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Research Article

The Unbalanced Linguistic Aggregation Operator in Group Decision Making

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Many linguistic aggregation methods have been proposed and applied in the linguistic decision-making problems. In practice, experts need to assess a number of values in a side of reference domain higher than in the other one; that is, experts use unbalanced linguistic values to express their evaluation for problems. In this paper, we propose a new linguistic aggregation operator to deal with unbalanced linguistic values in group decision making, we adopt 2-tuple representation model of linguistic values and linguistic hierarchies to express unbalanced linguistic values, and moreover, we present the unbalanced linguistic ordered weighted geometric operator to aggregate unbalanced linguistic evaluation values; a comparison example is given to show the advantage of our method.

1. Introduction

The need of uncertain linguistic information processing has become an important topic in many areas dealing with vague information such as in universal design used for developing products and environments that are usable by all people to the greatest extent possible; although a set of acknowledged principles has been developed and commonly used by industry and academia, it is difficult to quantitatively evaluate whether a product is indeed a good example of universal design, and linguistic evaluation approach for universal design can improve the shortcomings of traditional analytical hierarchy process methods, quantitative judgments, priority analysis, and aggregation performance [1]. Formally, the fuzzy logic framework and especially fuzzy sets themselves are not always easy to obtain from the linguistic value sets. That is why we choose to keep the words themselves without going through a fuzzy modeling [2]. In fact, linguistic value is the granularity of information that allows for a better approximation of the concepts when it is needed [3].

In linguistic group decision-making analysis, the problems are associated with the following [4]: (1) the choice of the linguistic value set with its semantic; (2) the choice of the aggregation operator of linguistic information; (3) the choice of the best alternatives. In the above-mentioned three steps, the aim of (1) consists of establishing the linguistic variable [5] or linguistic expression domain with a view to provide the linguistic performance values. In practice, fuzzy numbers or an ordered structure of linguistic values can be used to explain their semantic [5, 6]. The aim of (2) is to carry out the aggregation of linguistic information; there are many numeric aggregation operators [7–9] and linguistic aggregation operators [4, 10–13] to aggregate them. The aim of (3) consists of obtaining a collective performance value over each alternative and finding a solution set of alternatives. The solution set of alternatives is the best alternative that is the most satisfied alternative for the experts.

In most cases, information of group decision-making problems can be assessed in a qualitative form rather than a quantitative one, and experts use natural language instead of numerical values to express their evaluations of decision-making problems [4]; hence, we need the linguistic approach to solve group decision-making problems with linguistic assessment [3, 6, 10, 14]. Up to now, many linguistic approaches have been proposed and applied to solve problems with linguistic assessment, for example, personnel management [4, 12], web information processing [15, 16], and sensor evaluation and fuzzy risk analysis [17–24]. In fact, linguistic approaches are the core of computing with words proposed by Zadeh [25]. In [26], Herrera et al. make a review of the developments of Computing with Words in decision-making and explore different linguistic computational models that have been applied to the decision making field. In this paper, unbalanced linguistic values proposed in [27] are used to deal with group decision making, in practice, experts need to assess a number of values in a side of reference domain higher than in the other one; that is, experts use unbalanced linguistic values to express their evaluation for problems, to deal with unbalanced linguistic values in group decision making, and we adopt 2-tuple representation model of linguistic values [10] and linguistic hierarchies [4] to express unbalanced linguistic values; moreover, inspired by the ordered weighted geometric operator [28–30], we propose the unbalanced linguistic ordered weighted geometric operator to aggregate unbalanced linguistic evaluation values, and some interesting properties of the unbalanced linguistic ordered weighted geometric operator are also discussed.

The organization of this paper is as follows. In Section 2, we briefly make a review of the 2-tuple fuzzy linguistic representation model and unbalanced linguistic terms. In Section 3, we present the unbalanced linguistic ordered weighted geometric operator and some properties of the operator. An example is given in Section 4 to deal with a group decision-making problem with unbalanced linguistic evaluation values, and it is shown that our method is an alternative aggregation operator for linguistic evaluation approach. We conclude in Section 5.

2. Preliminaries

In this section, we briefly review the 2-tuple linguistic representation model and unbalanced linguistic values we refer to [10, 27] for more details.

