Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2009, Article ID 284368, 12 pages
doi:10.1155/2009/284368

Research Article

A Deflection-Based Bridge Diagnosis Method

Xiaofan Yang,[1] Chunming Zhang,[1] and Weimin Chen[2]

[1] *College of Computer Science, Chongqing University, Chongqing 400044, China*
[2] *Key Laboratory of Optoelectronic Technology and System of Educational Ministry of China, Chongqing University, Chongqing 400044, China*

Correspondence should be addressed to Xiaofan Yang, xf_yang1964@yahoo.com

Received 9 November 2008; Revised 8 March 2009; Accepted 9 June 2009

Recommended by Angelo Luongo

By exploiting the "correlation of deflections" phenomena, we propose a new bridge diagnosis method. First, we introduce the notion of deflection correlation (DC) graphs and propose a method for building a DC graph. Second, we present a new algorithm for locating the abnormal cliques. Finally, we demonstrate the potential utility of the new method by applying it to the simulated diagnosis of a real-life bridge.

1. Introduction

Structural health monitoring (SHM) means the process of monitoring the condition of a structure and detecting the damages occurring in the structure over the time. SHM is a highly interdisciplinary area of research. In particular, the detection of damages in structures involves multiple disciplines such as statistics, pattern recognition, and algorithm design.

In the past three decades, SHM has been applied to the health monitoring of grand bridges. In such applications, it is desired that the early stage damage in a bridge to be detected by examining changes in its measured responses. A large number of methods have been proposed for detecting the abnormal behaviors occurring in bridges. Now, let us make a survey on the state of the art of this area. Catbas et al. [1] suggested to evaluate bridge condition using two damage-sensitive features. Catbas et al. [2], Deraemaeker et al. [3], and Yan et al. [4, 5] studied how to diagnose structural damages under varying environmental conditions. Koh and Dyke [6] designed a bridge evaluation scheme by making use of the correlation of modal parameters. Lee et al. [7] presented a bridge detection method under loading. Lee and Yun [8] and Zhang [9] propose schemes of bridge detection using ambient vibration data. In addition, some advanced techniques, such as the time-series classification [10], the fuzzy reasoning [11, 12], the image processing [13], the interval analysis [14], the

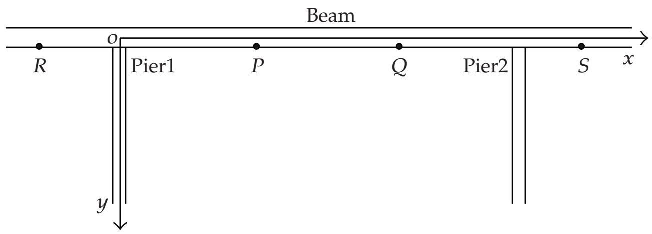

Figure 1: A bridge model, where the beam is horizontal.

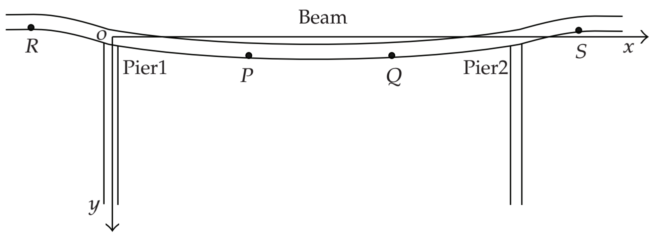

Figure 2: A bridge model, where the beam bends down.

multistage identification [15], and the neural network [16], were employed for the bridge detection purpose.

A real-life bridge is subject to varying environmental and operational conditions such as traffic, temperature, humidity, wind, and solar-radiation. These environmental effects also cause change in any well-defined pattern, which may mask the change caused by structural damage or by sensor failure [17]. So, the authors believe that the best approach is not to use a single feature but a suite of validated features depending on the structure and damage type. In this context, new effective bridge diagnosis methods need to be developed.

