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*Research Article*

# Stochastic Finite-Time Guaranteed Cost Control of Markovian Jumping Singular Systems

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The problem of stochastic finite-time guaranteed cost control is investigated for Markovian jumping singular systems with uncertain transition probabilities, parametric uncertainties, and time-varying norm-bounded disturbance. Firstly, the definitions of stochastic singular finite-time stability, stochastic singular finite-time boundedness, and stochastic singular finite-time guaranteed cost control are presented. Then, sufficient conditions on stochastic singular finite-time guaranteed cost control are obtained for the family of stochastic singular systems. Designed algorithms for the state feedback controller are provided to guarantee that the underlying stochastic singular system is stochastic singular finite-time guaranteed cost control in terms of restricted linear matrix equalities with a fixed parameter. Finally, numerical examples are given to show the validity of the proposed scheme.

## 1. Introduction

Singular systems are also referred to as descriptor systems or generalized state-space systems and describe a larger family of dynamic systems. The singular systems are applied to handle mechanical systems, electric circuits, interconnected systems, and so forth; see more practical examples in [1, 2] and the references therein. Many control problems have been extensively investigated, and results in state-space systems have been extended to singular systems, such as stability, stabilization, and robust control; for instance, see the references in [3–10]. Meanwhile, Markovian jumping systems are referred to as a special family of hybrid systems and stochastic systems, which are very appropriate to model plants whose structure is subject to random abrupt changes, such as stochastic failures and repairs of the components, changes in the interconnections of subsystems, and sudden environment changes; see the references in [11, 12]. The existing results for Markovian jumping systems include a large of variety of problems such as stochastic Lyapunov stability [13–16], sliding mode control [17, 18], the

$H^{\infty}$ control [19, 20], the $H^{\infty}$ filtering [12, 21], and so forth; for more results, the readers are to refer to [22–24] and the references therein.

In many practical applications, on the other hand, many concerned problems are the practical ones which described system state which does not exceed some bound over a time interval. Compared with classical Lyapunov asymptotical stability on which most results in the literature concentrated, finite-time stability (FTS) or short-time stability was studied to deal with the transient behavior of systems in finite time interval. Some earlier results on FTS can be found in [25–28]. Some appealing results were obtained to guarantee finite-time stability, finite-time boundedness, and finite-time stabilization of different systems including linear systems, nonlinear systems, and stochastic systems; for instance, see the papers in [29–35] and the references therein. However, to date and to the best of our knowledge, the problems of stochastic singular finite-time guaranteed cost control analysis of stochastic singular systems have not been investigated, although some studies on stochastic singular systems have been conduced recently; see the references [8–11, 15, 18]. We investigate finite-time guaranteed cost control of one class of continuous-time stochastic singular systems. Our results are totally different from those previous results. This motivates us for the study.

It is well known that linear matrix inequalities (LMIs) have viewed as a powerful formulation and design technique for a variety of linear control problems. Thus reducing a control design problem to an LMI can be considered as a practical solution to this problem [36]. At present, it is an important tool to address stability and stabilization, roust control, the $H^{\infty}$ filtering, guaranteed cost control, and so forth; see the references [2, 4–11, 13, 15] and the references therein. The novelty of our study is that stochastic finite-time stability, stochastic finite-time bounded and stochastic finite-time guaranteed cost control are investigated for one family of Markovian jumping singular systems with uncertain transition probabilities, parametric uncertainties, and time-varying norm-bounded disturbance. The main contribution of this paper is that sufficient conditions on stochastic singular finite-time guaranteed cost control are obtained for the class of stochastic singular systems and, a state feedback controller is designed to guarantee that the underlying stochastic singular system is stochastic singular finite-time guaranteed cost control in terms of restrict LMIs with a fixed parameter.

The rest of this paper is organized as follows. In Section 2 the problem formulation and some preliminaries are introduced. The results on stochastic singular finite-time guaranteed cost control are given in Section 3. Section 4 presents numerical examples to demonstrate the validity of the proposed methodology. Some conclusions are drawn in Section 5.

*Notations*

Throughout the paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the sets of $n$ component real vectors and $n \times m$ real matrices, respectively. The superscript $T$ stands for matrix transposition or vector. $E\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\wp$. In addition, the symbol $*$ denotes the transposed elements in the symmetric positions of a matrix, and $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix. $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote the smallest and the largest eigenvalue of matrix $P$, respectively. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

## 2. Problem Formulation

Let the dynamics of the class of Markovian jumping singular systems be described by the following:

$$E(r_t)\dot{x}(t) = [A(r_t) + \Delta A(r_t)]x(t) + [B(r_t) + \Delta B(r_t)]u(t) + [G(r_t) + \Delta G(r_t)]w(t), \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is system state, $u(t) \in \mathbb{R}^m$ is system input, $E(r_t)$ is a singular matrix with rank $E(r_t) = r_i < n$; $\{r_t, t \geq 0\}$ is continuous-time Markovian stochastic process taking values in a finite space $\mathcal{M} := \{1, 2, \ldots, N\}$ with transition matrix $\Gamma = (\pi_{ij})_{N\times N}$ and the transition probabilities are described as follows:

$$P_{ij} = Pr(r_{t+\Delta} = j \mid r_t = i) = \begin{cases} \pi_{ij}\Delta + o(\Delta), & \text{if } i \neq j, \\ 1 + \pi_{ij}\Delta + o(\Delta), & \text{if } i = j, \end{cases} \tag{2.2}$$

where $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$, $\pi_{ij}$ satisfies $\pi_{ij} \geq 0$ $(i \neq j)$, and $\pi_{ii} = -\sum_{j=1,\, j\neq i}^{N} \pi_{ij}$ for all $i, j \in \mathcal{M}$; $\Delta A(r_t)$, $\Delta B(r_t)$, and $\Delta G(r_t)$ are uncertain matrices and satisfy

$$[\Delta A(r_t), \Delta B(r_t), \Delta G(r_t)] = F(r_t)\Delta(r_t)[E_1(r_t), E_2(r_t), E_3(r_t)], \tag{2.3}$$

where $\Delta(r_t)$ is an unknown, time-varying matrix function and satisfies

$$\Delta^T(r_t)\Delta(r_t) \leq I, \quad \forall r_t \in \mathcal{M}. \tag{2.4}$$

Moreover, the disturbance $w(t) \in \mathbb{R}^p$ satisfies

$$\int_0^T w^T(t)w(t)dt < d^2, \quad d > 0, \tag{2.5}$$

and the matrices $A(r_t)$, $B(r_t)$, and $G(r_t)$ are coefficient matrix and of appropriate dimension for all $r_t \in \mathcal{M}$. In addition, we make the following assumption on uncertain transition probabilities in stochastic singular system (2.1).

