensors are reliable, and hence, one can weed the linearly dependent measurements without diminishing the ability to design reliable diagnostic algorithms.
2. The given pair (C, A) is completely observable.

We will design a Luenberger observer of the form:

$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x}),$$
$$\hat{y} = C\hat{x}.$$

With $\tilde{x} = x - \hat{x}$, as the state estimation error and $\tilde{y} = y - C\hat{x}$ as the output residual. We obtain the following error dynamics

$$\dot{\tilde{x}} = (A - LC)\tilde{x} + F\mu,$$
$$\tilde{y} = C\tilde{x}.$$

To illustrate the concept of a detection space, consider the case when only the ith fault occurs, *i.e.*, $\mu_i \neq 0$, while $\mu_j = 0, j \neq i$. The estimation error of the state evolves according to

$$\tilde{x}(t) \in \sum_{i=0}^{n-1} (A - LC)^i \mathbf{f}_i, \quad \text{with } \mathbf{f}_i = \mathrm{Im}[f_i] \text{ and } \tilde{x}(0) = 0$$

$$\text{and } \tilde{y}(t) \in C \sum_{i=0}^{n-1} (A - LC)^i \mathbf{f}_i.$$

This can be written in terms of the controllable subspace of $(A - LC, f_i)$ as

$$\tilde{x}(t) \in \langle A - LC | \mathbf{f}_i \rangle,$$
$$\tilde{y}(t) \in C\langle A - LC | \mathbf{f}_i \rangle.$$

There are two issues associated with the design of a detection filter:

1. The evolution of the output residual $\tilde{y}(t)$ must have one-to-one correspondence with the occurrence of a fault. This is achievable if the output residual evolves with time in a predetermined subspace specific to a fault. Clearly, such subspaces should not overlap; otherwise, simultaneous detection and isolation of multiple faults will not be possible.
2. At the same time, one should have the freedom to specify all the eigenvalues of $(A - LC)$ arbitrarily in the left half of the complex plane. This is required to tailor the speed of decay to meet the reliability specifications on the speed of detection/diagnosis.

Since fault isolation requires that output residuals evolve in mutually independent subspaces of the output space, this can happen only if the controllability subspaces associated with two different faults are linearly independent (otherwise, the assumption of observability will be violated since each controllability subspace is a $(A - LC)$ invariant subspace). If one were to look at each fault individually, it will make sense to determine the smallest controllability subspace for each fault, so that one can make the controllability subspaces corresponding to two different faults non-overlapping. As one changes L, the controllability subspaces change for each fault; in particular, if one places some eigenvalues of the observer, using an appropriate L, at the transmission zeros of the triplet (C, A, F), then the corresponding dimensions of the controllability subspaces, $\mathcal{W}_i^* = \langle A - LC|\mathbf{f}_i\rangle$, $i = 1, \ldots, m$ will be smallest and will be given by the components of vector relative degree corresponding to the triplet (F^T, A^T, C^T). If k_i is the ith component of the vector relative degree of the triplet (F^T, A^T, C^T), then the output residuals will evolve along $CA^{k_i-1}f_i$; since $\mathbf{f}_i$ is a one dimensional subspace, the image of $CA^{k_i-1}f_i$ is also a one-dimensional subspace. For clarity of notation, we will refer to $A^{k_i-1}f_i$ as $\bar{f}_i$. Clearly, a necessary condition for the output residuals to evolve in mutually non-overlapping subspaces is that the following decoupling matrix for the triplet (F^T, A^T, C^T) be non-singular:

$$D := [C\bar{f}_i \quad C\bar{f}_2 \quad \cdots \quad C\bar{f}_m].$$

This condition is referred to as output separability condition.

However, such a choice of L restricts the arbitrary assignability of eigenvalues of $(A - LC)$. In particular, it is a problem if any of the transmission zeros of the triplet (C, A, f_i) lie in the right half of the complex plane.

The problem of stabilizing the observer requires one to deal with a larger subspace than $\mathcal{W}_i^*$. The smallest subspace which overcomes this restriction is $\mathcal{T}_i^*$, which is referred to as the detection space of fault vector, $\mathbf{f}_i$. Since $C\mathcal{T}_i^*$ should be aligned along Cf_i, so that output separability condition is satisfied, $\mathcal{T}_i^*$ must include $\mathcal{W}_i^*$ and a subspace of the kernel of C; in particular, $\mathcal{T}_i^* = \mathcal{W}_i^* + \mathcal{V}_i^*$, where $\mathcal{V}_i^*$ is the space associated with the transmission zeros of (C, A, f_i).