The 2-tuple linguistic representation model was introduced by Herrera and Martínez [10]. Let $S = \{s_0, \ldots, s_g\}$ be the initial finite linguistic value set. Formally, the 2-tuple linguistic representation model is formed by (s_i, α), in which $s_i \in S(i \in \{0, 1, \ldots, g\})$ and $\alpha \in [-0.5, 0.5)$; that is, linguistic information is encoded in the space $S \times [-0.5, 0.5)$. Based on the representation (s_i, α), we can easily obtain the following symbolic translation of linguistic values from $\beta \in [0, g]$ to $S \times [-0.5, 0.5)$, that is, $\Delta : [0, g] \rightarrow S \times [-0.5, 0.5)$, $\beta \mapsto (s_i, \alpha)$, in which $i = \text{round}(\beta)$ ($\text{round}(\cdot)$ is the usual round operation), $\alpha = \beta - i \in [-0.5, 0.5)$. Intuitively, $\Delta(\beta) = (s_i, \alpha)$ expresses that s_i is the closest linguistic value to β, and α is the value of the symbolic translation. Additionally, there is a Δ^{-1} function such that from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g]$, that is, $\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g]$, $\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$. In fact, the model defines a set of transformation functions between linguistic values and 2-tuple linguistic representations as well as numeric values and 2-tuple linguistic representations. This allows us to easily process linguistic information by numeric value; for example, we have the following linguistic aggregation operators: Let a set of the 2-tuple linguistic representations be $x = \{(s_1, \alpha_1), \ldots, (s_n, \alpha_n)\}$, and let $W = \{w_1, \ldots, w_n\}$ be an associated weight such that $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$:

(1) the 2-tuple arithmetic mean operator $\overline{x}^e$: $\overline{x}^e = \Delta(\sum_{i=1}^{n} 1/n \times \Delta^{-1}(s_i, \alpha_i))$;

(2) the 2-tuple weighted aggregation operator $F^{\overline{w}}$: $F^{\overline{w}} = \Delta(\sum_{i=1}^{n} w_i \times \Delta^{-1}(s_i, \alpha_i))$;

(3) the 2-tuple ordered weighted aggregation operator F^e: $F^e((s_1, \alpha_1), \ldots, (s_n, \alpha_n)) = \Delta(\sum_{j=1}^{n} w_j \times \beta_j^*)$, in which β_j^* is the jth largest linguistic value of $\{\beta_i = \Delta^{-1}(s_i, \alpha_i) \mid i = 1, \ldots, n\}$.

Unbalanced linguistic values proposed in [27] are used to deal with scales for assessing preferences where the experts need to assess a number of values in a side of reference domain higher than in the other one. Generally, an unbalanced linguistic term set S has a minimum label, a maximum label, and a central label, and the remaining labels are nonuniformly and nonsymmetrically distributed around the central one on both left and right lateral sets; that is, we can represent S in the following form: $S = S_l \cup S_c \cup S_r$, in which, S_l contains all left lateral labels but the central label, S_c just contains the central label, and S_r contains all right lateral labels higher than the central label.

Example 2.1. $S = \{$none (N), low (L), medium (M), almost high (AH), high (H), quite high (QH), very high (VH), almost total (AT), total $(T)\}$ is an unbalanced linguistic term set, in which $S_l = \{N, L\}$, $S_c = \{M\}$, and $S_r = \{AH, H, QH, VH, AT, T\}$.

To obtain 2-tuple fuzzy linguistic representations of unbalanced linguistic values without loss of information, we need the concept of linguistic hierarchies [4], that is, $LH = \bigcup_t l(t, n(t))$ it takes into account a set of levels where each level is a linguistic term set with different granularity from the remaining levels of the hierarchy, where $l(t, n(t))$ is a linguistic hierarchy with t being a number that indicates the level of the hierarchy and $n(t)$ the granularity of the linguistic term set of t. Generally, the linguistic term set $S^{n(t+1)}$ of the level $t + 1$ is obtained from its predecessor $S^{n(t)}$ as $l(t, n(t)) \rightarrow l(t + 1, 2 \times n(t) - 1)$. In linguistic hierarchies LH, transformation function between labels from different levels to represent

2-tuple fuzzy linguistic representations is as follows [4]: for any linguistic levels t and t', $TF_{t'}^{t} : l(t,n(t)) \rightarrow l(t',n(t'))$ such that

$$TF_{t'}^{t}\left(s_i^{n(t)},\alpha^{n(t)}\right) = \Delta_{t'}\left(\frac{\Delta_t^{-1}\left(s_i^{n(t)},\alpha^{n(t)}\right)\times(n(t')-1)}{n(t)-1}\right). \tag{2.1}$$

By using linguistic hierarchies $LH = \bigcup_t l(t,n(t))$, we can obtain the 2-tuple fuzzy linguistic representation of each term of unbalanced linguistic term set in LH; here, we use the following example to explain the converting process, and we refer to [27] for more details.

