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Research Article

Photovoltaic Pumps: Technical and Practical Aspects for Applications in Agriculture

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The paper deals with a series of tests conducted on a PV-DC pump in Viterbo (42°24′ North, 12°06′ East). The tests lasted from January 2003 up to November 2004 and involved measurements of solar radiation, on both a horizontal surface and the tilted module surface, flow rates, volumes, and total dynamic heads. In total, up to 3000 data were collected every day whose analysis allowed us to find empirical relationships among system efficiencies, solar radiations, and total dynamic heads. In the second part of the paper we develop a simple method that allows both the assessment of performances of the whole system when installed in a different site from that in which the tests were performed and the optimal inclination angle of the panel to be determined in relation to annual or seasonal use (see irrigation).

1. Introduction

Solar photovoltaic (PV) systems have shown their potential in rural electrification projects around the world, especially concerning Solar Home Systems. With continuing price decreases of PV systems, other applications are becoming economically attractive, and growing experience is gained with the use of PV in such areas as social and communal services, agriculture and other productive activities, which can have a significant impact on rural development. There is still a lack of information, however, on the potential and limitations of such PV applications. Rural energy is generally recognized as an important element of rural socioeconomic development, not as an end in itself, but through the demand for the services made possible through energy inputs, such as potable water pumping

and extension of the day by lighting and cooking. As a general trend, an increasing energy demand—both in quantity and quality—is highly correlated with socioeconomic development.

This study is focused on solar photovoltaic (PV) systems, which can fulfil only a part of rural energy needs. As has been noted before, most PV programmes have given attention to the so called "Solar Home Systems" as the most proven of PV applications. With continuing advances in PV technology, decreasing prices and growing experience in the organizational aspects of introducing this new technology, many other applications of PV have shown their potential. This promises to open the door for a greater contribution of PV systems to rural development [1].

Our department took on two research problems:

- (i) to test the field performances of a commercialized solar pump that is sold with a photovoltaic panel,
- (ii) to estimate, once the system's operational characteristics were defined, what performances could be expected when the same complex would be located in such African countries as Ghana, Benin, and Burkina Faso.

These were the cues to set up a test bed for this type of equipment and to try to deepen our knowledge regarding a subject that is not without interest, also from an economic point of view. At present, the lack of electricity and high gas-oil costs (where and when available) are opening vast markets for these pumps in many developing countries, both Asian and African (particularly sub-Saharan).

The reasons for this increasing use are manifold, among them: easy and rapid installation, low and rare maintenance, the long service life of these types of photovoltaic panels, and the great increase in their efficiency [2, 3], with a vertiginous rapid descent of the panels' costs that in the last decade of the last century decreased—at equal power output—to 1/4 of their initial value [4–6].

Hence, there is a growing interest in this type of machinery and the promotion of their diffusion, especially in third world countries, by individual western countries like Germany and other northern European countries [7–10], as well by the UE, FAO, UNESCO, and the previously cited World Bank.

Nevertheless, they are not many works in the literature which deal specifically with tests run on photovoltaic pumps. In fact, with few exceptions (e.g., [11, 12]), the tests that exist concern either the solar panel and its performance or—as in the case of the advertising material supplied by the manufacturers—the solar pump for which it is provided, without many details, only the total head-flow rate curves obtained by coupling the pump to the electric network instead of to the photovoltaic panel [13, 14].

The tests conducted by *Argaw*—that, unlike the others, consider the whole panel/pump complex—were performed on community plants (in Brazil) and, therefore, were subject to a series of conditions that limited the possibility of the researchers to vary the operating conditions of the system (e.g., discharge and/or total head).

In this regard, a test bed was set up in the Hydraulic Laboratory of our department that essentially consists of a closed hydraulic circuit, complete with valves and measuring instruments (flow rates, total heads, and volumes), and that is equipped with the measuring instrumentation for photoelectric parameters (solar radiation, both on the horizontal plane and on the panel, intensity, and voltage of the electricity inside and outside the panel).

The present work is organised in two parts:

- (i) the first contains a description of the experimental equipment and a discussion of the results obtained by a long series of tests that lasted for about two years;
- (ii) the second is essentially theoretical and aimed at defining: first, a methodology that allows a technician to easily and reliably estimate the performances of the panel/pump system (the one we experienced or another whose operational characteristics are known), taking into consideration its installation in places different from Viterbo (42°24′ North; 12°06′ East) where the tests were performed; second, the optimum spatial position to be assigned to the panel depending on the use, annual, or seasonal (irrigation), required of the pump.

This work ends with a practical application that shows what results are to be expected in the case of installation of the experimented system in one of the African countries mentioned above.

2. Layout of the Experimental Installation and Measurement Techniques

2.1. Experimental Installation

The whole panel/pump system was installed in the experimental field of our department, which is situated within the Faculty of Agriculture. The pump—a "SOLAFLUX" made by FLUXINOS in Grosseto, central Italy—is a submerged piston pump of low power and fed by direct current (tension 20–70 V; intensity 1–4 Å.). Among the various models of pumps, we gave preference to the lower total head type, for which the manufacturer specifies the maximum available total head in 5.0 bars.

