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Research Article

An Inverse Eigenvalue Problem for Jacobi Matrices

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A kind of inverse eigenvalue problem is proposed which is the reconstruction of a Jacobi matrix by given four or five eigenvalues and corresponding eigenvectors. The solvability of the problem is discussed, and some sufficient conditions for existence of the solution of this problem are proposed. Furthermore, a numerical algorithm and two examples are presented.

1. Introduction

An $n \times n$ matrix J is called a Jacobi matrix if it is of the following form:

$$J = \begin{bmatrix} a_1 & b_1 & & & & \\ b_1 & a_2 & b_2 & & & \\ & b_2 & a_3 & b_3 & & \\ & & \ddots & \ddots & \ddots & \\ & & & b_{n-2} & a_{n-1} & b_{n-1} \\ & & & & b_{n-1} & a_n \end{bmatrix}, \quad b_i > 0. \tag{1.1}$$

A Jacobi matrix inverse eigenvalue problem, roughly speaking, is how to determine the elements of Jacobi matrix from given eigen data. This kind of problem has great value for many applications, including vibration theory and structural design, for example, the vibrating rod model [1, 2]. In recent years, some new results have been obtained on the construction of a Jacobi matrix [3, 4]. However, the problem of constructing a Jacobi matrix from its four or five eigenpairs has not been considered yet. The problem is as follows.

Problem 1. Given four different real scalars λ, μ, ξ, and η (supposed $\lambda > \mu > \xi > \eta$) and four real orthogonal vectors of size $n x = [x_1, x_2, \ldots, x_n]^T$, $y = [y_1, y_2, \ldots, y_n]^T$, $m = [m_1, m_2, \ldots, m_n]^T$, $r = [r_1, r_2, \ldots, r_n]^T$, finding a Jacobi matrix J of size n such that (λ, x), (μ, y), (ξ, m), and (η, r) are its four eigenpairs.

Problem 2. Given five different real scalars λ, μ, ν, ξ, and η (supposed $\lambda > \mu > \nu > \xi > \eta$) and five real orthogonal vectors of size $n x = [x_1, x_2, \ldots, x_n]^T$, $y = [y_1, y_2, \ldots, y_n]^T$, $z = [z_1, z_2, \ldots, z_n]^T$, $m = [m_1, m_2, \ldots, m_n]^T$, $r = [r_1, r_2, \ldots, r_n]^T$, finding a Jacobi matrix J of size n such that $(\lambda,\ x)$, $(\mu,\ y)$, $(\nu,\ z)$, $(\xi,\ m)$, and $(\eta,\ r)$ are its five eigenpairs.

In Sections 2 and 3, the sufficient conditions for the existence and uniqueness of the solution of Problems 1 and 2 are derived, respectively. Numerical algorithms and two numerical examples are given in Section 4. We give conclusion and remarks in Section 5.

2. The Solvability Conditions of Problem 1

Lemma 2.1 (see [5, 6]). *Given two different real scalars λ, μ (supposed $\lambda > \mu$) and two real orthognal vectors of size n, $x = [x_1,\ x_2, \ldots, x_n]^T$, $y = [y_1, y_2, \ldots, y_n]^T$, there is a unique Jacobi matrix J such that $(\lambda,\ x)$, $(\mu,\ y)$ are its two eigenpairs if the following condition is satisfied:*

$$\frac{d_k}{D_k} > 0, \quad (k = 1, 2, \ldots, n-1), \tag{2.1}$$

where

$$\begin{aligned} d_k &= \sum_{i=1}^{k} x_i y_i, \quad (k = 1, 2, \ldots, n), \\ D_k &= \begin{vmatrix} x_k & x_{k+1} \\ y_k & y_{k+1} \end{vmatrix} \neq 0, \quad (k = 1, 2, \ldots, n-1). \end{aligned} \tag{2.2}$$

And the elements of matrix J are

$$b_k = \frac{(\lambda - \mu) d_k}{D_k}, \quad (k = 1, 2, \ldots, n-1),$$

$$a_1 = \lambda - \frac{b_1 x_2}{x_1},$$

$$a_n = \lambda - \frac{b_{n-1}x_{n-1}}{x_n},$$

$$a_k = \begin{cases} \lambda - \dfrac{(b_{k-1}x_{k-1} + b_k x_{k+1})}{x_k}, & x_k \neq 0, \\ \mu - \dfrac{(b_{k-1}y_{k-1} + b_k y_{k+1})}{y_k}, & x_k = 0, \end{cases} \quad (k = 2, 3, \ldots, n-1). \tag{2.3}$$

From Lemma 2.1, we can see that under some conditions two eigenpairs can determine a unique Jacobi matrix. Therefore, for Problem 1, we only prove that the Jacobi matrices determined by (λ, x), (μ, y) and (ξ, m), (η, r) are the same.

