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Research Article

Transient Natural Convection in Porous Square Cavity Heated and Cooled on Adjacent Walls

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Transient natural convection in a square cavity filled with a porous medium is studied numerically. The cavity is assumed heated from one vertical wall and cooled at the top, while the other walls are kept adiabatic. The governing equations are solved numerically by a finite difference method. The effects of Rayleigh number on the initial transient state up to the steady state are investigated for Rayleigh number ranging from 10 to 2×10^2. The evolutions of flow patterns and temperature distributions were presented for Rayleigh numbers, $\mathrm{Ra} = 10^2$ and 10^3. It is observed that the time taken to reach the steady state is longer for low Rayleigh number and shorter for high Rayleigh number.

1. Introduction

Convective heat transfer has attracted significant attention due to wide applications in engineering such as operation of solar collectors, cooling systems in electronics equipments, and insulations of buildings. Many studies with application to the previous research areas may be found in the books by Nield and Bejan [1], Ingham and Pop [2], and Vafai [3].

In recent years, most researchers devoted their studies on natural convection related to either a vertically or horizontally imposed heat flux or temperature difference; see, for example, Bejan [4], Rasoul and Prinos [5], Goyeau et al. [6], Saeid and Pop [7], Baytas and Pop [8], and Zeng et al. [9]. Works on natural convection with differentially heated neighbouring walls include those of Ganzarolli and Milanez [10], Ameziani et al. [11], Ishihara et al. [12], Rahman and Sharif [13], Aydin and Yang [14], Ece and Büyük [15], Frederick and Berbakow [16], and Dalal and Das [17].

The heat transfer characteristics of natural convection in a square and rectangular cavity heated from one side and cooled from the ceiling have been studied by Aydin [18]

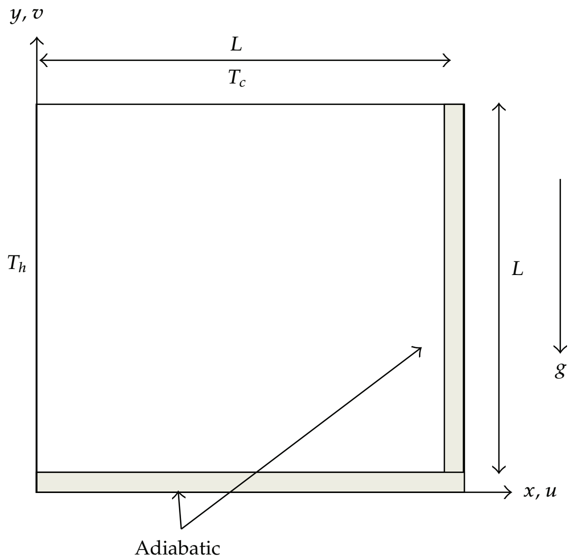


Figure 1: Schematic diagram of the physical model and coordinate system.

and Aydin et al. [19, 20]. Aydin and Ünal [21] conducted a numerical study on transient buoyant flow inside an air-filled 45°-inclined enclosure. Varol et al. [22] studied the natural convection in a porous enclosure divided by a triangular massive partition and heated and cooled on adjacent walls. In addition, Revnic et al. [23] investigated the natural convection in an inclined square cavity heated and cooled on adjacent walls and filled with a porous medium. Finally, we mention that Yesiloz and Aydin [24] studied experimentally and numerically the natural convection in an inclined quadrantal cavity heated from below and cooled on one side.

In this work, transient natural convection in a square cavity filled with a porous medium is studied numerically. The square enclosure is heated from one vertical side and cooled from the ceiling. The vertical wall is assumed heated to a uniform temperature or heat flux while the ceiling is cooled at a uniform temperature. The other vertical wall and the floor are adiabatic.

2. Mathematical Formulation

The schematic diagram of two-dimensional square cavity is shown in Figure 1. It is assumed that the left vertical wall is suddenly heated to the constant temperature or heat flux T_h and the top wall is suddenly cooled to the constant temperature T_c where $T_h > T_c$ by equal amount relative to an initially uniform temperature distribution. The right vertical and bottom walls are adiabatic.

