Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2010, Article ID 198943, 12 pages
doi:10.1155/2010/198943

Research Article

Some New Iterative Methods for Nonlinear Equations

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Received 1 October 2010; Accepted 20 December 2010

Academic Editor: Carlo Cattani

We suggest and analyze some new iterative methods for solving the nonlinear equations $f(x) = 0$ using the decomposition technique coupled with the system of equations. We prove that new methods have convergence of fourth order. Several numerical examples are given to illustrate the efficiency and performance of the new methods. Comparison with other similar methods is given.

1. Introduction

It is well known that a wide class of problem which arises in several branches of pure and applied science can be studied in the general framework of the nonlinear equations $f(x) = 0$. Due to their importance, several numerical methods have been suggested and analyzed under certain conditions. These numerical methods have been constructed using different techniques such as Taylor series, homotopy perturbation method and its variant forms, quadrature formula, variational iteration method, and decomposition method; see, for example, [1–19]. To implement the decomposition method, one has to calculate the so-called Adomian polynomial, which is itself a difficult problem. Other technique have also their limitations. To overcome these difficulties, several other techniques have been suggested and analyzed for solving the nonlinear equations. One of the decompositions is due to Daftardar-Gejji and Jafari [6]. In this paper, we use this decomposition method to construct some new iterative methods. To apply this technique, we first use the new series representation of the nonlinear function, which is obtained by using the quadrature formula and the fundamental theorem of calculus. We rewrite the nonlinear equation as a coupled system of nonlinear equations. Applying the decomposition of Daftardar-Gejji and Jafari [6], we are able to construct some new iterative methods for solving the nonlinear equations. Our method of construction of these iterative methods is very simple as compared with other methods. We also prove convergence of the proposed methods, which is of order four. Several numerical examples are given to illustrate the efficiency and the performance of the new iterative methods. Our results can be considered as an important improvement and refinement of the previously results.

2. Iterative Methods

Consider the nonlinear equation

$$f(x) = 0. \tag{2.1}$$

Using the quadrature formula and the fundamental theorem of calculus, (2.1) can be written as

$$f(x) = f(\gamma) + (x-\gamma)\left[\frac{f'(\gamma) + 2f'((\gamma+x)/2) + f'(x)}{4}\right] = 0, \tag{2.2}$$

where γ is an initial guess sufficiently close to α, which is a simple root of (2.1). We can rewrite the nonlinear equation (2.1) as a coupled system

$$f(\gamma) + (x-\gamma)\left[\frac{f'(\gamma) + 2f'((\gamma+x)/2) + f'(x)}{4}\right] + g(x) = 0, \tag{2.3}$$

$$g(x) = f(x) - f(\gamma) - (x-\gamma)\left[\frac{f'(\gamma) + 2f'((\gamma+x)/2) + f'(x)}{4}\right]. \tag{2.4}$$

From (2.3), we have

$$x = \gamma - 4\left[\frac{f(\gamma) + g(x)}{f'(\gamma) + 2f'((\gamma+x)/2) + f'(x)}\right] = c + N(x), \tag{2.5}$$

where

$$c = \gamma, \tag{2.6}$$

$$N(x) = -4\left[\frac{f(\gamma) + g(x)}{f'(\gamma) + 2f'((\gamma+x)/2) + f'(x)}\right]. \tag{2.7}$$

It is clear that the operator $N(x)$ is nonlinear. We now construct a sequence of higher-order iterative methods by using the decomposition technique, which is mainly due to Daftardar-Gejji and Jafari [6]. This decomposition of the nonlinear function $N(x)$ is quite different from that of Adomian decomposition. In this method, one does not have to calculate the so-called the Adomian polynomial, which is another advantage of this decomposition. The main idea of this technique is to look for a solution having the series form

$$x = \sum_{i=0}^{\infty} x_i. \tag{2.8}$$

The nonlinear operator N can be decomposed as

$$N(x) = N(x_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{j=0}^{i} x_j \right) - N\left(\sum_{j=0}^{i-1} x_j \right) \right\}. \tag{2.9}$$