*Assumption 1.* The jump rates of the visited modes from a given mode $i$ are assumed to satisfy

$$0 < \underline{\pi}_i \leq \pi_{ij} \leq \overline{\pi}_i, \quad \forall i, j \in \mathcal{M},\ i \neq j, \tag{2.6}$$

where $\underline{\pi}_i$ and $\overline{\pi}_i$ are known parameters for each mode or may represent the lower and upper bounds when all the jump rates are known, that is,

$$0 < \underline{\pi}_i = \min_{i,j\in\mathcal{M}}\{\pi_{ij} \neq 0, i \neq j\} \leq \overline{\pi}_i = \max_{i,j\in\mathcal{M}}\{\pi_{ij} \neq 0, i \neq j\}. \tag{2.7}$$

Moreover, let $N_i$ denote the number of visited modes from $i$ including the mode itself.

Consider a state feedback controller

$$u(t) = K(r_t)x(r_t), \tag{2.8}$$

where $\{K(r_t), r_t = i \in \mathcal{M}\}$ is a set of matrices to be determined later. The system (2.1) with the controller (2.8) can be written by the form of the control system as follows:

$$E(r_t)\dot{x}(t) = \overline{A}(r_t)x(t) + \overline{G}(r_t)w(t), \tag{2.9}$$

where $\overline{A}(r_t) = A(r_t) + \Delta A(r_t) + [B(r_t) + \Delta B(r_t)]K(r_t)$ and $\overline{G}(r_t) = G(r_t) + \Delta G(r_t)$.

*Definition 2.1* (see [12, regular and impulse free]). (i) The singular system with Markovian jumps (2.9) with $u(t) = 0$ is said to be regular in time interval $[0, T]$ if the characteristic polynomial $\det(sE(r_t) - A(r_t))$ is not identically zero for all $t \in [0, T]$.

(ii) The singular systems with Markovian jumps (2.9) with $u(t) = 0$ is said to be impulse free in time interval $[0, T]$, if $\deg(\det(sE(r_t) - A(r_t))) = \operatorname{rank}(E(r_t))$ for all $t \in [0, T]$.

*Definition 2.2* (stochastic singular finite-time stability (SSFTS)). The closed-loop singular system with Markovian jumps (2.9) with $w(t) = 0$ is said to be SSFTS with respect to $(c_1, c_2, T, R(r_t))$, with $c_1 < c_2$ and $R(r_t) > 0$, if the stochastic system is regular and impulse free in time interval $[0, T]$ and satisfies

$$E\{x^T(0)E^T(r_t)R(r_t)E(r_t)x(0)\} \leq c_1^2 \Longrightarrow E\{x^T(t)E^T(r_t)R(r_t)E(r_t)x(t)\} < c_2^2, \quad \forall t \in [0, T]. \tag{2.10}$$

*Definition 2.3* (stochastic singular finite-time boundedness (SSFTB)). The closed-loop singular system with Markovian jumps (2.9) which satisfies (2.5) is said to be SSFTB with respect to $(c_1, c_2, T, R(r_t), d)$, with $c_1 < c_2$ and $R(r_t) > 0$, if the stochastic system is regular and impulse free in time interval $[0, T]$ and condition (2.10) holds.

*Remark 2.4.* SSFTB implies that not only is the dynamical mode of the stochastic singular system finite-time bounded but also the whole mode of the stochastic singular system is finite-time bounded in that the static mode is regular and impulse free.

*Definition 2.5* (see [11, 13]). Let $V(x(t), r_t = i,\ t > 0)$ be the stochastic function; define its weak infinitesimal operator $\mathbb{L}$ of stochastic process $\{(x(t), r_t = i), t \geq 0\}$ by

$$\mathbb{L}V(x(t), r_t = i, t) = V_t(x(t), i, t) + V_x(x(t), i, t)\dot{x}(t, i) + \sum_{j=1}^{N} \pi_{ij} V(x(t), j, t). \tag{2.11}$$

Associated with this system (2.9) is the cost function

$$J_T(r_t) = E\left\{\int_0^T \left[x^T(t)R_1(r_t)x(t) + u^T(t)R_2(r_t)u(t)\right]dt\right\}, \tag{2.12}$$

where $R_1(r_t)$ and $R_2(r_t)$ are two given symmetric positive definite matrices for all $r_t = i \in \mathcal{M}$.

*Definition 2.6.* There exists a controller (2.8) and a scalar $\psi_0$ such that the closed-loop stochastic singular system with Markovian jumps (2.9) is SSFTB with respect to $(c_1, c_2, T, R(r_t), d)$ and the value of the cost function (2.12) satisfies $J_T(r_t) < \psi_0$ for all $r_t \in \mathcal{M}$; then stochastic singular system (2.9) is said to be stochastic singular finite-time guaranteed cost control. Moreover, $\psi_0$ is said to be a stochastic singular guaranteed cost bound, and the designed controller (2.8) is said to be a stochastic singular finite-time guaranteed cost controller for stochastic singular system (2.9).

In the paper, our main aim is to concentrate on designing a state feedback controller of the form (2.8) that renders the closed-loop stochastic singular system with Markovian jumps (2.9) stochastic singular finite-time guarantee cost control.

**Lemma 2.7** (see [36]). *For matrices $\Upsilon$, $D$, and $H$ of appropriate dimensions, where $\Upsilon$ is a symmetric matrix, then*

$$\Upsilon + DF(t)H + H^T F^T(t)D^T < 0 \tag{2.13}$$

*holds for all matrix $F(t)$ satisfying $F^T(t)F(t) \le I$ for all $t \in \mathbb{R}$, if and only if there exists a positive constant $\epsilon$, such that the following equality holds:*

$$\Upsilon + \epsilon DD^T + \epsilon^{-1}H^T H < 0. \tag{2.14}$$

## 3. Main Results

This section deals with the guaranteed cost SSFTB analysis and design for the closed-loop singular system with Markovian jumps (2.9).

**Theorem 3.1.** *The closed-loop singular system with Markovian jumps* (2.9) *is SSFTB with respect to $(c_1, c_2, T, R(r_t), d)$, if there exist a scalar $\alpha \ge 0$, a set of nonsingular matrices $\{P(i),\ i \in \mathcal{M}\}$ with $P(i) \in \mathbb{R}^{n\times n}$, sets of symmetric positive definite matrices $\{Q_1(i),\ i \in \mathcal{M}\}$ with $Q_1(i) \in \mathbb{R}^{n\times n}$, $\{Q_2(i),\ i \in \mathcal{M}\}$ with $Q_2(i) \in \mathbb{R}^{p\times p}$, and for all $r_t = i \in \mathcal{M}$ such that*

$$E(i)P^T(i) = P(i)E^T(i) \ge 0, \tag{3.1a}$$

$$\begin{bmatrix} \overline{A}(i)P^T(i) + P(i)\overline{A}^T(i) + \Gamma(i) & \overline{G}(i) \\ * & -Q_2(i) \end{bmatrix} < 0, \tag{3.1b}$$

$$P^{-1}(i)E(i) = E^T(i)R^{1/2}(i)Q_1(i)R^{1/2}(i)E(i), \tag{3.1c}$$

$$\max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_1(i))\}c_1^2 + \max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_2(i))\}d^2 < \min_{i\in\mathcal{M}}\{\lambda_{\min}(Q_1(i))\}c_2^2 e^{-\alpha T} \tag{3.1d}$$

