

Research Article

Four-Impulsive Rendezvous Maneuvers for Spacecrafts in Circular Orbits Using Genetic Algorithms

Denilson Paulo Souza dos Santos,¹ Antônio Fernando Bertachini de Almeida Prado,¹ and Guido Colasurdo²

¹ *Division of Space Mechanics and Control INPE, CP 515, 12227-310 São José dos Campos SP, Brazil*

² *Department of Aerospace and Mechanical Engineering, Università degli Studi di Roma "La Sapienza", Piazzale Aldo Moro 5, 00185 Roma, Italy*

Correspondence should be addressed to

Denilson Paulo Souza dos Santos, denilson2011b@hotmail.com

Received 1 December 2011; Accepted 27 January 2012

Academic Editor: Maria Zanardi

Copyright © 2012 Denilson Paulo Souza dos Santos et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Spacecraft maneuvers is a very important topic in aerospace engineering activities today. In a more generic way, a spacecraft maneuver has the objective of transferring a spacecraft from one orbit to another, taking into account some restrictions. In the present paper, the problem of rendezvous is considered. In this type of problem, it is necessary to transfer a spacecraft from one orbit to another, but with the extra constraint of meeting another spacecraft when reaching the final orbit. In particular, the present paper aims to analyze rendezvous maneuvers between two coplanar circular orbits, seeking to perform this transfer with lowest possible fuel consumption, assuming that this problem is time-free and using four burns during the process. The assumption of four burns is used to represent a constraint posed by a real mission. Then, a genetic algorithm is used to solve the problem. After that, a study is made for a maneuver that will make a spacecraft to encounter a planet, in order to make a close approach that will change its energy. Several simulations are presented.

1. Introduction

This paper aims to analyze optimal rendezvous maneuvers between two spacecrafts that are initially in circular coplanar orbits around the Earth. The main goal is to perform this transfer having the fuel consumption as a penalty function, so the minimization of this quantity is searched during the process of finding the solutions. The approach used here is to assume that the problem is time-free, which means that the time of the transfer is not important. The control assumed to perform this task is an engine that can deliver four burns. This assumption

is used to represent a common constraint posed by real missions. In the present paper, we are considering a generic problem, not a specific mission, but this type of constraint appears very often in space activities.

Then, a genetic algorithm is used in order to solve the problem. This type of approach represents a new alternative to solve this problem and can be used for comparisons with results obtained by standard procedures available in the literature, as shown in [1–28]. Preliminary studies showed that, in some situations, this algorithm can be faster in convergence and more accurate, while in some others, it is slower and presents less accuracy. A detailed comparison still has to be made to evaluate under which circumstances this algorithm can be more efficient. In any case, several kinds of missions can use the benefits of the techniques based on the genetic algorithm showed in this work. The main types are transference with free time (to change the orbit of the space vehicle without restrictions in the time required by the execution of the maneuver), “rendezvous” (when one desires that the space vehicle stands alongside another spacecraft), “flyby” (a mission to intercept another body, however without the objective to remain next to it), “swing-by” (a close approach to a celestial body to gain or lose energy), and so forth. But, in the present paper, only the rendezvous maneuver is considered.

2. Description of the Problem

The problem of orbital maneuvers has been studied in several published papers. Some of them are shown in [1–28]. The different approaches to solve this problem can be appreciated in those references. Some authors assumed that a low magnitude force is applied to the spacecraft during a finite time. This is the so-called continuous thrust approach. References [7, 10, 17, 28] have some details on this topic. As an alternative approach, the idea of an impulsive maneuver is also studied. In this situation, a high magnitude force is applied during a time that can be considered negligible. References [3, 5, 8, 27] used this important approach. More recently, two more ideas appeared in the literature to perform orbital maneuvers. The first one is the use of a close approach with a celestial body to change the orbit of a spacecraft. It is the swing-by maneuver. References that used this approach are [2, 13]. The second recent approach is the gravitational capture, where the force generated by the perturbation of a third body [14] can be used to decrease the fuel expenditure of a space maneuver. References [11, 12] have some details of this idea. Some publications cover all those topics in more details, like [6, 9, 15]. Studies more related to the research shown in the present paper are the ones considering the Lambert’s problem ([1, 16]), the rendezvous maneuver ([20–26]), and genetic algorithms itself ([18, 19]).

In the present research, in order to solve the transfer proposed here, the Lambert’s problem is used, in the way described below. The Lambert’s problem can be formulated as follows: “Find an unperturbed orbit, under the mathematical model given by a law that works with the inverse square of the distance (Newtonian formulation), that connects two given points P_1 and P_2 , with the transfer time (Δt) specified.” In the literature, several researchers have solved this problem by using distinct formulations. Reference [1] shows several of them. In this way, the parameters of the transfer orbit can be defined by

- (1) ν_1 is the true anomaly of the departure point P_1 on the initial orbit. $\nu_1 \in [0, 2\pi]$,
- (2) $\Delta\nu$ is the angular length of the transfer. $\Delta\nu \in [0, 2\pi]$,

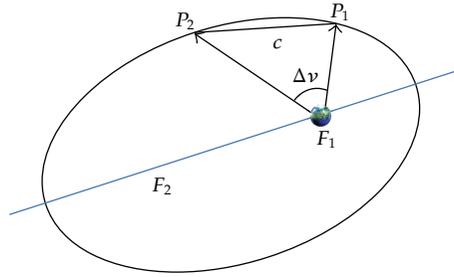


Figure 1: Instantaneous scenario of the problem.