Example 2.2. Let linguistic hierarchies be

$$\begin{aligned} LH &= l(1,3)\cup l(2,5)\cup l(3,9)\cup l(4,17) \\ &= \left\{s_0^3,s_1^3,s_2^3\right\}\cup\left\{s_0^5,s_1^5,\ldots,s_4^5\right\}\cup\left\{s_0^9,s_1^9,\ldots,s_8^9\right\} \\ &\quad \cup\left\{s_0^{17},s_1^{17},\ldots,s_{16}^{17}\right\}. \end{aligned} \tag{2.2}$$

$(s_5^9,0.3)$ is a 2-tuple fuzzy linguistic representation of level 3, and it is 2-tuple fuzzy linguistic representation in level 2 is

$$\begin{aligned} TF_2^3\left(s_5^9,0.3\right) &= \Delta_2\left(\frac{\Delta_3^{-1}(s_5^9,0.3)\times(5-1)}{9-1}\right) \\ &= \Delta_2\left(\frac{5.3\times 4}{8}\right) = \Delta_2(2.65) \\ &= \left(s_3^5,-0.35\right). \end{aligned} \tag{2.3}$$

For unbalanced linguistic term set $S = \{N,L,M,AH,H,QH,VH,AT,T\}$, (1) due to $n(2) = 5$ and $(n(2)-1)/2 = |S_l| = |\{N,L\}| = 2$, the representation of S_l is obtained from level 2 of LH as follows: $\{L \leftarrow s_1^5, N \leftarrow s_0^5\}$; (2) due to $n(3) = 9$, $n(4) = 17$, and $(n(3)-1)/2 = 4 < |S_r| = |\{AH,H,QH,VH,AT,T\}| = 6 < (n(4)-1)/2 = 8$, we use level 3 and level 4 to represent $S_r = \{AH,H,QH,VH,AT,T\}$, according to lab_3 and lab_4 defined in [27], and $\{AH,H\}$ and $\{QH,VH,AT,T\}$ are represented in level 3 and level 4, respectively, $\{AH \leftarrow s_5^9, H \leftarrow s_6^9\}$, $\{QH \leftarrow s_{13}^{17}, VH \leftarrow s_{14}^{17}, AT \leftarrow s_{15}^{17}, T \leftarrow s_{16}^{17}\}$; (3) according to density and bridging representation gaps defined in [27], the upside and the downside of the central label M are represented in level 2 and 3 of LH by means of $\overline{s_2^5}$ and $\underline{s_4^9}$, respectively. The upside and the downside of the label H are represented in level 3 and 4 of LH by means of $\overline{s_6^9}$ and $\underline{s_{12}^{17}}$, respectively; (4) the final 2-tuple fuzzy linguistic representations of S in LH are $S_l : \{N \leftarrow s_0^5, L \leftarrow s_1^5\}$, $S_c : \{M \leftarrow \overline{s_2^5} \cup \underline{s_4^9}\}$, $S_r : \{AH \leftarrow s_5^9, H \leftarrow \overline{s_6^9} \cup \underline{s_{12}^{17}}, QH \leftarrow s_{13}^{17}, VH \leftarrow s_{14}^{17}, AT \leftarrow s_{15}^{17}, T \leftarrow s_{16}^{17}\}$.

Formally, for any 2-tuple fuzzy linguistic representation (s_i,α_i) $(s_i \in S, \alpha_i \in [-0.5,0.5))$, it can be converted by the following unbalanced linguistic transformation functions in LH

and vice versa, that is, $\mathcal{LH} : S \times [-0.5, 0.5) \rightarrow LH \times [-0.5, 0.5)$, $(s_i, \alpha_i) \mapsto (s_{I(i)}^{G(i)}, \alpha_i)$ such that $s_{I(i)}^{G(i)} \in LH$. $\mathcal{LH}^{-1} : LH \times [-0.5, 0.5) \rightarrow S \times [-0.5, 0.5)$, $(s_k^{n(t)}, \alpha_k) \mapsto (s_i, \lambda)$, for example, $\mathcal{LH}(H, 0.3) = (s_6^9, 0.3)$, $\mathcal{LH}^{-1}(s_{13}^{17}, -0.2) = (QH, -0.2)$.

3. The Unbalanced Linguistic Ordered Weighted Geometric Operator

To aggregate unbalanced linguistic values, we present the unbalanced linguistic ordered weighted geometric operator [20], which is inspired by 2-tuple fuzzy linguistic representation model [27] and the ordered weighted geometric operator [29].