*hold, where $\Gamma(i) = \sum_{j=1}^{N}\pi_{ij}P(i)P^{-1}(j)E(j)P^T(i) + P(i)[R_1(i) + K^T(i)R_2(i)K(i)]P^T(i) - \alpha E(i)P^T(i)$. Moreover, a stochastic singular finite-time guaranteed cost bound for the stochastic singular system can be chosen as $\psi_0 = e^{\alpha T}\max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_1(i))c_1^2\} + \max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_2(i))d^2\}$.*

*Proof.* Firstly, we prove that the singular system with Markovian jumps (2.9) is regular and impulse free in time interval $[0, T]$. By Schur complement and noting condition (3.1b), we have

$$\overline{A}(i)P^T(i) + P(i)\overline{A}^T(i) + (\pi_{ii} - \alpha)E(i)P^T(i) < -\sum_{j=1,\, j\neq 1}^{N}\pi_{ij}P(i)P^{-1}(j)E(j)P^T(i) \le 0. \tag{3.2}$$

Now, we choose nonsingular matrices $M(i)$ and $N(i)$ such that

$$M(i)E(i)N(i) = \operatorname{diag}\{I_{r_i}, 0\}, \qquad M(i)\overline{A}(i)N(i) = \begin{bmatrix} A_{11}(i) & A_{12}(i) \\ A_{21}(i) & A_{22}(i) \end{bmatrix},$$
$$M(i)P(i)N^{-T}(i) = \begin{bmatrix} P_{11}(i) & P_{12}(i) \\ P_{21}(i) & P_{22}(i) \end{bmatrix}. \tag{3.3}$$

Then, we have

$$E(i) = M^{-1}(i)\operatorname{diag}\{I_{r_i}, 0\}N^{-1}(i), \qquad P(i) = M^{-1}(i)\begin{bmatrix} P_{11}(i) & P_{12}(i) \\ P_{21}(i) & P_{22}(i) \end{bmatrix}N^{T}(i). \tag{3.4}$$

From (3.1a) and (3.4), one can obtain

$$\begin{aligned} &\left(M^{-1}(i)\operatorname{diag}\{I_{r_i}, 0\}N^{-1}(i)\right)\left(M^{-1}(i)\begin{bmatrix} P_{11}(i) & P_{12}(i) \\ P_{21}(i) & P_{22}(i) \end{bmatrix}N^{T}(i)\right)^{T} \\ &\quad = \left(M^{-1}(i)\begin{bmatrix} P_{11}(i) & P_{12}(i) \\ P_{21}(i) & P_{22}(i) \end{bmatrix}N^{T}(i)\right)\left(M^{-1}(i)\operatorname{diag}\{I_{r_i}, 0\}N^{-1}(i)\right)^{T} \geq 0. \end{aligned} \tag{3.5}$$

Computing the above condition (3.5) and noting that $P(i)$ is nonsingular matrix, one can obtain from (3.3) and (3.4) that $P_{11}(i) = P_{11}^{T}(i) \geq 0$, $P_{21}(i) = 0$ and $\det(P_{22}(i)) \neq 0$ for all $i \in \mathcal{M}$. Thus, we have

$$E(i)P^{T}(i) = P(i)E^{T}(i) = M^{-1}(i)\begin{bmatrix} P_{11}(i) & 0 \\ 0 & 0 \end{bmatrix}M^{-T}(i) \geq 0. \tag{3.6}$$

Pre- and post-multiplying (3.2) by $M(i)$ and $M^{T}(i)$, respectively, and noting the equality (3.6), this results in the following matrix inequality:

$$\begin{bmatrix} \star & \star \\ \star & A_{22}(i)P_{22}^{T}(i) + P_{22}(i)A_{22}^{T}(i) \end{bmatrix} < 0, \tag{3.7}$$

where the star $\star$ will not be used in the following discussion. By Schur complement, we have $A_{22}(i)P_{22}^{\mathrm{T}}(i) + P_{22}(i)A_{22}^{T}(i) < 0$. Therefore $A_{22}(i)$ is nonsingular, which implies that the closed-loop continuous-time singular system with Markovian jumps (2.9) is regular and impulse free in time interval $[0, T]$.

Let us consider the quadratic Lyapunov-Krasovskii functional candidate as $V(x(t), i) = x^{T}(t)P^{-1}(i)E(i)x(t)$ for stochastic singular system (2.9). Computing $\mathbb{L}V$ the derivative of $V(x(t),i)$ along the solution of system (2.9) and noting the condition (3.1a), we obtain

$$\mathbb{L}V(x(t),i) = \xi^T(t)\begin{bmatrix} P^{-1}(i)\overline{A}(i) + \overline{A}^T(i)P^{-T}(i) + \sum_{j=1}^{N}\pi_{ij}P^{-1}(j)E(j) - \alpha P^{-1}(i)E(i) & P^{-1}(i)\overline{G}(i) \\ * & 0 \end{bmatrix}\xi(t), \tag{3.8}$$

where $\xi(t) = [x^T(t), w^T(t)]^T$. Pre- and postmultiplying (3.1b) by $\operatorname{diag}\{P^{-1}(i), I\}$ and $\operatorname{diag}\{P^{-T}(i), I\}$, respectively, we obtain

$$\begin{bmatrix} P^{-1}(i)\overline{A}(i) + \overline{A}^T(i)P^{-T}(i) + \sum_{j=1}^{N}\pi_{ij}P^{-1}(j)E(j) & \\ +R_1(i) + K^T(i)R_2(i)K(i) - \alpha P^{-1}(i)E(i) & P^{-1}(i)\overline{G}(i) \\ * & -Q_2(i) \end{bmatrix} < 0. \tag{3.9}$$

Noting that $R_1(i)$ and $R_2(i)$ are two symmetric positive definite matrices for all $i \in \mathcal{M}$, thus, from (3.8) and (3.9), we have

$$E\{\mathbb{L}V(x(t),i)\} < \alpha E\{V(x(t),i)\} + w^T(t)Q_2(i)w(t). \tag{3.10}$$

Further, (3.10) can be rewritten as

$$E\left\{e^{-\alpha t}\mathbb{L}V(x(t),i)\right\} < e^{-\alpha t}w^T(t)Q_2(i)w(t). \tag{3.11}$$

Integrating (3.11) from 0 to $t$, with $t \in [0,T]$ and noting that $\alpha \geq 0$, we obtain

$$e^{-\alpha t}E\{V(x(t),i)\} < E\{V(x(0), i = r_0)\} + \int_0^t e^{-\alpha\tau}w^T(\tau)Q_2(i)w(\tau)d\tau. \tag{3.12}$$