- (3) a_t is the semimajor axis of the transfer orbit. Note that, for each pair of departure and arrival points, a minimum value a_{\min} exists for a_t . Two transfer orbits can be found for the same value of a_t , depending on the sense of the transfer.

The parameter a_t is usually replaced by a different parameter y . The advantage of this substitution is that the new variable has values between 0 and 1. The relationship is shown below [19]

$$a_t = \frac{a_{\min}}{4y(1-y)}. \quad (2.1)$$

The parameters ν_1 and $\Delta\nu$ determine the position of the points P_1 and P_2 that can be related to the radius vectors \vec{r}_1 and \vec{r}_2 . Any permitted value of the parameter y determines univocally one transfer orbit. These parameters are, from the point of view of the genetic optimizer, the genes of the members of the population.

The genetic algorithm searches for the best solution among a number of possible solutions, represented by vectors in the solution space. To find a solution is to look for some extreme value (minimum or maximum) in the solution space. The fitness of each individual is represented by the total velocity impulse ΔV required to perform the orbital transfer. The total impulse is given by the sum of the single impulses ΔV_i provided in each thrust point in order to pass from an orbital arc to the following one. It corresponds to the velocity difference at the relevant thrust point.

The positions of the thrust points and the parameters of the transfer orbit are obtained using as input the three genes, that is, the parameters previously chosen. The velocities at the thrust points, before and after firing the engine, are easily computed, and it provides the total velocity impulse, which is the measurement of the individual fitness. The evolutionary process will select individuals with the genes corresponding to the optimal maneuver. Figure 1 shows an instantaneous scenario of the problem.

Note that ν_1 is the true anomaly of the point P_1 on the initial orbit; ν_2 is the true anomaly of the point P_2 on the final orbit; $\Delta\nu$ is the angular length of the transfer; the orientation of the transfer orbit is defined by the angle ω between its axis and the axis of the initial orbit; c is the distance between P_1 and P_2 (2.4); F_i are the focus of the ellipse.

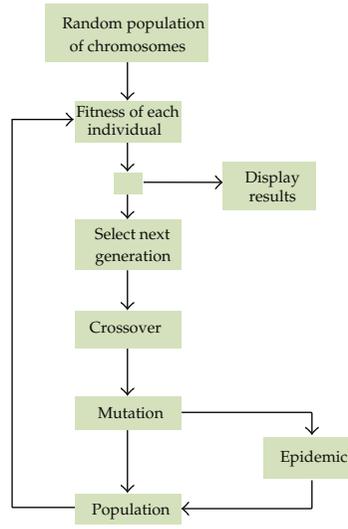


Figure 2: The genetic algorithm.

2.1. The Genetic Algorithm

The procedure starts with a random population of up to 800 individuals. The initial population is generated randomly, and consider its characteristics distance and angles according to the constrains of each variable. The vectors \vec{x} are assembled according to the allowed boundary condition. Then, the fitness of each individual is verified, following the criteria of the objective function, which is to minimize the fuel consumption (measured by the ΔV) found by solving the Lambert's problem. So, the best individuals are selected to go to the next generation, parents, and children. The procedure of crossover is then applied, as well as a mutation to insert diversity in the population (Figure 2).

The random variables used for the implementation of the algorithm are $\vec{x} = (\Delta\theta_1, \Delta\theta_2, R_1, R_2, \gamma_1, \gamma_3)$. Those symbols have the meaning that $\theta_i = \nu_i - \omega$ is the true anomaly of the P_i points that determine the transfer orbit, as shown in Figure 3; R_i determines the radius vector (position) in each thrust; the γ (2.7) are the angles between $(F_1, \widehat{P_1}, P_2)$ (see Figure 3 again).

Eventually, there are epidemics, with the goal of inserting diversity and reducing the elitism. After that, a new population is created, and the procedure is started again, finishing after n attempts. The block diagram of the genetic algorithm (Figure 2) shows the procedures followed to solve the problem. More details of the genetic algorithm can be obtained in [18, 19].

2.2. Selecting the Next Generation and Performing the Crossover and the Epidemic Process

The selection of the new generation is made after the analysis of each individual by measuring its objective function (Fitness). The ones with better values for this measurement are selected to undergo a process of crossing or reproduction (crossover), where parents are selected, and the children of this intersection are raised (Figure 2). When the population is too

uniform, measured by the values of their objective functions, part of the population suffers an epidemic process, where many individuals are killed and replaced by others using again a random process, to insert diversity in the population and to prevent premature convergence to local optimal values. The crossover starts by separating the chromosomes of the parents in two parts. After this separation, the first part of the parent 1 is combined with the second part of the parent 2, and the first part of the parent 2 is combined with the second part of the parent 1. In this way, a second generation is created. See [18, 19] for more details.

2.3. Chromosome

The chromosome representation is vital for a genetic algorithm (GA), because it is the way that we translate the information from the problem to a format that can be handled by the computer. This representation is completely arbitrary, so it varies according to the choice made by each developer, without any kind of obligation to adopt any representation available in the literature. This is a very important point to emphasize. The vast majority of researchers use the binary representation for this problem because it is the simplest one. In fact, many people, when they imagine a GA, quickly make an association with binary chromosomes (used to facilitate the crossing). However, other formulations using real chromosomes, modifying the way of performing crossover, get satisfactory results [18]. In this paper, each gene is chosen to be a real number between 0 and 1, being generated in a binary form and then converted in a real number. The value of the corresponding parameter is $X_i = X_i^{\min} + u_i(X_i^{\max} - X_i^{\min})$, where X_i^{\min} and X_i^{\max} are the minimum and maximum values of those variables, which means that they are the boundary conditions. The main reason to use the binary approach is to validate this usual approach, in GA problems, in the particular type of problem considered here. References [18, 19], that studied this same problem using GA, used different approaches, so the validation of the binary approach was considered important.