The photovoltaic panel, supplied by the same company, was produced by HELIOS TECHNOLOGY; it has a surface S of $2.8 \,\mathrm{m}^2$ and was mounted, on the supplier's instructions, with its surface directed south (azimuth = 180°) and inclination β directed to the horizontal plane (in other terms, β is the angle between the normal line to the panel surface and the vertical line of the site) equal to the latitude φ of Viterbo ($42^\circ24'$).

Panel performances were recorded by means of the followings tools, all having a current exit from 4 to 20 mA for connection to the datalogger (see below) used for data management and memorization:

- (i) 2 silicon pyranometers: the first one in a horizontal plane for measuring global solar radiation R_h on the horizontal plane and the second one, lined up with the panel, for measuring global radiation R_{β} on the panel itself; the radiations measured were those with wavelengths between 0.3 and 2 μ m;
- (ii) 2 platinum thermometers: the first one for measuring standard atmosphere temperature (therefore placed, according to the law, inside a special ventilated protection) and the second one for measuring panel temperature, and therefore glued to panel's back side;
- (iii) 1 voltmeter and 1 ammeter, both with a precision of 0.1% for measuring, respectively, current voltage (up to 100 V) and intensity (up to 10 A) that, on leaving the panel, feed the pump.

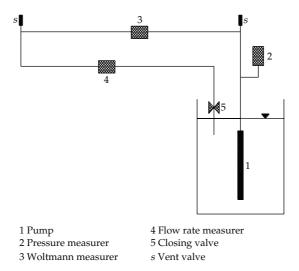


Figure 1: Hydraulic layout.

The hydraulic scheme is shown in Figure 1. The pump was connected to a plastic pipe (PEAD, Φ 25 mm; PFA 16 bars) that started from a plastic tank and then returned to the same tank; the following tools, all with a current exit from 4 to 20 mA, were inserted on the pipe for the connection to the datalogger:

- (i) 1 GEMS piezoelectric pressure transducer, with +/-0.25% precision, set immediately after the pump, to measure pressures up to 10 bars (operating temperatures -25/+85°C);
- (ii) 1 electromagnetic flow measurer (operating temperatures –20 +150°C, PN 40 bars), equipped with a signal converter (accuracy 0.5%), for connection to the datalogger; it allowed the measurement of the instant flow rates and even total volumes.

A "Woltmann" type volumetric measurer, for checking the data recorded with the electromagnetic flow measurer described above, completed the equipment.

The experimental data, recorded by the datalogger, were transmitted to our department by GSM modem. The datalogger used had 12 analogical and 2 digital channels and the capability of memorizing up to 62,000 data. It was set in a closed container with an emergency battery system and the equipment necessary to transfer the data. The datalogger, pyranometers, thermometers, voltmeter, and ammeter alimentations were fed by a small 20 W photovoltaic panel.

Measurements were recorded by the instruments every 5 seconds and then averaged over a time arc of 2 minutes for a variable daily duration dependant on the insolation hours of the period considered, that is, (in our tests) 12 hours (from 6.00 am to 6.00 pm) for the winter months and 16 hours (from 5.00 am to 9.00 pm) for the others.

The tests carried out can be divided into two series:

(i) in the first series, tests were conducted by modifying—within the total head limits foreseen by the manufacturer—the relationships between flow rate *Q* and total head *H* that characterized the hydraulic circuit. This was achieved by manually varying the opening degree of the closing valve, as shown in the scheme of Figure 1, after

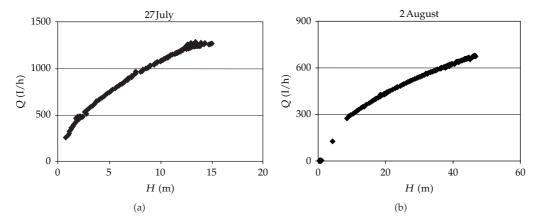


Figure 2: Daily *H-Q* relationships.

sunset when the pump was at rest. For every day, therefore, we have a unique relationship between among *Q* and *H*; some examples are in Figure 2;

(ii) the second series was obtained by modifying the hydraulic scheme with the insertion of a sustaining valve into the pipe that was able to assure a constant total head—fixed by the operator—at the pump exit although the flow rate was variable. This change was implemented for a double purpose to test the reliability of the system under conditions of operation similar to real ones, to avoid the introduction—as in the first test series—of an averaged value of the total head into the formulas that concern the daily performances evaluation.

In this work we refer mostly to this second series of tests. The results obtained with the first series of tests that have been the subject of a previous publication [15] will be briefly summarized in the following section.

2.2. First Tests Results and Data Processing

As a premise, is should be noted that the performances of these types of pumps are influenced negatively by the presence of cloudiness, especially if intermittent. That is because the pump is forced to continuously "stop and go" with consequent dispersion of the experimental data and, what is more important, with consequent diminution of efficiency, given the system's response time, both electric (condensers) and mechanic.

This is clearly visible in Figure 3 where two diagrams are shown: one (June, 20th) relative to a cloudless day, the other (June, 29th) relative to a cloudy day. The following can be seen in every diagram: in abscissas, time and minutes of the measurement (hhmm); in ordinates, the values of the radiation R_{β} (W m⁻²) incident on the panel plane and of the flow rate Q (L/h).