Definition 3.1. An ordered weighted geometric operator of dimension n is a mapping $g : (R^+)^n \rightarrow R^+$ that has a weighting vector $W = (w_1, w_2, \ldots, w_n)$ associated with it, with $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$, such that $g(a_1, a_2, \ldots, a_n) = \prod_{j=1}^n b_j^{w_j}$, where b_j is the jth largest linguistic value of the a_i $(i = 1, \ldots, n)$.

Definition 3.2. Assume that the set of unbalanced linguistic values $V = \{s_i \mid i = 1, 2, \ldots, n\}$ will be aggregated, in which $s_i \in S = \{s_0, s_1, \ldots, s_m\}$ $(m \geq n)$ is an unbalanced linguistic value, a weighting vector is $W = (w_1, w_2, \ldots, w_n)$ with $w_i \in [0, 1]$, and $\sum_{i=1}^n w_i = 1$. Then the unbalanced linguistic ordered weighted geometric operator (the ULOWG operator) is defined as follows:

$$\begin{aligned} f_{\text{ULOWG}}(V) &= f_{\text{ULOWG}}(\{s_i \mid i = 1, 2, \ldots, n\}) \\ &= f_{\text{ULOWG}}\left(\left\{TF_{t_0}^{t_i}(\mathcal{LH}(s_i)) \mid i = 1, \ldots, n\right\}\right) \\ &= \mathcal{LH}^{-1}\left(s_k^{n(t_0)}, \alpha_k\right), \end{aligned} \tag{3.1}$$

where t_i is the level of $\mathcal{LH}(s_i)$ in LH, t_0 is a level of LH, fixed by users, $s_k^{n(t_0)} \in S^{n(t_0)} \subset LH$ and $\alpha_k \in [-0.5, 0.5)$ such that

$$k + \alpha_k = \prod_{i=1}^n \left(\triangle_{t_0}^{-1}\left(TF_{t_0}^{t_i}(\mathcal{LH}(s_i))\right)\right)^{w_i}, \tag{3.2}$$

where for any $i \in \{1, \ldots, n\}$, $\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_{i-1}))) \geq \triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_i)))$; that is, $TF_{t_0}^{t_i}(\mathcal{LH}(s_i))$ is the ith largest linguistic value in $\{TF_{t_0}^{t_i}(\mathcal{LH}(s_i)) \mid i = 1, 2, \ldots, n\}$.

Remark 3.3. In Definition 3.2, $TF_{t_0}^{t_i}(\mathcal{LH}(s_i))$ means that unbalanced linguistic values are represented at level t_0 of LH, and t_0 is decided by users. In fact, by using $TF_{t_0}^{t_i}(\cdot)$ and $\mathcal{LH}(\cdot)$, S is converted in $S^{n(t_0)}$ of LH. By using $\mathcal{LH}^{-1}(s_k^{n(t_0)}, \alpha_k)$, aggregation result $f_{\text{ULOWG}}(V)$ is converted to the 2-tuple fuzzy linguistic representation of unbalanced linguistic value.

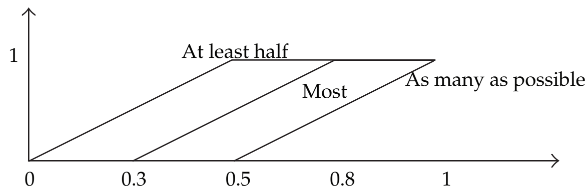

Figure 1: The functions of $Q(x, a, b)$.

In Definition 3.2, a natural question is how to obtain the associated weighting vector W. Following Yager's ideas [31] on quantifier-guided aggregation, we could compute the weighting vector of an ULOWG operator using a linguistic quantifier Q [31] as follows

$$w_i = Q\left(\frac{i}{r}, a, b\right) - Q\left(\frac{i-1}{r}, a, b\right), \quad i = 1, \ldots, r. \tag{3.3}$$

$$Q(x, a, b) = \begin{cases} 0, & \text{if } x < a, \\ \dfrac{x-a}{b-a}, & \text{if } a \leq x < b, \\ 1, & \text{if } x \geq b, \end{cases} \tag{3.4}$$

where $x, a, b \in [0, 1]$. Some examples of $Q(x, a, b)$ (see Figure 1) are most, at least half, and as many as possible; their parameters (a, b) are $(0.3, 0.8)$, $(0, 0.5)$, and $(0.5, 1)$, respectively.

