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Research Article

Global Synchronization in Complex Networks with Adaptive Coupling

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Global synchronization in adaptive coupling networks is studied in this paper. A new simple adaptive controller is proposed based on a concept of asymptotically stable led by partial state variables. Under the proposed adaptive update law, the network can achieve global synchronization without calculating the eigenvalues of the outer coupling matrix. The update law is only dependent on partial state variables of individual oscillators. Numerical simulations are given to show the effectiveness of the proposed method, in which the unified chaotic system is chosen as the nodes of the network with different topologies.

1. Introduction

Synchronization in complex networks of identical chaotic oscillators has been studied extensively and deeply in various fields of science and engineering in the last few years [1–18]. Some effective methods have been proposed to investigate the stability of the synchronous state of linearly coupled networks. In [2], Pecora and Carroll developed a useful approach, called the Master Stability Function, to the local synchronization for any linear coupling networks. Based on the calculation of the eigenvalues of the outer coupling matrix, this method has been widely used in local stability studies of synchronization in linearly coupled complex networks [3–6]. Global synchronization based on the eigenvalues of the outer coupling matrix was also obtained for undirected [1] and directed [7] networks. These studies show that both local and global synchronization mainly depend upon the eigenvalues of the outer coupling matrix. However, the eigenvalues can be calculated only for simple coupling schemes. For more complicated networks, it becomes a difficult task.

In [8–10], an alternate way is developed to achieve network synchronization, which does not depend on explicit knowledge of the eigenvalues of the outer coupling matrix. This approach combines the Lyapunov function approach with graph theoretical reasoning. It guarantees global synchronization, not just local stability, and is also valid for time-varying networks. In [11–18], adaptive control strategy is used to guarantee local and global synchronization in complex networks. Especially in [11], Zhang et al. present a concept of x_k-leading asymptotically stable, and study the synchronization in complex networks with adaptive coupling. In [12], Huang studies the global synchronization in an adaptive weighted network, but the coupling strengths between two connected nodes are vector functions.

In [11], however, leading asymptotically stable is achieved by only one state variable. In actual fact, some systems are difficult to meet this condition. Those systems are asymptotically stable led by more than one state variable; moreover, the coupling strengths are not vector functions in practice. Inspired by the upper notions, this paper presents the concept of x_k $(k = 1, 2, \ldots, s)(s \leq n)$ leading asymptotically stable which means that the system achieves asymptotically stable led by partial state variables and studies the global synchronization in an adaptive coupling network whose coupling strengths are not vector function. By using Lyapunov's direct method, under a simple adaptive update law, such network can achieve global synchronization finally. The synchronization strategy is setting adaptive coupling strength between two connected nodes, and the adaptive update law of a_{ij} is only relation to partial state variables of connected nodes i and j. The regular star coupled network and ring coupled network are simulated, in which the unified chaotic system is chosen as the dynamical node in the network. The simulation results also show the effectiveness of the proposed adaptive method.

Another active research named consensus or agreement considers the information exchange between the connected agents [19–22]. The aim is to set consensus protocols or coupling scheme to reach agreement on graph and the network could have switching topology [20, 21]. Of course, the consensus state is usually time-invariant [20], such as, average-consensus, max-consensus, min-consensus, and so on. To some extent, consensus is the special case of synchronization but quite independently studied.

2. Network Model and Preliminaries

Consider an undirected complex dynamical network consisting of N identical coupled nodes. The dynamical behavior of the network can be described by the following ordinary differential equations:

$$\dot{x}_i = F(x_i) + \sum_{j=1}^{N} a_{ij}(t)\Gamma x_j, \quad i = 1, 2, \ldots, N, \tag{2.1}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in R^n$ is the state vector representing the state variables of node i. $F(x_i) = (F_1(x_i), F_2(x_i), \ldots, F_n(x_i))^T$ is a smooth nonlinear vector-valued function. $\Gamma = \mathrm{diag}(\gamma_1, \ldots, \gamma_{k_0}, 0, \ldots, 0) \in M_n(R)$ is the inner coupling matrix, where $\gamma_i > 0$ $(i = 1, 2, \ldots, k_0)$ are some constants.