The results obtained with the first series of test will be synthesized below.

As usual, the panel/pump system's efficiency is defined as the ratio between useful power and absorbed power; in the latter case, to obtain this we must calculate the incident radiation that is given by R_{β} (W m⁻²) times the panel surface S (m²).

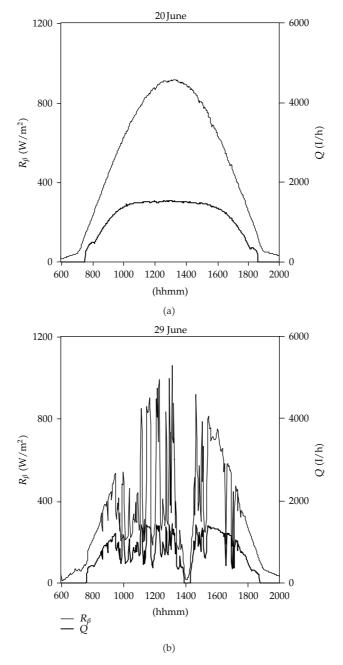


Figure 3: Examples of daily radiation and flow rates.

Being Q in L/h and $S = 2 \text{ m}^2$, in the formulas we have the theoretical expression:

$$\eta = 9.81 \left(\frac{(Q/3600)H}{R_{\beta}S} \right) = 9.73 \cdot 10^{-4} \frac{QH}{R_{\beta}}.$$
(2.1)

Introducing the measured values (16,000 for every considered greatness) of Q, H, and R_{β} into (2.1), many values of the output can be calculated. The empirical relationship obtained by regression analysis of η , H, and R_{β} is

$$\eta = 0.0047 \frac{H^{0,577}}{R_{\beta}^{0,02}} \tag{2.2}$$

with $R^2 = 0.934$.

Using the same data, it is also possible, always through regression, to obtain the empirical relationship that exists among Q, H, and R_{β} :

$$Q = 4.92 \frac{R_{\beta}^{0.98}}{H^{0.42}} \tag{2.3}$$

with R^2 equal to 0.89.

The two previous expressions were obtained independently; nevertheless, it is important, as a proof of the validity of the results obtained, that (2.3) can be obtained by replacing in (2.2) the expression of η as given by (2.1).

Concerning daily performances, if H_m is the daily average of total head, we calculated through summations of $Q \cdot \Delta T$ and $R_\beta \cdot \Delta T$ extended to the day duration (T in hours), respectively, daily pumped volumes w (m^3 day⁻¹) and global energy E_β incident daily on the panel unity area ($Wh \, m^{-2} \, day^{-1}$). The analysis of regression led to the following empirical expression:

$$w = 0.0033 \frac{E_{\beta}^{1,27}}{H_m^{0,44}} \tag{2.4}$$

with $R^2 = 0.976$.

As H varies continuously during the day (Figure 2), when we try to pass from instantaneous values to daily values—as in the case of the evaluation of the daily lifted volumes w—we are forced to introduce an averaged value H_m of H, (2.4), which could arouse perplexities regarding the possibility of the practical use of the same (2.4).

Hence, as previously stated, the hydraulic circuit was modified by introducing an automatic pressure regulation valve able to assure a constant total head at the pump exit equal to that established by the operator.

Therefore, we began a second series of test whose results are shown in the following paragraph.

2.3. Second Tests Results and Data Processing

This second and conclusive test series lasted from June to November 2004; the results were basically identical to those previously described since there were no substantial changes. Indeed, we modified, again with the pump at rest, the opening degree of the sustaining pressure valve which allowed the total head to be kept constant for several consecutive days. On the other hand, the solar incidental radiation changed—that is, the power input—affecting

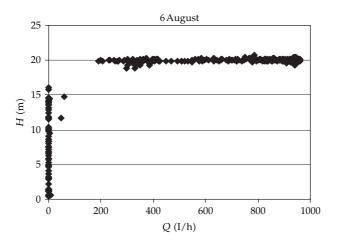


Figure 4: Example of (H; Q) daily data with the automatic sustaining valve.

the flow rate. Typically, the behaviour of the automatic valve was more than satisfactory, even if it required continuous surveillance. Indeed—probably due to resonance phenomena—sometimes when the pump started, the whole system became unstable; usually, but not always, the problem resolved itself after few seconds; otherwise, the test had to be ended.

The field of total heads investigated varied from a 7 to 50 meter water column.

As an example, the relationship between Q and H for August, 6th, is visible in Figure 4.

As you can see, total head remained virtually constant throughout the day, if we exclude the points on the left of the figure which are characterized by almost null discharges. The explanation of this phenomenon must be sought in two possible reasons:

- (i) the first, already present in the preliminary tests, requires that there is an incidental radiation threshold R_{β} greater than 100–150 W/m² for the whole system to be functioning. This is particularly evident in Figure 3, in which the lower Q curves begin later and end before of above R_{β} curves;
- (ii) the second, peculiar to this second series of test, is connected with the previously described instabilities that raise the value limit of R_{β} up to around 200–250 W m⁻², over which the experimental data become reliable.