Applying Darcy's flow model and the Boussinesq approximation, the governing equations are as follows [7]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{g\beta K}{\nu}\frac{\partial T}{\partial x},$$
$$\sigma\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{2.1}$$

where u and v are the velocity components along the x- and y-axes, respectively, T is the fluid temperature, and the definitions of the other symbols are mentioned in the Nomenclature. Equations (2) are subject to the following boundary conditions:

$$\begin{aligned}
&u(x,y,0) = v(x,y,0) = 0, && T(x,y,0) = T_0,\\
&u(0,y,t) = v(0,y,t) = 0, && T(0,y,t) = T_h(y),\\
&u(L,y,t) = v(L,y,t) = 0, && \frac{\partial T(L,y,t)}{\partial x} = 0,\\
&u(x,0,t) = v(x,0,t) = 0, && \frac{\partial T(x,0,t)}{\partial y} = 0,\\
&u(x,L,t) = v(x,L,t) = 0, && T(x,L,t) = T_c(x).
\end{aligned} \tag{2.2}$$

Using the stream functions ψ defined by $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$, and nondimensional variables:

$$X = \frac{x}{L}, \qquad Y = \frac{y}{L}, \qquad \theta = \frac{T - T_0}{T_h - T_c}, \qquad \Psi = \frac{\psi}{\alpha}, \qquad \tau = \frac{\alpha t}{\sigma L^2}, \tag{2.3}$$

where $T_0 = (T_h + T_c)/2$, the governing equations (2) can be written in dimensionless forms:

$$\frac{\partial^2\Psi}{\partial X^2} + \frac{\partial^2\Psi}{\partial Y^2} = -\mathrm{Ra}\frac{\partial\theta}{\partial X}, \tag{2.4}$$

$$\frac{\partial\theta}{\partial\tau} + \frac{\partial\Psi}{\partial Y}\frac{\partial\theta}{\partial X} - \frac{\partial\Psi}{\partial X}\frac{\partial\theta}{\partial Y} = \frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}, \tag{2.5}$$

where Ra is the Rayleigh number defined as

$$\mathrm{Ra} = \frac{g\beta K\left(\overline{T_h} - T_c\right)L}{\nu\alpha}. \tag{2.6}$$

The boundary conditions (2.2) can be written as

$$\Psi(X,Y,0)=0,\qquad \theta(X,Y,0)=0, \tag{2.7}$$

$$\Psi(0,Y,\tau)=0,\qquad \theta(0,Y,\tau)=0.5, \tag{2.8}$$

$$\Psi(1,Y,\tau)=0,\qquad \frac{\partial\theta(1,Y,\tau)}{\partial X}=0, \tag{2.9}$$

$$\Psi(X,0,\tau)=0,\qquad \frac{\partial\theta(X,0,\tau)}{\partial Y}=0, \tag{2.10}$$

$$\Psi(X,1,\tau)=0,\qquad \theta(X,1,\tau)=-0.5. \tag{2.11}$$

The local Nusselt numbers along the hot and cold wall are given, respectively, by [23]

$$\begin{aligned} \mathrm{Nu}_h(Y) &= -2\frac{\partial\theta}{\partial X}(0,Y),\\ \mathrm{Nu}_c(X) &= -2\frac{\partial\theta}{\partial Y}(X,1), \end{aligned} \tag{2.12}$$

and the average Nusselt numbers from the heated wall $\mathrm{Nu}_h(Y)$ and cold wall $\mathrm{Nu}_c(X)$ are, respectively, defined as

$$\begin{aligned} \overline{\mathrm{Nu}_h} &= \int_0^1 \mathrm{Nu}_h(Y)\,dY,\\ \overline{\mathrm{Nu}_c} &= \int_0^1 \mathrm{Nu}_c(X)\,dX. \end{aligned} \tag{2.13}$$

3. Numerical Scheme

The coupled system of (2.4) and (2.5) subject to boundary conditions (2.7)–(2.11) was solved numerically using a finite difference method. The alternating direction implicit method was applied for discretizing the equations. The unknowns Ψ and θ were calculated until the following convergence criterion was fulfilled:

$$\frac{\sum_{i,j}\left|\zeta_{i,j}^{n+1}-\zeta_{i,j}^{n}\right|}{\sum_{i,j}\left|\zeta_{i,j}^{n+1}\right|}\le\epsilon, \tag{3.1}$$

where ζ is either Ψ or θ, n represents the iteration number, and ϵ is the convergence criterium. In this study, the convergence criterion was set at $\epsilon=10^{-5}$. In order to validate the computation coding, accuracy test had been performed for $\mathrm{Ra}=10^2$ and $\mathrm{Ra}=10^3$ for several grid sizes. Due to lack of suitable results from literature, we only compared our results with those from Revnic et al.[23] as shown in Table 1. There has been found a reasonably good agreement between the results obtained using 101×101 and results using Richardson

Table 1: Comparison of the average Nusselt number, $\overline{\mathrm{Nu}}$, with results from [23] for heated and cold walls for Ra = 10^2.