The outer coupling matrix $A(t) = (a_{ij}(t))_{N\times N}$ represents not only the topological structure of the network, but also the weight strength, in which $a_{ij}(t)$ is defined as follows: if there is a connection between node i and node $j(j \neq i)$, then $a_{ij}(t) = a_{ji}(t) > 0$; otherwise,

$a_{ij}(t) = a_{ji}(t) = 0 (j \neq i)$. Suppose network (2.1) is diffusively coupled, that is, the diagonal elements of matrix $A(t)$ are defined by

$$a_{ii}(t) = -\sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij}(t), \quad i = 1, 2, \ldots, N. \tag{2.2}$$

Here the coupling strengths depend on the corresponding two nodes (i.e., $a_{ij}(t)$ depends on subscripts i and j) and time t. Simply speaking, our network is a weighted network. For convenience, we replace $a_{ij}(t)$ with a_{ij} in the following.

Definition 2.1. An equilibrium point $x = 0$ of a dynamical system $\dot{x} = f(x)$, $x = (x_1, x_2, \ldots, x_n)^T \in R^n$ is $x_k (k = 1, 2, \ldots, s)$ $(s \leq n)$ leading asymptotically stable if the asymptotical stability of $x_k = 0$ $(k = 1, 2, \ldots, s)$ can lead the asymptotical stability of $x_k = 0$ $(k = 1, 2, \ldots, n)$.

Remark 2.2. The x_k-leading asymptotically stable in [11] is a special case of $s = 1$ in our definition.

Letting $e_{ij} = (e_{ij1}, e_{ij2}, \ldots, e_{ijn})^T = x_j - x_i$, it is obvious that $e_{ii} = 0$, $e_{ji} = -e_{ij}$, and the error system is

$$\dot{e}_{ij} = F(x_j) - F(x_i) + \sum_{k=1}^{N} (a_{jk}\Gamma e_{jk} - a_{ik}\Gamma e_{ik}), \quad i, j = 1, 2, \ldots, N. \tag{2.3}$$

The coupled network (2.1) is said to achieve global synchronization if

$$\lim_{t \longrightarrow \infty} \|x_i(t) - x_j(t)\| = 0, \quad \text{that is, } e_{ij} \longrightarrow 0, \ (t \longrightarrow \infty), \ i, j = 1, 2, \ldots, N, \tag{2.4}$$

from arbitrary initial values.

3. Main Result

Now we can drive the following main result.

Theorem 3.1. *Suppose that system* (2.3) *satisfies the following conditions.*

(I) *Nonlinear function* $F: R^n \to R^n$ *satisfies the uniform Lipschitz condition, that is, for all* x, y, *there exists a constant* $l > 0$ *satisfying* $\|F(x) - F(y)\| \leq l\|x - y\|$.

(II) *Equilibrium point* $e_{ij} = 0$ *is* $e_{ijk} (k = 1, 2, \ldots, s; i, j = 1, 2, \ldots, N) (s \leq k_0)$ *leading asymptotically stable.*

(III) *The nonzero weight strength* a_{ij} $(i \neq j)$ *satisfy the following adaptive update law*

$$\dot{a}_{ij} = k_{ij} e_{ij}^T P \Gamma e_{ij} = k_{ij} (x_i - x_j)^T P \Gamma (x_i - x_j). \tag{3.1}$$

Then network (2.1) *with coupling strength* (3.1) *will achieve global synchronization as* $t \to \infty$, *where* $k_{ij} > 0$ *are some arbitrary constants,* $P = \operatorname{diag}(\underbrace{1,\ldots,1}_{s},0,\ldots,0) \in M_n(R)$.

Proof. Let the Lyapunov function be

$$V = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} e_{ij}^T P e_{ij} + \frac{N}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{k_{ij}}\left(a_{ij}-c\right)^2, \tag{3.2}$$

where c is a large positive constant. Obviously, it is positive definite about $e_{ijk}(k = 1,2,\ldots,s)$. The time derivative of V along the trajectory of the system (2.1) with coupling strength (3.1) is given by