Deflections are one of the most important physical quantities characterizing the change of a bridge. Commonly, a number of deflection checkpoints on the beam of a bridge are specified and, for each checkpoint, a dedicated sensor is assigned to measure its deflection values periodically. For our diagnosis purpose, a bridge will be regarded as being composed of a number of *cliques*, where each clique is composed of a deflection checkpoint and its dedicated sensor. This paper addresses the following diagnosis problem.

Starting with a set of measurements of the deflection data, identify those cliques that behave abnormally.

Once a diagnosis result is achieved, each clique that has been diagnosed as abnormal will need a separate examination to see whether the checkpoint is damaged or the sensor fails to work properly.

The diagnosis method to be proposed is inspired by the "correlation of deflections" phenomenon, which is explained as follows. Some types of bridges, especially cable-stayed bridges and continuous rigid-frame bridges, have integral beams. Consider such a bridge model with four deflection checkpoints, P, Q, R, and S on the beam plus an $x - y$ coordinate system (see Figure 1). Depending on the traffic and environmental conditions, the beam may bend down or bend up. In the former case, P and Q should assume positive deflection values simultaneously, while R and S should take on negative deflection values simultaneously (see Figure 2). In the latter case, the converse is true (see Figure 3).

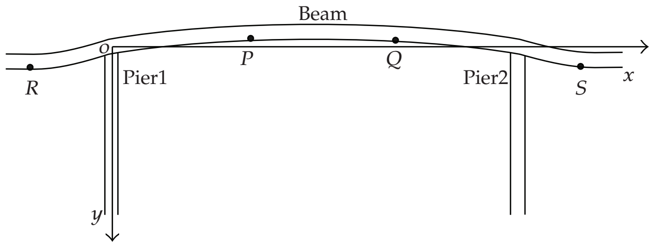

Figure 3: A bridge model, where the beam bends up.

We will treat the aforementioned bridge diagnosis problem following the idea of pattern recognition [18] and exploiting the above mentioned "correlation of deflections" phenomenon. Our method is composed of two phases.

Phase I. Starting with a set of deflection measurements that were previously acquired when the bridge was in its healthy status, establish a reference pattern of the bridge.

Phase II. Using a set of recently acquired deflection data, a current pattern of this bridge is obtained. By comparing the current pattern with the reference pattern, identify a set of abnormal cliques.

For our purpose, a weighted graph, known as the *deflection correlation graph* (DC graph, for short), is defined, which will be taken as the reference pattern. A method is proposed for building a DC graph of a bridge. On this basis, we present a time-effective diagnosis algorithm. The proposed method is then applied to the simulated diagnosis of a real-life bridge, where the cliques with distorted deflection data are identified correctly. This exhibits the potential utility of this method.

The subsequent materials are organized this way. Section 2 introduces some fundamental knowledge. Section 3 defines the DC graph of a bridge, and describes a method of building a DC graph, which is used to form two DC graphs of a real-life bridge system. In Section 4, a diagnosis algorithm is presented, which is then applied to identify those abnormal parts of a real-life bridge provided some deflection data are distorted. Finally, Section 5 briefly summarizes the work of this paper and indicates some issues that are worth further study.

2. Fundamentals

A *graph* is defined as an ordered pair of sets, $G = (V, E)$, where elements in V are referred as *nodes*, elements in E are referred to as *edges*, and every edge is a set of two distinct nodes [19]. Graphs are a class of mathematical models for characterizing binary relations between objects.

An edge-labeled graph is a graph with edges being labeled with intervals. An edge-labeled graph can be represented by an edge-labeled drawing or by a table. For instance, the edge-labeled graph $G = (V, E, \text{label})$, where

$$V = \{v_1, v_2, v_3, v_4, v_5\}, \qquad E = \{\{v_1, v_2\}, \{v_1, v_5\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_4, v_5\}\},$$

$$\text{label}(\{v_1, v_2\}) = [0.82, 0.90], \qquad \text{label}(\{v_1, v_5\}) = [0.33, 0.36],$$

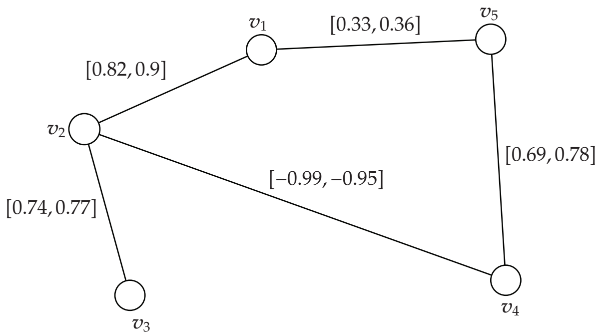

Figure 4: The drawing of an edge-labeled graph.