We will use $\mathbf{m}$ to mean the set $\{1, 2, \ldots, m\}$.

Output Separability We say a family of subspaces $\{\mathcal{W}_i^*,\ i \in \mathbf{m}\}$ is *output separable* if $C\mathcal{W}_i^* \cap (\sum_{j \neq i} C\mathcal{W}_j^*) = 0$, $i \in \mathbf{m}$. Since $C\mathcal{T}_i^* = C\mathcal{W}_i^*$, a set of detection spaces is output separable iff the matrix D given below is non-singular:

$$D := [CA^{k_1-1}f_1 \quad CA^{k_2-1}f_2 \quad \cdots \quad CA^{k_m-1}f_m].$$

Example Consider the following system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -120 & -74 & -15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mu_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mu_2$$

$$y_i = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

For this system, the matrix, D, can be shown to be singular and the faults are not output separable.

Output separability is a necessary condition for detecting and isolating multiple faults in a LTI system simultaneously. One requires an additional condition – mutual detectability of faults.

We denote $\mathcal{W}^* = \sum_{i=1}^{m} \mathcal{W}_i^*$, and $\mathcal{T}^*$ as the infimal (C, A) unobservable subspace that contains $\mathcal{W}^*$. In particular, $\mathcal{T}^* = \mathcal{W}^* + \mathcal{V}^*$, where $\mathcal{V}^*$ is the space associated with the transmission zeros of (C, A, F). Clearly, $\mathcal{T}_1^* \oplus \mathcal{T}_2^* \oplus \cdots \oplus \mathcal{T}_m^* \subseteq \mathcal{T}^*$. The direct sum of individual detection spaces may not necessarily equal $\mathcal{T}^*$, because two faults can occur simultaneously and in a certain proportion so that the corresponding output will be identically zero for appropriate initial conditions and will decay for all other initial conditions, if the observer is stabilized. In order to stabilize the observer and have arbitrary assignability, a mutual detectability condition must be satisfied, see [5].

Mutual Detectability The detection spaces $\{\mathcal{T}_i^*, i \in \mathbf{m}\}$ are defined to be *mutually detectable* if and only if

$$\mathcal{T}_1^* \oplus \mathcal{T}_2^* \oplus \cdots \oplus \mathcal{T}_m^* = \mathcal{T}^*.$$

A consequence of mutual detectability is that, for square minimal systems, $\dim(\mathcal{T}_1^*) + \dim(\mathcal{T}_2^*) + \cdots + \dim(\mathcal{T}_m^*) = n$. If faults are mutually detectable, they are automatically output separable, but the converse may not hold; we illustrate this point by the following example.

$$\text{Let } A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad \text{and} \quad F = \begin{bmatrix} -4 & 0 \\ 0 & -3 \\ -2 & 3 \end{bmatrix}$$

For this example, one can show that

$$\bar{F} = F = \begin{bmatrix} -4 & 0 \\ 0 & -3 \\ -2 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} -6 & 3 \\ -2 & 0 \end{bmatrix}.$$

The fault vectors are clearly output separable. Let $\ker(C) = \text{span}(u) = \text{span}([1 \quad 1 \quad -1]^T)$. There are only three possibilities for detection spaces, $\mathcal{T}_1^* = \text{span}\{f_1\}$, $\mathcal{T}_2^* = \text{span}\{f_2, u\}$, or $\mathcal{T}_2^* = \text{span}\{f_2\}$, $\mathcal{T}_1^* = \text{span}\{f_1, u\}$, or $\mathcal{T}_2^* = \text{span}\{f_2\}$, $\mathcal{T}_1^* = \text{span}\{f_1\}$. The first two choices are ruled out since, $(A - LC)u = Au = f_1 + f_2 + 2u$, indicating that at least one of the detection spaces from the first two choices will not be $(A - LC)$ invariant for any choice of L. Should one proceed to make $\bar{f}_1, \bar{f}_2$ detection spaces with an appropriate L, by virtue of their being $(A - LC)$ invariant,

$$(A - LC)[\bar{f}_1 \quad \bar{f}_2 \quad u] = [\bar{f}_1 \quad \bar{f}_2 \quad u] \begin{bmatrix} \lambda_1 & 0 & 1 \\ 0 & \lambda_2 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

In this case, only λ_1, λ_2 can be chosen freely, while the third eigenvalue of the observer is fixed at 2.