Example 3.4. We select Yager's linguistic quantifier *most*, that is, $Q(x, a, b) = Q(x, 0.3, 0.8)$, according to (3.3), $r = 3$, and

$$\begin{aligned} w_1 &= Q\left(\frac{1}{3}, 0.3, 0.8\right) - Q\left(\frac{0}{3}, 0.3, 0.8\right) = \frac{1}{15}, \\ w_2 &= Q\left(\frac{2}{3}, 0.3, 0.8\right) - Q\left(\frac{1}{3}, 0.3, 0.8\right) = \frac{2}{3}, \\ w_3 &= Q\left(\frac{3}{3}, 0.3, 0.8\right) - Q\left(\frac{2}{3}, 0.3, 0.8\right) = \frac{4}{15}, \end{aligned} \tag{3.5}$$

so weighting vectors $W = (1/15, 2/3, 4/15)$.

Let $S = \{N, L, M, AH, H, QH, VH, AT, T\}$ be a set of unbalanced linguistic values. Suppose that $V = \{AH, QH, H\}$ will be aggregated, and a weighting vector is $W = (1/15, 2/3, 4/15)$. In this example, we select $t_0 = 3$ and density$_{S_R}$ is extreme [27]; hence,

$$\begin{aligned} TF_3^{t_i}(\mathcal{LH}(AH)) &= TF_3^3\left(s_5^9\right) = s_5^9, \\ TF_3^{t_i}(\mathcal{LH}(QH)) &= TF_3^4\left(s_{13}^{17}\right) = \left(s_6^9, 0.5\right), \\ TF_3^{t_i}(\mathcal{LH}(H)) &= TF_3^3\left(s_6^9\right) = s_6^9, \end{aligned} \tag{3.6}$$

and $\triangle_3^{-1}(s_6^9, 0.5) = 6.5 > \triangle_3^{-1}(s_6^9) = 6 > \triangle_3^{-1}(s_5^9) = 5$; according to (3.2), we have $k + \alpha_k = 6.5^{1/15} \times 6^{2/3} \times 5^{4/15} \doteq 5.7485$, that is,

$$\begin{aligned} f_{\text{ULOWG}}(V) &= f_{\text{ULOWG}}(\{AH, QH, H\}) \\ &\doteq \mathcal{LH}^{-1}\left(s_6^9, -0.2542\right) = (H, -0.2542). \end{aligned} \tag{3.7}$$

Proposition 3.5. *Let the set of unbalanced linguistic values be $V = \{s_i \mid i = 1, 2, \ldots, n\}$, and a weighting vector is $W = (w_1, \ldots, w_n)$ with $w_i \in [0,1]$ and $\sum_{i=1}^n w_i = 1$.*

(1) *If $w_1 = 1$ and $w_2 = \cdots = w_n = 0$, then $f_{\text{ULOWG}}(s_1, \ldots, s_n) = \max\{s_1 \ldots, s_n\}$;*

(2) *if $w_n = 1$ and $w_1 = \cdots = w_{n-1} = 0$, then $f_{\text{ULOWG}}(s_1, \ldots, s_n) = \min\{s_i \ldots, s_n\}$;*

(3) *if $w_1 = w_n = 0$, then f_{ULOWG} reduces to the linguistic Olympic operator; that is, the smallest and largest linguistic values are deleted from linguistic evaluation values;*

(4) *if for any $i \in \{1, \ldots, n\}$, $w_i = 1$, then*

$$f_{\text{ULOWG}}(\{s_1 \ldots, s_n\}) = s_i. \tag{3.8}$$

Proof. (1) According to (3.2), we have $k + \alpha_k = \prod_{i=1}^n (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_i))))^{w_i} = (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_1))))^{w_1} \times (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_2))))^{w_2} \times \cdots \times (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_n))))^{w_n} = (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_1))))^1 \times (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_2))))^0 \times \cdots \times (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_n))))^0 = (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_1))))$, where $(\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_1))))$ is the largest linguistic value of the $\{TF_{t_0}^{t_i}(\mathcal{LH}(s_i)) \mid i = 1, 2, \ldots, n\}$; then, we can get

$$\begin{aligned} f_{\text{ULOWG}}(s_1, \ldots, s_n) &= \mathcal{LH}^{-1}(\triangle_{t_0}(k + a_k)) = \mathcal{LH}^{-1}\left(\triangle_{t_0}\left(\left(\triangle_{t_0}^{-1}\left(TF_{t_0}^{t_i}(\mathcal{LH}(s_1))\right)\right)\right)\right) \\ &= \max\{s_1 \ldots, s_n\}. \end{aligned} \tag{3.9}$$

(2) According to (3.2), we have $k + \alpha_k = \prod_{i=1}^n (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_i))))^{w_i} = (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_1))))^{w_1} \times (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_2))))^{w_2} \times \cdots \times (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_n))))^{w_n} = (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_1))))^0 \times (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_2))))^0 \times \cdots \times (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_n))))^1 = (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_n))))$, where $(\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_n))))$ is the smallest linguistic value of the $\{TF_{t_0}^{t_i}(\mathcal{LH}(s_i)) \mid i = 1, 2, \ldots, n\}$; then, we can get

$$f_{\text{ULOWG}}(s_1, \ldots, s_n) = \mathcal{LH}^{-1}(\triangle_{t_0}(k + a_k)) = \min\{s_1 \ldots, s_n\}. \tag{3.10}$$

(3) It is obvious.

