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Multiperiod Hydrothermal Economic Dispatch by an Interior Point Method

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This paper presents an interior point algorithm to solve the multiperiod hydrothermal economic dispatch (HTED). The multiperiod HTED is a large scale nonlinear programming problem. Various optimization methods have been applied to the multiperiod HTED, but most neglect important network characteristics or require decomposition into thermal and hydro subproblems. The algorithm described here exploits the special bordered block diagonal structure and sparsity of the Newton system for the first order necessary conditions to result in a fast efficient algorithm that can account for all network aspects. Applying this new algorithm challenges a conventional method for the use of available hydro resources known as the peak shaving heuristic.

Key words: Power systems, economic dispatch, interior point methods

AMS Subject Classification: 90C51, 65F99, 65F50

1 INTRODUCTION

Hydro generators play an important role in a deregulated environment due to their fast response times on both start-up and in changing output. The determination of economic hydro and thermal generation levels that satisfy both physical and operational constraints is known as hydrothermal economic dispatch (HTED). Traditional HTED algorithms [6, 8] are largely static since the problem is decomposed by time period, and the solutions are coordinated over the time horizon using heuristics. In addition, they do not account for important network constraints.

The multiperiod HTED is a dynamic problem, that determines optimal hydro and thermal generation settings over a time horizon. Recent approaches to the HTED are introduced in Refs. [4] and [10] which both include more complete network models and allow for the dynamic nature of the multiperiod HTED. In Ref. [4] a successive approximation algorithm is used to find an approximate solution. In Ref. [10] the problem is decomposed into thermal and hydro subproblems with solutions coordinated through a set of Lagrange multipliers.

The interior point algorithm presented in this paper produces a fast, efficient solution without requiring decomposition and coordination. The algorithm takes advantage of the

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special bordered block diagonal structure and sparsity of the Newton system for the first order necessary conditions.

The algorithm was tested using several different hydro systems of varying sizes. The results include consideration of different models for hydro generator water discharge rate. The optimal dispatch for nonlinear discharge rates distributes the hydro generation throughout the time horizon rather than using the hydro primarily in the peak demand periods. Allocating hydro primarily to the peak demand periods is a common heuristic for HTED known as peak shaving. The results presented here challenge the optimality of this heuristic.

The formulation of the multiperiod HTED and the derivation of the interior point algorithm are presented in Section 2. Numerical results are presented in Section 3 and conclusions are drawn in Section 4.

2 PROBLEM FORMULATION

The economic dispatch problem is given in general form as

$$
\begin{aligned}
&\min_{p_{i,t},p^h_{i,t}} \sum_{t=1}^{N_t}\sum_{i=1}^{N_g} C_i(p_{i,t}) \\
&\text{subject to} \\
&\sum_{i=1}^{N_g} p_{i,t} + \sum_{i=1}^{N_h} p^h_{i,t} = d_t && t = 1,\dots,N_t \\
&\underline{p}_i \le p_{i,t} \le \overline{p}_i && i = 1,\dots,N_g; \\
& && t = 1,\dots,N_t \\
&\underline{p}^h_i \le p^h_{i,t} \le \overline{p}^h_i && i = 1,\dots,N_n; \\
& && t = 1,\dots,N_t \\
&\underline{f}_l \le \sum_{i=1}^{N_g} \Gamma_{l,i} p_{i,t} + \sum_{i=1}^{N_h} \Gamma_{l,i} p^h_{i,t} \le \overline{f}_l && l = 1,\dots,N_l \\
&\sum_{i=1}^{N_t} \gamma(p^h_{i,t}) \le \overline{g}_i && i = 1,\dots,N_h
\end{aligned}
\tag{1}
$$