Table 1: The table of an edge-labeled graph.

	v_1	v_2	v_3	v_4	v_5
v_1		.82 .90			.33 .36
v_2			.74 .77		−.99 − .95
v_3					
v_4					.69 .78
v_5					

$$\text{label}(\{v_2, v_3\}) = [0.74, 0.77],$$
$$\text{label}(\{v_2, v_4\}) = [-0.99, -0.95], \quad \text{label}(\{v_4, v_5\}) = [0.69, 0.78] \tag{2.1}$$

can be represented by an edge-labeled drawing (Figure 4) or by a table (Table 1).

Consider a pair of random variables, X and Y. Let $\rho_{X,Y}$ denote their *correlation coefficient*. That is,

$$\rho_{X,Y} \triangleq \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}. \tag{2.2}$$

If we have a series of n measurements of X and Y written as x_i and y_i where $i = 1, 2, \ldots, n$, then the *Pearson product-moment correlation coefficient*, which is defined as

$$r_{X,Y} \triangleq \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \sqrt{n \sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2}}, \tag{2.3}$$

can be used to estimate the correlation coefficient of X and Y. The Pearson correlation coefficient is also known as the *sample correlation coefficient*.

3. Deflection Correlation Graph

For clarity, we call the sensor dedicated to deflection checkpoint P as sensor P, and we call the corresponding clique as clique P.

3.1. Definition of a Deflection Correlation Graph

For clique P, let $X_P(n)$ denote the measured deflection value at checkpoint P at time n. Then, $X_P = \{X_P(n) : n = 1, 2, \ldots\}$ forms a *time series* [20]. Let $x_P(n)$ denote the measurement of $X_P(n)$. In this paper, we assume that the following reasonable hypothesis holds.

(H) *Suppose clique P is healthy. Then,* X_P *is a weak stationary process* [20].

In the case that cliques P and Q are healthy, it follows from hypothesis (H) that the correlation coefficients $\rho_{X_P(n),X_Q(n)}$ assume a common value for all values of n. In what follows, we will use the symbol ρ_{X_p,X_Q} to denote this common value. Clearly, ρ_{X_p,X_Q} characterizes an internal connection between the two checkpoints and, hence, is useful for diagnosis purpose. Unfortunately, the precise value of ρ_{X_p,X_Q} is not practically available.

Given a collection of historical data $H = \{x_P(i) : P \text{ is a checkpoint}, s \le i \le t\}$. For every pair of checkpoints, P and Q, we can fetch a number of pieces of the form $H' = \{x_P(i), x_Q(i) : s_1 \le i \le t_1\}$ and, for each such piece, we can calculate a Pearson correlation coefficient according to the formula

$$r_{X_P,X_Q,H'} \triangleq \frac{(t_1 - s_1 + 1)\sum_{i=s_1}^{t_1} x_i y_i - \sum_{i=s_1}^{t_1} x_i \sum_{i=s_1}^{t_1} y_i}{\sqrt{(t_1 - s_1 + 1)\sum_{i=s_1}^{t_1} x_i^2 - \left(\sum_{i=s_1}^{t_1} x_i\right)^2}\sqrt{(t_1 - s_1 + 1)\sum_{i=s_1}^{t_1} y_i^2 - \left(\sum_{i=s_1}^{t_1} y_i\right)^2}}. \tag{3.1}$$

Thereby, we can get a smallest interval in which at least $\alpha \times 100 \times \%$ Pearson correlation coefficients fall. This interval can also characterize the internal connection between the two checkpoints. Now, let us introduce the following definition.

Definition 3.1. Given a collection of historical data H for a bridge. Consider a pair of checkpoints, P and Q, on a bridge. Let $0 \le \alpha \le 1$.