2.4. Objective Function

Most of the selection techniques used in this procedure require comparisons of the fitness to decide which solutions should be propagated to the next generation. Normally, the fitness has a direct relation with the value of the objective function, according to the rule that better values of the objective function generate higher values of the fitness parameter. When the genetic algorithm calls the objective function, it transfers an array of parameters that specify the selected solution. This selection parameter must not be changed in any way by the objective function. Genetic algorithms are based on biological evolution, and they are able to identify and explore environmental factors to converge to optimal solutions, or approximately optimal global levels. Then, the fitness of each individual can be computed by using the five data that define the problem ($a_1, e_1, a_2, e_2, \Delta\omega$, the first one being unit because of the normalization of the variables) and the three genes ($v_1, \Delta v, y$) that characterize the individual. Then, we can obtain several important parameters [15]. The true anomaly of the arrival point is given by

$$v_i = v_{i-1} + \Delta v. \quad (2.2)$$

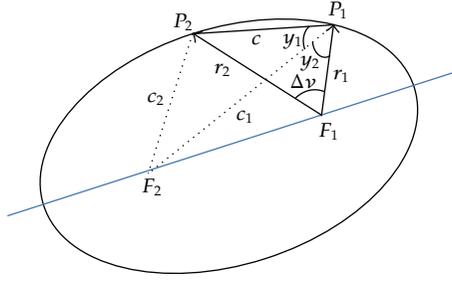


Figure 3: Geometry of the problem and the angles involved in the problem.

The radii of the departure and arrival point are given by

$$\begin{aligned} r_1 &= \frac{a_1(1 - e_1^2)}{1 + e_1 \cos \nu_1}, \\ r_2 &= \frac{a_2(1 - e_2^2)}{1 + e_2 \cos \nu_2}. \end{aligned} \quad (2.3)$$

The distance between P_1 and P_2 is

$$c = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos \Delta\nu}. \quad (2.4)$$

The semimajor axis of the transfer orbit is

$$a_{\min} = \frac{r_1 + r_2 + c}{4}. \quad (2.5)$$

The distances c_1 and c_2 of P_1 and P_2 from the vacant focus F_2 can be specified by the equations

$$c_i = 2a - r_i. \quad (2.6)$$

Figure 3 shows a description of several important variables.

The angles can be calculated by ($y = y_1 + y_2$),

$$\begin{aligned} y &= \arccos\left(\frac{r_1^2 - r_2^2 + c^2}{2r_1c}\right), \\ y_1 &= \arccos\left(\frac{c_1^2 - c_2^2 + c^2}{2c_1c}\right). \end{aligned} \quad (2.7)$$

The eccentricity of the transfer orbit is given by

$$e_t = \frac{\sqrt{c_1^2 + r_1^2 - 2c_1r_1 \cos \gamma_2}}{2a_t}. \quad (2.8)$$

The true anomaly θ_1 of the P_1 on the transfer orbit is

$$\begin{aligned} r_1 &= \frac{a_t(1 - e_t^2)}{1 + e_t \cos \theta_1}, \\ \theta_1 &= \arccos\left(\frac{a_t(1 - e_t^2) - r_1}{r_1 e_t}\right). \end{aligned} \quad (2.9)$$

The argument of perigee for the transfer orbit is

$$\omega = \nu_1 - \theta_1, \quad (2.10)$$

which is the angle between the perigees of the transfer and the initial orbits.

Now that the geometry of the maneuver has been shown, it is possible to calculate the radial and the tangential components of the spacecraft velocity before and after both impulses, what permits the computation of the total ΔV , which has been assumed to be the measurement of the individual fitness.

2.5. Normalization

Nondimensional variables are used in the procedure. They are shown below

$$\begin{aligned} r &= \frac{\tilde{r}}{\tilde{a}_1}, \\ v &= \frac{\tilde{v}}{\sqrt{\mu/\tilde{a}_1}}. \end{aligned} \quad (2.11)$$

The distance and velocity units for the normalized variables are the semimajor axis of the initial orbit and the velocity on a circular orbit with the same energy as the initial one. So, the reference time is $\Delta t = \sqrt{\tilde{a}_1^3/\mu}$.