The following theorem gives a sufficient condition for the uniqueness of the solution of Problem 1.

Theorem 2.2. *Problem 1 has a unique solution if the following conditions are satisfied:*

(i) $(\lambda - \mu)d_k^{(1)}/D_k^{(1)} = (\lambda - \xi)d_k^{(2)}/D_k^{(2)} = (\lambda - \eta)d_k^{(3)}/D_k^{(3)} > 0$;

(ii) *if* $x_k = 0$, *then* $(\lambda - \mu)d_j^{(1)}/D_j^{(1)} = (\mu - \xi)d_j^{(4)}/D_j^{(4)} = (\mu - \eta)d_j^{(5)}/D_j^{(5)}$, $j = k, k-1$, *where*

$$\begin{aligned} d_k^{(1)} &= \sum_{i=1}^{k} x_i y_i, \quad & d_k^{(2)} &= \sum_{i=1}^{k} x_i m_i, \quad & d_k^{(3)} &= \sum_{i=1}^{k} x_i r_i, \\ d_k^{(4)} &= \sum_{i=1}^{k} y_i m_i, \quad & d_k^{(5)} &= \sum_{i=1}^{k} y_i r_i, \quad & d_k^{(6)} &= \sum_{i=1}^{k} m_i r_i, \end{aligned} \quad (k = 1, 2, \ldots, n), \tag{2.4}$$

$$\begin{aligned} D_k^{(1)} &= \begin{vmatrix} y_k & y_{k+1} \\ x_k & x_{k+1} \end{vmatrix}, \quad & D_k^{(2)} &= \begin{vmatrix} m_k & m_{k+1} \\ x_k & x_{k+1} \end{vmatrix}, \quad & D_k^{(3)} &= \begin{vmatrix} r_k & r_{k+1} \\ x_k & x_{k+1} \end{vmatrix}, \\ D_k^{(4)} &= \begin{vmatrix} m_k & m_{k+1} \\ y_k & y_{k+1} \end{vmatrix}, \quad & D_k^{(5)} &= \begin{vmatrix} r_k & r_{k+1} \\ y_k & y_{k+1} \end{vmatrix}, \quad & D_k^{(6)} &= \begin{vmatrix} r_k & r_{k+1} \\ m_k & m_{k+1} \end{vmatrix}, \end{aligned} \quad (k = 1, 2, \ldots, n-1). \tag{2.5}$$

Proof. According to Lemma 2.1, under certain condition, (λ, x) and (μ, y), (λ, x) and (ξ, m), (λ, x) and (η, r) can determine one unique Jacobi matrix, denoted J, J', J'', respectively. Their elements are as follows:

$$b_k = \frac{(\lambda - \mu) d_k^{(1)}}{D_k^{(1)}}, \quad (k = 1, 2, \dots, n-1),$$
$$a_1 = \lambda - \frac{b_1 x_2}{x_1},$$
$$a_n = \lambda - \frac{b_{n-1} x_{n-1}}{x_n}, \tag{2.6}$$
$$a_k = \begin{cases} \lambda - \dfrac{(b_{k-1} x_{k-1} + b_k x_{k+1})}{x_k}, & x_k \neq 0, \\ \mu - \dfrac{(b_{k-1} y_{k-1} + b_k y_{k+1})}{y_k}, & x_k = 0, \end{cases} \quad (k = 2, 3, \dots, n-1),$$

$$b'_k = \frac{(\lambda - \xi) d_k^{(2)}}{D_k^{(2)}}, \quad (k = 1, 2, \dots, n-1),$$
$$a'_1 = \lambda - \frac{b_1 x_2}{x_1},$$
$$a'_n = \lambda - \frac{b_{n-1} x_{n-1}}{x_n}, \tag{2.7}$$
$$a'_k = \begin{cases} \lambda - \dfrac{(b_{k-1} x_{k-1} + b_k x_{k+1})}{x_k}, & x_k \neq 0, \\ \xi - \dfrac{(b_{k-1} m_{k-1} + b_k m_{k+1})}{m_k}, & x_k = 0, \end{cases} \quad (k = 2, 3, \dots, n-1),$$