Combining (2.5), (2.8), and (2.9), we have

$$\sum_{i=0}^{\infty} x_i = c + N(x_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{j=0}^{i} x_j \right) - N\left(\sum_{j=0}^{i-1} x_j \right) \right\}. \tag{2.10}$$

Thus, we have the following iterative scheme:

$$\begin{aligned} x_0 &= c, \\ x_1 &= N(x_0), \\ x_2 &= N(x_0 + x_1) - N(x_0), \\ &\vdots \\ x_{m+1} &= N\left(\sum_{j=0}^{m} x_j \right) - N\left(\sum_{j=0}^{m-1} x_j \right), \quad m = 1, 2, \dots \end{aligned} \tag{2.11}$$

Then,

$$x_1 + x_2 + \cdots + x_{m+1} = N(x_0 + x_1 + \cdots + x_m), \quad m = 1, 2, \dots, \tag{2.12}$$

$$x = c + \sum_{i=1}^{\infty} x_i. \tag{2.13}$$

It can be shown that the series $\sum_{i=0}^{\infty} x_i$ converges absolutely and uniformly to a unique solution of (2.6). see [6].

From (2.7) and (2.12), we have

$$x_0 = c = \gamma. \tag{2.14}$$

From (2.4), (2.8) and using the idea of Yun [19], we obtain

$$g(x_0) = 0,$$
$$x_1 = N(x_0) = -4\left[\frac{f(\gamma) + g(x_0)}{f'(\gamma) + 2f'((\gamma + x_0)/2) + f'(x_0)}\right] = -4\left[\frac{f(\gamma)}{f'(\gamma) + 2f'((\gamma + x_0)/2) + f'(x_0)}\right]. \tag{2.15}$$

Note that x is approximated by

$$X_m = x_0 + x_1 + x_2 + \cdots + x_m, \tag{2.16}$$

where $\lim_{m\to\infty} X_m = x$.

For $m = 0$,

$$x \approx X_0 = x_0 = c = \gamma. \tag{2.17}$$

For $m = 1$,

$$x \approx X_1 = x_0 + x_1 = \gamma - 4\left[\frac{f(\gamma)}{f'(\gamma) + 2f'((\gamma + x_0)/2) + f'(x_0)}\right] = x_0 - \frac{f(x_0)}{f'(x_0)}. \tag{2.18}$$

This formulation allows us to suggest the following one-step iterative method for solving the nonlinear equation (2.1).

Algorithm 2.1. For a given x_0, compute the approximate solution x_{n+1} by the following iterative scheme:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad f'(x_n) \neq 0, \quad n = 0, 1, 2, \ldots. \tag{2.19}$$

It is a well-known Newton method for solving nonlinear equations (2.1), which has second-order convergence.

From (2.1), we have

$$x_0 + x_1 - \gamma = -\frac{f(\gamma)}{f'(\gamma)}. \tag{2.20}$$

From (2.4), (2.8) and using the idea of Yun [19], we have

$$\begin{aligned}
g(x_0+x_1) &= f(x_0+x_1) - f(\gamma) - (x_0+x_1-\gamma)\left[\frac{f'(\gamma)+2f'((\gamma+x_0+x_1)/2)+f'(x_0+x_1)}{4}\right] \\
&= f(x_0+x_1) - f(\gamma) + \frac{f(\gamma)}{4f'(\gamma)}\left[f'(\gamma)+2f'\left(\frac{\gamma+x_0+x_1}{2}\right)+f'(x_0+x_1)\right], \\
x_1+x_2 = N(x_0+x_1) &= -4\left[\frac{f(\gamma)+g(x_0+x_1)}{f'(\gamma)+2f'((\gamma+x_0+x_1)/2)+f'(x_0+x_1)}\right] \\
&= -\frac{f(\gamma)}{f'(\gamma)} - \frac{4f(x_0+x_1)}{f'(\gamma)+2f'((\gamma+x_0+x_1)/2)+f'(x_0+x_1)}.
\end{aligned} \tag{2.21}$$

For $m = 2$,

$$\begin{aligned}
x \approx X_2 = x_0+x_1+x_2 &= c + N(x_0+x_1) \\
&= \gamma - \frac{f(\gamma)}{f'(\gamma)} - \frac{4f(x_0+x_1)}{f'(\gamma)+2f'((\gamma+x_0+x_1)/2)+f'(x_0+x_1)}.
\end{aligned} \tag{2.22}$$

Using this relation, we can suggest the following two-step iterative method for solving nonlinear equation (2.1).