Noting that $\alpha \geq 0$, $t \in [0,T]$, and condition (3.1c), we have

$$\begin{aligned} E\left\{x^T(t)P^{-1}(i)E(i)x(t)\right\} &= E\{V(x(t),i)\} \\ &< e^{\alpha t}E\{V(x(0), i = r_0)\} + e^{\alpha t}\int_0^t e^{-\alpha\tau}w^T(\tau)Q_2(i)w(\tau)d\tau \\ &\leq e^{\alpha t}\left\{\max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_1(i))\}c_1^2 + \max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_2(i))\}d^2\right\}. \end{aligned} \tag{3.13}$$

Taking into account that

$$\begin{aligned} E\left\{x^T(t)P^{-1}(i)E(i)x(t)\right\} &= E\left\{x^T(t)E^T(i)R^{1/2}(i)Q_1(i)R^{1/2}(i)E(i)x(t)\right\} \\ &\geq \min_{i\in\mathcal{M}}\{\lambda_{\min}(Q_1(i))\}E\left\{x^T(t)E^T(i)R(i)E(i)x(t)\right\}, \end{aligned} \tag{3.14}$$

we obtain

$$\begin{aligned} E\left\{x^T(t)E^T(i)R(i)E(i)x(t)\right\} &\leq \max_{i\in\mathcal{M}}\left\{\lambda_{\max}\left(Q_1^{-1}(i)\right)\right\}\mathrm{E}\left\{x^T(t)P(i)E(i)x(t)\right\} \\ &< e^{\alpha T}\frac{\max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_1(i))\}c_1^2 + \max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_2(i))\}d^2}{\min_{i\in\mathcal{M}}\{\lambda_{\min}(Q_1(i))\}}. \end{aligned} \tag{3.15}$$

Therefore, it follows that condition (3.1d) implies $E\{x^T(t)E^T(r_t)R(r_t)E(r_t)x(t)\} \leq c_2^2$ for all $t \in [0, T]$.

Once again from (3.8) and (3.9), we can easily obtain

$$\mathbb{L}V(x(t), i) < \alpha V(x(t), i) + w^T(t)Q_2(i)w(t) - \left[x^T(t)R_1(i)x(t) + u^T(t)R_2(i)u(t)\right]. \tag{3.16}$$

Further, (3.16) can be represented as

$$\mathbb{L}\left[e^{-\alpha t}V(x(t), i)\right] < e^{-\alpha t}w^T(t)Q_2(i)w(t) - e^{-\alpha t}\left[x^T(t)R_1(i)x(t) + u^T(t)R_2(i)u(t)\right]. \tag{3.17}$$

Integrating (3.17) from 0 to $T$, we have

$$\begin{aligned} &\int_0^T e^{-\alpha t}\left[x^T(t)R_1(i)x(t) + u^T(t)R_2(i)u(t)\right]dt \\ &\quad< \int_0^T e^{-\alpha t}w^T(t)Q_2(i)w(t)dt - \int_0^T \mathbb{L}\left[e^{-\alpha t}V(x(t), i)\right]dt. \end{aligned} \tag{3.18}$$

Using the Dynkin formula and the fact that the system (2.9) is regular and impulse free, we obtain

$$\begin{aligned} &E\left\{\int_0^T e^{-\alpha t}\left[x^T(t)R_1(i)x(t) + u^T(t)R_2(i)u(t)\right]dt\right\} \\ &\quad< \int_0^T e^{-\alpha t}w^T(t)Q_2(i)w(t)dt - E\left\{\int_0^T \mathbb{L}\left[e^{-\alpha t}V(x(t), i)\right]dt\right\}. \end{aligned} \tag{3.19}$$

Noting that $\alpha \geq 0$ and $R_1(i)$ and $R_2(i)$ are two given symmetric positive definite matrices for all $i \in \mathcal{M}$, thus, we have

$$\begin{aligned} J_T(i) &= E\left\{\int_0^T \left[x^T(t)R_1(i)x(t) + u^T(t)R_2(i)u(t)\right]dt\right\} \\ &\leq e^{\alpha T}E\left\{\int_0^T e^{-\alpha t}\left[x^T(t)R_1(i)x(t) + u^T(t)R_2(i)u(t)\right]\mathrm{d}t\right\} \\ &< e^{\alpha T}\left\{\int_0^T e^{-\alpha t}w^T(t)Q_2(i)w(t)dt - E\left\{\int_0^T \mathbb{L}\left[e^{-\alpha t}V(x(t),i)\right]dt\right\}\right\} \\ &\leq e^{\alpha T}\left\{\max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_1(i))\}c_1^2 + \max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_2(i))\}d^2\right\}. \end{aligned} \tag{3.20}$$

Thus, one can obtain that the cost function

$$J_T(i) < \psi_0 = e^{\alpha T}\left\{\max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_1(i))\}c_1^2 + \max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_2(i))\}d^2\right\} \tag{3.21}$$

holds for all $i \in \mathcal{M}$. This completes the proof of the theorem. □

**Corollary 3.2.** *The singular system with Markovian jumps* (2.9) *with $w(t) = 0$ is SSFTS with respect to $(c_1, c_2, T, R(r_t))$, if there exist a scalar $\alpha \geq 0$, a set of nonsingular matrices $\{P(i), i \in \mathcal{M}\}$ with $P(i) \in \mathbb{R}^{n\times n}$, a set of symmetric positive definite matrices $\{Q_1(i), i \in \mathcal{M}\}$ with $Q_1(i) \in \mathbb{R}^{n\times n}$, and for all $r_t = i \in \mathcal{M}$ such that* (3.1a), (3.1c) *and*

$$\overline{A}(i)P^T(i) + P(i)\overline{A}^T(i) + \Gamma(i) < 0, \tag{3.22a}$$

$$\max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_1(i))\}c_1^2 < \min_{i\in\mathcal{M}}\{\lambda_{\min}(Q_1(i))\}c_2^2e^{-\alpha T} \tag{3.22b}$$

*hold, where $\Gamma(i) = \sum_{j=1}^N \pi_{ij}P(i)P^{-1}(j)E(j)P^T(i) + P(i)[R_1(i) + K^T(i)R_2(i)K(i)]P^T(i) - \alpha E(i)P^T(i)$. Moreover, a guaranteed cost bound for stochastic singular system can be chosen as $\psi_0 = \max\{e^{\alpha T}\lambda_{\max}(Q_1(i))c_1^2,\ i \in \mathcal{M}\}$.*

By Lemma 2.7, Theorem 3.1, and using matrix decomposition novelty, we can obtain the following theorem.