where

- N_t is the number of time periods
- N_g is the number of thermal generators
- N_h is the number of hydro generators
- N_l is the number of transmission lines in the network
- $p_{i,t}$ is the generation level of thermal generator i at time t
- $p^h_{i,t}$ is the generation level of hydro generator i at time t
- $C_i(p_{i,t})$ are the thermal generation costs, usually assumed quadratic
- $\underline{p}_i$ and $\overline{p}_i$ are the maximum and minimum thermal generation limits
- $\underline{p}^h_i$ and $\overline{p}^h_i$ are the maximum and minimum hydro generation limits
- $\underline{f}_l$,$\overline{f}_l$, are the maximum and minimum capacities of transmission line l
- $\Gamma_{l,i}$ determines the power flow in transmission line l as a function of generator output via a DC load flow
- $\gamma(p^h_{i,t})$ represents the water discharge rate of hydro generator i as a function of generation level
- $\overline{g}_i$ represents the maximum water capacity available for hydro generator i

Assuming the generation costs $C_i(p_{i,t})$ are quadratic, this is a quadratic programming problem with both equality and inequality constraints. The number of variables is equal to the product of the number of generators and the number of time intervals, $(N_g + N_h) \times N_t$. It is important to note that in the absence of the water capacity constraints on the hydro generators, the problem could be decomposed by time period and solved as N_t individual problems with $N_g + N_h$ variables. For any practical number of generators and time periods, the multiperiod HTED is extremely large and requires special solution methods.

While most HTED algorithms rely on decomposing the problem into hydro and thermal subproblems and using various methods for coordinating the solutions, a careful use of linear algebra to exploit the problem structure and sparse matrix methods to expedite the computations produces an efficient interior point algorithm that does not require decomposition. In addition, the algorithm can be easily adapted to include nonlinear constraints associated with the optimal power flow problem by a method such as Ref. [5].

The interior point algorithm developed here for the multiperiod HTED is based on the following methodology

1. Convert all inequality constraints to equality constraints by introducing nonnegative slack variables.
2. Form the Lagrangian for the entire problem. A logarithmic barrier function is used to enforce the nonnegativity of the slack variables.
3. Differentiate to form the Karush-Kuhn-Tucker (KKT) or first order necessary conditions.
4. Iteratively find an approximate solution to the KKT conditions using Newton's method.

To simplify the derivation of the KKT conditions, it is useful to re-write the problem using vector notation. Letting $p_t = [p_{1,t}, p_{2,t}, \ldots, p^h_{1,t}, \ldots, p^h_{N_h,t}]^{\mathrm{T}}$ for $t = 1, 2, \ldots, N_t$, $p = [p_1, p_2, \ldots, p_{N_t}]^{\mathrm{T}}$, $F = [\Gamma_{l,i}]$, $\overline{g} = [\overline{g}_1, \overline{g}_2, \ldots, \overline{g}_{N_h}]$ and assuming the thermal generation costs are quadratic and hydro generation incurs no cost, we can formulate the multiperiod HTED in the following way

$$
\begin{array}{ll}
\displaystyle\min_{p_t} \sum_{t=1}^{N_T} \frac{1}{2} p_t^{\mathrm{T}} Q p_t + c^{\mathrm{T}} p_t & \\
\text{subject to} & \\
e^{\mathrm{T}} p_t = d_t & t = 1, \ldots, N_t \\
\underline{p} \le p_t \le \overline{p} & t = 1, \ldots, N_t \\
\underline{f} \le F p_t \le \overline{f} & t = 1, \ldots, N_t \\
\gamma(p) \le \overline{g} &
\end{array} \qquad (2)
$$

where e is a vector consisting entirely of ones. Adding nonnegative slack variables $s_{1,t}$, $s_{2,t}$, $s_{3,t}$, $s_{4,t}$, s_5 results in time following system for $t = 1, \ldots, N_t$

$$
\begin{array}{l}
\displaystyle\min_{p_t} \frac{1}{2} p_t^{\mathrm{T}} Q p_t + c^{\mathrm{T}} p_t \\
\text{subject to} \\
e^{\mathrm{T}} p_t = d_t \\
\quad p_t = \overline{p} - s_{1,t} \\
\quad p_t = \underline{p} + s_{2,t}
\end{array}
$$

$$\begin{aligned} Fp_t &= \overline{f} - s_{3,t} \\ Fp_t &= \underline{f} + s_{4,t} \\ \gamma(p) &= \overline{g} - s_5 \\ s_{1,t}, s_{2,t}&, s_{3,t}, s_{4,t}, s_5 \geq 0 \end{aligned} \tag{3}$$