Algorithm 2.2. For a given x_0, compute the approximate solution x_{n+1} by the iterative following scheme:

$$\begin{aligned}
y_n &= x_n - \frac{f(x_n)}{f'(x_n)}, \\
x_{n+1} &= y_n - \frac{4f(y_n)}{f'(x_n)+2f'((x_n+y_n)/2)+f'(y_n)}, \quad n = 0, 1, 2, \ldots.
\end{aligned} \tag{2.23}$$

From (2.22), we obtain

$$x_0+x_1+x_2-\gamma = -\frac{f(\gamma)}{f'(\gamma)} - \frac{4f(x_0+x_1)}{f'(\gamma)+2f'((\gamma+x_0+x_1)/2)+f'(x_0+x_1)}. \tag{2.24}$$

From (2.4), (2.8) and using the idea of Yun [19], we get

$$\begin{aligned}
&g(x_0+x_1+x_2)\\
&\quad= f(x_0+x_1+x_2)-f(\gamma)-(x_0+x_1+x_2-\gamma)\\
&\qquad\times\left[\frac{f'(\gamma)+2f'((\gamma+x_0+x_1+x_2)/2)+f'(x_0+x_1+x_2)}{4}\right]\\
&\quad= f(x_0+x_1+x_2)-f(\gamma)\\
&\qquad-\frac{1}{4}\left(-\frac{f(\gamma)}{f'(\gamma)}-\frac{4f(x_0+x_1)}{f'(\gamma)+2f'((\gamma+x_0+x_1)/2)+f'(x_0+x_1)}\right)\\
&\qquad\times\left[f'(\gamma)+2f'\left(\frac{\gamma+x_0+x_1+x_2}{2}\right)+f'(x_0+x_1+x_2)\right],\\
&x_1+x_2+x_3\\
&\quad= N(x_0+x_1+x_2)=-4\left[\frac{f(\gamma)+g(x_0+x_1+x_2)}{f'(\gamma)+2f'((\gamma+x_0+x_1+x_2)/2)+f'(x_0+x_1+x_2)}\right]\\
&\quad= -\frac{f(\gamma)}{f'(\gamma)}-\frac{4f(x_0+x_1)}{f'(\gamma)+2f'((\gamma+x_0+x_1)/2)+f'(x_0+x_1)}\\
&\qquad-\frac{4f(x_0+x_1+x_2)}{f'(\gamma)+2f'((\gamma+x_0+x_1+x_2)/2)+f'(x_0+x_1+x_2)}.
\end{aligned}\tag{2.25}$$

For $m=3$,

$$\begin{aligned}
x\approx X_3 = x_0+x_1+x_2+x_3 &= c+N(x_0+x_1+x_2)\\
&=\gamma-\frac{f(\gamma)}{f'(\gamma)}-\frac{4f(x_0+x_1)}{f'(\gamma)+2f'((\gamma+x_0+x_1)/2)+f'(x_0+x_1)}\\
&\quad-\frac{4f(x_0+x_1+x_2)}{f'(\gamma)+2f'((\gamma+x_0+x_1+x_2)/2)+f'(x_0+x_1+x_2)}.
\end{aligned}\tag{2.26}$$

Using this formulation, we can suggest the following three-step iterative method for solving nonlinear equation (2.1).

Algorithm 2.3. For a given x_0, compute the approximate solution x_{n+1} by the iterative following scheme.

$$
\begin{aligned}
y_n &= x_n - \frac{f(x_n)}{f'(x_n)},\\
z_n &= y_n - \frac{4f(y_n)}{f'(x_n) + 2f'((x_n + y_n)/2) + f'(y_n)},\\
x_{n+1} &= z_n - \frac{4f(z_n)}{f'(x_n) + 2f'((x_n + z_n)/2) + f'(z_n)}, \quad n = 0, 1, 2, \ldots.
\end{aligned} \tag{2.27}
$$

3. Convergence Analysis

In this section, we consider the convergence criteria of the iterative methods developed in Section 2. In a similar way, one can consider the convergence of other algorithms.