(4) If for any $i \in \{1, \ldots, n\}$, $w_i = 1$; thus, for any $j \neq l$, $w_j = 0$, then we have $k + \alpha_k = (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_l))))$

$$f_{\text{ULOWG}}(s_1, \ldots, s_n) = \mathcal{LH}^{-1}(\triangle_{t_0}(k + a_k)) = \mathcal{LH}^{-1}(\triangle_{t_0}(k + a_k)) = s_l. \tag{3.11}$$

□

Proposition 3.6. *Assume that a set of unbalanced linguistic values $V = \{s_i \mid i = 1, \ldots, n\}$ will be aggregated and a weighting vector is $W = (w_1, w_2, \ldots, w_n)$ with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$. The* ULOWG *operator satisfies the following:*

(1) $\min\{s_i \mid i = 1, \ldots, n\} \le f_{\text{ULOWG}}(\{s_i \mid i = 1, \ldots, n\}) \le \max\{s_i \mid i = 1, \ldots, n\}$;

(2) f_{ULOWG} *is idempotent; that is, if* $s_1 = s_2 = \cdots = s_n$, *then* $f_{\text{ULOWG}}(\{s_1, \ldots, s_n\}) = s_1$;

(3) f_{ULOWG} *is monotone in relation to the input values* s_i; f_{ULOWG} *is commutative;*

(4) f_{ULOWG} *reduces to the linguistic geometric mean if* $w_i = 1/n$ *for all* $i = 1, \ldots, n$, *that is,*

$$\begin{gathered} f_{\text{ULOWG}}(\{s_1, \ldots, s_n\}) = (s_k, \alpha_k), \\ k + \alpha_k = \sqrt[n]{\prod_{i=1}^{n} \triangle_{t_0}^{-1}\left(TF_{t_0}^{t_i}(\mathcal{LH}(s_i))\right)}. \end{gathered} \tag{3.12}$$

Proof. (1) For any $V = \{s_i \mid i = 1, \ldots, n\}$, $W = (w_1, w_2, \ldots, w_n) k + \alpha_k = \prod_{i=1}^{n} (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_i))))^{w_i}$; according to Proposition 3.5 (1) and (2), we know that $(\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_n)))) \le \prod_{i=1}^{n} (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_i))))^{w_i} \le (\triangle_{t_0}^{-1}(TF_{t_0}^{t_i}(\mathcal{LH}(s_1))))$, that is, $\min\{s_i \mid i = 1, \ldots, n\} \le f_{\text{ULOWG}}(\{s_i \mid i = 1, \ldots, n\}) \le \max\{s_i \mid i = 1, \ldots, n\}$.

(2) Due to $s_1 = s_2 = \cdots = s_n$, and according to (3.2), we have

$$\begin{aligned} k + \alpha_k &= \prod_{i=1}^{n} \left(\triangle_{t_0}^{-1}\left(TF_{t_0}^{t_i}(\mathcal{LH}(s_i))\right)\right)^{w_i} = \left(\triangle_{t_0}^{-1}\left(TF_{t_0}^{t_i}(\mathcal{LH}(s_1))\right)\right)^{w_1 + w_2 + \cdots + w_n} \\ &= \left(\triangle_{t_0}^{-1}\left(TF_{t_0}^{t_i}(\mathcal{LH}(s_1))\right)\right), \end{aligned} \tag{3.13}$$

then, $f_{\text{ULOWG}}(s_1, \ldots, s_n) = \mathcal{LH}^{-1}(\triangle_{t_0}(k + a_k)) = \mathcal{LH}^{-1}(\triangle_{t_0}(k + a_k)) = s_l$.