Regarding the elaborations that follow reference will be made only to the measurements obtained with $R_{\beta} > 250 \,\mathrm{W\,m^{-2}}$. In doing so there was obviously some data manipulation, in particular concerning general daily evaluations, but, to our mind, this is virtually of no importance for the reasons that follow:

- (i) in the field $R_{\beta} \le 250 \,\mathrm{W}\,\mathrm{m}^{-2}$ values fall lower, and, therefore, there are less-significant values of Q (and of course, of R_{β});
- (ii) in the absence of clouds, modest values of R_{β} occur only in the morning and in the evening and, therefore, for a small part of the daily time of the pump's functioning.

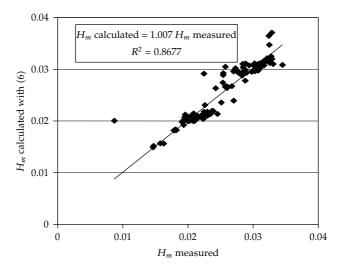


Figure 5: Comparison between measured efficiencies and those calculated with (2.6).

To obtain a theoretical expression for daily efficiency, η_m is defined as the ratio between the pump's average power output and the panel's average power input; proceeding as in (2.1), we have

$$\eta_m = 9.73 \cdot 10^{-4} \frac{Q_m H}{R_{\beta,m}},\tag{2.5}$$

where Q_m is the average daily flow rate, $R_{\beta,m}$ is the average daily panel radiation, and H is total head (that was kept constant during the entire day for the tests of this series).

Once calculated, though (2.5) represents the experimental values of efficiency, from the regression of η_m on H and $R_{\beta,m}$ we obtained an expression analogous to (2.2) but with the exponent of $R_{\beta,m}$ so small that it was almost equal to zero. Therefore, for practical purposes, this parameter is irrelevant, and the expression can be written as

$$\eta_m = 0.0048\sqrt{H}. (2.6)$$

In Figure 5, the values of η_m are reported, calculated by means of (2.6) versus the η_m measured values; as can be seen, (2.6) succeeds in interpreting the experimental data in a satisfactory way. Especially since it gives results that are practically identical to (2.2) which was obtained starting from instantaneous values of the parameters involved. In fact, if considering only the daily average data, we calculate the efficiency with both the formulas obtaining the relationship

$$\eta = 1.02\eta_m \tag{2.7}$$

with $R^2 = 0.98$.

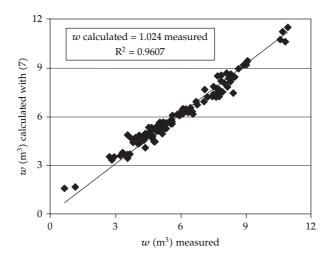


Figure 6: Comparison between measured volumes and those calculated with (2.8).

Considering now the daily lifted volumes, w, the relationship that best interprets the experimental data is

$$w = 0.0049 \frac{E_{\beta}}{\sqrt{H}},\tag{2.8}$$

where, as always, E_{β} is the global daily energy incident on panel unit area obtained, as previously stated, from summations of $R_{\beta} \cdot \Delta T$ (obviously, between E_{β} and $R_{\beta,m}$ there is the relation: $E_{\beta} = T \cdot R_{\beta,m}$) values extended to T.

In Figure 6, that has no need of any comments, the w values are shown calculated with (2.8) versus those measured.

Even in this case, (2.8) would have been directly derived from (2.5). In effect, replacing (2.5) with (2.6) we have

$$Q_m = 4.95 \frac{R_{\beta,m}}{\sqrt{H}} \tag{2.9}$$

from which (2.8) is immediately obtained if we multiply both the members for T—day duration by hours—and if we remember that w is in m^3 while the product $Q_m \cdot T$ is in litres.

Before proceeding further, a brief comment, throughout the whole trial period taken into account, the values of E_{β} ranged between 1,500 and 7,500 Wh m⁻² day⁻¹. This means that with, for example, $H=30\,\mathrm{m}$, the tested pump (that of those produced by the supplier company can be classified as of medium power) would be able to raise from 1.5 to 6 m³ day⁻¹. This performance—not exceptional if compared with traditional pumps—is mainly caused by efficiencies η_m of the whole panel/pump complex that, in line with those of similar installations [12], are around 2-3%. In particular, according to our measurements, this fact is mainly due to the panel and, to a lesser extent, to the electric feeding circuit that, in our tests, was able to feed the pump with electric power equal to 6-7% of the incidental solar power.

3. Operation Forecasts and Performances Optimization

Regarding forecasts, the problem can be posed in these terms: knowing operating characteristics of the whole panel/pump system—those of the one we tested or others of different manufacturers—assesses its performance when it is installed in locations different from where tested. In concrete, we have to estimate, the total head H being fixed, the volumes and/or the averages discharges lifted up daily or monthly or yearly (even the possibility of estimating "instantaneous" flow rates could exist. But it would be necessary to obtain hourly distribution of solar radiation R_{β} , starting from values of E_{β} , which would implicate the use of rather complex procedures to achieve an estimation that would usually be of limited practical interest). To this end, in order to use relations such as (2.8) and (2.9), the values of E_{β} , incidental energy on panel, or of $R_{\beta,m}$, average radiation on the same panel, must be known, being climatological parameters that are variable from day to day and from place to place.