3. Numerical Solutions

Several maneuvers were simulated with the procedure developed here, using the genetic algorithm. Then, the equivalent Hohmann maneuvers were calculated to provide a level of comparison. The idea is not to find a transfer that has a smaller total ΔV , when compared to the Hohmann transfers, but to try to minimize the difference in costs, assuming that the engine of the spacecraft has a limitation that does not allow two impulsive maneuvers to be performed. In theory, for the cases simulated here, the two impulses maneuvers always have a lower consumption. So, the idea is to find the best maneuver that has four impulses, in order to compare with other works [18, 19]. The number of impulses is a parameter that can be modified in the input data of the algorithm to be useful for other applications. The results shown in the present paper always consider a rendezvous maneuver between two spacecrafts, where the radius of the orbit of the first spacecraft is $r_o = 1$, and several values were used for the radius of the spacecraft that is in the final orbit (see Table 1). The genetic

Table 1: Rendezvous between coplanar circular orbits showing the values of the ΔV for each burn and the total expenditure (ΔV_T). The results for the Hohmann are included for comparison.

n°	Simulation ($r_o = 1$)	Cost using the genetic algorithm				$\Delta V_T = \sum_{i=1}^4 \Delta V_i$	Hohmann transfer $\Delta V_{HT} = \sum_{i=1}^2 \Delta V_i$	$\Delta V_T - \Delta V_{HT}$
		ΔV_1	ΔV_2	ΔV_3	ΔV_4			
1	$r_f = 1.2$	0.104455	0.118925	0.159234	0.154917	0.537531	0.087000	0.450500
2	$r_f = 1.5$	0.044987	0.118996	0.165893	0.189594	0.519469	0.181600	0.337900
3	$r_f = 1.6$	0.028621	0.120939	0.163349	0.204023	0.516932	0.206600	0.310300
4	$r_f = 1.8$	0.000000	0.131481	0.169234	0.225721	0.526435	0.249300	0.277100
5	$r_f = 1.9$	0.000000	0.127047	0.178111	0.233180	0.538338	0.267700	0.270600
6	$r_f = 2$	0.008136	0.117267	0.185660	0.242748	0.553811	0.284500	0.269300
7	$r_f = 2.5$	0.134621	0.000000	0.222416	0.298130	0.655167	0.349600	0.205300
8	$r_f = 3$	0.171286	0.000000	0.242839	0.369209	0.783334	0.393800	0.271100
9	$r_f = 5$	0.172864	0.310322	0.000000	0.423150	0.906336	0.480000	0.426300
10	$r_f = 10$	0.004673	0.000000	0.372418	0.311566	0.688657	0.499300	0.189400

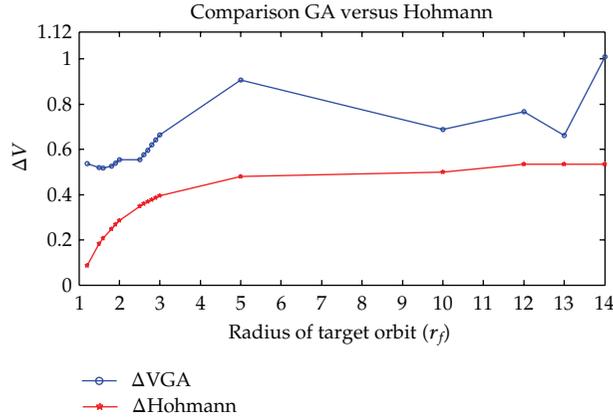


Figure 4: Comparison between the ΔV_{GA} and $\Delta V_{Hohmann}$.

algorithm provided satisfactory solutions, when compared with the solutions of the literature [18], as shown in Table 1. The population is composed by 800 individuals, and up to 400 generations of individuals were used.

The results indicated that the maneuvers using the GA with 4 impulses do not provide savings over the Hohmann transfer for all cases simulated (see Figure 4 and Table 1), as expected and explained before, but it minimizes the difference in costs for the assumed four impulsive maneuvers. Figure 4 shows all the details for this comparison.

Figures 5, 6, 7, 8, and 9, as well as Table 1, show a series of maneuvers. In general, an impulse is applied in the initial orbit (ΔV_1), generating the first elliptical transfer arc, and then, according to the procedure, the second impulse is applied (ΔV_2), leading to another elliptical transfer orbit. The third point of burn will happen (ΔV_3) to put the spacecraft in the last transfer arc, and, finally, the last impulse (ΔV_4) is applied to locate the vehicle in the desired orbit. The total consumption is the sum of all the intermediate impulses, and it is named ΔV_T (Table 1). This total consumption serves as an index of measurement and comparison between the methods. In other words, the information of the extra cost is due to

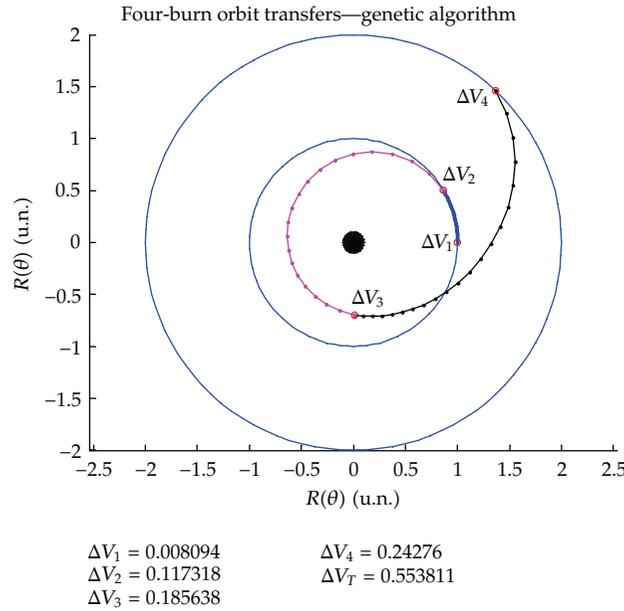


Figure 5: Four-burn orbit transfers for simulation 6.