Reference	Mesh size	$\overline{\mathrm{Nu}_h}$	$\overline{\mathrm{Nu}_c}$
Revnic et al. [23]	26 × 26	8.2636	8.2240
	51 × 51	9.2162	9.2008
	101 × 101	10.1319	10.1247
	Richardson extrapolation	10.4371	10.4326
Present results	26 × 26	8.5553	8.2786
	51 × 51	9.3885	9.2271
	101 × 101	10.2297	10.1383

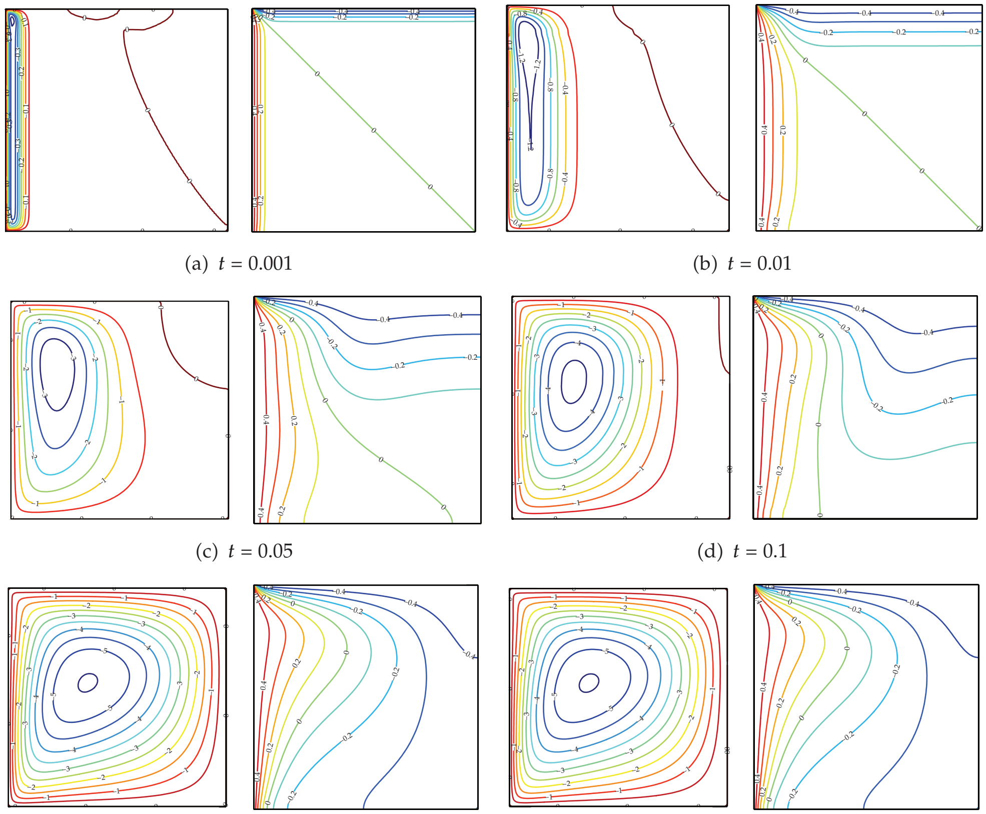
extrapolation and grid size 101 × 101 as reported by Revnic et al. [23]. We noticed that further refinement in grid size would increase the average Nusselt numbers. For Ra = 10^2, Ra = 10^3 when the grid refined from 101 × 101 to 141 × 141 the increase in Nusselt number was 3.9 percent and 1.6 percent, respectively. However, in view of maximum value of stream function, $\Psi_{\max}$, an increase from 101 × 101 to 141 × 141 nodal points would increase the $\Psi_{\max}$ less than 0.3 percent for both Ra. Therefore, in a way to compromise the accuracy and cost in calculations, the grid size 101 × 101 was used in all the calculations.