Theorem 3.1. *Let $\alpha \in I$ be a simple zero of sufficiently differentiable function $f : I \subseteq R \to R$ for an open interval I. If x_0 is sufficiently close to α, then the iterative methods defined by Algorithm 2.3 has fourth-order convergence.*

Proof. Let α be a simple zero of f. Then, by expanding $f(x_n)$ and $f'(x_n)$ in Taylor's Series about α, we have

$$f(x_n) = f'(\alpha)\left[e_n + c_2e_n^2 + c_3e_n^3 + c_4e_n^4 + O\left(e_n^5\right)\right], \tag{3.1}$$

$$f'(x_n) = f'(\alpha)\left[1 + 2c_2e_n + 3c_3e_n^2 + 4c_4e_n^3 + 5c_5e_n^4 + O\left(e_n^5\right)\right], \tag{3.2}$$

where $c_k = (1/k!)(f^{(k)}(\alpha)/f'(\alpha))$, $k = 2, 3, \ldots$ and $e_n = x_n - \alpha$.

From (3.1) and (3.2), we have

$$\frac{f(x_n)}{f'(x_n)} = e_n - c_2e_n^2 + 2\left(c_2^2 - c_3\right)e_n^3 + \left(7c_2c_3 - 4c_2^3 - 3c_4\right)e_n^4 + O\left(e_n^5\right). \tag{3.3}$$

From (3.3), we get

$$y_n = \alpha + c_2e_n^2 + 2\left(c_3 - c_2^2\right)e_n^3 - \left(7c_2c_3 - 4c_2^3 - 3c_4\right)e_n^4 + O\left(e_n^5\right). \tag{3.4}$$

Expanding $f(y_n), f'(y_n), f'((x_n+y_n)/2)$ in Taylor's Series about α and using (3.4), we have

$$f(y_n) = f'(\alpha)\left[c_2 e_n^2 + \left(-2c_2^2 + 2c_3\right)e_n^3 + \left(5c_2^3 - 7c_2c_3 + 3c_4\right)e_n^4 + O\left(e_n^5\right)\right], \tag{3.5}$$

$$f'(y_n) = f'(\alpha)\left[1 + 2c_2^2 e_n^2 + 4\left(c_2c_3 - c_2^3\right)e_n^3 + \left(-11c_2^2c_3 + 6c_2c_4 + 8c_2^4\right)e_n^4 + O\left(e_n^5\right)\right], \tag{3.6}$$

$$\begin{aligned} f'\left(\frac{x_n+y_n}{2}\right) = f'(\alpha)\Bigg[&1 + c_2e_n + \left(c_2^2 + \frac{3}{4}c_3\right)e_n^2 + \left(\frac{7}{2}c_2c_3 + \frac{1}{2}c_4 - 2c_2^3\right)e_n^3 \\ &+\left(\frac{9}{2}c_2c_4 + 4c_2^4 - \frac{37}{4}c_2^2c_3 + 3c_3^2 + \frac{5}{16}c_5\right)e_n^4 + O\left(e_n^5\right)\Bigg]. \end{aligned} \tag{3.7}$$

From (3.2), (3.6), and (3.7), we have

$$\begin{aligned} &f'(x_n) + 2f'\left(\frac{x_n+y_n}{2}\right) + f'(y_n) \\ &\quad = f'(\alpha)\Bigg[4 + 4c_2e_n + \left(4c_2^2 + \frac{9}{2}c_3\right)e_n^2 + \left(11c_2c_3 + 5c_4 - 8c_2^3\right)e_n^3 \\ &\qquad +\left(6c_3^2 - \frac{59}{2}c_2^2c_3 + 16c_2^4 + \frac{45}{8}c_5 + 15c_2c_4\right)e_n^4 + O\left(e_n^5\right)\Bigg]. \end{aligned} \tag{3.8}$$