The Lagrangian with logarithmic barrier function for the system (3) is

$$\begin{aligned} L = & \sum_{t=1}^{N_T} \frac{1}{2} p_t^{\mathrm{T}} Q p_t + c^{\mathrm{T}} p_t \\ & + \sum_{t=1}^{N_T} \left\{ \lambda_t (e^{\mathrm{T}} p_t - d_t) + y_{1,t}^{\mathrm{T}} (p_t - \overline{p} + s_{1,t}) \right\} \\ & + \sum_{t=1}^{N_T} \left\{ y_{2,t}^{\mathrm{T}} (-p_t + \underline{p} + s_{2,t}) + y_{3,t} (Fp_t - \overline{f} + s_{3,t}) \right\} \\ & + \sum_{t=1}^{N_T} \left\{ y_{4,t}^{\mathrm{T}} (-Fp_t + \underline{f} + s_{4,t}) \right\} + y_5^{\mathrm{T}} (\gamma(p) - \overline{g} + s_5) \\ & - \sum_{i=1}^{N_t} \mu \left\{ \sum_{k=1}^{n_1} \ln \left(s_{1,t}^k \right) + \sum_{k=1}^{n_2} \ln \left(s_{2,t}^k \right) \right\} \\ & - \sum_{i=1}^{N_t} \mu \left\{ \sum_{k=1}^{n_3} \ln \left(s_{3,t}^k \right) + \sum_{k=1}^{n_4} \ln \left(s_{4,t}^k \right) \right\} \\ & - \mu \sum_{k=1}^{n_5} \ln \left(s_5^k \right) \end{aligned} \tag{4}$$

where λ, $y_{1,t}$, $y_{2,t}$, $y_{3,t}$, $y_{4,t}$ and y_5 are nonnegative Lagrange multipliers, μ is a nonnegative barrier parameter, and $n_1, n_2, \ldots, n_5$ are the length of vectors $s_{1,t}, \ldots, s_5$. The first order necessary (KKT) conditions for system (4) are

$$\begin{aligned} Qp_t + c + e\lambda_t + y_{1,t} - y_{2,t} + F^{\mathrm{T}} y_{3,t} - F^{\mathrm{T}} y_{4,t} + G_{p_t} y_5 &= 0 \\ e^{\mathrm{T}} p_t - d_t &= 0 \\ p_t - \overline{p} + s_{1,t} &= 0 \\ -p_t + \underline{p} + s_{2,t} &= 0 \\ Fp_t - \overline{f} + s_{3,t} &= 0 \\ -Fp_t + \underline{f} + s_{4,t} &= 0 \\ \gamma(P) - \overline{g} + s_5 &= 0 \\ y_{i,t} - \frac{\mu}{s_{i,t}} &= 0 \\ y_5 - \frac{\mu}{s_5} &= 0 \end{aligned} \tag{5}$$

where Gp_t is the Jacobian matrix for $\gamma(p)$. The resulting Newton system for the approximate solution of (5) is

$$Q\Delta p_t + e\Delta\lambda_t + \Delta y_{1,t} - \Delta y_{2,t} + F^{\mathrm{T}} \Delta y_{3,t} - F^{\mathrm{T}} \Delta y_{4,t} + G_{p_t} \Delta y_5 = b_p \tag{6}$$