**Theorem 3.3.** *There exists a state feedback controller $u = K(r_t)x(t)$ with $K(r_t) = L^T(r_t)P^{-T}(r_t)$, $r_t = i \in \mathcal{M}$ such that the closed-loop stochastic singular system with Markovian jumps* (2.9) *is SSFTB with respect to $(c_1, c_2, T,\ R(r_t), d)$, if there exist a scalar $\alpha \geq 0$, a set of positive matrices $\{X(i), i \in \mathcal{M}\}$ with $X(i) \in \mathbb{R}^{n\times n}$, a set of symmetric positive definite matrices $\{Q_2(i), i \in \mathcal{M}\}$*

*with* $Q_2(i) \in \mathbb{R}^{p\times p}$, *and a set of matrices* $\{Y(i), i \in \mathcal{M}\}$ *with* $Y(i) \in \mathbb{R}^{n\times(n-r_i)}$, *two sets of positive scalars* $\{\sigma_i, i \in \mathcal{M}\}$ *and* $\{\epsilon_i, i \in \mathcal{M}\}$, *for all* $r_t = i \in \mathcal{M}$ *such that* (3.1d) *and*

$$0 \le E(i)P^T(i) = P(i)E^T(i) = E(i)N(i)X(i)N^T(i)E^T(i) \le \sigma_i I, \tag{3.23a}$$

$$\begin{bmatrix} \Omega_{11}(i) & G(i) & P(i) & L(i) & \Omega_{15}(i) & U_i \\ * & -Q_2(i) & 0 & 0 & E_3^T(i) & 0 \\ * & * & -R_1^{-1}(i) & 0 & 0 & 0 \\ * & * & * & -R_2^{-1}(i) & 0 & 0 \\ * & * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & * & -W_i \end{bmatrix} < 0 \tag{3.23b}$$

*hold, where* $\Omega_{11}(i) = P(i)A^T(i) + L(i)B^T(i) + (P(i)A^T(i) + L(i)B^T(i))^T + \epsilon_i F(i)F^T(i) - [(N_i-1)\underline{\pi}_i + \alpha]P(i)E^T(i)$, $\Omega_{15}(i) = P(i)E_1^T(i) + L(i)E_2^T(i)$, $U_i = [\sqrt{\overline{\pi}_i}P(i), \dots, \sqrt{\overline{\pi}_i}P(i)]$, $W_i = \operatorname{diag}\{P^T(1) + P(1) - \sigma_1 I, \dots, P^T(i-1) + P(i-1) - \sigma_{i-1} I, P^T(i+1) + P(i+1) - \sigma_{i+1} I, \dots, P^T(N) + P(N) - \sigma_N I\}$, $P(i) = E(i)N(i)X(i)N^T(i) + M^{-1}(i)Y(i)\Upsilon^T(i)$, $M(i)E(i)N(i) = \operatorname{diag}\{I_{r_i}, 0\}$, $\Upsilon(i) = N(i)[0, I_{n-r_i}]^T$, *and* $Q_1(i) = R^{-1/2}(i)M^T(i)X^{-1}(i)M(i)R^{-1/2}(i)$. *Moreover,* $X(i)$ *and* $Y(i)$ *are from the form* (3.35). *Furthermore, a stochastic singular finite-time guaranteed cost bound for stochastic singular system can be chosen as*

$$\begin{aligned} \psi_0 = e^{\alpha T}\Big\{ &\max_{i\in\mathcal{M}}\Big\{\lambda_{\max}\Big(R^{-1/2}(i)M^T(i)X^{-1}(i)M(i)R^{-1/2}(i)\Big)\Big\}c_1^2 \\ &+\max_{i\in\mathcal{M}}\Big\{\lambda_{\max}(Q_2(i))d^2\Big\}\Big\}. \end{aligned} \tag{3.24}$$

*Proof.* We firstly prove that condition (3.23b) implies condition (3.1b). By condition (3.23a), we have

$$P^{-1}(j)E(j) \le \sigma_j P^{-1}(j)P^{-T}(j), \quad \forall j \in \mathcal{M}. \tag{3.25}$$

Using Assumption 1, we obtain

$$\pi_{ii}P(i)E^T(i) = -\sum_{j=1,\, j\ne i}^{N} \pi_{ij}P(i)E^T(i) \le -(N_i-1)\underline{\pi}_i P(i)E^T(i), \tag{3.26a}$$

$$\sum_{j=1,\, j\ne i}^{N} \pi_{ij}\sigma_j P^{-1}(j)P^{-T}(j) \le \sum_{j=1,\, j\ne i}^{N} \overline{\pi}_i\sigma_j P^{-1}(j)P^{-T}(j). \tag{3.26b}$$

Thus the inequality

$$\begin{aligned}
\sum_{j=1, j\neq i}^{N} \pi_{ij} P(i) P^{-1}(j) E(j) P^{T}(i) &\leq \sum_{j=1, j\neq i}^{N} \pi_{ij} \sigma_j P(i) P^{-1}(j) P^{-T}(j) P^{T}(i) \\
&\leq \sum_{j=1, j\neq i}^{N} \overline{\pi}_i \sigma_j P(i) P^{-1}(j) P^{-T}(j) P^{T}(i) \\
&\leq U_i V_i^{-1} U_i^{T}
\end{aligned} \tag{3.27}$$

holds, where $U_i = [\sqrt{\overline{\pi}_i} P(i), \ldots, \sqrt{\overline{\pi}_i} P(i)]$ and

$$V_i = \operatorname{diag}\left\{\sigma_1^{-1} P^{T}(1) P(1), \ldots, \sigma_{i-1}^{-1} P^{T}(i-1) P(i-1), \sigma_{i+1}^{-1} P^{T}(i+1) P(i+1), \ldots, \sigma_N^{-1} P^{T}(N) P(N)\right\}. \tag{3.28}$$

Noting that the inequality $\sigma_i^{-1} P^{T}(i) P(i) \geq P^{T}(i) + P(i) - \sigma_i I$ holds for each $i \in \mathcal{M}$. Thus we have

$$\sum_{j=1, j\neq i}^{N} \pi_{ij} P(i) P^{-1}(j) E(j) P^{T}(i) \leq U_i W_i^{-1} U_i^{T}, \tag{3.29}$$

where $W_i = \operatorname{diag}\{P^{T}(1) + P(1) - \sigma_1 I, \ldots, P^{T}(i-1) + P(i-1) - \sigma_{i-1} I,\ P^{T}(i+1) + P(i+1) - \sigma_{i+1} I, \ldots, P^{T}(N) + P(N) - \sigma_N I\}$.

Therefore, a sufficient condition for (3.1b) to guarantee is that

$$\Xi(i) := \begin{bmatrix} \overline{\Psi}(i) & \overline{G}(i) \\ * & -Q_2(i) \end{bmatrix} < 0, \tag{3.30}$$

where $\overline{\Psi}(i) = \overline{A}(i) P^{T}(i) + P(i) \overline{A}^{T}(i) + P(i)[R_1(i) + K^{T}(i) R_2(i) K(i)] P^{T}(i) + U_i W_i^{-1} U_i^{T} - [(N_i - 1)\underline{\pi}_i + \alpha] E(i) P^{T}(i)$.