3.1. Theory

Some databases currently exist that give the values, averaged over long series of years, of global energy E_h that reaches the unit area of a horizontal surface for various places in the world. In this paper we will refer mainly to ESRA [16] commissioned by the European Commission and edited by a team of universities and organizations of our continent. The years of observation are ten in number; the countries considered are those with latitudes φ between 30° (Morocco, Tunisia, and Middle-east) and 60° (Baltic countries). The parameters considered are numerous (the CD-ROM that accompanies the two-volume text contains, in addition to the measured data, those derived from them (yearly and monthly averages, e.g.) that proved to be very useful to verify our elaborations.) (temperature, pressure, rainfall, etc.); among the ones that concern us are the daily values of energy E_h and of brightness index K_t that depends on the presence of clouds and that will be defined later.

In Italy, the Central Bureau of Agricultural Ecology (UCEA) operates a database commissioned by the Ministry of Agriculture that, among other things, provides daily values of E_h for thirty national locations.

In any case, if we do not use software programs to estimate w or Q, that is, in order to use (2.8) or (2.9), it is necessary to obtain the values of E_{β} or $R_{\beta,m}$ from the E_h values. The procedure to be followed, which is quite long, is shown below and is furnished with diagrams and tips to make it easier and also to allow us to make choices that are more appropriate in relation to the optimal photovoltaic panel inclination.

In general, E_h is the sum of three components:

$$E_h = E_{h,d} + E_{h,r} + E_{h,df}, (3.1)$$

where $E_{h,d}$ is the direct radiation energy incident on a horizontal unit area with a precise incidence angle; $E_{h,r}$ is the reflected radiation energy on the same unit area that comes from the ground and land objects; $E_{h,df}$ is the diffuse radiation energy on the same unit area that comes reflected from the sky and clouds, after reflection and dispersion in the atmosphere.

In any case, if we are not in the presence of snowy mantles (very reflecting), generally $E_{h,r}$ is very small compared to the other two terms; furthermore, it depends on local situations

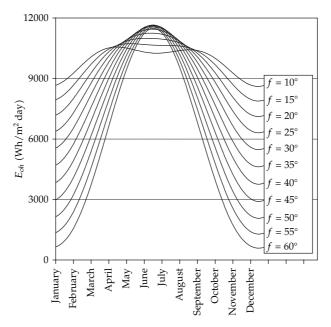


Figure 7: E_{oh} annual trend.

that are evidently not possible to take into account. Therefore in our elaborations, we make reference to the simplified relationship:

$$E_h = E_{h,d} + E_{h,df}. ag{3.2}$$

As known, with the purpose to optimize performances, photovoltaic panels are not horizontally disposed, and thus, as previously mentioned, we are forced to deduce the E_{β} values by starting from the corresponding E_h ones. In fact, the procedure to follow is rather laborious because the two components that form E_h by (3.2) vary according to laws when β varies, and, therefore, we need to decompose E_h into $E_{h,d}$ and $E_{h,df}$; separately calculate the $E_{\beta,d}$ and $E_{\beta,df}$ values that these two parameters assume on the panel plane, and, finally, by the sum of these two, come to E_{β} .

The proportions between the two components, from which E_h is constituted by (3.2), exclusively depend on cloudiness and, therefore, on the so-called brightness coefficient:

$$K_t = \frac{E_h}{E_{oh}},\tag{3.3}$$

where E_{oh} is the maximum global radiation available. This represents the theoretical limit of E_h in ideal atmospheric conditions and depends only on spatial latitude φ and on time, that is, on angle δ (declination) which, in the moment, the sunrays form with the equatorial plane.

Following the procedure recommended by the ESRA, we calculated and reported in Figure 7 the annual trend of E_{oh} for different latitudes (one curve for each). As visible in the same figure, as φ decreases, passing from 60° in Finland to 10°–15° in African countries,

the diagram tends to flatten so that, for practical purposes, for $\varphi < 10^{\circ}$, E_{oh} can be assumed constant and equal to $10 \, \text{kWh m}^{-2} \, \text{day}^{-1}$.

By (3.3), knowing E_h from the database and E_{oh} determined by the graphic of Figure 7, it is possible to obtain K_t and, consequently, the value of $E_{h,df}/E_h$ by means of the following relations:

$$\frac{E_{h,df}}{E_h} = 0.14\tag{3.4'}$$

if $K_t > 0.75$;

$$\frac{E_{h,df}}{E_h} = 1 - 0.273K_t + 2.45K_t^2 - 11.95K_t^3 + 9.39K_t^4 \tag{3.4}$$

if $K_t \le 0.75$.

By calculating $E_{h,df}$ with the previous relations, by (3.2), it is possible to determine direct radiation $E_{h,d}$.