Construction of a Detection Filter If the set of faults are mutually detectable, one way to compute the detection space for $\mathbf{f}_i$, is to construct a *maximal rank matrix*, U_i, such that:

$$AU_i = U_i\bar{X}_i + A^{k_i-1}f_i\bar{C}_i,$$
$$CU_i = 0,$$

for some $\bar{X}_i$, $\bar{C}_i$. Once U_i, is computed, $\mathcal{T}_i^*$ is the column space of $T_i := [U_i \ \bar{f}_i]$. One can express AU_i, as T_iX_i, where $X_i = [\bar{X}_i^T \ \bar{C}_i^T]^T$. Clearly, $\mathcal{T}_i^*$ contains $\mathcal{W}_i^*$ and the space associated with the transmission zeros of f_i. Define $n_i := \text{rank}(T_i)$ and L to be

$$L := ([A\bar{f}_1 \quad A\bar{f}_2 \quad \cdots \quad A\bar{f}_m] - [T_1 \quad T_2 \quad \cdots \quad T_m])\bar{L}, \text{ where} \tag{3}$$

$$\bar{L} := \begin{bmatrix} L_1 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & L_2 & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & L_m \end{bmatrix} D^{-1}, \tag{4}$$

and the column vectors, L_i are of size n_i. With this choice of L, one can show that $(A - LC)T_i = T_i(A_i - L_iC_i)$, where $A_i = [X_i \ 0]$ and $C_i = [0 \ 0 \cdots 1]$. One can show that (A_i, C_i) is a completely observable pair and L_i can be chosen to arbitrarily assign the spectrum of $(A - LC)$ restricted to $\mathcal{T}_i^*$.

Example Consider the following problem from [7],

$$\dot{x} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2,$$
$$y = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix} x.$$

For this problem, one can show that

$$T_1 = [\bar{f}_1] \Rightarrow T_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad T_2 = [U_2 \quad \bar{f}_2] \Rightarrow T_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 3 \end{bmatrix}, \text{ and,}$$
$$\bar{L} = \begin{bmatrix} -2 & 0 \\ 0 & -12 \\ 0 & -7 \end{bmatrix}, \quad L = \begin{bmatrix} 1.5 & 1.5 \\ -5.5 & 6.5 \\ -21.0 & 21.0 \end{bmatrix},$$

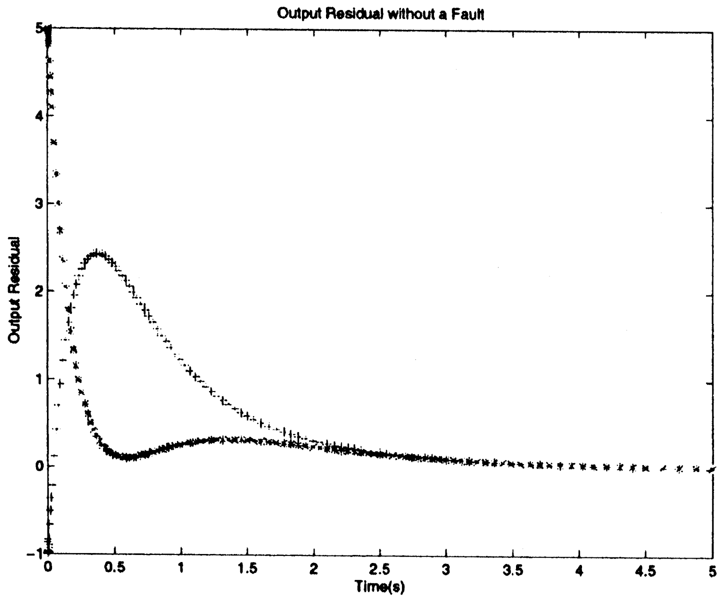

FIGURE 1 Output residual without any fault.

to place the eigen values of the observer at −2, −3, −4. On implementing the detection filter we get the error dynamics as

$$\dot{\tilde{x}} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -10 & 1 \\ 0 & -42 & 3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2, \text{ and}$$

$$\tilde{y} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \tilde{x}.$$

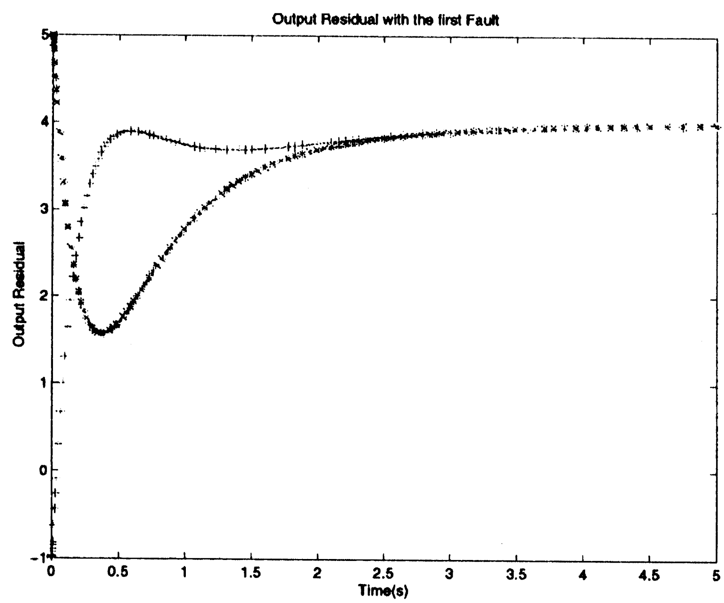

FIGURE 2 Output residual with first fault.