Noting that

$$\Xi(i) = \begin{bmatrix} \overline{\Psi}_0(i) + P(i)\left(R_1(i) + K^{T}(i) R_2(i) K(i)\right) P^{T}(i) + U_i W_i^{-1} U_i^{T} & G(i) \\ * & -Q_2(i) \end{bmatrix} + \Xi_1(i), \tag{3.31}$$

where

$$\begin{aligned}
\Xi_1(i) &= \begin{bmatrix} (\Delta A(i) + \Delta B(i) K(i)) P^{T}(i) + \left((\Delta A(i) + \Delta B(i) K(i)) P^{T}(i)\right)^{T} & \Delta G(i) \\ * & 0 \end{bmatrix} \\
&= \begin{bmatrix} F(i) \\ 0 \end{bmatrix} \Delta(i) \left[E_{12}(i) P^{T}(i) \quad E_3(i)\right] + \begin{bmatrix} P(i) E_{12}^{T}(i) \\ E_3^{T}(i) \end{bmatrix} \Delta^{T}(i) \left[F^{T}(i) \quad 0\right],
\end{aligned} \tag{3.32}$$

and $\overline{\Psi}_0(i) = \widetilde{A}(i)P^T(i) + P(i)\widetilde{A}^T(i) - [(N_i - 1)\underline{\pi}_i + \alpha]P(i)E^T(i)$, $E_{12}(i) = E_1(i) + E_2(i)K(i)$, $\widetilde{A}(i) = A(i) + B(i)K(i)$.

By Lemma 2.7, we have

$$\begin{aligned}
\Xi(i) \leq{}& \begin{bmatrix} \overline{\Psi}_0(i) + P(i)\big(R_1(i) + K^T(i)R_2(i)K(i)\big)P^T(i) + U_iW_i^{-1}U_i^T & G(i) \\ * & -Q_2(i) \end{bmatrix} \\
&+ \epsilon_i \begin{bmatrix} F(i)F^T(i) & 0 \\ * & 0 \end{bmatrix} + \epsilon_i^{-1} \begin{bmatrix} P(i)E_{12}^T(i) \\ E_3^T(i) \end{bmatrix} \begin{bmatrix} E_{12}(i)P^T(i) & E_3(i) \end{bmatrix} \\
={}& \begin{bmatrix} \overline{\Psi}_0(i) + \epsilon_i F(i)F^T(i) & G(i) \\ * & -Q_2(i) \end{bmatrix} - \Lambda_i \Gamma^{-1}(i) \Lambda_i^T, \\
:={}& \overline{\Xi}(i),
\end{aligned} \tag{3.33}$$

where $\Gamma(i) = \operatorname{diag}\{-R_1^{-1}(i), -R_2^{-1}(i), -\epsilon_i I, -W_i\}$ and $\Lambda_i = \begin{bmatrix} P(i) & P(i)K^T(i) & P(i)E_{12}^T(i) & U_i \\ 0 & 0 & E_3^T(i) & 0 \end{bmatrix}$.

By Schur complement, $\overline{\Xi}(i) < 0$ holds if and only if the following inequality:

$$\begin{bmatrix}
\Omega_{11}(i) & G(i) & P(i) & P(i)K^T(i) & P(i)E_{12}^T(i) & U_i \\
* & -Q_2(i) & 0 & 0 & E_3^T(i) & 0 \\
* & * & -R_1^{-1}(i) & 0 & 0 & 0 \\
* & * & * & -R_2^{-1}(i) & 0 & 0 \\
* & * & * & * & -\epsilon_i I & 0 \\
* & * & * & * & * & -W_i
\end{bmatrix} < 0 \tag{3.34}$$

holds, where $\Omega_{11}(i) = \widetilde{A}(i)P^T(i) + P(i)\widetilde{A}^T(i) + \epsilon_i F(i)F^T(i) - [(N_i - 1)\underline{\pi}_i + \alpha]P(i)E^T(i)$ and $\widetilde{A}(i) = A(i) + B(i)K(i)$.

Thus, let $L(i) = P(i)K^T(i)$, and noting that $E(i)P^T(i) = P(i)E^T(i)$, from (3.34), it is easy to obtain that condition (3.23b) implies condition (3.1b).

Noting that $E(i)$ is a singular matrix with rank $E(i) = r_i$ for every fixed $r_t = i \in \mathcal{M}$, thus there exist two nonsingular matrices $M(i)$ and $N(i)$, which satisfy $M(i)E(i)N(i) = \operatorname{diag}\{I_{r_i}, 0\}$. Let $\overline{P}(i) = M(i)P(i)N^{-T}(i)$; by the proof of Theorem 3.1, we obtain that $\overline{P}(i)$ is of the following form $\begin{bmatrix} P_{11}(i) & P_{12}(i) \\ 0 & P_{22}(i) \end{bmatrix}$, where $P_{11}(i) \geq 0, P_{12}(i) \in \mathbb{R}^{r\times(n-r_i)}, P_{22}(i) \in \mathbb{R}^{(n-r_i)\times(n-r_i)}$. Denote $\Upsilon(i) = N(i)[0, I_{n-r_i}]^T$. Then we have rank $\Upsilon(i) = n - r_i, E(i)\Upsilon(i) = 0$ and

$$\begin{aligned} P(i) &= M^{-1}(i)\begin{bmatrix} P_{11}(i) & P_{12}(i) \\ 0 & P_{22}(i) \end{bmatrix} N^T(i) \\ &= \left(M^{-1}(i)\begin{bmatrix} I_{r_i} & 0 \\ 0 & 0 \end{bmatrix} N^{-1}(i)\right)\left(N(i)X(i)N^T(i)\right) + \left(M^{-1}(i)Y(i)\right)\left(\begin{bmatrix} 0 & I_{n-r_i} \end{bmatrix} N^T(i)\right) \\ &= E(i)N(i)X(i)N^T(i) + M^{-1}(i)Y(i)\Upsilon^T(i), \end{aligned} \tag{3.35}$$

where $X(i) = \mathrm{diag}\{P_{11}(i), \Theta(i)\}$ and $Y(i) = [P_{12}^T(i), P_{22}^T(i)]^T$.

Let $Q_1(i) = R^{-1/2}(i)M^T(i)X^{-1}(i)M(i)R^{-1/2}(i)$, one can see that $P(i) = E(i)N(i)X(i)N^T(i) + M^{-1}(i)Y(i)\Upsilon^T(i)$ satisfies $P(i)E^T(i) = E(i)P^T(i) = E(i)N(i)X(i)N^T(i)E^T(i)$ and (3.1c) holds.

From the proof of Theorem 3.1 and noting that $Q_1(i) = R^{-1/2}(i)M^T(i)X^{-1}(i)M(i)R^{-1/2}(i)$, we can obtain $J_T(i) < \psi_0 = e^{\alpha T}\{\max_{i\in\mathcal{M}}\{\lambda_{\max}(R^{-1/2}(i)M^T(i)X^{-1}(i)M(i)R^{-1/2}(i))\}c_1^2 + \max_{i\in\mathcal{M}}\{\lambda_{\max}(Q_2(i))\}d^2\}$ for all $i \in \mathcal{M}$. This completes the proof of the theorem. □

By Theorem 3.1, Corollary 3.2, and Theorem 3.3, we have the following corollary.