4. Results and Discussion

The main objective of the present work is to investigate the effects of Rayleigh number, Ra, on natural convection in an enclosure heated and cooled on adjacent walls and filled with a porous medium. The computations were carried out for Ra = 10 to 2 × 10^3. At first, the left and top walls were cooled and the fluid was at a uniform temperature and motionless in the enclosure. After sudden heating on the left wall, the fluid near the hot wall began to rise and flow along the cooled top wall. Then, the flow reached the other side of the wall and moved downward to the bottom wall. Finally, after passing the adiabatic walls, the flow reached the hot wall and completed the flow cycle. Due to buoyancy force, the fluid moved from the left region to the right region of the cavity, yielding a clockwise circulation cell inside the enclosure.

Figure 2 shows the transient result of the flow and temperature field for Ra = 10^2. Initially, at $t = 0$, there was no motion in the enclosure. At $t = 0.001$, a single clockwise rotating cell appeared close to the hot wall. At this moment, the convective motion was not fully developed. Thus the isotherms crowded and formed a boundary layer parallel to the isothermal walls, implying that the conduction was the dominant heat transfer mechanism. After a short time, the flow grew in magnitude to their steady-state condition. As time increased ($t = 0.1$), temperature had been distributed from the left to the right wall; therefore the streamlines intensity increased and the center of cell moved to the center of the enclosure. Meanwhile the isotherms formed thermal boundary layer along the hot and cold wall. This indicated that convection occurred and dominated the heat transfer in the enclosure. As time progressed longer ($t = 0.8$), no significant difference was observed in the flow and temperature fields, which indicated that the flow had reached the steady-state regime.

The transient development of flow structure for Ra = 10^3 is illustrated in Figure 3. At the beginning, the flow structure was almost similar to that of the case Ra = 10^2. However, after a short time ($t = 0.05$), the streamline had been extended throughout the cavity and



(a) $t = 0.001$ (b) $t = 0.01$

(c) $t = 0.05$ (d) $t = 0.1$

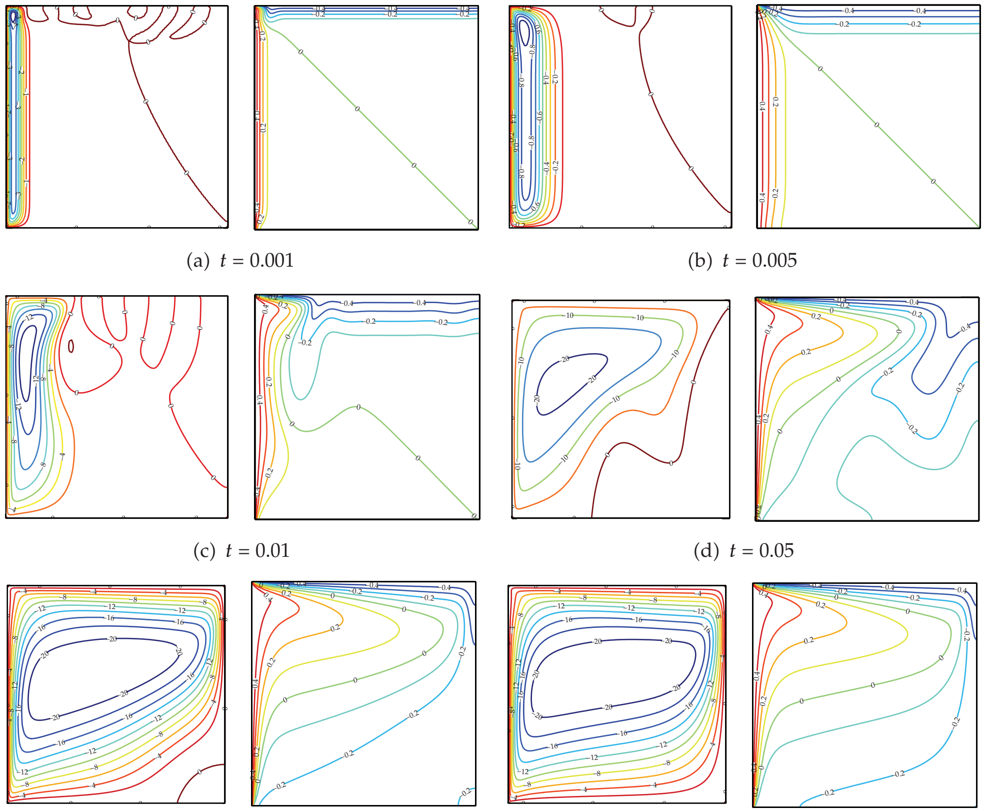
(e) $t = 0.5$ (f) $t = 0.8$

Figure 2: Time history for streamlines (left) and isotherms (right) for Ra = 100.

convection became more important. In addition, for $t > 0.1$, the flow was then going to attain the steady-state condition. The thermal boundary layer was more pronounced and grew faster than the lower Ra case.