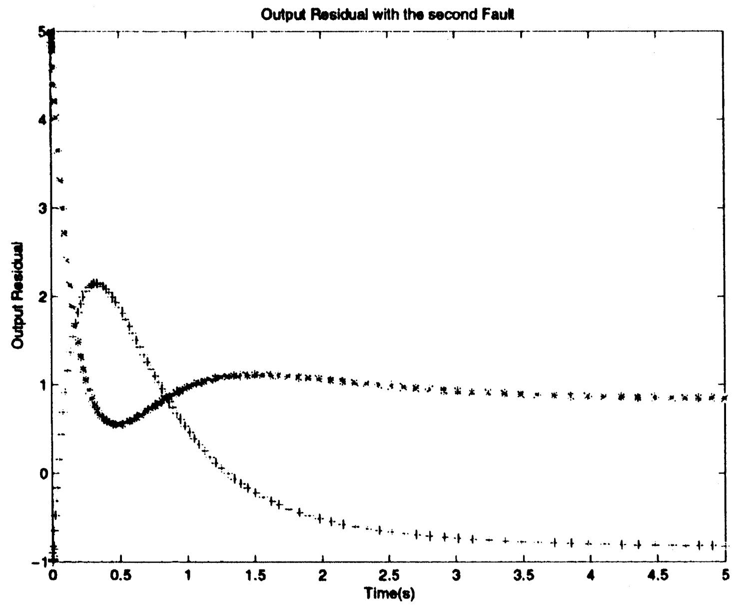

FIGURE 3 Output residual with second fault.

Simulation Results We simulate fault inputs as constant functions of time. We consider two different cases – in the first case, only the first fault occurs; in the second case, only the second fault occurs. Figure 1 shows that, in the absence of the faults, the closed loop is asymptotically stable. Figure 2 shows that, if the fault appears through the channel f_1, then the output residual evolves in the output space along a fixed direction $Cf_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Figure 3 shows that, if the fault appears through the channel f_2, then the output residual evolves in the output space along a fixed direction $Cf_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

3 DECENTRALIZED DETECTION FILTER DESIGN

Motivation This paper draws inspiration from the problem of detecting and isolating faults in each of the vehicles in a platoon. The platoon of vehicles interact with each other by employing a "look-ahead" vehicle following algorithm. Information such as velocity and acceleration are communicated between the vehicles and are available for synthesizing the control laws in the platoon. A fault in the communication or in any subsystem of a vehicle affects its following vehicles and can potentially lead to accidents. It is, therefore, desirable to construct identical, local detection filters for each of the vehicles so that fault detection and isolation can be achieved in a decentralized manner.

In this paper, we will restrict ourselves to actuation faults; in fact, we even model a communication failure between ith and $(i+1)$th systems in the collection as an actuator failure in the $(i+1)$th system, since it affects the control law for the $(i+1)$th system. We construct a fault detection and isolation system for the collection by designing a detection filter for each of the systems and by allowing the exchange of state estimates amongst the systems to account for their interactions.

Due to the process of information exchange between detection filters, a fault in any system gets propagated. As a result, one must take into account the following issues:

1. In the absence of any fault, there will be state estimation errors stemming from the inexact knowledge of the initial state of any system. This can lead to propagation of errors from one system to another and can lead to erroneous diagnosis. Therefore, such propagation of estimation errors must be attenuated for reliability and for localizing the fault isolation.
   The attenuation of state estimation errors in the absence of faults is important for the scalability of the detection filter algorithm. It is important in automobile or formation flying applications, that no matter what the size of the vehicle string be, stringing the "local" detection filters together should not result in either an erroneous or a delayed diagnosis.
   In this paper, this issue is formulated via a $\mathcal{H}_\infty$ constraint on the estimation error propagation transfer functions, which will be discussed in detail, in this section.
2. In the presence of a fault, there must be a one-to-one correspondence between the evolution of residual and the occurrence of a fault (this includes propagated fault). The diagnosis problem is harder; one must now identify the system in which a fault occurs and isolate the exact fault in that system.