(3) Assume that $W = (w_1, w_2, \ldots, w_n)$ denotes the weight of $\{s_1, s_2, \ldots s_n\}$ and $W = (w'_1, w'_2, \ldots, w'_n)$ denotes the weight of $\{s'_1, s'_2, \ldots s'_n\}$. If for any $i \in N$, $s_i \le s'_i$, then according to (3.2),

$$k + \alpha_k = \prod_{i=1}^{n} \left(\triangle_{t_0}^{-1}\left(TF_{t_0}^{t_i}(\mathcal{LH}(s_i))\right)\right)^{w_i} \le k' + \alpha'_k = \prod_{i=1}^{n} \left(\triangle_{t_0}^{-1}\left(TF_{t_0}^{t_i}(\mathcal{LH}(s'_i))\right)\right)^{w_i}, \tag{3.14}$$

that is,

$$f_{\text{ULOWG}}(s_1, \ldots, s_n) = \mathcal{LH}^{-1}(\triangle_{t_0}(k + a_k)) \le f_{\text{ULOWG}}(s'_1, \ldots, s'_n) = \mathcal{LH}^{-1}(\triangle_{t_0}(k' + a'_k)). \tag{3.15}$$

(4) Since $\{s'_1, s'_2, \ldots s'_n\}$ is the permutation of the $\{s_1, s_2, \ldots s_n\}$, so we can get the same sort result $\{b_1, b_2, \ldots, b_n\}$, that is to say, b_j is the jth largest element in the set $\{s'_1, s'_2, \ldots s'_n\}$ and $\{s_1, s_2, \ldots s_n\}$. According to Definition 3.2,

$$k + \alpha_k = \prod_{i=1}^{n} \left(\triangle_{t_0}^{-1}\left(TF_{t_0}^{t_i}(\mathcal{LH}(s_i))\right)\right)^{w_i} = k' + \alpha'_k = \prod_{i=1}^{n} \left(\triangle_{t_0}^{-1}\left(TF_{t_0}^{t_i}(\mathcal{LH}(s'_i))\right)\right)^{w_i}, \tag{3.16}$$

Table 1: The four alternatives (x_i) and consultancy departments (d_i).

Alternatives	x_1 UNIX	x_2 WINDOWS-NT	x_3 AS/400	x_4 VMS
Departments	d_1 Cost analysis	d_2 System analysis	d_3 Risk analysis	d_4 Technology analysis

Table 2: Evaluations provided by four experts.

Alternatives	Experts			
	e_1	e_2	e_3	e_4
x_1	L	AH	H	H
x_2	AH	H	QH	L
x_3	AH	L	AH	M
x_4	M	H	AH	M

that is,

$$f_{\text{ULOWG}}(s_1, \dots, s_n) = f_{\text{ULOWG}}(s'_1, \dots, s'_n). \tag{3.17}$$

(5) It is obviously set up. □

4. Comparison between ULOWA Operator and $F^{\overline{w}}$ Operator, and F^e Operator

In this section, we introduced an application in group decision-making. In the following example, we used the ULOWA operator, ULOWA operator and F^e operator to choose the better alternative, respectively. Then, we analyze the difference of the ULOWA operator, $F^{\overline{w}}$ operator, and F^e operator based on evaluation results.

Example 4.1. A distribution company needs to upgrade its computing system, so it hires a consulting company to survey the different possibilities existing on the market, to decide which is the best option for its needs. The options (alternatives) and consultancy departments are shown in Table 1. In each of the departments, there is one expert providing evaluation for each alternative (shown in Table 2); these evaluations are assessed in the initial finite unbalanced linguistic value set $S = \{\text{none } (N), \text{low } (L), \text{medium } (M), \text{almost high } (AH), \text{high } (H), \text{quite high } (QH), \text{very high } (VH), \text{almost total } (AT), \text{total } (T)\}$.

In this example, a linguistic hierarchy is $LH = l(1,3) \cup l(2,5) \cup l(3,9) \cup l(4,17) = \{s_0^3, s_1^3, s_2^3\} \cup \{s_0^5, s_1^5, \dots, s_4^5\} \cup \{s_0^9, s_1^9, \dots, s_8^9\} \cup \{s_0^{17}, s_1^{17}, \dots, s_{16}^{17}\}$ of Example 2.2. We select $t_0 = 3$, and density$_{S_R}$ is extreme, according to Table 2

$$TF_3^{t_i}(\mathcal{LH}(L)) = TF_3^3\left(s_1^5\right) = \left(s_2^9, 0\right),$$

$$TF_3^{t_i}(\mathcal{LH}(M)) = TF_3^3\left(s_4^9\right) = \left(s_4^9, 0\right),$$

$$TF_3^{t_i}(\mathcal{LH}(AH)) = TF_3^3\left(s_5^9\right) = \left(s_5^9, 0\right),$$

$$TF_3^{t_i}(\mathcal{LH}(H)) = TF_3^3\left(s_6^9\right) = \left(s_6^9, 0\right),$$
$$TF_3^{t_i}(\mathcal{LH}(QH)) = TF_3^4\left(s_{13}^{17}\right) = \left(s_6^9, 0.5\right). \tag{4.1}$$

According to (3.3), we can get the weighting vector $W = (0, 0.4, 0.5, 0.1)$ (1) Evaluations based on $F^{\overline{w}}$.