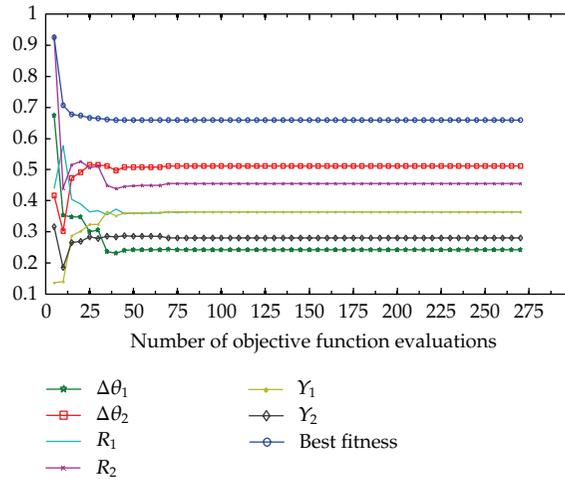


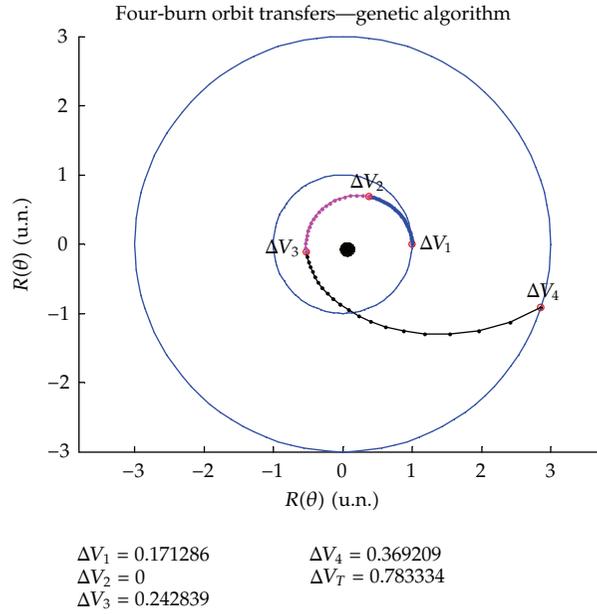
Figure 6: The variables of the problem using the method of genetic algorithm and the best fitness for simulation 6.

the fact that a two-impulse maneuver is not possible and a detailed vision of the best four-impulse strategy is generated by the GA.

The variables of the problem are $\vec{x} = (\Delta\theta_1, \Delta\theta_2, R_1, R_2, y_1, y_3)$ (see Figures 5 and 7). In each new generation of the population, the individuals are approaching the values suggested by the algorithm, converging to a solution of the problem. The best fitness values of the parameters show the convergence to the optimal value. Table 2 shows a detailed view of the maneuver, explaining all the intermediate Keplerian orbits obtained.

Table 2: The Keplerian elements of the intermediate orbits for the case where $r_f = 2$.

Orbit	a	E	ω	V_c
0	1	0	0	1
1	1.067591	0.247661	4.720904	0.967826586
2	1.553574	0.460257	5.401124	0.802294893
3	1.553574	0.460257	5.401124	0.802294893
4	2	0		0.707106781

**Figure 7:** Four-burn orbit transfers for simulation 8.

Simulation 8 and Figure 7 show some new results that confirm that the use of the procedure with four impulses provides results with higher consumption than the bi-impulsive maneuver (Table 1), but that minimizes the four-impulsive burn technique.

This study can also be applied to find orbital maneuvers that search for the minimum fuel consumption for a spacecraft that leaves one celestial body and goes back to this same body (Figures 9 and 10). This question is of great importance for missions whose objective is to shift the position of the satellite in a given orbit, without changing the other orbital elements. Prado and Broucke [1] also studied this problem using the Lambert method, under different circumstances.

3.1. The Swing-By Maneuvers

The next step is to use the algorithm developed here to study a maneuver that will make a spacecraft to encounter a planet, in order to make a close approach that will change its energy. This problem can be seen as a rendezvous problem, where the second spacecraft, the one to be reached, is a planet and not a space vehicle. Using this approach, a transfer maneuver using

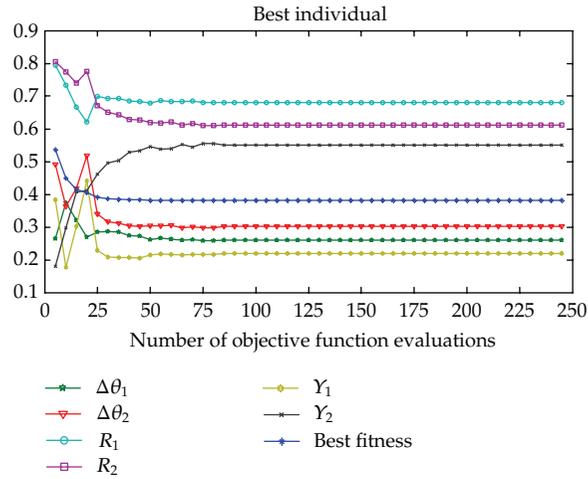


Figure 8: The variables of the problem using the method of genetic algorithm and the best fitness for simulation 8.