$$b''_k = \frac{(\lambda - \eta) d_k^{(3)}}{D_k^{(3)}}, \quad (k = 1, 2, \dots, n-1),$$
$$a''_1 = \lambda - \frac{b_1 x_2}{x_1},$$
$$a''_n = \lambda - \frac{b_{n-1} x_{n-1}}{x_n}, \tag{2.8}$$
$$a''_k = \begin{cases} \lambda - \dfrac{(b_{k-1} x_{k-1} + b_k x_{k+1})}{x_k}, & x_k \neq 0, \\ \eta - \dfrac{(b_{k-1} r_{k-1} + b_k r_{k+1})}{r_k}, & x_k = 0, \end{cases} \quad (k = 2, 3, \dots, n-1).$$

From the conditions, we have

$$b_k = b'_k = b''_k > 0, \quad k = 1, 2, \dots, n-1. \tag{2.9}$$

If $x_k \neq 0$, we have $a_k = a'_k = a''_k$; if $x_k = 0$,

$$\frac{(\lambda-\mu)d_k^{(1)}}{D_k^{(1)}} = \frac{(\mu-\xi)d_k^{(4)}}{D_k^{(4)}},$$
$$\frac{(\lambda-\mu)d_{k-1}^{(1)}}{D_{k-1}^{(1)}} = \frac{(\mu-\xi)d_{k-1}^{(4)}}{D_{k-1}^{(4)}},$$
$$\frac{(\lambda-\mu)d_k^{(1)}}{D_k^{(1)}} = \frac{(\mu-\eta)d_k^{(4)}}{D_k^{(4)}},$$
$$\frac{(\lambda-\mu)d_{k-1}^{(1)}}{D_{k-1}^{(1)}} = \frac{(\mu-\eta)d_{k-1}^{(4)}}{D_{k-1}^{(4)}}. \tag{2.10}$$

Since (2.6), we have

$$b_k D_k^{(4)} = (\mu-\xi)d_k^{(4)},$$
$$b_{k-1} D_{k-1}^{(4)} = (\mu-\xi)d_{k-1}^{(4)}. \tag{2.11}$$

That is,

$$(\mu-\xi)y_k m_k + b_{k-1}D_{k-1}^{(4)} - b_k D_k^{(4)} = 0. \tag{2.12}$$

Since $D_k^{(i)} \neq 0$ and $x_k = 0$, we have $y_k \neq 0, m_k \neq 0$.

$D_{k-1}^{(4)} = m_{k-1}y_k - m_k y_{k-1}, D_k^{(4)} = m_k y_{k+1} - m_{k+1}y_k$ replacing $D_{k-1}^{(4)}, D_k^{(4)}$ in (2.12), then we have

$$\mu - \frac{(b_{k-1}y_{k-1} + b_k y_{k+1})}{y_k} = \xi - \frac{(b_{k-1}m_{k-1} + b_k m_{k+1})}{m_k}. \tag{2.13}$$

Thus, if $x_k = 0$, we also have $a_k = a'_k$. In the same way, we have $a_k = a''_k$. Then, $a_k = a'_k = a''_k$.

Therefore,

$$J = J' = J'' \tag{2.14}$$

with four eigenpairs (λ, x), (μ, y), (ξ, m), and (η, r). □

3. The Solvability Conditions of Problem 2

Lemma 3.1 (see [7]). *Given three different real scalars λ, μ, ν (supposed $\lambda > \mu > \nu$) and three real orthogonal vectors of size n $x = [x_1, x_2, \ldots, x_n]^T, y = [y_1, y_2, \ldots, y_n]^T, z = [z_1, z_2, \ldots, z_n]^T$, there is a unique Jacobi matrix J such that $(\lambda, x), (\mu, y), (\nu, z)$ are its three eigenpairs if the following conditions are satisfied:*

(i) $(\lambda-\mu)d_k^{(1)}/D_k^{(1)} = (\lambda-\nu)d_k^{(2)}/D_k^{(2)} > 0$;