$$e^{\mathrm{T}} \Delta p_t = b_\lambda \tag{7}$$

$$\Delta p_t + \Delta s_{1,t} = b_{y_1} \tag{8}$$

$$-\Delta p_t + \Delta s_{2,t} = b_{y_2} \tag{9}$$

$$F\Delta p_t + \Delta s_{3,t} = b_{y_3} \tag{10}$$

$$-F\Delta p_t + \Delta s_{4,t} = b_{y_4} \tag{11}$$

$$G_{p_t}\Delta p_t + \Delta s_5 = b_{y_5} \tag{12}$$

$$S_{i,t}\Delta Y_{i,t} + Y_{i,t}\Delta S_{i,t} = b_s \tag{13}$$

$$S_5\Delta Y_5 + Y_5\Delta S_5 = b_{s_5} \tag{14}$$

where $S_{i,t} = \operatorname{diag}(s_{i,t})$, $Y_{i,t} = \operatorname{diag}(y_{i,t})$, $S_5 = \operatorname{diag}(s_5)$, $Y_5 = \operatorname{diag}(y_5)$ and

$$\begin{aligned}
b_p &= -Qp_t - c - e\lambda_t - y_{1,t} + y_{2,t} - F^{\mathrm{T}}y_{3,t} + F^{\mathrm{T}}y_{4,t} - G_{p_t}y_5 \\
b_\lambda &= -e^{\mathrm{T}}p_t + d_t \\
b_{y_1} &= -p_t + \overline{p} - s_{1,t} \\
b_{y_2} &= p_t - \underline{p} - s_{2,t} \\
b_{y_3} &= -Fp_t + \overline{f} - s_{3,t} \\
b_{y_4} &= Fp_t - \underline{f} - s_{4,t} \\
b_{y_5} &= G_{p_t}p_t + \overline{g} - s_5 \\
b_s &= \mu e - S_{i,t}Y_{i,t} \\
b_{s_5} &= \mu e - S_5 Y_5
\end{aligned} \tag{15}$$

We can eliminate $\Delta s_{i,t}$ and Δs_5 using (13) and (14). Then

$$\Delta s_{i,t} = Y_{i,t}^{-1}(b_s - S_{i,t}\Delta y_{i,t}) \tag{16}$$

and

$$\Delta s_5 = Y_5^{-1}(b_{s_5} - S_5\Delta y_5). \tag{17}$$

The resulting system is

$$Q\Delta p_t + e\Delta\lambda_t + \Delta y_{1,t} - \Delta y_{2,t} + F^{\mathrm{T}}\Delta y_{3,t} - F^{\mathrm{T}}\Delta y_{4,t} + G_{p_t}\Delta y_5 = b_p \tag{18}$$

$$e^{\mathrm{T}}\Delta p_t = b_\lambda \tag{19}$$

$$\Delta p_t - Y_{1,t}^{-1}S_{1,t}\Delta y_{1,t} = \tilde{b}_{y_1} \tag{20}$$

$$-\Delta p_t - Y_{2,t}^{-1}S_{2,t}\Delta y_{2,t} = \tilde{b}_{y_2} \tag{21}$$

$$F\Delta p_t - Y_{3,t}^{-1}S_{3,t}\Delta y_{3,t} = \tilde{b}_{y_3} \tag{22}$$

$$-F\Delta p_t - Y_{4,t}^{-1}S_{4,t}\Delta y_{4,t} = \tilde{b}_{y_4} \tag{23}$$

$$G_{p_t}\Delta p_t - Y_5^{-1}S_5\Delta y_5 = \tag{24}$$

where

$$\begin{aligned}
\tilde{b}_{y_1} &= b_{y1} - \mu Y_{1,t}^{-1}e \\
\tilde{b}_{y_2} &= b_{y2} - \mu Y_{2,t}^{-1}e \\
\tilde{b}_{y_3} &= b_{y3} - \mu Y_{3,t}^{-1}e \\
\tilde{b}_{y_4} &= b_{y4} - \mu Y_{4,t}^{-1}e \\
\tilde{b}_{y_5} &= b_{y5} - \mu Y_5^{-1}e
\end{aligned} \tag{25}$$