The next step is to transform $E_{h,d}$ and $E_{h,df}$ values into those of $E_{\beta,d}$ and $E_{\beta,df}$ that the two energies, respectively, direct and diffused, assume on the unity area and on an angle β with respect to the horizon. The formulas to be used, in the case of a panel oriented, as always, south for the northern and vice versa for the southern hemisphere, are

$$\frac{E_{\beta,d}}{E_{h,d}} = \frac{\cos(\varphi \mp \beta)\cos\delta \cdot \sin\omega_s + \omega_s \sin(\varphi \mp \beta)\sin\delta}{\cos\varphi \cdot \cos\delta \cdot \sin\omega_s + \omega_s \cdot \sin\varphi \cdot \sin\delta},$$
(3.5')

$$\frac{E_{\beta,df}}{E_{h,df}} = \frac{1 + \cos \beta}{2}.\tag{3.5"}$$

Referring to (3.5'), it must be noted that a minus sign for the northern hemisphere and a plus sign for the other must be used; furthermore, in addition to the symbols already known, ω_s appears, which is function of δ and φ .

To simplify the calculations, even in this case, it was possible to develop graphs (Figure 8) that, for the values assigned to K_t , allow the daily values of $E_{\beta}/E_h = (E_{\beta}, d + E_{\beta,df})/E_h$ to be estimated for several latitudes. To draw them, in this last equation we had to replace the expressions of $E_{\beta,d}$ and of $E_{\beta,df}$ obtained from (3.5') and (3.5") in the numerator; use (3.2) and (3.4") and (3.4") to write $E_{h,d}$ and $E_{h,df}$ as functions of E_h ; reduce the parameters expressing β as function of φ . As will be explained in the next section, this last relationship was obtained by determining mathematically the optimal value β^* of β that, for a given φ , is able to maximize the total energy reaching the panel. In doing so, two possible uses of the pump were taken into account: annual, which, of course, is the most usual, and seasonal, which is in connection with irrigation that lasts from May to September in our climate.

3.2. Best Panel Tilting

Obviously, the best panel position would be, in principle, that for which every day the sun's rays are at astronomical noon, exactly perpendicular to the panel itself, which implies a daily update of its inclination β . If the values of β , in function of the different days of the year,

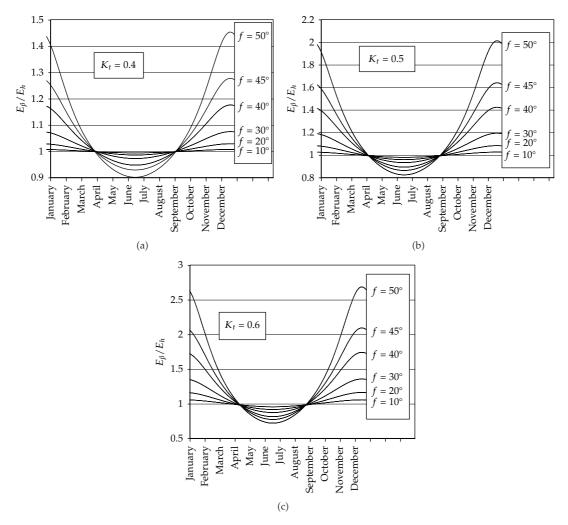


Figure 8: Annual trend of E_{β}/E_h ratio.

are plotted in a graph, we obtain, for a fixed latitude, a sinusoidal curve that is symmetric around its lowest point—which is the summer solstice in mid-June—with a maximum at the winter solstice in mid-December. For instance for a latitude like that of Viterbo ($\varphi = 42^{\circ}$, approximately) β would rise from 17° at the winter solstice to 63° at the summer solstice.

Indeed, if you do not use the solar tracking that has been mentioned in a previous note, to give the panel constant inclination, it is usual to refer to a kind of average value choosing $\beta = \varphi$, which implies that the condition of perpendicularity of the sunrays is exactly verified only in correspondence to the two equinoctial astronomical noons.

This practical rule is common and is very simple, yet it implies that the energy that annually reaches the panel is not the maximum possible but is lower by about 4–5% in the case of annual utilization and, on the basis of our evaluations, to a greater extent in the case of seasonal use (irrigation) of the pump.

Therefore, even if it is known that the influence of β is rather modest, we checked whether there are equally easy alternative rules to optimize the position of the panel in both utilization cases.

To this end, in the sum $E_{\beta} = E_{\beta,d} + E_{\beta,df}$, we replaced the expressions of $E_{\beta,d}$ and $E_{\beta,df}$, obtainable by (3.5') and (3.5"), and tried to find mathematically the inclination values β_a^* and β_s^* able to maximize the integrals of E_{β} extended to the entire year or to the months from May to September, respectively.

Assuming that $K_{t,a}$ is the annual average value of K_t , for annual use we obtained the relationship

$$\beta_a^* = 0.01 \cdot \varphi^2 + 0.4 \cdot \varphi \tag{3.6}$$

that is valid for $0.4 < K_{t,a} < 0.6$ which, in any case, is the range where the values of $K_{t,a}$ usually fall. In fact, from the data reported by the ESRA—and also from another database found on the web (So.Da, Joint Research Centre)— $K_{t,a}$ varies between 0.39–0.42 for northern countries and 0.53–0.60 for Mediterranean countries (Morocco, the Middle-East, etc.) and north Africa.