(ii) *if* $x_k = 0$, $(\lambda-\mu)d_j^{(1)}/D_j^{(1)} = (\mu-\nu)d_j^{(3)}/D_j^{(3)}$, $j = k, k-1$, *where*

$$d_k^{(1)} = \sum_{i=1}^{k} x_i y_i, \qquad d_k^{(2)} = \sum_{i=1}^{k} x_i z_i, \qquad d_k^{(3)} = \sum_{i=1}^{k} y_i z_i,$$
$$D_k^{(1)} = \begin{vmatrix} y_k & y_{k+1} \\ x_k & x_{k+1} \end{vmatrix}, \qquad D_k^{(2)} = \begin{vmatrix} z_k & z_{k+1} \\ x_k & x_{k+1} \end{vmatrix}, \qquad D_k^{(3)} = \begin{vmatrix} z_k & z_{k+1} \\ y_k & y_{k+1} \end{vmatrix}, \qquad (k = 1, 2, \ldots, n-1). \tag{3.1}$$

And the elements of matrix J are

$$b_k = \frac{(\lambda-\mu)d_k}{D_k} \quad (k = 1, 2, \ldots, n-1),$$
$$a_1 = \lambda - \frac{b_1 x_2}{x_1},$$
$$a_n = \lambda - \frac{b_{n-1}x_{n-1}}{x_n}, \tag{3.2}$$
$$a_k = \begin{cases} \lambda - \dfrac{(b_{k-1}x_{k-1} + b_k x_{k+1})}{x_k}, & x_k \neq 0, \\ \mu - \dfrac{(b_{k-1}y_{k-1} + b_k y_{k+1})}{y_k}, & x_k = 0, \end{cases} \qquad (k = 2, 3, \ldots, n-1).$$

From Lemma 3.1, we can see that under some conditions three eigenpairs can determine a unique Jacobi matrix. Therefore, for Problem 2, we only prove that the Jacobi matrices determined by (λ, x), (μ, y), (ν, z); (λ, x), (μ, y), (ξ, m), (λ, x), (μ, y), (η, r) are the same.

The following theorem gives a sufficient condition for the uniqueness of the solution of Problem 2.

Theorem 3.2. *Problem 2 has a unique solution if the following conditions are satisfied:*

(i) $(\lambda-\mu)d_k^{(1)}/D_k^{(1)} = (\lambda-\nu)d_k^{(2)}/D_k^{(2)} = (\lambda-\xi)d_k^{(3)}/D_k^{(3)} = (\lambda-\eta)d_k^{(4)}/D_k^{(4)} > 0$;

(ii) if $x_k = 0$, then $(\lambda-\mu)d_j^{(1)}/D_j^{(1)} = (\mu-\nu)d_j^{(5)}/D_j^{(5)} = (\mu-\xi)d_j^{(6)}/D_j^{(6)} = (\mu-\eta)d_j^{(7)}/D_j^{(7)}$, $j = k, k-1$, where

$$\begin{gathered}
d_k^{(1)} = \sum_{i=1}^{k} x_i y_i, \qquad d_k^{(2)} = \sum_{i=1}^{k} x_i z_i, \qquad d_k^{(3)} = \sum_{i=1}^{k} x_i m_i, \\
d_k^{(4)} = \sum_{i=1}^{k} x_i n_i, \qquad d_k^{(5)} = \sum_{i=1}^{k} y_i z_i, \qquad d_k^{(6)} = \sum_{i=1}^{k} y_i m_i, \\
d_k^{(7)} = \sum_{i=1}^{k} y_i n_i, \qquad d_k^{(8)} = \sum_{i=1}^{k} z_i m_i, \qquad d_k^{(9)} = \sum_{i=1}^{k} z_i n_i, \\
d_k^{(10)} = \sum_{i=1}^{k} m_i n_i, \quad (k = 1, 2, \ldots, n) \\
D_k^{(1)} = \begin{vmatrix} y_k & y_{k+1} \\ x_k & x_{k+1} \end{vmatrix}, \qquad D_k^{(2)} = \begin{vmatrix} z_k & z_{k+1} \\ x_k & x_{k+1} \end{vmatrix}, \qquad D_k^{(3)} = \begin{vmatrix} m_k & m_{k+1} \\ x_k & x_{k+1} \end{vmatrix}, \\
D_k^{(4)} = \begin{vmatrix} n_k & n_{k+1} \\ y_k & y_{k+1} \end{vmatrix}, \qquad D_k^{(5)} = \begin{vmatrix} z_k & z_{k+1} \\ y_k & y_{k+1} \end{vmatrix}, \qquad D_k^{(6)} = \begin{vmatrix} m_k & m_{k+1} \\ y_k & y_{k+1} \end{vmatrix}, \\
D_k^{(7)} = \begin{vmatrix} n_k & n_{k+1} \\ y_k & y_{k+1} \end{vmatrix}, \qquad D_k^{(8)} = \begin{vmatrix} m_k & m_{k+1} \\ z_k & z_{k+1} \end{vmatrix}, \qquad D_k^{(9)} = \begin{vmatrix} n_k & n_{k+1} \\ z_k & z_{k+1} \end{vmatrix}, \\
D_k^{(10)} = \begin{vmatrix} n_k & n_{k+1} \\ m_k & m_{k+1} \end{vmatrix}, \quad (k = 1, 2, \ldots, n-1).
\end{gathered} \tag{3.3}$$