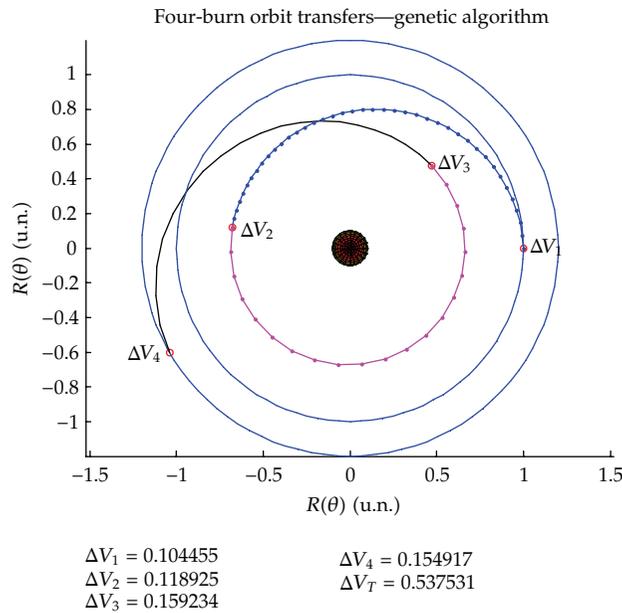


Figure 9: Four-burn orbit transfers for simulation 1.

an impulsive engine with four burns is followed by a gravity-assisted maneuver to send the spacecraft further in the solar system. This technique will reduce the cost of an interplanetary mission. This is a standard procedure in orbital maneuvers, and a more detailed description is available in references [2, 9]. In this case, the system consists of three bodies:

- (1) the body M_1 , with finite mass, situated in the center of mass of the Cartesian system of reference;

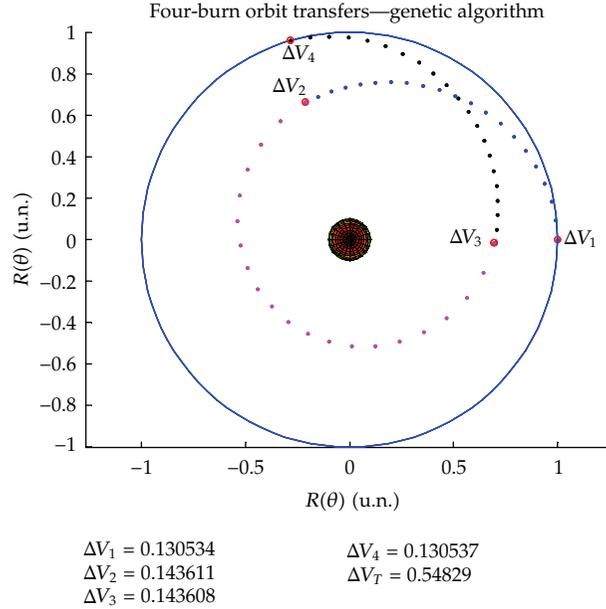


Figure 10: Transfer maneuver from one orbit back to the same orbit.

- (2) M_2 , a smaller body, that can be a planet or a satellite of M_1 , in a Keplerian orbit around M_1 ;
- (3) a body M_3 , a space vehicle with infinitesimal mass, traveling in a Keplerian orbit around M_1 , when it passes close to M_2 .

This close approach changes the orbit of M_3 and, by the hypothesis assumed for the problem, it is considered that the orbits of M_1 and M_2 do not change. Using the “patched conics” approximation, the equations that quantify those changes are available in the literature [9].

The standard maneuver can be identified by the following three parameters (Figure 11):

- (i) $|\vec{V}_\infty|$, the magnitude of the velocity of the spacecraft with respect to M_2 when approaching the celestial body;
- (ii) r_p , the distance between the spacecraft and the celestial body during the closest approach;
- (iii) Ψ_A , the angle the approach.

Having those variables, it is possible to obtain δ , half of the total deflection angle, by using the equation [2]

$$\delta = \arcsin\left(\frac{1}{1 + (r_p V_\infty^2 / \mu_2)}\right). \quad (3.1)$$

Note that V_2 is the velocity of the celestial body with respect to the main body and V_p is the velocity of the smaller mass when passing by the periapsis. A complete description

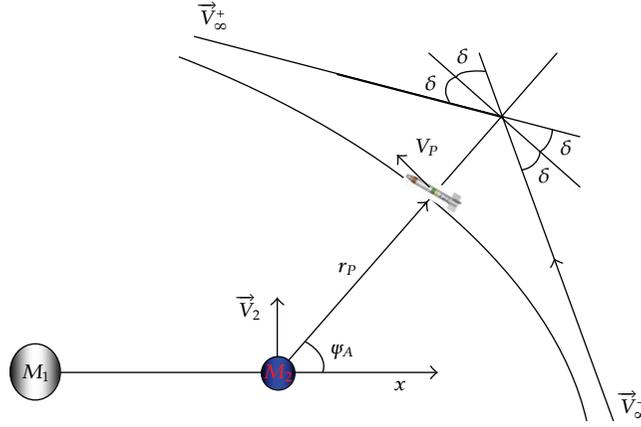


Figure 11: The parameters of the swing-by maneuver.

of this maneuver and the derivation of the equations can be found in Prado [9]. The final equations are reproduced below

$$\begin{aligned}\Delta E &= -2V_2V_\infty \sin(\delta) \sin(\Psi_A), \\ \Delta C &= \frac{-2V_2V_\infty \sin(\delta) \sin(\Psi_A)}{\omega_2},\end{aligned}\quad (3.2)$$

where ω_2 is the angular velocity of the motion of the primaries, ΔE is the variation of energy, ΔC is the variation of the angular momentum, and ΔV is the variation of the magnitude of the velocity due to the swing-by. For the ΔV , we have the equation

$$\Delta V = \left| \Delta \vec{V} \right| = 2 \left| \vec{V}_\infty \right| \sin(\delta) = 2V_\infty \sin(\delta). \quad (3.3)$$

The gravity-assisted maneuver (swing-by) can provide a considerable change of the velocity and energy of the spacecraft, reducing the costs of the mission. During this approach, the spacecraft will be transferred to another orbit of interest of the mission. The dynamics used to solve this problem is the traditional model given by the ‘‘Patched Conics,’’ so it is assumed that all three bodies involved are points of mass and do not suffer external disturbances. The variations given by the swing-by, in terms of velocity variation (ΔV) and energy variation (ΔE), can now be obtained.