With the exception of the equatorial areas, where it is worthwhile to keep the panel horizontal or nearly so, from (3.6) we can deduce that a practical rule is to adopt an inclination equal to φ minus 6°–7°, for latitudes between 10° and 55°.

Greater advantages, of 10–12% (even of 17% for northern-Europe) and also greater corrections to the rule that would require $\beta = \varphi$, occur when we consider a primarily seasonal pump utilization. In effect, if we consider the countries for which we have an irrigation season covering the months from May to September—and therefore those between 30° of latitude of the African Mediterranean countries and 53° of the northern Germany—following the previously described procedure we obtained the empirical relationship:

$$\beta_S^* = 052 \cdot \varphi - 13.2 \tag{3.6'}$$

which is valid, like (3.6), for $0.4 \le K_{t,s} \le 0.6$. As for $K_{t,a}$, even the usual values of $K_{t,s}$ are within these limits. In effect, from the dataset of the ESRA, we can see that $K_{t,s}$ is only 12–13% greater than correspondent $K_{t,a}$.

From (3.6'), we can deduce that in practice we can obtain the optimal panel inclination by subtracting 13° from half the latitude value. This implies very low β_s^* values, from 2° to 14° for latitudes 30° $\leq \varphi \leq$ 53°, and in particular, for Italian countries (37° $\leq \varphi \leq$ 46°), an almost horizontal panel disposition ($\beta_s^* = 4^\circ - 10^\circ$).

One last note. In the case of a utilization primarily finalized, but not exclusively, for agricultural uses, it would be worthwhile to increase the panel inclination at the end of the irrigation season. It is a very simple operation that, with the help of an inclinometer, can be performed in few minutes and that involves an increase in energy E_{β} reaching the panel in the remaining months of the year by some percentages points (from October to April). In such a case, our elaborations, carried out with the same previously adopted procedure, show that angle β should be raised to a value *equal to latitude* φ *plus* 6°–8° at the end of the irrigation season.

All the aforementioned conclusions have been verified through simulations. In particular, reference has been made to the ESRA CD-ROM, for locations and for the years (from 1981 to 1990) of its database, furnishes the values of E_{β} related to the various months,

Month	E_h	$R_{h,m}$	E_{eta}	$R_{eta,m}$	w	Q_m
Jan	5766	501	5905	513	5.28	464
Feb	5994	512	6090	520	5.45	470
Mar	5917	495	5946	497	5.32	449
Apr	5624	460	5590	457	5.00	413
May	5433	436	5357	430	4.79	389
Jun	4640	369	4556	363	4.08	328
Jul	3900	312	3838	307	3.43	277
Aug	3623	294	3591	291	3.21	263
Sep	4003	332	4003	332	3.58	300
Oct	5352	453	5411	458	4.84	414
Nov	5771	499	5897	510	5.28	461
Dec	5686	497	5840	510	5.22	461

Table 1: Average value simulations for Tamalè.

for each β value assigned; this allows us to proceed by trials to the determination of the β value that maximizes the sum of E_{β} extended to the period May–September.

Analogous checks have been performed for (3.5') and (3.5'') even if the presence of some sites on the web which give the value of β_a^* must be cited. Of particular interest—see bibliography—are the European Commission Joint Research Centre and the So.Da. Project websites.

4. A Case Study

As a practical application of our work and in response to a specific request from the firm, we have estimated the performances of a pump of the type that we tested in view of its installation in the Tamalè area (central Ghana; $\varphi = 9^{\circ}41'$) assuming the total head being $H = 30 \,\mathrm{m}$.

From the So.Da. Project website, monthly mean values of K_t were derived for the years from 1997 to 2004; they ranged from about 0.40 in the summer months to 0.63–0.65 in the period from November to February, with an average annual $K_{t,a}$ value = 0.54.

An inclination $\beta_a^* = 5^\circ$ was assigned to the panel according to (3.6), and in Table 1 the following are reported:

- (i) in the first and second column the monthly average values of E_h and $R_{h,m}$, respectively;
- (ii) in columns three and four, the corresponding values of E_{β} and $R_{\beta,m}$, found—for $\varphi \approx 10^{\circ}$ and $K_t \equiv K_{t,a} \approx 0.5$ —by means of the graph in Figure 8;
- (iii) in the last two columns, the monthly average values of w and Q estimated, respectively, by (2.8) and (2.9).

The same procedure is to be followed, obviously, in the case we wish to expand the estimates, going beyond the simple monthly averages. For example, with reference to the month of January, we found that, over the arc of the years of observation, the daily values of $R_{\beta,m}$ ranged between 439 and 544 W m⁻² so, by (2.9), they led to values of average daily flow rates of between 396 and 491 L h⁻¹. These Q_m would correspond to possible peaks of maximum discharge of the order of 1200–1300 L h⁻¹ in the hottest hours if we consider that,

on cloudless days, the hourly distribution of the solar radiation presents peaks of about 2.5–3 times the average value.