Proof. According to Lemma 3.1, under certain condition, (λ, x), (μ, y), (ν, z); (λ, x), (μ, y), (ξ, m), (λ, x), (μ, y), (η, r) can determine one unique Jacobi matrix, denoted J, J', J'', respectively. Their elements are as follows: □

$$\begin{gathered}
b_k = \frac{(\lambda-\mu)d_k^{(1)}}{D_k^{(1)}} \quad (k = 1, 2, \ldots, n-1), \\
a_1 = \lambda - \frac{b_1 x_2}{x_1}, \\
a_n = \lambda - \frac{b_{n-1} x_{n-1}}{x_n}, \\
a_k = \begin{cases} \lambda - \dfrac{(b_{k-1}x_{k-1} + b_k x_{k+1})}{x_k}, & x_k \neq 0, \\[2ex] \mu - \dfrac{(b_{k-1}y_{k-1} + b_k y_{k+1})}{y_k}, & x_k = 0, \end{cases} \qquad (k = 2, 3, \ldots, n-1),
\end{gathered}$$

$$
\begin{gathered}
b_k' = \frac{(\lambda-\mu)d_k^{(1)}}{D_k^{(1)}}, \quad (k=1,2,\dots,n-1), \\
a_1' = \lambda - \frac{b_1 x_2}{x_1}, \\
a_n' = \lambda - \frac{b_{n-1}x_{n-1}}{x_n}, \\
a_k' = \begin{cases} \lambda - \dfrac{(b_{k-1}x_{k-1}+b_k x_{k+1})}{x_k}, & x_k \neq 0, \\ \mu - \dfrac{(b_{k-1}y_{k-1}+b_k y_{k+1})}{y_k}, & x_k = 0, \end{cases} \quad (k=2,3,\dots,n-1), \\
b_k'' = \frac{(\lambda-\mu)d_k^{(1)}}{D_k^{(1)}}, \quad (k=1,2,\dots,n-1), \\
a_1'' = \lambda - \frac{b_1 x_2}{x_1}, \\
a_n'' = \lambda - \frac{b_{n-1}x_{n-1}}{x_n}, \\
a_k'' = \begin{cases} \lambda - \dfrac{(b_{k-1}x_{k-1}+b_k x_{k+1})}{x_k}, & x_k \neq 0, \\ \mu - \dfrac{(b_{k-1}y_{k-1}+b_k y_{k+1})}{y_k}, & x_k = 0, \end{cases} \quad (k=2,3,\dots,n-1),
\end{gathered}
\tag{3.4}
$$

From conditions (i) and (ii) we have obviously

$$
b_k = b_k' = b_k'' > 0, \quad k=1,2,\dots,n-1, \quad a_k = a_k' = a_k''. \tag{3.5}
$$

Therefore,

$$
J = J' = J'' \tag{3.6}
$$

with five eigenpairs (λ, x), (μ, y), (ν, z), (ξ, m), and (η, r).

4. Numerical Algorithms and Examples

The process of the proof of the theorem provides us with a recipe for finding the solution of Problem 1 if it exists.

From Theorem 2.2, we propose a numerical algorithm for finding the unique solution of Problem 1 as follows.

Algorithm 1. **Input.** The real numbers $\lambda > \mu > \xi > \eta$ and mutually orthogonal vectors x, y, m, r.

Output. The symmetric Jacobi matrix having the eigenpairs $(\lambda, x), (\mu, y), (\xi, m), (\eta, r)$:

(1) compute $d_k^{(1)}, d_k^{(2)}, d_k^{(3)}, d_k^{(4)}, d_k^{(5)}, d_k^{(6)}$ and $D_k^{(1)}, D_k^{(2)}, D_k^{(3)}, D_k^{(4)}, D_k^{(5)}, D_k^{(6)}$;

(2) if any one of $D_k^{(1)}, D_k^{(2)}, D_k^{(3)}, D_k^{(4)}, D_k^{(5)}, D_k^{(6)}$ is zero, the Problem 1 can not be solved by this method;

(3) for $k = 1, 2, \ldots, n-1$.