The entire Newton system can now be written in matrix from in the following way

$$\begin{pmatrix} A_1 & 0 & 0 & \cdots & 0 & (G_{p_t}^1)^{\mathrm{T}} \\ 0 & A_2 & 0 & 0 & \cdots & (G_{p_t}^2)^{\mathrm{T}} \\ & & \ddots & & & \\ & & & & A_{N_T} & (G_{p_t}^{N_t})^{\mathrm{T}} \\ G_{p_t}^1 & G_{p_t}^2 & & & G_{p_t}^{N_t} & D_5 \end{pmatrix} \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_{N_t} \\ \Delta y_5 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{N_t} \\ \tilde{b}_{y_5} \end{pmatrix} \tag{26}$$

where A_i is the symmetric matrix

$$A_i = \begin{pmatrix} Q & e & I & -I & F^{\mathrm{T}} & -F^{\mathrm{T}} \\ e & 0 & 0 & 0 & 0 & 0 \\ I & 0 & D_1 & 0 & 0 & 0 \\ -I & 0 & 0 & D_2 & 0 & 0 \\ F & 0 & 0 & 0 & D_3 & 0 \\ -F & 0 & 0 & 0 & 0 & D_4 \end{pmatrix} \tag{27}$$

$$\Delta_i = [\Delta P_i\ \Delta\lambda_i\ \Delta y_{1,i}\ \Delta y_{2,i}\ \Delta y_{3,i}\ \Delta y_{4,i}]^{\mathrm{T}}$$

$$b_i = [b_p\ b_\lambda\ \tilde{b}_{y_1}\ \tilde{b}_{y_2}\ \tilde{b}_{y_3}\ \tilde{b}_{y_4}]^{\mathrm{T}}$$

$D_i = Y_{i,t}^{-1}S_{i,t}$, $D_5 = Y_5^{-1}S_5$ and $G_{p_t}^i$ is the appropriate block of G_{p_t} padded with zeros, *i.e.*

$$G_{p_t} = \left[\tilde{G}_{p_t}^1\ \tilde{G}_{p_t}^2 \cdots \tilde{G}_{p_t}^{N_t}\right]$$

and

$$G_{p_t}^i = \left[\tilde{G}_{p_t}^i\ :\ 0\ :\ 0\ :\cdots:\ 0\right].$$

Carefully examining the structure of system (26) we can see that it is a bordered block diagonal system. In addition, the system is sparse. Efficient solution requires taking advantage of these two aspects of the system.

By elimination, system (26) reduces to

$$\left[D_5 - \sum_{i=1}^{N_t}\left(G_{p_t}^i\right)^{\mathrm{T}} A_i^{-1} G_{p_t}^i\right]\Delta y_5 = \tilde{b}_{y_5} - \sum_{i=1}^{N_t}\left(G_{p_t}^i\right)^{\mathrm{T}} A_i^{-1} b_i \tag{28}$$

It is important to notice that

$$\Delta_i = A_i^{-1} b_i$$

would be the solution to system (26) in the absence of the hydro energy constraint

$$\gamma(p) \leq \bar{g}. \tag{29}$$

Therefore, the Eq. (28) can be solved efficiently in the following way

- Solve thermal economic dispatch for each time period to determine $A_i^{-1} b_i$. This is accomplished by factoring $A_i = L_i D_i L_i^{\mathrm{T}}$ and using forward-backward substitution.
- Form the sum $\sum_{i=1}^{N_t} (G_{p_t}^i)^{\mathrm{T}} A_i^{-1} b_i$. Since $G_{p_t}^i$ is sparse, a sparse matrix multiplication is employed.
- Form the product

$$(G_{P_t}^i)^{\mathrm{T}} A_i^{-1} G_{p_t}^i = (G_{p_t}^i)^{\mathrm{T}} L_i^{-T} D_i^{-1} L_i G_{p_t}^i = V^{\mathrm{T}} D_i^{-1} V$$

  by first forming V column by column using forward substitution in the equation $L_i V = G_{p_t}^i$. Sum the contributions over time period to find $\sum_{i=1}^{N_t} (G_{p_t}^i)^{\mathrm{T}} A_i^{-1} G_{p_t}^i$
- Solve the system (28) for Δy_5. The system is sparse, so sparse matrix methods are used.
- Substitute Δy_5 in system (26) and solve for Δ_i, $i = 1, \ldots, N_t$

The interior point algorithm combines the solution of system (26) with a suitable choice of step size and barrier parameter μ in an iterative process. The interior point algorithm consists of the following steps.