(1) An H-based *α-deflection correlation interval* (*α-DC interval*) of P and Q, denoted $I_{H,\alpha}(P,Q)$, is defined as an interval in which $r_{X_p,X_Q,H'}$ falls with probability $\ge \alpha$. For diagnosis purpose, it is appropriate to take $\alpha \in \{0.90, 0.95\}$.

(2) An H-based *α-deflection correlation degree* (*α-DC degree*) of P and Q is defined as

$$dc_{H,\alpha}(P,Q) \triangleq 1 - \frac{1}{2} \times \text{length}(I_{H,\alpha}(P,Q)). \tag{3.2}$$

Clearly, $0 \le dc_{H,\alpha}(P,Q) \le 1$. The nearer to one $dc_{H,\alpha}(P,Q)$ is close, the more strongly correlated P and Q will be with each other.

Definition 3.2. Given a collection of historical data H for a bridge. Let $0 \le \alpha, \beta \le 1$. An *$H$-based$(\alpha,\beta)$-deflection correlation graph*((α,β)-DC graph) of the bridge is defined as an edge-labeled graph $G = (V, E, \text{label})$, where V is the set of all cliques of the bridge,

$$E \triangleq \left\{\{P,Q\} : dc_{H,\alpha}(P,Q) \ge \beta\right\}, \qquad \text{label}(\{P,Q\}) \triangleq I_{H,\alpha}(P,Q). \tag{3.3}$$

INPUT: V: the set of all deflection checkpoints on a bridge;
H: a set of deflection data acquired in the infancy of the bridge;
α, β: $0 \leq \alpha, \beta < 1$,
OUTPUT: $G = (V, E, \text{label})$: an (α, β)-DC graph of the bridge.
{
(1) initially, E is empty;
(2) choose a real number ε, $0 < \varepsilon < 1$;
/* the meaning of ε is clear according to statement 8; */
(3) choose two integers r and l;
/* the meanings of r and l are clear according to statements 5-6; */
(4) for every pair of checkpoints, P and Q,
{
/* sentences 5-6 calculate r sample correlated coefficients between P and Q; */
(5) randomly select r pairs of length-l sequences from H, one for P, and the other for Q;
(6) for each pair of selected sequences, calculate a sample correlated coefficient;
/* sentence 7 gets an initial estimate of $I_{H,\alpha}(P, Q)$; */
(7) find the smallest interval $[a, b]$ so that the r sample correlated coefficients obtained previously fall in it with frequency $\geq \alpha$;
/* sentence 8 gets a final estimate of $I_{H,\alpha}(P, Q)$ by eliminating the randomness of the initial estimate; */
(8) $a \leftarrow a - \varepsilon$; $b \leftarrow b + \varepsilon$;
/* sentence 9 adds a legal edge in E; */
(9) if $1 - (1/2)(b - a) \geq 1 - \beta$, then add in E the edge $\{P, Q\}$ with label $[a, b]$;
}
}

Algorithm 1: BUILD_DC.

For diagnosis purpose, we will take $\beta \in \{0.90, 0.95\}$. Figure 1 gives an exemplar $(0.95, 0.95)$ DC graph.

In its infancy, a bridge can be regarded as healthy. So, the deflection data acquired during the infancy can be utilized to build a DC graph of a bridge. This DC graph will be taken as the reference pattern.

3.2. Construction of a Deflection Correlation Graph

Based on the previous discussions, let us describe a method for creating a deflection correlation graph.

3.3. An Example

The Chongqing MaSangXi Grand Bridge is a cable-suspension bridge across the Yangtze River, which is about 1.1 kilometers in length, and has an Asia-longest main span of 360 meters (Figure 5). The monitoring system for this bridge holds 22 deflection checkpoints numbered as $n_1, \ldots, n_{11}, s_1, \ldots, s_{11}$ (Figure 6) and has employed the photoelectronic imaging technique to enhance the measurement precision.

Since the MaSangXi grand bridge was put into use in 2001, the deflection data on each checkpoint have been successively acquired at an interval of 30 minutes. For the purpose of simulated diagnosis, we selected a set H of 24 000 complete deflection records from these historical data.