From (3.5) and (3.8), we obtain

$$\begin{aligned} &\frac{4f(y_n)}{f'(x_n) + 2f'((x_n+y_n)/2) + f'(y_n)} \\ &\quad = c_2e_n^2 + \left(-3c_2^2 + 2c_3\right)e_n^3 + \left(3c_4 + 7c_2^3 - \frac{81}{8}c_2c_3\right)e_n^4 + O\left(e_n^5\right). \end{aligned} \tag{3.9}$$

From (3.4) and (3.9), we have

$$z_n = \alpha + c_2^2e_n^3 + \left(-3c_2^3 + \frac{25}{8}c_2c_3\right)e_n^4 + O\left(e_n^5\right). \tag{3.10}$$

Expanding $f(z_n), f'(z_n), f'((x_n+z_n)/2)$ in Taylor's Series about α and using (3.10), we obtain

$$f(z_n) = f'(\alpha)\left[c_2^2e_n^3 + \left(-3c_2^3 + \frac{25}{8}c_2c_3\right)e_n^4 + O\left(e_n^5\right)\right], \tag{3.11}$$

$$f'(z_n) = f'(\alpha)\left[1 + 2c_2^3e_n^3 + \left(-6c_2^4 + \frac{25}{4}c_2^2c_3\right)e_n^4 + O\left(e_n^5\right)\right], \tag{3.12}$$

$$f'\left(\frac{x_n+z_n}{2}\right) = f'(\alpha)\left[1 + c_2e_n + \frac{3}{4}c_3e_n^2 + \left(\frac{1}{2}c_4 + c_2^3\right)e_n^3 + \left(-3c_2^4 + \frac{37}{8}c_2^2c_3 + \frac{5}{16}c_5\right)e_n^4 + O\left(e_n^5\right)\right]. \tag{3.13}$$

From (3.2), (3.12), and (3.13), we have

$$f'(x_n)+2f'\left(\frac{x_n+z_n}{2}\right)+f'(z_n)$$
$$=f'(\alpha)\left[4+4c_2e_n+\frac{9}{2}c_3e_n^2+\left(5c_4+4c_2^3\right)e_n^3+\left(-12c_2^4+\frac{31}{2}c_2^2c_3+\frac{45}{8}c_5\right)e_n^4+O\left(e_n^5\right)\right]. \tag{3.14}$$

From (3.11) and (3.14), we obtain

$$\frac{4f(z_n)}{f'(x_n)+2f'((x_n+z_n)/2)+f'(z_n)}=c_2^2e_n^3+\left(-4c_2^3+\frac{25}{8}c_2c_3\right)e_n^4+O\left(e_n^5\right). \tag{3.15}$$

From (3.10) and (3.15), we have

$$x_{n+1}=\alpha+c_2^3e_n^4+O\left(e_n^5\right). \tag{3.16}$$

Thus, we have

$$e_{n+1}=c_2^3e_n^4+O\left(e_n^5\right). \tag{3.17}$$

Error equation (3.17) shows that the Algorithm 2.3 is fourth-order convergent. □

4. Numerical Results

We now present some examples to illustrate the performance of the newly developed two-step and three-step iterative methods in this paper. We compare Newton method (NM), method of M. A. Noor et al. [9] (NNT), method of Chun [3] (CM), Algorithm 2.2 (NR1), and the Algorithm 2.3 (NR2) introduced in this paper. We used $\varepsilon = 10^{-15}$. The following stopping criteria is used for computer programs:

(i) $|x_{n+1}-x_n|<\varepsilon$,

(ii) $|f(x_n)|<\varepsilon$.

The computational order of convergence p approximated by means of

$$p\approx\frac{\ln(|x_{n+1}-x_n|/|x_n-x_{n-1}|)}{\ln(|x_n-x_{n-1}|/|x_{n-1}-x_{n-2}|)}. \tag{4.1}$$

Table 1: Numerical examples.