The steady-state condition for the cavity with different values of Ra is shown in Figure 4. For Ra = 10, it was observed that flow exhibited a unicellular recirculation pattern and the corresponding isotherm formed a diagonally symmetric structure indicating that the heat transfer mechanism was still under influence of conduction. As the convective motion increased with the increasing in Ra, the colder flow tended to occupy the lower part of the enclosure and the temperature gradient was more severe near the heated vertical wall and cooled ceiling. Therefore, a thermal boundary layers were observed near the heated and cooled walls.

Figure 5 depicts the transient responses of the average Nusselt number, $\overline{\mathrm{Nu}_h}$, on the hot wall for different values of the Rayleigh number. Initially, the value of $\overline{\mathrm{Nu}_h}$ was large because of the high-temperature gradient close to the hot wall. For Ra = 10, the $\overline{\mathrm{Nu}_h}$ decreased monotonously with time until it converged to a steady-state value. This behavior was related to conduction as the dominant heat transfer mechanism for low Ra value. However, for Ra = 10^2, $\overline{\mathrm{Nu}_h}$ dropped to a minimum value first and then increased continuously to a steady-state value. Similar behavior could be observed for Ra > 10^2, dropping to a minimum and



(a) $t = 0.001$

(b) $t = 0.005$

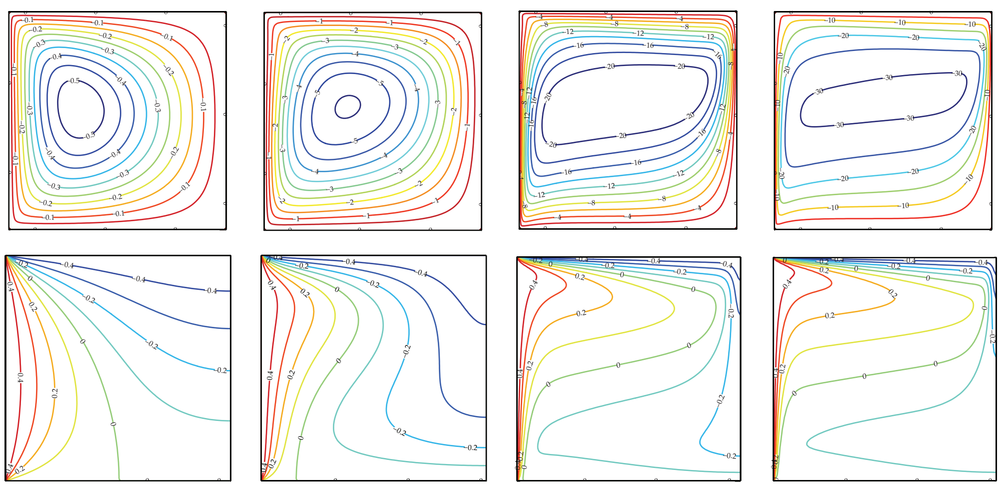
(c) $t = 0.01$

(d) $t = 0.05$

(e) $t = 0.1$

(f) $t = 0.2$

Figure 3: Time history for streamlines (left) and isotherms (right) for Ra = 1000.