Figure 5: A picture of the Chongqing MaSangXi Grand Bridge.

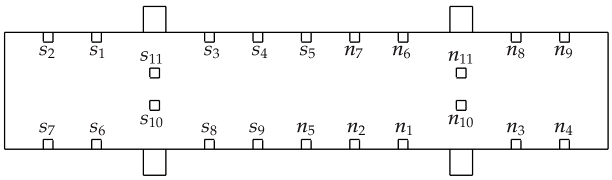

Figure 6: Layout of 22 deflection checkpoints.

In our simulation experiments, We tried 1000 combinations of the values of the five parameters, $\alpha, \beta, \varepsilon, r$, and l. Below are the reports of two such experiments.

Experiment 3.3. Set $(\alpha, \beta) = (0.90, 0.95)$ and choose $(\varepsilon, r, l) = (0.01, 2000, 100)$, the BUILD_DC algorithm built a $(0.90, 0.95)$-DC graph, which is shown in Table 2.

Experiment 3.4. Set $(\alpha, \beta) = (0.95, 0.95)$ and choose $(\varepsilon, r, l) = (0.01, 2000, 100)$, the BUILD_DC algorithm created a $(0.95, 0.95)$-DC graph, which is shown in Table 3.

4. A Bridge Diagnosis Method

In this section, we will propose a bridge diagnosis algorithm based on the reference pattern previously defined.

4.1. Abnormal Cliques

First, let us describe a kind of abnormal behaviors of cliques.

Definition 4.1. Consider a bridge. Let H be a collection of infancy deflection data. Let $G = (V, E, \text{label})$ be an H-based (α, β)-*DC* graph of a bridge. Let $\widetilde{H}$ be a collection of recent data. Let $0 < \theta < 1$. A clique P is θ-*significantly abnormal* if there is another clique Q such that $r_{X_P, X_Q, \widetilde{H}'} \in I_{H,\alpha}(P, Q)$ with frequency $\leq \alpha - \theta$.

It can be seen that the larger θ is, the more seriously abnormal a clique will be.

Table 2: A (0.90, 0.95)–DC graph of the MaSangXi grand bridge.

	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}	n_{11}	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
n_1		.982 .999			.935 .985	.915 .976	.864 .948					−.954 −.891	−.967 −.915			.883 .967			.923 .970	.930 .974		
n_2					.913 .974	.884 .962						−.964 −.907	−.972 −.924			.908 .979			.936 .978	.938 .976		
n_3				.950 .980																		
n_4																						
n_5						.935 .983	.920 .968					−.943 −.851	−.947 −.860				−.931 −.833		.845 .943	.848 .946		
n_6						.	.938 .980															
n_7																						
n_8									.977 .994													
n_9																						
n_{10}																						
n_{11}																						
s_1													.984 .998				.938 .980	.891 .977	−.957 −.883	−.953 −.869		
s_2																	.915 .971	.887 .974	−.965 −.898	−.963 −.884		
s_3																						
s_4																.770 .857						
s_5																			.937 .980	.947 .987		
s_6																		.902 .986				
s_7																						
s_8																				.984 .999		
s_9																						
s_{10}																						
s_{11}																						

Table 3: A (0.95, 0.95)–DC graph of the MaSangXi grand bridge.

	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}	n_{11}	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
n_1		.986 .998			.942 .985	.923 .972	.867 .946					−.954 −.899	−.964 −.919			.891 .965			.928 .970	.926 .964		
n_2					.919 .971	.870 .949						−.952 −.902	−.974 −.926			.902 .961			.937 .979	.944 .976		
n_3				.952 .979																		
n_4																						
n_5						.941 .986	.916 .965					−.954 −.870	−.933 −.857				−.921 −.835		.858 .948	.848 .935		
n_6						.	.945 .979															
n_7																						
n_8									.979 .996													
n_9																						
n_{10}																						
n_{11}																						
s_1													.987 .999				.941 .977	.909 .981	−.959 −.893	−.942 −.880		
s_2																	.910 .967	.905 .973	−.953 −.892	−.964 −.890		
s_3																						
s_4																.788 .868						
s_5																			.937 .977	.955 .986		
s_6																		.915 .986				
s_7																						
s_8																				.985 .999		
s_9																						
s_{10}																						
s_{11}																						