**Corollary 3.4.** *There exists a state feedback controller $u = K(r_t)x(t)$ with $K(r_t) = L^T(r_t)P^{-T}(r_t)$, $r_t = i \in \mathcal{M}$ such that the closed-loop stochastic singular system with Markovian jumps* (2.9) *with $w(t) = 0$ is SSFTS with respect to $(c_1, c_2, T, R(r_t))$, if there exist a scalar $\alpha \geq 0$, a set of positive matrices $\{X(i), i \in \mathcal{M}\}$ with $X(i) \in \mathbb{R}^{n\times n}$, and a set of matrices $\{Y(i), i \in \mathcal{M}\}$ with $Y(i) \in \mathbb{R}^{n\times(n-r_i)}$, two sets of positive scalars $\{\sigma_i, i \in \mathcal{M}\}$ and $\{\epsilon_i, i \in \mathcal{M}\}$, for all $r_t = i \in \mathcal{M}$ such that* (3.22b), (3.23a) *and*

$$\begin{bmatrix} \overline{\Phi}_{11}(i) & P(i) & L(i) & \overline{\Phi}_{14}(i) & U_i \\ * & -R_1^{-1}(i) & 0 & 0 & 0 \\ * & * & -R_2^{-1}(i) & 0 & 0 \\ * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & -W_i \end{bmatrix} < 0 \tag{3.36}$$

*holds, where $\overline{\Phi}_{11}(i) = P(i)A^T(i) + L(i)B^T(i) + (P(i)A^T(i) + L(i)B^T(i))^T + \epsilon_i F(i)F^T(i) - [(N_i - 1)\underline{\pi}_i - \alpha]P(i)E^T(i)$, $\overline{\Phi}_{14}(i) = P(i)E_1^T(i) + L(i)E_2^T(i)$. Furthermore, the other matrical variables are the same as Theorem 3.3, and a guaranteed cost bound for the stochastic singular system can be chosen as*

$$\psi_0 = \max\left\{e^{\alpha T}\lambda_{\max}\left(R^{-1/2}(i)M^T(i)X^{-1}(i)M(i)R^{-1/2}(i)\right)c_1^2,\ i \in \mathcal{M}\right\}. \tag{3.37}$$

*Remark 3.5.* It is easy to check that condition (3.1d) and (3.22b) can be guaranteed by imposing the conditions, respectively,

$$\eta_1 I < R^{1/2}(i) M^{-1}(i) X(i) M^{-T}(i) R^{1/2}(i) < I, \tag{3.38a}$$

$$\eta_3 I < Q_2(i) < \eta_2 I, \tag{3.38b}$$

$$\begin{bmatrix} e^{-\alpha T} c_2^2 - d^2 \eta_2 & c_1 \\ c_1 & \eta_1 \end{bmatrix} > 0, \tag{3.38c}$$

$$\eta_1 I < R^{1/2}(i) M^{-1}(i) X(i) M^{-T}(i) R^{1/2}(i) < I, \tag{3.39a}$$

$$\begin{bmatrix} e^{-\alpha T} c_2^2 & c_1 \\ c_1 & \eta_1 \end{bmatrix} > 0. \tag{3.39b}$$

In addition, conditions (3.23b) and (3.36) are not strict LMIs; however, once we fix parameter $\alpha$, conditions (3.23b) and (3.36) can be turned into LMIs-based feasibility problem.

*Remark 3.6.* From the above discussion, one can see that the feasibility of conditions stated in Theorem 3.3 and Corollary 3.4 can be turned into the following LMIs-based feasibility problem with a fixed parameter $\alpha$, respectively,

$$\begin{aligned} \min \quad & c_2^2 \\ & X(i), Y(i), L(i), Q_2(i), \epsilon_i, \sigma_i, \eta_1, \eta_2, \eta_3 \\ \text{s.t.} \quad & (3.23\text{a}),\ (3.23\text{b}), \text{and}\ (3.38\text{a})\text{–}(3.38\text{c}) \end{aligned} \tag{3.40}$$

$$\begin{aligned} \min \quad & c_2^2 \\ & X(i), Y(i), L(i), \epsilon_i, \sigma_i, \eta_1 \\ \text{s.t.} \quad & (3.23\text{a}), (3.36), \text{and}\ (3.39\text{a})\text{-}(3.39\text{b}). \end{aligned} \tag{3.41}$$

*Remark 3.7.* If $\alpha = 0$ is a solution of feasibility problem (3.41), then the closed-loop stochastic singular system with Markovian jumps (2.9) with $w(t) = 0$ is SSFTS with respect to $(c_1, c_2, T, R(r_t))$ and is also stochastically stable.

## 4. Numerical Examples

*Example 4.1.* Consider a two-mode Markovian jumping singular system (2.1) with

*Mode 1.*

$$E(1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A(1) = \begin{bmatrix} 2.6 & 1 & 1 \\ -1 & 3 & 0 \\ -1 & -1 & 0 \end{bmatrix}, \quad B(1) = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$F(1) = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad E_1(1) = \begin{bmatrix} 0.03 & 0 & 0.2 \\ 0.01 & 0.2 & 0 \\ 0.3 & 0 & 0.1 \end{bmatrix}, \tag{4.1}$$
$$E_2(1) = \begin{bmatrix} 0.06 & 0 & 0.02 \\ 0.01 & 0.1 & 0 \\ 0.04 & 0 & 0.5 \end{bmatrix}, \quad E_3(1) = \begin{bmatrix} 0.01 \\ 0.01 \\ 0 \end{bmatrix}, \quad G(1) = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix},$$

*Mode 2.*

$$E(2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A(2) = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}, \quad B(2) = \begin{bmatrix} 0.5 & 0.1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$
$$F(2) = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad E_1(2) = \begin{bmatrix} 0.02 & 0 & 0.2 \\ 0.01 & 0.2 & 0 \\ 0.1 & 0 & 0.5 \end{bmatrix}, \tag{4.2}$$
$$E_2(2) = \begin{bmatrix} 0.04 & 0 & 0.01 \\ 0.01 & 0.1 & 0 \\ 0.3 & 0 & 0.1 \end{bmatrix}, \quad E_3(2) = \begin{bmatrix} 0.04 \\ 0.01 \\ 0.3 \end{bmatrix}, \quad G(2) = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix},$$

and $d = 2, \Delta(i) = \operatorname{diag}\{r_1(i), r_2(i), r_3(i)\}$, where $r_j(i)$ satisfies $|r_j(i)| \leq 1$ for all $i = 1, 2$ and $j = 1, 2, 3$.

The switching between the two modes is described by the transition rate matrix $\Gamma = \left[\begin{smallmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{smallmatrix}\right]$. The lower and upper bounds parameters of $\pi_{ij}$ for all $i, j \in \mathcal{M}$ are given in Table 1.