From Section 2, we know that if the subsystem is observable and the faults are mutually detectable, then, one can design a detection filter which guarantees closed loop stability and directional properties.

System Dynamics Consider a string of LTI systems modeled by the following system of differential equations:

$$\dot{x}_i(t) = Ax_i(t) + B(z_{i-1} + v_i) + F\bar{\mu}_i, \tag{5}$$
$$y_i(t) = Cx_i(t), \quad i = 1, 2, \ldots, \tag{6}$$

with the following structure for interconnections: $z_i = Kx_i$, where for the ith subsystem $x_i(t) \in \Re^n$ represents the state, $y_i(t) \in \Re^r$ is the output and $\mu_i(t) \in \Re^r$ is the fault of the ith subsystem at time t. The notation is that $x_0 \equiv 0$ for all time t. We consider an infinite string so as to address the issue of scalability of the detection filter algorithm.

The matrix $F = [f_1, f_2, \ldots, f_r]$ contains the fault channels through which the faults $\bar{\mu}_i = [\mu_{1,i}, \mu_{2,i}, \ldots, \mu_{r,i}]$ appear in the ith subsystem. The vector v_i represents those communication failures which get propagated through the interconnection dynamics as actuation faults.

We make the following assumptions about the ith subsystem,

- The system $(A, (B\ F), C)$ is minimal.
- The fault channels (B, F) are mutually detectable.

Since (A, C) observable, we design a detection filter (a Luenberger observer with directional properties) for each system with a detection gain L:

$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + B\hat{z}_{i-1}(t) + L(y_i - C\hat{x}_i(t)) \tag{7}$$

and obtain the error dynamics as

$$\dot{\tilde{x}}_i = (A - LC)\tilde{x}_i + B(\tilde{z}_{i-1} + v_i) + F\bar{\mu}_i, \tag{8}$$
$$\tilde{y}_i = C\tilde{x}_i, \tag{9}$$
$$\tilde{z}_i = K\tilde{x}_i, \tag{10}$$

where $\tilde{x}_i$ is the state estimation error of the ith system.

We consider two problems: the first problem is of general interest and deals with Decentralized Scalable Observer Design (DSOD) for the collection of systems considered here; the second problem deals with the Decentralized Scalable Detection Filter Design (DSDFD). For both problems, we are looking for a Luenberger of the form given by Eq. (7); hence, we will assume that the term $\hat{z}_{i-1}$ is communicable to the ith system.

Problem Statement for DSOD Find an observer gain matrix, L, so that $A - LC$ is Hurwitz and the following error propagation constraint is met:

$$\|K(jwI - A + LC)^{-1}B\|_\infty < 1. \tag{11}$$

Problem Statement for DSDFD Find a detection filter gain matrix, L, so that $A - LC$ is Hurwitz and the following error propagation constraint is met:

$$\|K(jwI - A + LC)^{-1}B\|_\infty < 1. \tag{12}$$

Remarks

1. The DSOD problem deals with decentralized estimation of states in the absence of faults and is primarily concerned with the amplification of state estimation errors. The DSDFD problem further requires a one-to-one correspondence between the evolution of the output residuals and the occurrence of a fault.
2. The $\hat{H}_\infty$ constraint $\|K(jwI - A + LC)^{-1}B\|_\infty < 1$ ensures that the state estimation errors do not anmplify as they propagate through the interaction terms. In fact, if L is chosen such that $(A - LC)$ is Hurwitz and $\|K(jwI - A - LC)^{-1}B\|_\infty < 1$, then, in the absence of any faults, one can show the following [9, 10]:
   (a) Given $c > 0$, there exists a $\delta > 0$ such that
   $$\sup_i \|\tilde{x}_i(0)\| < \delta \Rightarrow \sup_{t\geq 0} \sup_i \|\tilde{x}_i(t)\| < \varepsilon,$$
   and furthermore,
   (b) $\lim_{t\to\infty} \|\tilde{x}_k(t)\| = 0$ for all k.
   In other words, if the initial state estimation errors are small, the state estimation errors continue to remain small in the absence of any faults; furthermore, they will decay to zero asymptotically in time.
3. A solution to the DSOD problem is readily obtained from the dual of a standard result in $\mathcal{H}_\infty$ literature: $\exists L$ such that
   (a) $(A - LC)$ is Hurwitz, and
   (b) $\|K(jwI - A + LC)^{-1}B\|_\infty < 1$, iff $\exists P = P^T > 0$ such that
   $$AP + PA^T + P(K^TK - C^TC)P + BB^T < 0.$$
   The observer gain is synthesized as $L = PC^T$.
   The solution to the DSDFD problem is tricky, because L must be of a certain form (given by Eqs. (3) and (4)) to ensure one-to-one correspondence between the evolution of the residual and the occurrence of a fault; the form for L is given in Section 2, where the free parameters, L_i, are chosen to assign the eigenvalues of $(A_i - L_iC_i)$.