INPUT: (V, E, label): a (α, β)-DC graph of a bridge;
$\widetilde{H}$: a set of recent data;
θ: a real number, $0 < \theta < 1$.
{
OUTPUT: F: the collection of all θ-significantly abnormal cliques.
(1) initially, F is empty;
(2) choose two integers r and l;
(3) for every pair of checkpoints, P and Q,
{
/* sentences 4-5 calculate r sample correlated coefficients between P and Q; */
(4) randomly select r pairs of length-l sequences from $\widetilde{H}$, one for P, and the other for Q;
(5) for each pair of selected sequences, calculate a sample correlated coefficient;
/* sentence 6 calculates the frequency at which these r sample correlated coefficients fall in $I_H(P, Q)$; */
(6) num $\leftarrow$ the number of these r sample correlated coefficients that fall in $I_H(P, Q)$;
/* sentence 7 performs diagnosis; */
(7) if $\text{num}/r < \alpha - \theta$, then add P and Q to F;
}
}.

Algorithm 2: DIAG_BRIDGE.

4.2. Description of the Diagnosis Algorithm

We are ready to present a diagnosis algorithm for bridges.

4.3. An Application

By applying the new diagnosis method, we performed simulated diagnosis of the MaSangXi grand bridge for 1000 times. Below is the three-stage report of one of these 1000 diagnosis processes.

Stage I. Get a reference pattern. We choose the $(0.95, 0.95)$-DC graph given in Table 3 as our reference pattern.

Stage II. Create an abnormal pattern. Starting with the data collection H given in Subsection 3.3, we create a collection $\widetilde{H}$ of distorted deflection data according to the following four rules.

Rule 1. Replace the data on checkpoints n_3 with the Gaussian noise with zero mean and a standard deviation of 0.01.

Rule 2. Replace the deflection data on checkpoints n_7 with the Rayleigh noise with zero mean and a standard deviation of 0.01.

Rule 3. Replace the deflection data on checkpoints s_4 with the Rician noise with zero mean and a standard deviation of 0.01.

Rule 4. Keep intact all of the other deflection data.

Stage III. Perform diagnosis. By setting $r = 200$, $l = 100$, and executing algorithm DIAG_BRIDGE on $(V, E, \text{label}, \widetilde{H}, 0.05)$, we get the set of 0.05 significantly abnormal cliques, $F = \{n_1, n_3, n_4, n_5, n_6, n_7, s_4, s_5\}$, which does contain the three cliques with distorted data.

Among the 1000 experiments, there are up to 967 times for which all of the distorted cliques were successfully isolated. This justifies the potential utility of the proposed diagnosis algorithm.

5. Conclusions

We have proposed a deflection-based bridge diagnosis method, and we have justified the potential utility of this method by applying it to the simulated diagnosis of a real-life bridge.

Toward this direction, some issues are worth further study. First, the proposed diagnosis method needs to be improved because it may identify some healthy nodes as being “abnormal.” Second, it is important to make clear how the diagnosis outcome is affected by the parameters involved in the algorithm. Finally, it is necessary to incorporate some other important factors (say, temperature, strain) into the diagnosis scheme.

Acknowledgments

This work is supported by 863 Program of China (2006AA04Z433), Natural Science Foundation of China (10771227), and Program for New Century Excellent Talent of Educational Ministry of China (NCET-05-0759).