**Table 1:** Partially known rate parameters.

| Parameters | Lower bound | Upper bound |
|---|---|---|
| $\pi_{12}$ | 1 | 1.1 |
| $\pi_{21}$ | 2 | 2.2 |

Then, we choose $R_1(1) = R_1(2) = R_2(1) = R_2(2) = R(1) = R(2) = I_3$, $T = 1.5$, $c_1 = 1$, $\alpha = 2$. Using the LMI control toolbox of Matlab, we can obtain from Theorem 3.3 that the optimal value $c_2 = 20.6686$, $\psi_0 = 426.2786$, and

$$\begin{gathered} X(1) = \begin{bmatrix} 0.0946 & -0.0244 & 0 \\ -0.0244 & 0.0748 & 0 \\ 0 & 0 & 0.5271 \end{bmatrix}, \qquad X(2) = \begin{bmatrix} 0.0544 & -0.0025 & 0 \\ -0.0025 & 0.0806 & 0 \\ 0 & 0 & 0.5271 \end{bmatrix}, \\ Y(1) = \begin{bmatrix} -0.0181 \\ -0.0025 \\ 0.1558 \end{bmatrix}, \qquad Y(2) = \begin{bmatrix} -0.1182 \\ 0.0329 \\ 0.3341 \end{bmatrix}, \\ L(1) = \begin{bmatrix} 0.1041 & -0.9367 & -0.8087 \\ -0.9746 & 0.9310 & 0.0596 \\ 0.0383 & -0.0023 & -0.4082 \end{bmatrix}, \qquad L(2) = \begin{bmatrix} -0.3732 & -0.0971 & 0.0419 \\ -0.8239 & -0.9822 & 0.0579 \\ -0.1311 & -0.9817 & -0.0439 \end{bmatrix}, \\ \eta_1 = 0.0541, \qquad \eta_2 = 0.6948, \qquad \eta_3 = 0.2301, \\ \epsilon_1 = 0.1571, \qquad \epsilon_2 = 0.5278, \qquad \sigma_1 = 0.1110, \\ \sigma_2 = 0.0809, \qquad Q_2(1) = 0.6920, \qquad Q_2(2) = 0.6875. \end{gathered} \tag{4.3}$$

Then, we can obtain the following state feedback controller gains:

$$K(1) = \begin{bmatrix} -2.4102 & -13.8074 & 0.2455 \\ -7.3133 & 10.0632 & -0.0146 \\ -9.6821 & -2.4455 & -2.6195 \end{bmatrix}, \qquad K(2) = \begin{bmatrix} -8.1944 & -10.3144 & -0.3925 \\ -8.6928 & -11.2534 & -2.9386 \\ 0.5215 & 0.7875 & -0.1313 \end{bmatrix}. \tag{4.4}$$

Furthermore, let $R_1(1) = R_1(2) = R_2(1) = R_2(2) = R(1) = R(2) = I_3$, $T = 1.5$, $c_1 = 1$; by Theorem 3.3, the optimal bound with minimum value of $c_2^2$ relies on the parameter $\alpha$. We can find feasible solution when $0.37 \le \alpha \le 12.92$. Figure 1 shows the optimal value with different value of $\alpha$. When $\alpha = 1.4$, it yields the optimal value $c_2 = 18.3686$ and $\psi_0 = 337.0518$. Then, by

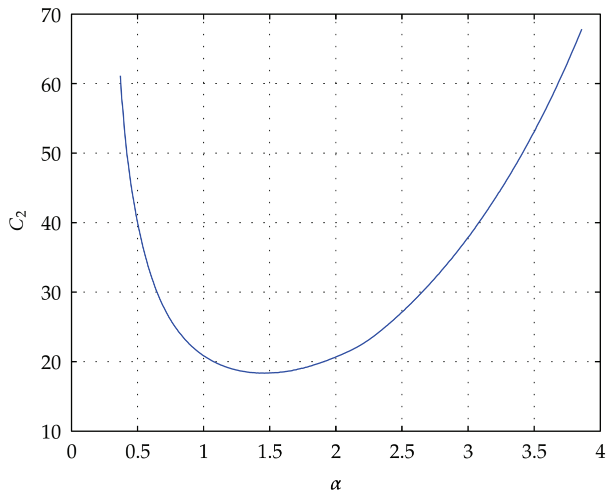


**Figure 1:** The local optimal bound of $c_2$.

using the program *fminsearch* in the optimization toolbox of Matlab starting at $\alpha = 1.4$, the locally convergent solution can be derived as

$$K(1) = \begin{bmatrix} -2.9698 & -17.3253 & 0.4007 \\ -9.0355 & 12.3614 & -0.0178 \\ -11.8034 & -2.3818 & -3.3381 \end{bmatrix}, \qquad K(2) = \begin{bmatrix} -10.4290 & -12.9186 & -1.0405 \\ -16.3971 & -19.4281 & -4.6741 \\ 2.9505 & 4.3393 & -0.3515 \end{bmatrix}, \quad (4.5)$$

with $\alpha = 1.4217$ and the optimal value $c_2 = 18.3341$ and $\psi_0 = 336.0016$.

*Remark 4.2.* From the above example and Remark 3.6, condition (3.23b) in Theorem 3.3 is not strict in LMI form; however, one can find the parameter $\alpha$ by an unconstrained nonlinear optimization approach, which a locally convergent solution can be obtained by using the program *fminsearch* in the optimization toolbox of Matlab.

*Example 4.3.* Consider a two-mode stochastic singular system (2.1) with $w(t) = 0$ and

$$A(1) = \begin{bmatrix} -2.6 & 1 & 1 \\ -1 & 3 & 0 \\ 1 & -1 & 0 \end{bmatrix}, \qquad A(2) = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}. \quad (4.6)$$

In addition, the transition rate matrix and the other matrices parameters are the same as Example 4.1.

Then, let $R_1(1) = R_1(2) = R_2(1) = R_2(2) = R(1) = R(2) = I_3$, $T = 1.5$, $c_1 = 1$. By Corollary 3.4, the optimal bound with minimum value of $c_2^2$ relies on the parameter $\alpha$. We can find feasible solution when $0 \le \alpha \le 13.37$. Thus the above system is stochastically stable, and when $\alpha = 0$, it yields the optimal value $c_2 = 2.7682$, $\psi_0 = 7.6608$, and the following optimized state feedback controller gains

$$K(1) = \begin{bmatrix} -0.3633 & -7.2605 & 0.1285 \\ -3.7567 & 6.3517 & 0.0046 \\ -4.2329 & -0.6284 & -2.0607 \end{bmatrix}, \qquad K(2) = \begin{bmatrix} -3.7840 & -7.5189 & -0.0097 \\ -4.1361 & -9.5627 & -2.6484 \\ 0.0089 & 0.0198 & -0.0046 \end{bmatrix}. \tag{4.7}$$

## 5. Conclusions

In this paper, we deal with the problem of stochastic finite-time guaranteed cost control of Markovian jumping singular systems with uncertain transition probabilities, parametric uncertainties, and time-varying norm-bounded disturbance. Sufficient conditions on stochastic singular finite-time guaranteed cost control are obtained for the class of stochastic singular systems. Designed algorithms for the state feedback controller are provided to guarantee that the underlying stochastic singular system is stochastic singular finite-time guaranteed cost control in terms of restricted linear matrix equalities with a fixed parameter. Numerical examples are also presented to illustrate the validity of the proposed results.

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