Figure 12 shows the maneuver obtained by the genetic algorithm. The spacecraft comes from an initial orbit with radius $r_o = 1$ u.a., which represents the position of the Earth’s heliocentric system, in astronomical units. It means that the spacecraft starts from the Earth. Then, it performs a maneuver with 4 impulses, using three elliptic intermediate transfer orbits, and finally it arrives in an orbit with $r_f = 5.202803$ u.a. (Jupiter). At this moment, it realizes a maneuver of Swing-by with the planet Jupiter. Note that the gain in velocity was $\Delta V_{SB} = 1.104368$ and the gain in energy was $\Delta E = 2.017347$. During this approach, the space vehicle place itself in another orbit of the interest of the mission. In this mission, the participation of the GA is to find the best procedure to make the spacecrafts reach the planet

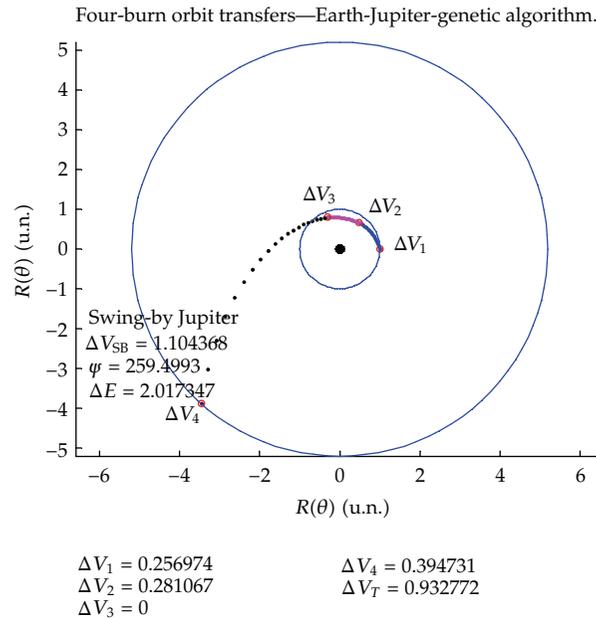


Figure 12: Simulation of a maneuvers leaving the Earth ($r_o = 1$) and reaching the orbit of Jupiter ($r_f = 5.202803$), performing a swing-by maneuver on the planet to gain velocity and energy.

Jupiter. From this point, standard procedures of interplanetary trajectories can complete the mission.

4. Conclusion

Based on the analysis of the results obtained, the genetic algorithm implemented here shows that this technique brings good results for the proposed four impulsive rendezvous maneuvers, when compared with the ones obtained by the traditional impulsive methods. It means that it can be used in real cases, specially when a bi-impulsive transfer is not possible due to the limitations of the engine of the spacecraft. The procedure is also effective in maneuvering the spacecraft from one body back to the same body, that is, making it leaving and returning to the same orbit.

The results indicate that the maneuver using the genetics algorithm with four impulses does not provide better fuel consumption in any case simulated, since the bi-impulsive maneuver is better in this situation, but the method proves to be efficient in minimizing the four impulsive maneuvers. It is necessary to take into account that, in many cases, the limitations of the propellers of the spacecraft require that the maneuver has to be performed using several impulses, passing through intermediate orbits to reach the target.

Then, we studied a maneuver where the goal is to send a spacecraft to encounter the planet Jupiter to make a swing-by maneuver. The algorithm worked well in finding a good solution for this problem.

In general, the proposed technique can be used when a rendezvous maneuver is required between two given circular orbits for a spacecraft that has an engine that requires the application of four impulses.

In the future, it is possible to apply this technique in three dimensions, in maneuvers that requires more impulses, and also in maneuvers to avoid collisions between a spacecraft and asteroids.

Acknowledgments

This work was accomplished with the support of São Paulo State Science Foundation (FAPESP) under Contract 2009/16517-7 and National Institute for Space Research, INPE, Brazil.