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N}\sum_{j=1}^{N} e_{ij}^T P \dot{e}_{ij} + N\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{k_{ij}}\left(a_{ij}-c\right)\dot{a}_{ij} \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N} e_{ij}^T P\left(F(x_j)-F(x_i)\right) + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\left(a_{jk}e_{ij}^T P\Gamma e_{jk} - a_{ik}e_{ij}^T P\Gamma e_{ik}\right) + N\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{k_{ij}}\left(a_{ij}-c\right)\dot{a}_{ij}.\end{aligned} \tag{3.3}$$

From condition (I), the first sum $S_1 = \sum_{i=1}^{N}\sum_{j=1}^{N} e_{ij}^T P(F(x_j)-F(x_i)) \le \sum_{i=1}^{N}\sum_{j=1}^{N} l e_{ij}^T P e_{ij}$ holds. And the second sum can be calculated as follows. Since $e_{ij}^T = -e_{ji}^T$, we can see that

$$S_2 = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\left(a_{jk}e_{ij}^T P\Gamma e_{jk} - a_{ik}e_{ij}^T P\Gamma e_{ik}\right) = -\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\left(a_{jk}e_{ji}^T P\Gamma e_{jk} + a_{ik}e_{ij}^T P\Gamma e_{ik}\right). \tag{3.4}$$

Renaming the summation index i by j in the second term and vice versa, the second term becomes identical to the first and we obtain

$$S_2 = -2\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} a_{jk}e_{ji}^T P\Gamma e_{jk}. \tag{3.5}$$

Using $e_{jj} = 0$, we get

$$S_2 = -2\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{j} a_{jk}e_{ji}^T P\Gamma e_{jk} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=j}^{N} a_{jk}e_{ji}^T P\Gamma e_{jk}\right). \tag{3.6}$$

Renaming j by k in the second term and vice versa, and using the symmetry of $A(t)$, we obtain

$$\begin{aligned} S_2 &= -2\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{j} a_{jk}e_{ji}^T P\Gamma e_{jk} + \sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{j=k}^{N} a_{jk}e_{ki}^T P\Gamma e_{kj}\right) \\ &= -2\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{j} a_{jk}e_{ji}^T P\Gamma e_{jk} + \sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{j=k}^{N} a_{jk}e_{ik}^T P\Gamma e_{jk}\right) = -2\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{j} a_{jk}\left(e_{ji}^T + e_{ik}^T\right)P\Gamma e_{jk}. \end{aligned} \tag{3.7}$$

Since $e_{ji}^T + e_{ik}^T = x_i^T - x_j^T + x_k^T - x_i^T = e_{jk}^T$ and the symmetry of $A(t)$, we get

$$S_2 = -2\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{j} a_{jk}e_{jk}^T P\Gamma e_{jk} = -2N\sum_{j=1}^{N}\sum_{k=1}^{j} a_{jk}e_{jk}^T P\Gamma e_{jk} = -N\sum_{j=1}^{N}\sum_{k=1}^{N} a_{jk}e_{jk}^T P\Gamma e_{jk}. \tag{3.8}$$

So

$$\begin{aligned} \dot{V} &\le \sum_{i=1}^{N}\sum_{j=1}^{N} l e_{ij}^T P e_{ij} - N\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{ij}^T P\Gamma e_{ij} + N\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{k_{ij}}(a_{ij}-c)\dot{a}_{ij} \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N} l e_{ij}^T P e_{ij} - N\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{ij}^T P\Gamma e_{ij} + N\sum_{i=1}^{N}\sum_{j=1}^{N}(a_{ij}-c)e_{ij}^T P\Gamma e_{ij} = \sum_{i=1}^{N}\sum_{j=1}^{N} e_{ij}^T[lP - cNP\Gamma]e_{ij}. \end{aligned} \tag{3.9}$$

Because of $s \le k_0$, for large constant c, it holds that $lP - cNP\Gamma = \mathrm{diag}(c_1, c_2, \ldots, c_s, 0, \ldots, 0) \in M_n(R)$, where $c_i < 0$ $(i = 1, 2, \ldots, s)$. So $\dot{V}$ is negative definite about $e_{ijk}(k = 1, 2, \ldots, s)$. According to Lyapunov's direct method, starting with arbitrary initial values, $e_{ijk} \to 0$ $(t \to \infty)$ $(k = 1, 2, \ldots, s)$. Since equilibrium point $e_{ij} = 0$ of system (2.3) is $e_{ijk}(k = 1, 2, \ldots, s)$ $(s \le k_0)$ leading asymptotically stable, so network (2.1) can reach global synchronization as $t \to \infty$. This completes the proof. □

Remark 3.2. Condition (I) is easily satisfied if $\partial F_i/\partial x_j$ $(i, j = 1, 2, \ldots, n)$ are bounded. And a number of chaotic systems satisfy this condition, such as Rössler system and unified chaotic system.