2.1 Algorithm (Interior Point Optimization)

$l = 0$
Initialize p_t, λ, $y_{i,t}$, $s_{i,t}$, y_5, s_5 and barrier parameter μ
while not converged

$l = l + 1$
Solve system (26) for Δp_t, $\Delta \lambda_t$, $\Delta y_{i,t}$ and Δy_5
Solve Eqs. (16) and (17) for $\Delta s_{i,t}$ and Δs_5
Select appropriate step size $\alpha \in (0, 1)$
Update all variables

$$\begin{aligned}
p_t^l &= p_t^{l-1} + \alpha \Delta p_t \\
\lambda^l &= \lambda^{l-1} + \alpha \Delta \lambda \\
y_{i,t}^l &= y_{i,t}^{l-1} + \alpha \Delta y_{i,t} \\
y_5^l &= y_5^{l-1} + \alpha \Delta y_5 \\
s_{i,t}^l &= s_{i,t}^{l-1} + \alpha \Delta s_{i,t} \\
s_5^l &= s_5^{l-1} + \alpha \Delta s_5
\end{aligned}$$

Compute complementarity gap

$$\delta = s_l^{\mathrm{T}} \pi_l$$

Update barrier parameter μ
end while

The convergence criterion is based on the fact that at the optimal solution the complementarity gap $\delta = 0$. The algorithm terminates when δ is less than the desired tolerance. At each iteration the barrier parameter $\mu = \beta\delta$ where β is a problem dependent constant.

3 NUMERICAL RESULTS

A number of computational experiments were run to test the multiperiod HTED and examine the effect of different models for water discharge rates on the resulting dispatch. The basic systems consisted of two thermal units and one hydro unit with a 24 hour scheduling period. Data for the thermal units was based on Ref. [1] and is shown in Table I. The transmission system is derived from the IEEE 30 bus test case, with 30 buses and 41 transmission lines.

A variety of hydro units were considered with characteristics derived from Ref. [9]. A quadratic function is a very good approximation for the hydro generation discharge curve [7]. Both linear and quadratic models for discharge rates were considered. The model takes the form

$$\gamma(p_{i,t}^h) = a_h (p_{i,t}^h)^2 + b_h p_{i,t}^h + c_h.$$

Coefficients a_h, b_h, c_h were estimated by least squares regression on the data. Coefficients for the quadratic model can be found in Table II. Linear coefficients are given in Table III.

Results were produced for three test systems consisting of the 2 thermal units combined with each of the three hydro units. The results were similar for all three test system. Figures 1 and 2 show the results of the multiperiod HTED for the test system including the two thermal units and hydro generator 3. The dispatch depicted in Figure 1 uses the linear model for the

TABLE I Thermal Unit Characteristics.

Unit	$\underline{p_i}$ *(MWh)*	$\overline{p_i}$ *(MWh)*	a_i *(\$/MWh²)*	b_i *(\$/MWh)*	c_i *(\$)*
1	100	1000	0.0	2.0	0.00375
2	30	200	0.0	3.75	0.0175

TABLE II Coefficients for Quadratic Hydro Discharge Curve.

Unit	a_h *(acre-ft/MWh²)*	b_h *(acre-ft/MWh)*	c_h *(acre-ft)*
Hydro 1	712.55	0.739499	0.00247689
Hydro 2	7.91027	0.395815	0.011795
Hydro 3	4633.69	−0.331932	0.00590656

TABLE III Coefficients for Linear Hydro Discharge Curve.