	IT	x_n	$f(x_n)$	δ	p
$f_1, \quad x_0 = -1$					
NM	7	1.40449164821534122603508681779	$-1.04e-50$	$7.33e-26$	2.00003
NNT	5	1.40449164821534122603508681779	0	$4.86e-29$	3.16501
CM	5	1.40449164821534122603508681779	0	$1.31e-17$	2.85844
NR1	5	1.40449164821534122603508681779	0	$3.19e-32$	3.03300
NR2	4	1.40449164821534122603508681779	0	$1.50e-25$	4.31447
$f_2, \; x_0 = 2$					
NM	6	0.25753028543986076045536730494	$2.93e-55$	$9.10e-28$	2.00050
NNT	5	0.25753028543986076045536730494	$1.00e-59$	$1.77e-24$	2.82952
CM	4	0.25753028543986076045536730494	0	$9.46e-29$	4.57143
NR1	4	0.25753028543986076045536730494	$-3.70e-52$	$2.24e-17$	3.57234
NR2	4	0.25753028543986076045536730494	$-1.00e-59$	$3.55e-43$	4.25114
$f_3, \; x_0 = 3.5$					
NM	8	2	$2.06e-42$	$8.28e-22$	2.00025
NNT	5	2	0	$3.45e-24$	2.83484
CM	5	2	0	$2.74e-24$	3.53144
NR1	6	2	0	$1.66e-40$	2.99063
NR2	5	2	0	$2.14e-42$	3.86697
$f_4, x_0 = 1.5$					
NM	7	2.15443469003188372175929356652	$2.06e-54$	$5.64e-28$	2.00003
NNT	5	2.15443469003188372175929356652	$1.00e-58$	$4.70e-43$	2.65300
CM	5	2.15443469003188372175929356652	$1.00e-58$	$1.57e-22$	3.48932
NR1	5	2.15443469003188372175929356652	$-8.00e-59$	$1.45e-35$	3.01710
NR2	4	2.15443469003188372175929356652	$-8.00e-59$	$3.79e-28$	4.20825
$f\_5, x\_0 = -2$					
NM	9	−1.20764782713091892700941675836	$-2.27e-40$	$2.73e-21$	2.00085
NNT	5	−1.20764782713091892700941675836	$8.00e-59$	$1.53e-32$	2.22201
CM	6	−1.20764782713091892700941675836	$-1.10e-58$	$2.15e-36$	3.88967
NR1	6	−1.20764782713091892700941675836	$-2.65e-56$	$8.33e-20$	3.01400
NR2	5	−1.20764782713091892700941675836	$-1.10e-58$	$2.34e-20$	4.04259
$f\_6, x\_0 = 3.5$					
NM	13	3	$1.52e-47$	$4.21e-25$	2.00023
NNT	7	3	0	$1.65e-30$	2.38562
CM	8	3	0	$2.12e-23$	3.68024
NR1	9	3	0	$1.24e-37$	2.99410
NR2	7	3	0	$4.33e-23$	3.84449

We consider the following nonlinear equations as test problems which are the same as M. Aslam Noor and K. Inayat Noor [10].

$$\begin{aligned} f_1(x) &= \sin^2 x - x^2 + 1, \\ f_2(x) &= x^2 - e^x - 3x + 2, \\ f_3(x) &= (x-1)^3 - 1, \\ f_4(x) &= x^3 - 10, \\ f_5(x) &= xe^{x^2} - \sin^2 x + 3\cos x + 5, \\ f_6(x) &= e^{x^2+7x-30} - 1. \end{aligned} \tag{4.2}$$

5. Conclusion

In this paper, we have considered one-step, two-step, and three-step iterative methods for solving nonlinear equations by using a different decomposition technique. Our method of derivation of the iterative methods is very simple as compared with the Adomian decomposition methods. From the Table 1, it is obvious that three-step method introduced in this paper performs better than the fourth-order method of Chun [3]. Using the technique and idea of this paper, one can suggest and analyze higher-order multistep iterative methods for solving nonlinear equations as well as system of nonlinear equations. It is an open problem to extend the technique and ideas of this paper for solving the obstacle problems associated with the variational inequalities and related problems see [20–23] and the references therein. This is another direction for future research.

Acknowledgments

The authors would like to thank Dr. S. M. Junaid Zaidi, Rector, CIIT, for providing excellent research facilities. This research is supported by the Visiting Professor Program of King Saud University, Riyadh, Saudi Arabia and Research Grant no. KSU.VPP.108.

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