References

[1] F. N. Catbas, M. Gul, and J. L. Burkett, “Conceptual damage-sensitive features for structural health monitoring: laboratory and field demonstrations,” *Mechanical Systems and Signal Processing*, vol. 22, no. 7, pp. 1650–1669, 2008.
[2] F. N. Catbas, M. Susoy, and D. M. Frangopol, “Structural health monitoring and reliability estimation: long span truss bridge application with environmental monitoring data,” *Engineering Structures*, vol. 30, no. 9, pp. 2347–2359, 2008.
[3] A. Deraemaeker, E. Reynders, G. De Roeck, and J. Kullaa, “Vibration-based structural health monitoring using output-only measurements under changing environment,” *Mechanical Systems and Signal Processing*, vol. 22, no. 1, pp. 34–56, 2008.
[4] A.-M. Yan, G. Kerschen, P. De Boe, and J.-C. Golinval, “Structural damage diagnosis under varying environmental conditions—part I: a linear analysis,” *Mechanical Systems and Signal Processing*, vol. 19, no. 4, pp. 847–864, 2005.
[5] A.-M. Yan, G. Kerschen, P. De Boe, and J.-C. Golinval, “Structural damage diagnosis under varying environmental conditions—part II: local PCA for non-linear cases,” *Mechanical Systems and Signal Processing*, vol. 19, no. 4, pp. 865–880, 2005.
[6] B. H. Koh and S. J. Dyke, “Structural health monitoring for flexible bridge structures using correlation and sensitivity of modal data,” *Computers and Structures*, vol. 85, no. 3-4, pp. 117–130, 2007.
[7] J. W. Lee, J. D. Kim, C. B. Yun, J. H. Yi, and J. M. Shim, “Health-monitoring method for bridges under ordinary traffic loadings,” *Journal of Sound and Vibration*, vol. 257, no. 2, pp. 247–264, 2002.
[8] J. J. Lee and C. B. Yun, “Damage diagnosis of steel girder bridges using ambient vibration data,” *Engineering Structures*, vol. 28, no. 6, pp. 912–925, 2006.
[9] Q. W. Zhang, “Statistical damage identification for bridges using ambient vibration data,” *Computers and Structures*, vol. 85, no. 7-8, pp. 476–485, 2007.
[10] E. P. Carden and J. M. W. Brownjohn, “ARMA modelled time-series classification for structural health monitoring of civil infrastructure,” *Mechanical Systems and Signal Processing*, vol. 22, no. 2, pp. 295–314, 2008.
[11] Y. Cheng and H. G. Melhem, “Monitoring bridge health using fuzzy case-based reasoning,” *Advanced Engineering Informatics*, vol. 19, no. 4, pp. 299–315, 2005.
[12] Z. Zhao and C. Chen, “A fuzzy system for concrete bridge damage diagnosis,” *Computers and Structures*, vol. 80, no. 7-8, pp. 629–641, 2002.
[13] G. Fu and A. G. Moosa, “Structural damage diagnosis using high resolution images,” *Structural Safety*, vol. 23, no. 4, pp. 281–295, 2001.
[14] O. García, J. Vehí, J. C. Matos, A. A. Henriques, and J. R. Casas, “Structural assessment under uncertain parameters via interval analysis,” *Journal of Computational and Applied Mathematics*, vol. 218, no. 1, pp. 43–52, 2008.
[15] J. M. Ko, Z. G. Sun, and Y. Q. Ni, “Multi-stage identification scheme for detecting damage in cable-stayed Kap Shui Mun Bridge,” *Engineering Structures*, vol. 24, no. 7, pp. 857–868, 2002.

[16] J. J. Lee, J. W. Lee, J. H. Yi, C. B. Yun, and H. Y. Jung, "Neural networks-based damage detection of bridges considering errors in baseline finite element models," *Journal of Sound and Vibration*, vol. 280, no. 3, pp. 555–578, 2005.
[17] J. M. Ko and Y. Q. Ni, "Technology developments in structural health monitoring of large-scale bridges," *Engineering Structures*, vol. 27, no. 12, pp. 1715–1725, 2005.
[18] A. R. Webb, *Statistical Pattern Recognition*, John Wiley & Sons, Chichester, UK, 2nd edition, 2002.
[19] R. Diestel, *Graph Theory*, vol. 173 of *Graduate Texts in Mathematics*, Springer, Berlin, Germany, 3rd edition, 2005.
[20] J. D. Hamilton, *Time Series Analysis*, Princeton University Press, Princeton, NJ, USA, 1994.