Solution to the DSDFD Problem A solution to this problem may not exist, if there is insufficient information. The insolvability of the DSDFD problem manifests in two forms: first, the mutual detectability condition may not be met, and second, the error propagation constraint may not be satisfiable. Since we have assumed that the mutual detectability condition is met by every system in the collection, we provide a sufficient condition for the solvability of the DSDFD problem in terms of Riccati inequalities that reflect the error propagation constraints.

Our approach is to construct the detection spaces for the ith system, so as to satisfy the directionality constraints on the output evolution in the event of a fault occurrence and then see if the stabilization and error propagation constraints can be met.

Let the column space of the matrix $T_j, j = 1, \ldots, r+1$ denote the detection space for the jth fault. We will treat a fault in communicating $\hat{z}_{i-1}$ as the $(r+1)$th actuation fault of the ith system in the collection.

Define the matrix T as $T = [T_1, T_2, \ldots, T_{r+1}]$. Since $(A - LC)T_j = T_j(A_j - L_jC_j)$, one can write $[A - LC]T = T\Lambda$, where

$$\Lambda = \begin{bmatrix} A_1 - L_1C_1 & 0 & 0 & \cdots \\ 0 & A_2 - L_2C_2 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_{r+1} - L_{r+1}C_{r+1} \end{bmatrix}.$$

The above equation may be rewritten as:

$$\Lambda = \underbrace{\begin{bmatrix} A_1 & 0 & 0 & \cdots \\ 0 & A_2 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_{r+1} \end{bmatrix}}_{\bar{A}} - \underbrace{\begin{bmatrix} L_1 & 0 & 0 & \cdots \\ 0 & L_2 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & L_{r+1} \end{bmatrix}}_{\bar{L}} \underbrace{\begin{bmatrix} C_1 & 0 & 0 & \cdots \\ 0 & C_2 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{r+1} \end{bmatrix}}_{\bar{C}}$$

The question is whether $L_1, \ldots, L_{r+1}$ can be chosen such that Λ is Hurwitz and the $\hat{H}_\infty$ constraint is satisfied.

To answer this question, we use a similarity transformation; since $\Lambda = T^{-1}(A - LC)T$, the condition for non-amplification of state estimation errors may be expressed as,

$$\| \underbrace{KT}_{\bar{K}}(jwI - \Lambda)^{-1} \underbrace{T^{-1}B}_{\bar{B}} \|_\infty < 1, \text{ or,}$$
$$\|\bar{K}(jwI - \bar{A} + \bar{L}\bar{C})^{-1}\bar{B}\|_\infty < 1.$$

Since Λ is block diagonal, this condition may be expressed as

$$\left\| \sum_{p=1}^{r+1} \bar{K}_p(jwI - A_p + L_pC_p)^{-1}\bar{B}_p \right\|_\infty < 1,$$

where $K = [\bar{K}_1 \cdots \bar{K}_{r+1}]$ and $\bar{B}^T = [\bar{B}_1^T \cdots \bar{B}_{r+1}^T]^T$.

A sufficient condition for solvability of DSDFD is the following: If for some $\gamma_1, \ldots, \gamma_{r+1}$ such that $\gamma_1 + \cdots + \gamma_{r+1} < 1$, there exist symmetric positive definite matrices, $P_i, i = 1, \ldots, r+1$ satisfying the inequalities:

$$A_i P_i + P_i A_i^T + P_i\left(\bar{K}_i^T \bar{K}_i - \frac{\bar{C}_i^T \bar{C}_i}{\gamma_i^2}\right) P_i + \bar{B}_i \bar{B}_i^T < 0, \quad i = 1, \ldots, r+1.$$

The corresponding gains are given by $L_i = P_i \bar{C}_i^T / \gamma_i, i = 1, \ldots, r+1$ and the detection filter is constructed using Eqs. (3) and (4).

The solvability of the set of Riccatti inequalities is linked to the information available for diagnosis; in particular, if the entire state information is available, then the set of Riccatti inequalities is always solvable.