References

- [1] A. F. B. A. Prado and R. A. Broucke, "Study of h enon's orbit transfer problem using the lambert algorithm," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 17, no. 5, pp. 1075–1081, 1993.
- [2] D. P. S. Santos, A. F. B. A. P. Prado, and E. M. Rocco, "The use of consecutive collision orbits to obtain swing-by maneuvers," in *Proceedings of the 56th International Astronautical Congress*, Fukuoka, Japan, October 2005.
- [3] F. W. Gobetz and J. R. Doll, "A survey of impulsive trajectories," *AIAA Journal*, vol. 7, pp. 801–834, 1969.
- [4] R. H. Goddard, "A method of reaching extreme altitudes," *Smithsonian Institute Public Miscelanea Collect*, vol. 71, no. 2, pp. 809–811, 1920.
- [5] W. Hohmann, *Die Erreichbarkeit Der Himmelskorper*, Oldenbourg, Munique, 1925.
- [6] J. P. Marec, *Optimal Space Trajectories*, Elsevier, New York, NY, USA, 1979.
- [7] D. P. S. Santos, L. Casalino, G. Colasurdo, and A. F. B. A. P. Prado, "Optimal trajectories using gravity assisted maneuver and solar electric propulsion (SEP) towards near-earth-objects," in *Proceedings of the 4th WSEAS International Conference on Applied and Theoretical Mechanics (Mechanics '08)*, pp. 62–68, Cairo, Egypt, December 2008.
- [8] E. M. Rocco, A. F. B. A. Prado, and M. L. O. Souza, "Bi-impulsive orbital transfers between non-coplanar orbits with time limit," in *Applied Mechanics in the Americas*, D. Pamplona, C. Steele, H. I. Weber, P. B. Gonalves, I. Jasiuk, and L. Bevilacqua, Eds., vol. 6, pp. 259–262, 1999.
- [9] A. F. B. A. Prado, *Optimal transfer and swing-by orbits in the two- and three- body problems*, Ph.D. thesis, Department of Aerospace Engineering and Engineering Mechanics—University of Texas, 1993.
- [10] A. A. Sukhanov and A. F. B. A. Prado, "Constant tangential low-thrust trajectories near on oblate planet," *Journal of Guidance, Control, and Dynamics*, vol. 24, no. 4, pp. 723–731, 2001.
- [11] A. F. B. A. Prado, "Numerical and analytical study of the gravitational capture in the bicircular problem," *Advances in Space Research*, vol. 36, no. 3, pp. 578–584, 2005.
- [12] A. F. B. A. Prado, "Numerical study and analytic estimation of forces acting in ballistic gravitational capture," *Journal of Guidance, Control, and Dynamics*, vol. 25, no. 2, pp. 368–375, 2002.
- [13] A. F. B. A. Prado and R. Broucke, "Transfer orbits in restricted problem," *Journal of Guidance Control and Dynamics*, vol. 18, no. 3, pp. 593–598, 1995.
- [14] A. F. B. A. Prado, "Third-body perturbation in orbits around natural satellites," *Journal of Guidance, Control, and Dynamics*, vol. 26, no. 1, pp. 33–40, 2003.
- [15] V. A. Chobotov, *Orbital Motion*, American Institute of Aeronautics and Astronautics, 2nd edition, 1996.
- [16] J. E. Prussing, "Geometrical interpretation of the angles α and β in lambert's problem," *Journal of Guidance, Control, and Dynamics*, vol. 2, no. 5, pp. 442–443, 1979.
- [17] J. E. Prussing, "Equation for optimal power-limited spacecraft trajectories," *Journal of Guidance, Control, and Dynamics*, vol. 16, no. 2, pp. 391–393, 1993.
- [18] M. Rosa Sentinella and L. Casalino, "Genetic algorithm and indirect method coupling for low-thrust. Trajectory optimization," AIAA 064468, 2006.
- [19] F. Cacciatore and C. Toglia, "Optimization of orbital trajectories using genetic algorithms," *Journal of Aerospace Engineering, Sciences and Applications*, vol. 1, no. 1, pp. 58–69, 2008.
- [20] J. E. Prussing and J. H. Chiu, "Optimal multiple-impulse time-fixed rendezvous between circular orbits," *Journal of Guidance, Control, and Dynamics*, vol. 9, no. 1, pp. 17–22, 1986.
- [21] B. H. Billik and H. L. Roth, "Studies relative to rendezvous between circular orbits," *Astronautica Acta*, vol. 13, 1967.

- [22] J. E. Prussing, "Optimal four-impulse fixed-time rendezvous in the vicinity of a circular orbit," *AIAA Journal*, vol. 7, no. 5, pp. 928–935, 1969.
- [23] J. E. Prussing, "Optimal two-and three-impulse fixed-time rendezvous in the vicinity of a circular orbit," *AIAA Journal*, vol. 8, no. 7, pp. 1221–1228, 1970.
- [24] H. Shen and P. Tsiotras, "Optimal two-impulse rendezvous using multiple-revolution Lambert solutions," *Journal of Guidance, Control, and Dynamics*, vol. 26, no. 1, pp. 50–61, 2003.
- [25] H. Shen and P. Tsiotras, "Optimal two-impulse rendezvous between two circular orbits using multiple revolution Lambert's solutions," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, 1999.
- [26] Y. Z. Luo, G. J. Tang, and H. Y. Li, "Optimization of multiple-impulse minimum-time rendezvous with impulse constraints using a hybrid genetic algorithm," *Aerospace Science and Technology*, vol. 10, no. 6, pp. 534–540, 2006.
- [27] E. M. Rocco, A. F. B. A. Prado, M. L. O. Souza, and J. E. Baldo, "Optimal bi-impulsive non-coplanar maneuvers using hyperbolic orbital transfer with time constraint," *Journal of Aerospace Engineering, Sciences and Applications*, vol. 1, no. 2, pp. 43–51, 2008.
- [28] D. P. S. Santos, A. F. B. A. Prado, L. Casalino, and G. Colasurdo, "Optimal Trajectories towards near-earth-objects using solar electric propulsion (SEP) and gravity assisted maneuver," *Journal of Aerospace Engineering, Sciences and Applications*, vol. 1, no. 2, pp. 51–64, 2008.



Hindawi

Submit your manuscripts at
<http://www.hindawi.com>