Remark 3.3. In practice, we can set $k_{ij} = k$, and the adaptive update law is $\dot{a}_{ij} = k e_{ij}^T P e_{ij} = k(x_i - x_j)^T P(x_i - x_j)$.

Remark 3.4. By (3.1), the nonzero coupling strength $a_{ij}(t) \to C_{ij}$ when the network achieves synchronization, where C_{ij} are some positive constants.

Remark 3.5. Our synchronization strategy is setting adaptive coupling strengths between two nodes to achieve global synchronization, and our adaptive update law of a_{ij} is only relation to partial state variables of connected nodes i and j.

4. Illustrative Example

To demonstrate the theoretical result in Section 3, two regular networks with the unified chaotic system are chosen as simulation examples in this section.

The unified chaotic [23] system is described by

$$\begin{aligned} \dot{x}_1 &= (25a+10)(x_2-x_1), \\ \dot{x}_2 &= (28-35a)x_1+(29a-1)x_2-x_1x_3, \\ \dot{x}_3 &= x_1x_2-\frac{(a+8)x_3}{3}. \end{aligned} \tag{4.1}$$

When $a \in [0,1]$, it has well-known chaotic behavior.

We consider $\Gamma = \operatorname{diag}(\gamma_1, \gamma_2, \gamma_3)$, $\gamma_i > 0$ $(i = 1,2,3)$, that is,

$$\begin{aligned} \dot{x}_{i1} &= (25a+10)(x_{i2}-x_{i1})+\gamma_1\sum_{j=1}^{N}c_{ij}a_{ij}x_{j1}, \\ \dot{x}_{i2} &= (28-35a)x_{i1}+(29a-1)x_{i2}-x_{i1}x_{i3}+\gamma_2\sum_{j=1}^{N}c_{ij}a_{ij}x_{j2}, \quad (i=1,2,\ldots,N), \\ \dot{x}_{i3} &= x_{i1}x_{i2}-\frac{(a+8)x_{i3}}{3}+\gamma_3\sum_{j=1}^{N}c_{ij}a_{ij}x_{j3}. \end{aligned} \tag{4.2}$$

The error system is

$$\begin{aligned} \dot{e}_{ij1} &= (25a+10)(e_{ij2}-e_{ij1})+\gamma_1\sum_{k=1}^{N}(c_{jk}a_{jk}e_{jk1}-c_{ik}a_{ik}e_{ik1}), \\ \dot{e}_{ij2} &= (28-35a)e_{ij1}+(29a-1)e_{ij2}-x_{j1}x_{j3}+x_{i1}x_{i3}+\gamma_2\sum_{k=1}^{N}(c_{jk}a_{jk}e_{jk2}-c_{ik}a_{ik}e_{ik2}) \quad (i,j=1,2,\ldots,N), \\ \dot{e}_{ij3} &= x_{j1}x_{j2}-x_{i1}x_{i2}-\frac{(a+8)e_{ij3}}{3}+\gamma_3\sum_{k=1}^{N}(c_{jk}a_{jk}e_{jk3}-c_{ik}a_{ik}e_{ik3}). \end{aligned} \tag{4.3}$$

In order to use our Theorem, we need the following Lemma.

Lemma 4.1. *For $a \in [0,1]$, system (4.3) is $e_{ijk}(k=1,2)$ leading asymptotically stable.*

Proof. If $e_{ijk} = 0$ $(k = 1,2)$ for all i and j, choose the following Lyapunov function:

$$V = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}e_{ij3}^2. \tag{4.4}$$