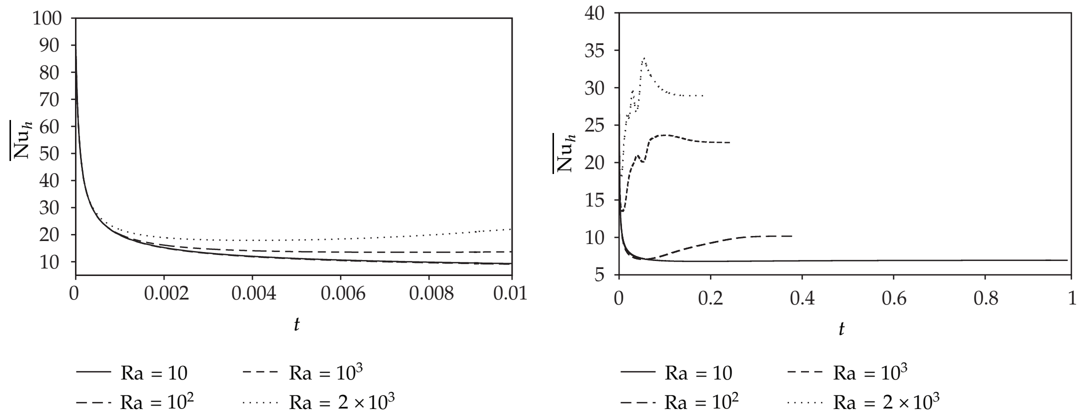
(a) Ra = 10

(b) Ra = 100

(c) Ra = 1000

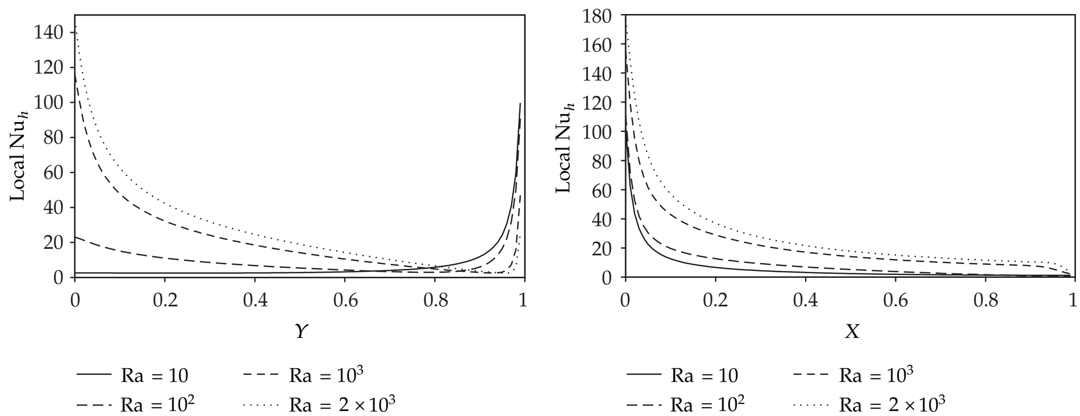
(d) Ra = 2000

Figure 4: Streamlines (top) and isotherms (bottom) at steady-state for difference Ra.



(a) (b)

Figure 5: Variations of the average Nusselt number on the hot wall with time, $t = 0$ to $t = 1.0$.



(a) (b)

Figure 6: Variations of the local Nu_h and local Nu_c on the hot and cold wall, respectively, for different Ra at steady-state condition.

increasing with some oscillations to a steady-state value. As we observed, the steady-state value was reached faster with the increasing in Ra. This was a consequence of increased convective motion with the increasing on Raleigh number.

Figure 6 illustrates the variation of the local Nusselt number Nu_h along the hot wall (left) and Nu_c for cold wall (right) at the steady-state regime for different Ra. It was observed that, for all Ra considered, except for Ra $= 10$, the Nu_h decreased to a minimum value first, followed by a sharp increase near the top wall. For all values of Ra, the Nu_c decreased continuously along the ceiling to the adiabatic right wall.

5. Conclusion

The present study considers a transient natural convection in a two-dimensional square cavity filled with a porous medium heated from the left vertical wall and cooled from the top wall, while the other walls are kept adiabatic. The effect of Rayleigh number on the transient thermal behavior was investigated. Initially, the heat transfer process was characterized by pure conduction. However, after a short time and for Rayleigh number $\mathrm{Ra} > 10$, the convection dominated the flow motion in the enclosure. Increasing the Rayleigh number could lead to more convective motion and shorter time to reach a steady-state condition.

Nomenclature

g: Magnitude of gravitational acceleration
K: Permeability of the porous medium
L: Cavity height/width
Nu: Nusselt number
Ra: Rayleigh number for porous medium
t: Time
T: Fluid temperature
u, v: Velocity components along x- and y-axes, respectively
U, V: Nondimensional along X- and Y-axes, respectively
x, y: Cartesian coordinates
X, Y: nondimensional Cartesian coordinates.

Greek Symbols

α: Effective thermal diffusivity
β: Coefficient of thermal expansion
θ: Nondimensional temperature
κ: Wave number
ν: Kinematic viscosity of the fluid
σ: Ratio of composite material heat capacity to convective fluid heat capacity
τ: Non-dimensional time
ψ: Stream function
Ψ: Nondimensional stream function.

Subscripts and Superscripts

c: Cold wall
h: Hot wall.

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