5. Conclusions

Thanks to an opportunity offered to us by company we became interested in photovoltaic pumps and, in particular, in the development of a "test bench" that could test the performances of a whole panel/pump complex. It is not, to our knowledge, a kind of installation that is frequently created because even if it is quite easy to find information on photovoltaic panel and pump performances produced by different companies, these data are obtained from tests that were performed on the two components separately, that is, the panel only or solely the pump, and not with the whole complex consisting of both. The results are quite misleading because the companies test their pumps inside establishments, coupling them to the electric network and not to a photovoltaic panel. The reasons for this state of things are probably many; among them there are certainly economic reasons related to both the costs of instruments and, above all, the necessarily long duration of the tests.

Our tests have clarified the operating characteristics of the panel/pump system in the sense that they allowed us to find relationships that link flow rates and daily pumped volumes to the total head and to the radiation incident on the panel. These results, however, depending on solar radiation, have only a local validity in that they are useful only for latitudes equal to those of Viterbo (42°24′ North) and on a panel inclination equal to that latitude, as the general routine.

Therefore, we have developed—providing also a practical application—a simple methodology that can allow

- (1) reliable assessment of the performances of the panel/pump complex tested in Viterbo or of another complex whose operating characteristics are known and that is called upon to operate in areas of different geographic coordinates;
- (2) a more informed choice of the panel inclination that maximizes the energy incident, both in the case of annual operation of the pump or in the case of such predominantly seasonal usage as irrigation, for example.

Regarding the panel/pump system we have tested: on the one hand, it proved to be reliable during the whole long period of our tests, and, on the other, it also highlighted the limitations typical of all these kinds of devices [12] that use photovoltaic energy.

Efficiencies equal to 2–3% are certainly not thrilling; nevertheless, they are not so small that an installation of this type is not able to satisfy modest demands, both in western countries and, above all—as in the case study related to Ghana—in those third world countries where there are often no alternatives.

Of course, despite the progress in recent times, a great deal of work has to be done; just think that the main reason for this state of affairs lies in the efficiency of the panel whose value is currently about 10–12%. But this is precisely the reason that leads to hope in the future; PV technology is relatively new and, therefore, as such, is an evolving area in which further progress can be made (in 1980, the average efficiency of the panels was approximately 3%). The cost, even in terms of human lives, of traditional energy sources such as oil has reached such levels that it is reasonable to expect that, even due to the recent pushing exerted by the United States of President Obama, also the EU will develop an even stronger interest in this area which could lead to an increased flow of resources, both human and financial.

Symbols

For all the parameters we adopted international system units, or ones accepted by it (like bar), with the exception of flow rates Q expressed, for obviously practical reasons, in liters per hour.

- E_{0h} : Maximum specific energy (referred to unit area) of daily global radiation incident on the horizontal plane, expressed in Wh m⁻² day⁻¹
- E_h : Specific energy (referred to unit area) of daily global radiation incident on the horizontal plane, expressed in Wh m⁻² day⁻¹
- $E_{h,d}$: Specific energy (referred to unit area) of daily direct radiation incident on the horizontal plane, expressed in Wh m⁻² day⁻¹
- $E_{h,df}$: Specific energy (referred to unit area) of daily diffuse radiation incident on the horizontal plane, expressed in Wh m⁻² day⁻¹
- $E_{h,r}$: Specific energy (referred to unit area) of daily reflected radiation incident on the horizontal plane, expressed in Wh m⁻² day⁻¹
- E_{β} : Specific energy (referred to unit area) of daily global radiation incident on the panel plane, expressed in Wh m⁻² day⁻¹
- $E_{\beta,d}$: Specific energy (referred to unit area) of daily direct radiation incident on the panel plane, expressed in Wh m⁻² day⁻¹
- $E_{\beta,df}$: Specific energy (referred to unit area) of daily diffuse radiation incident on the panel plane, expressed in Wh m⁻² day⁻¹
- H: Total head pump (m)
- H_m : Average daily total head pump (m)
- K_t : Brightness index, adimensional
- $K_{t,a}$: Average yearly brightness index
- $K_{t,s}$: Average seasonal brightness index
- Q: Flow rate (L h⁻¹)
- Q_m : Average daily flow rate (L h⁻¹)
- R_h : Global radiation incident on the horizontal plane (W m⁻²)
- $R_{h,m}$: Average daily global radiation incident on the horizontal plane (W m⁻²)
- R_{θ} : Global radiation incident on the panel plane (W m⁻²)
- $R_{\beta,m}$: Average daily global radiation incident on the panel plane (W m⁻²)
- R^2 : Correlation coefficient
- S: Panel surface (m^2)
- *T*: Time duration (hours)
- w: Volume daily pumped, expressed in m³ day⁻¹
- β : Panel tilt angle; that is, the angle between the normal line to the surface of the panel and the vertical line of the site (°)
- β^* : Best panel tilt angle (°)
- β_a^* : Best panel tilt angle in the case of annual use (°)
- β_s^* : Best panel tilt angle in the case of seasonal use (°)
- δ : Declination angle (°)
- η : Panel/pump complex efficiency
- η_m : Average daily panel/pump system efficiency

Φ: Latitude (°)

 ω_s : Examination site hourly angle at sunset or sunrise.

Authors' Contribution

The author's contributions to this paper can be considered equal.

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