Example We consider a platoon of N vehicles following a lead vehicle on a highway. Longitudinal control laws for each vehicle in a platoon, say the ith vehicle, use the lead vehicle's velocity (v_l) and acceleration (a_l) in addition to the relative velocity of the ith vehicle and preceding velocity ($v_i - v_{i-1}$), acceleration of the ($i - 1$)th vehicle (a_{i-1}) and the distance between the ith vehicle and the ($z - 1$)th vehicle. Each vehicle in the platoon is assigned a slot of length α_i; the abscissa of the rear bumper of the ith vehicle with respect to a fixed point O on the road is denoted by x_i; for $i = 1, 2, \ldots, N$. ε_i denotes the deviation of the ith vehicle's position from its assigned position. Hence, we have $\varepsilon_i = x_i - x_{i-1} + \alpha_i$ for $i = 1, 2, \ldots, N$. α_i is the desired intervehicular distance. Each vehicle is equipped with sensors and communication links to measure position, velocity and acceleration. We use the linear vehicle longitudinal dynamics developed in [15]:

$$\dddot{x}_i = u_i. \tag{13}$$

The above differential equation can be written in a state space form:

$$\begin{bmatrix} \dot{x}_i \\ \dot{v}_i \\ \dot{a}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_i \\ v_i \\ a_i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_i + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mu_i$$

We choose the control law μ_i as,

$$u_i = -k_1 \varepsilon_i - k_2 \dot{\varepsilon}_i + k_a \ddot{\varepsilon}_i - k_3(\dot{x}_i - \dot{x}_l) + k_l a_i \tag{14}$$

where the subscript l refers the lead vehicle in the platoon. k_1, k_2, k_3, k_l, k_a are design constants. $x_i(t)$, $\dot{x}_i(t)$, $\ddot{x}_i(t)$ refers to the position, velocity and acceleration of the ith vehicle at time t. We express the longitudinal dynamics shown in the Eq. (10), in terms the position error and its higher derivatives as, $\dddot{\varepsilon}_i = \dddot{x}_i - \dddot{x}_{i-1}$; Substituting Eq. (11) into Eq. (10) and use the definition of spacing error, ε_i, we have,

$$\dddot{\varepsilon}_i = -k_1 \varepsilon_i - (k_2 + k_3)\dot{\varepsilon}_i + k_a \ddot{\varepsilon}_i + k_1 \varepsilon_{i-1} + k_2 \dot{\varepsilon}_{i-1} - k_a \ddot{\varepsilon}_{i-1} \tag{15}$$

We choose the design parameters from [15] and model the communication failures as actuator failures. In addition, we add an extra fault input so as to make the system square. Writing the Eq. (12) in state space we get,

$$\begin{bmatrix} \dot{\varepsilon}_i \\ \ddot{\varepsilon}_i \\ \dddot{\varepsilon}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -120 & -74 & -15 \end{bmatrix} \begin{bmatrix} \varepsilon_i \\ \dot{\varepsilon}_i \\ \ddot{\varepsilon}_i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left([120 \quad 49 \quad 15] \begin{bmatrix} \varepsilon_{i-1} \\ \dot{\varepsilon}_{i-1} \\ \ddot{\varepsilon}_{i-1} \end{bmatrix} + \mu_{i,1} \right)$$

$$+ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} (\mu_{i,2} - \mu_{i-1,2}) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \mu_{i,3}$$

$$y_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_i \\ \dot{\varepsilon}_i \\ \ddot{\varepsilon}_i \end{bmatrix}$$

The fault vectors in the system are:

$$f_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad f_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad f_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

The fault vector, f_1, is associated with internal faults. The fault vector, f_2, is associated with communication failure. The three faults are output separable and can also shown to be mutually detectable. The detection spaces associated with the faults is given by

$$T = [\bar{T}_1 \quad T_2 \quad T_3] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

From the invariance property of the detection space we have, $[A - LC]T = T[\bar{A} - \bar{L}\bar{C}]$. The condition for non-amplification of state estimation errors may be written as:

$$\left\| [120 \quad 49 \quad 15] \left(j\omega I - \begin{bmatrix} \bar{l}_1 & 0 & 0 \\ 0 & \bar{l}_2 & 0 \\ 0 & 0 & \bar{l}_3 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\|_\infty \leq 1,$$

$$\Rightarrow \lim_{\omega \to \infty} \frac{15}{|(j\omega - \bar{l}_3)|} < 1$$

Therefore, we choose $\bar{l}_1 = -10$, $\bar{l}_2 = -5$ and $\bar{l}_3 = -20$. We get the detection gain L as,