Unit	b_h *(acre-ft/MWh)*	c_h *(acre-ft)*
Hydro 1	3.354	81.3003
Hydro 2	0.915	3.64
Hydro 3	6.67	2717.07

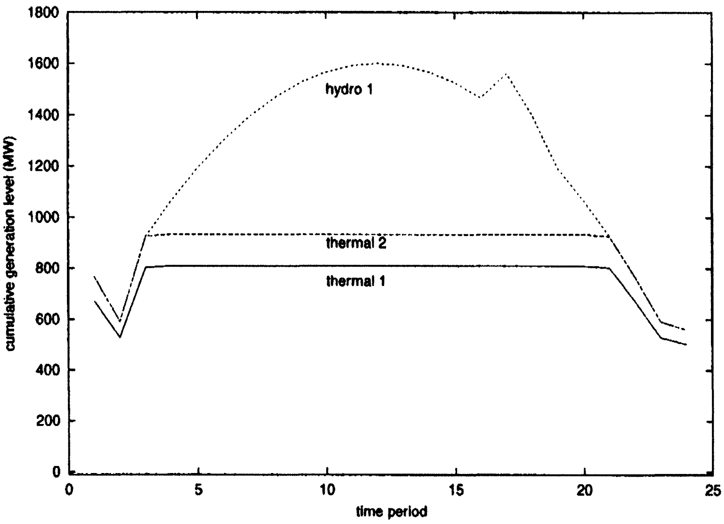

FIGURE 1 Economic dispatch with hydro unit 3: linear discharge rate.

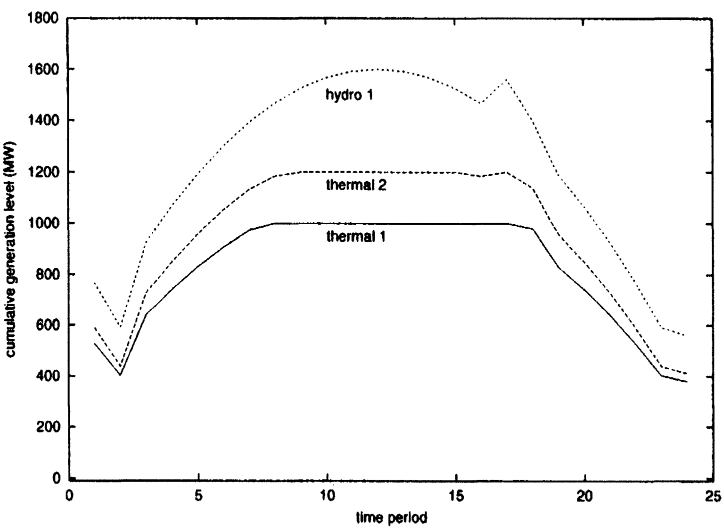

FIGURE 2 Economic dispatch with hydro unit 3: quadratic discharge rate.

hydro discharge rate with coefficients from Table III. The dispatch in Figure 2 shows results for a quadratic discharge model with coefficients from Table II.

As Figure 1 illustrates, the linear model allocates the available hydro generation to the peak demand periods, an effect known as peak shaving. Peak shaving is a well-known hydro-thermal scheduling characteristic based on the heuristic that since thermal generation costs increase with generation level, it is most economical to use hydro energy during the peak. Peak shaving algorithms were developed in Refs. [2, 3]. The validity of this concept was first investigated in Ref. [11].

In Figure 2 the peak shaving characteristic is no longer evident. The optimal dispatch produced with the quadratic discharge model utilizes the hydro generation more evenly throughout the dispatch horizon. Since the quadratic models the hydro discharge more accurately, the results provide further evidence of the nonoptimality of the peak-shaving heuristic.

4 CONCLUSION

An interior point algorithm for the multiperiod HTED has been presented. The algorithm has the ability to account for all transmission network constraints and does not require decomposition into thermal and hydro subproblems.

Results were generated for several test systems with hydro units of varying sizes. A comparison of linear and quadratic models for the hydro unit discharge rate shows when the discharge rate is accurately modeled, the hydro generation does not follow a peak shaving pattern. This result challenges the use of the peak shaving heuristic to decouple the hydro and thermal units in HTED algorithms.

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