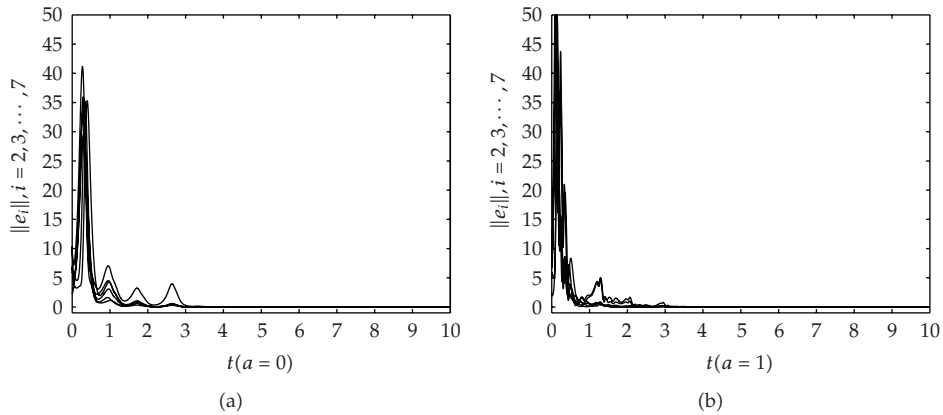

Figure 1: Synchronization errors $\|e_i\|$ $(i = 2, 3, \ldots, 7)$ of the star coupled network.

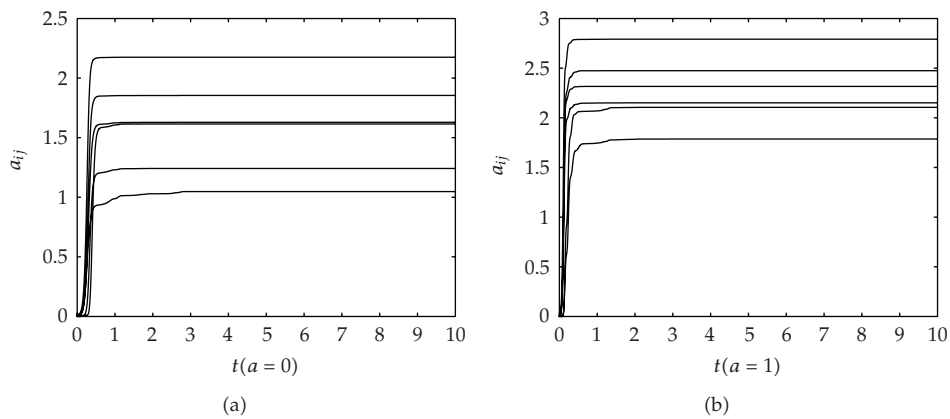

Figure 2: Weight strength a_{ij} evolving for the star coupled network.

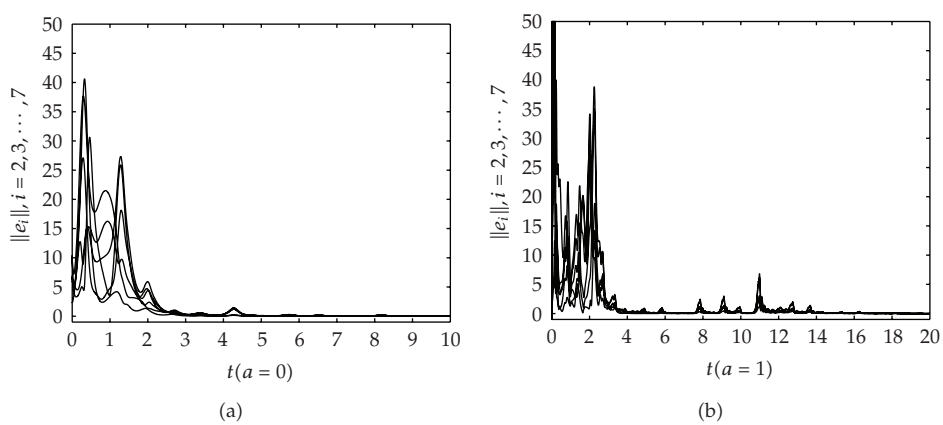

Figure 3: Synchronization errors $\|e_i\|$ $(i = 2, 3, \ldots, 7)$ of the ring coupled network.