$$L = \begin{bmatrix} 10 & 1 & 0 \\ 0 & 5 & 1 \\ -120 & -74 & 5 \end{bmatrix}$$

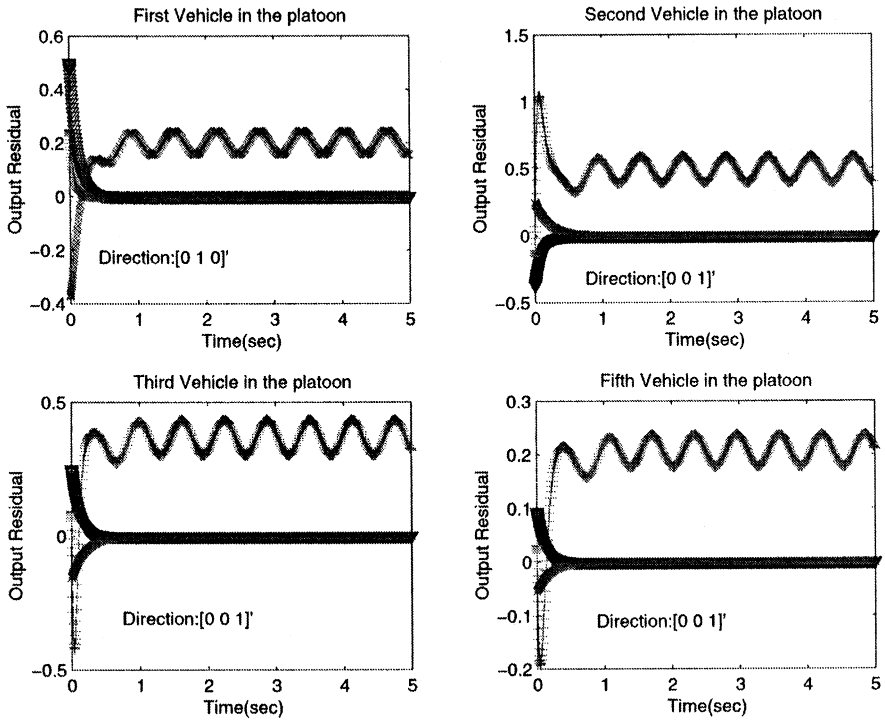

FIGURE 4 Estimation error of different vehicles with the first fault in the first vehicle.

and the error dynamics as

$$\dot{\tilde{x}}_i = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -20 \end{bmatrix} \tilde{x}_i + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mu_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [\,120 \quad 49 \quad 15\,] \tilde{x}_{i-1} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \mu_3$$

$$\tilde{z}_i = \begin{bmatrix} 120 \\ 49 \\ 15 \end{bmatrix} \tilde{x}_i$$

3.1 Simulation Results

We consider a string of 5 vehicles in the platoon. We consider a scenario, where there is an internal fault in the first vehicle appearing through the fault channel f_1. The magnitude of the fault is $1 + 0.5\sin(10t)\,\mathrm{m/s^2}$. This corresponds to the case when the actuation system fails or produces extraneous forces in the first vehicle.

The remaining vehicles in the substring do not have any internal faults. The internal fault of the first vehicle gets propagated through the communicated estimates of the state of the first vehicle through the interconnection matrix, B. Since each vehicle has three states, the initial estimation error in the jth state of the ith vehicle is purposely chosen to be $(i+j)(-0.5)^{i+j}$, so that on the plot, one can see the estimation errors of all states of all vehicles.

The purpose of this simulation is to demonstrate that the fault in first vehicle is isolated and that the state estimation errors satisfy the condition of non-amplification of errors, thereby localizing the fault in the first system.

We observe in the Figure 4 that in the event the internal fault occurs in first vehicle through the channel f_1; the estimation error in relative velocity is about 0.2 m/sec. the output residuals associated with the first vehicle propagate in the direction $Cf_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. We also observer in the Figure 4 that the internal fault gets propagated through the channel, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and output residual associated with the acceleration of each vehicle which gets affected due the interconnection gets attenuated upstream. In fact, the residuals in acceleration in the second, third and fifth vehicles, from the Figure 4, are respectively about 0.5, 0.4 and $0.2\,\mathrm{m/s^2}$. The negative eigenvalues of L ensure that in the absence of faults, closed loop of each subsystem is asymptotically stable. Therefore, we see that the detection filter so designed has the properties of the detection filter and also ensures stability in the interconnections.

Acknowledgements

The authors thank Professor S. P. Bhattacharyya of the Electrical Engineering department at Texas A&M University for his lectures on Geometric Control Theory which have helped us immensely in understanding some of the papers in the literature. The authors acknowledge the financial support of Allied Signal Truck Brake Systems, Inc., Elyria, OH, and of the Southwest University Transportation Center at TTI, College Station.

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