(a) When $x_k = 0$, if

$$\frac{(\lambda-\mu)d_j^{(1)}}{D_j^{(1)}} = \frac{(\mu-\xi)d_j^{(4)}}{D_j^{(4)}} = \frac{(\mu-\eta)d_j^{(5)}}{D_j^{(5)}}, \quad j = k, k-1, \tag{4.1}$$

then

$$b_k = \frac{(\lambda-\mu)d_k^{(1)}}{D_k^{(1)}}, \qquad a_k = \mu - \frac{(b_{k-1}y_{k-1} + b_k y_{k+1})}{y_k}. \tag{4.2}$$

Otherwise, Problem 1 has no solution.

(b) When $x_k \neq 0$, if

$$\frac{(\lambda-\mu)d_k^{(1)}}{D_k^{(1)}} = \frac{(\lambda-\xi)d_k^{(2)}}{D_k^{(2)}} = \frac{(\lambda-\eta)d_k^{(3)}}{D_k^{(3)}} > 0, \tag{4.3}$$

then

$$b_k = \frac{(\lambda-\mu)d_k^{(1)}}{D_k^{(1)}}, \qquad a_k = \lambda - \frac{(b_{k-1}x_{k-1} + b_k x_{k+1})}{x_k}. \tag{4.4}$$

Otherwise, Problem 1 has no solution;

(4) $a_n = \lambda - b_{n-1}x_{n-1}/x_n$.

Note that we can also propose a numerical algorithm from Theorem 3.2. Because of the limitation of space, we don't describe it here in detail.

Now we give two numerical examples here to illustrate that the results obtained in this paper are correct.

Example 4.1. Given four real numbers $\lambda = 3$, $\mu = 2$, $\xi = 1$, $\eta = 0.2679$, and the four vectors $x = [1,1,0,-1,-1]^T$, $y = [1,0,-1,0,1]^T$, $m = [1,-1,0,1,-1]^T$, $r = [1,-\sqrt{3},2,-\sqrt{3},1]^T$, it is easy to verify that these given data satisfy the conditions of the Theorem 2.2. After calculating on the microcomputer through making program of Algorithm 1, we have a unique Jacobi matrix:

$$J = \begin{bmatrix} 2 & 1 & & & \\ 1 & 2 & 1 & & \\ & 1 & 2 & 1 & \\ & & 1 & 2 & 1 \\ & & & 1 & 2 \end{bmatrix}. \tag{4.5}$$

Example 4.2. Given five real numbers $\lambda = 7.543$, $\mu = -3.543$, $\nu = 2$, $\xi = 4.296$, and $\eta = -0.296$, and the five vectors: $x = [0.1913, 0.3536, 0.4619, 0.5000, 0.4619, 0.3536, 0.1913]^T$, $y = [0.1913, -0.3536, 0.4619, -0.5000, 0.4619, -0.3536, 0.1913]^T$, $z = [0.5000, 0, -0.5000, 0, 0.5000, 0, -0.5000]^T$, $m = [0.4619, 0.3536, -0.1913, 0.5000, -0.1913, 0.3536, 0.4619]^T$, and $r = [0.4619, -0.3536, -0.1913, 0.5000, -0.1913, -0.3536, 0.4619]^T$, it is easy to verify that these given numbers can not satisfy the conditions of the Theorem 2.2 but Theorem 3.2. After calculating on the microcomputer through making program of Theorem 3.2, we have a Jacobi matrix:

$$J = \begin{bmatrix} 2 & 3 & & & & & \\ 3 & 2 & 3 & & & & \\ & 3 & 2 & 3 & & & \\ & & 3 & 2 & 3 & & \\ & & & 3 & 2 & 3 & \\ & & & & 3 & 2 & 3 \\ & & & & & 3 & 2 \end{bmatrix}. \tag{4.6}$$

5. Conclusion and Remarks

As a summary, we have presented some sufficient conditions, as well as simple methods to construct a Jacobi matrix from its four or five eigenpairs. Numerical examples have been given to illustrate the effectiveness of our results and the proposed method. Also, the idea in this paper may provide some insights for other banded matrix inverse eigenvalue problems.

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