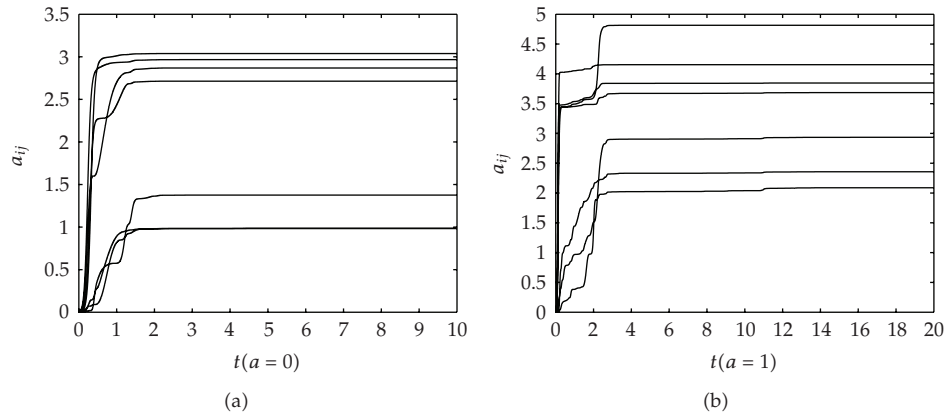

Figure 4: Weight strength a_{ij} evolving for the ring coupled network.

The derivative of V along with the solution of (4.2) is

$$\dot{V} = \sum_{i=1}^{N}\sum_{j=1}^{N} e_{ij3}\dot{e}_{ij3} = \sum_{i=1}^{N}\sum_{j=1}^{N}\left[-\frac{(a+8)e_{ij3}^2}{3}\right] + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\left(a_{jk}e_{ij3}e_{jk3} - a_{ik}e_{ij3}e_{ik3}\right). \tag{4.5}$$

Similar to the calculation of S_2, we can get $\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}(a_{jk}e_{ij3}e_{jk3} - a_{ik}e_{ij3}e_{ik3}) < 0$ and for all $a \in [0,1]$, $\dot{V} < 0$. So it is said that $e_{ij3} \to 0$ $(t \to \infty)$ $(i, j = 1, 2, \ldots, N)$.

If we set $\dot{a}_{ij} = k_{ij}\gamma_1(x_{i1} - x_{j1})^2 + k_{ij}\gamma_2(x_{i2} - x_{j2})^2$, the network (4.3) satisfies the conditions of the Theorem and will achieve global synchronization finally. □

Remark 4.2. According to Remark 3.3, we choose

$$\dot{a}_{ij} = ke_{ij}^T P e_{ij} = k\left(x_{i1} - x_{j1}\right)^2 + k\left(x_{i2} - x_{j2}\right)^2, \tag{4.6}$$

in our simulation, and $\Gamma = \mathrm{diag}(1, 2, 3)$, $k = 0.01$, $N = 7$.

We consider two types of regular network: star coupled network and ring coupled network. Note that our adaptive strategy is also valid for other type of complex networks.

(1) Star coupled network

Figure 1 shows the synchronization errors $\|e_i\| = \|x_i - x_1\|$ $(i = 2, 3, \ldots, 7)$ of the star coupled network when $a = 0$ and $a = 1$, respectively. From Figure 1, we can see that the synchronization errors evolve and converge to zero. In Figure 2, we plot the curve of the weight strength a_{ij} of $a = 0$ and $a = 1$. We can see that a_{ij} converges to some constants, respectively.

(2) Ring coupled network

Figure 3 shows the synchronization errors $\|e_i\| = \|x_i - x_1\|$ $(i = 2, 3, \ldots, 7)$ of the ring coupled network when $a = 0$ and $a = 1$, respectively. It is said that the network achieves global synchronization finally. Figure 4 shows that the weight strength a_{ij} of $a = 0$ and $a = 1$ converges to some constants, respectively.

5. Conclusion

In this paper, we presented a new concept of asymptotically stable led by partial state variables and designed a simple adaptive controller for global synchronization in weight complex dynamical networks. For this network, we proved by using Lyapunov's direct method that the states of such complex network can achieve global synchronization finally under our adaptive update law. Our synchronization strategy is to set adaptive coupling strength, and our adaptive update law of a_{ij} is only relation to partial state variables of connected nodes i and j. The complex networks with star coupled and ring coupled are simulated, in which the unified chaotic system is chosen as the dynamical node of the network. The simulation results also show the effectiveness of our method.

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