If the weighting vector $W = (0, 0.4, 0.5, 0.1)$ is for four experts and associated with their evaluations, then $F^{\overline{w}}$ can be used to evaluate alternatives; for example, for alternative x_1, we have

$$\begin{aligned} F^{\overline{w}}(L, AH, H, H) &= \Delta\Big(0 \times \Delta^{-1}\left(s_2^9, 0\right) + 0.4 \times \Delta^{-1}\left(s_5^9, 0\right) + 0.5 \times \Delta^{-1}\left(s_6^9, 0\right) \\ &\quad + 0.1 \times \Delta^{-1}\left(s_6^9, 0\right)\Big) \\ &= \Delta(0 \times 2 + 0.4 \times 5 + 0.5 \times 6 + 0.1 \times 6) \\ &= \Delta(5.6) = \left(s_6^9, -0.4\right) = (H, -0.4). \end{aligned} \tag{4.2}$$

(2) Evaluations based on F^e.

When the weighting vector $W = (0, 0.4, 0.5, 0.1)$ is for four experts and associated with particular ordered positions rather than their evaluations, then F^e can be used to evaluate alternatives, for example, for alternative x_1, we have

$$\begin{aligned} F^{e}(L, AH, H, H) &= \Delta\Big(0 \times \Delta^{-1}\left(s_6^9, 0\right) + 0.4 \times \Delta^{-1}\left(s_6^9, 0\right) + 0.5 \times \Delta^{-1}\left(s_5^9, 0\right) \\ &\quad + 0.1 \times \Delta^{-1}\left(s_2^9, 0\right)\Big) \\ &= \Delta(0 \times 6 + 0.4 \times 6 + 0.5 \times 5 + 0.1 \times 2) \\ &= \Delta(5.1) = \left(s_5^9, 0.1\right) = (AH, 0.1). \end{aligned} \tag{4.3}$$

(3) Evaluations based on f_{ULOWG}.

When the weighting vector $W = (0, 0.4, 0.5, 0.1)$ is for four experts and associated with particular ordered positions rather than their evaluations, f_{ULOWG} can also be used to evaluate alternatives; for example, for alternative x_1, we have

$$\begin{aligned} f_{\mathrm{ULOWG}}(L, AH, H, H) &= \Delta\Big(\left(\Delta^{-1}\left(s_6^9, 0\right)\right)^{0} \times \left(\Delta^{-1}\left(s_6^9, 0\right)\right)^{0.4} \times \left(\Delta^{-1}\left(s_5^9, 0\right)\right)^{0.5} \\ &\quad \times \left(\Delta^{-1}\left(s_2^9, 0\right)\right)^{0.1}\Big) \\ &= \Delta\left(6^0 \times 6^{0.4} \times 5^{0.5} \times 2^{0.1}\right) \\ &\doteq \Delta(5.3555) = \left(s_5^9, 0.3555\right) = (AH, 0.3555). \end{aligned} \tag{4.4}$$

Table 3: Evaluations of alternatives based on $F^{\overline{w}}$, F^e, and f_{ULOWG}.

Alternatives	Operators		
	$F^{\overline{w}}$	F^e	f_{ULOWG}
x_1	$(H, -0.4)$	$(AH, 0.1)$	$(AH, 0.3555)$
x_2	$(AH, -0.4)$	$(H, 0)$	$(AH, 0.3282)$
x_3	$(M, -0.2)$	$(M, -0.2)$	$(H, -0.4925)$
x_4	$(AH, 0.1)$	$(H, -0.4)$	$(H, -0.4982)$
The best alternative	x_4	x_1	x_1

Evaluations of all alternatives based on $F^{\overline{w}}$, F^e, and f_{ULOWG} are shown in Table 3, we notice that using operator F^e and f_{ULOWG} the same results are obtained, that is, the best alternative is x_1. However, evaluations of f_{ULOWG} are smaller than evaluations of F^e correspondingly. Similarly to F^e operator, the ULOWG operator has many interesting properties. So, the ULOWA operator is a alternative aggregation operator to hand multiple attribute group decision making.

5. Conclusion

From the practical point of view, group decision making is associated with multiple information sources fusion. In this paper, we propose the ULOWG operator to solve linguistic group decision-making problems, and the ULOWG operator has many interesting properties, hence, the ULOWG operator is an alternative linguistic aggregation operator in linguistic decision-making problems.

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