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Research Article

Boundary Layer Flow of Second Grade Fluid in a Cylinder with Heat Transfer

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An analysis is carried out to obtain the similarity solution of the steady boundary layer flow and heat transfer of a second grade through a horizontal cylinder. The governing partial differential equations along with the boundary conditions are reduced to dimensionless form by using the boundary layer approximation and applying suitable similarity transformation. The resulting nonlinear coupled system of ordinary differential equations subject to the appropriate boundary conditions is solved by homotopy analysis method (HAM). The effects of the physical parameters on the flow and heat transfer characteristics of the model are presented. The behavior of skin friction coefficient and Nusselt numbers is studied for different parameters.

1. Introduction

Due to wide range of applications in coating of wires and polymer fiber spinning, the concept of heat convection in the cylinders has been a field of interest for many theoretical and experimental researchers. Buchlin [1] examined the natural and forced convective heat transfer along vertical slender cylinder and also the case of two cylinders. In his analysis, obtained results indicated that the convective heat transfer has a strong dependence on the curvature of the cylinder and its misalignment with the main flow. The natural convection flow from an isothermal circular cylinder of viscous fluid having temperature-dependent viscosity has been tackled by Molla et al. [2]. The problem of mixed convection boundary layer flow over a vertical circular cylinder with prescribed surface heat flux has been studied by Bachok and Ishak [3]. They focused on the suction/injection effects on the flow field and the heat transfer rate at the surface by considering the free stream velocity and the surface

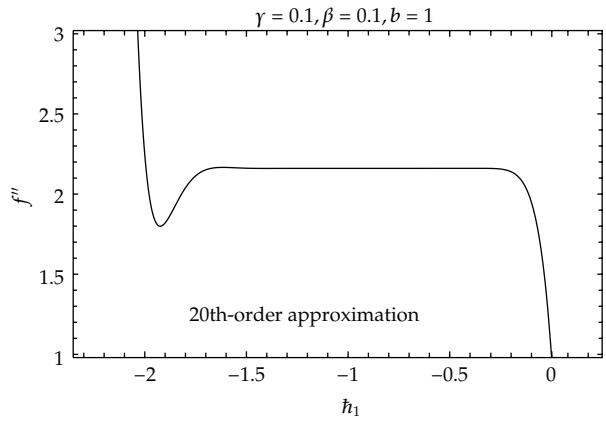

Figure 1: $\hbar$-curve for f.

heat flux to be linear functions of the distance from the leading edge. Further, Heckel et al. [4] have examined the problem of mixed convective flow of a viscous fluid along a vertical slender cylinder with variable surface temperature. They assumed the temperature to be varying arbitrarily with the axial coordinate. Later on Na [5] has investigated the effect of wall conduction on the heat transfer of the natural convection over a vertical slender hollow circular cylinder. Recently Ahmed et al. [6] have numerically examined the problem of laminar free convection boundary layer flow over horizontal cylinders of elliptic cross-section having constant surface heat flux for both the blunt (major axis of the cylinder is in the horizontal direction) and slender (major axis of the cylinder is in the vertical direction) orientations. They showed that the local skin friction coefficient is an increasing function of the ratio of the length of the major axis to that of the minor axis of the elliptic cylinder. Moreover, the combined heat and mass transfer along a vertical moving cylinder with a free stream for uniform wall temperature and heat flux was countered by Takhar et al. [7]. Later on, in his paper Cheng [8] studied the natural convection heat transfer problem through a horizontal elliptical cylinder in a Newtonian fluid with constant heat flux and temperature-dependent internal heat generation. He concluded that an increase in the aspect ratio decreases (increases) the average surface temperature of the elliptical cylinder with blunt (slender) orientation and that the average surface temperature of the elliptical cylinder with slender orientation is less than that due to the blunt orientation.

The works cited above are all carried out for Newtonian fluids; however, some literature concerning non-Newtonian fluids is also available. In a very recent paper Chang [9] has presented the numerical treatment of the flow and heat transfer characteristics of forced convection in a micropolar fluid flow along a vertical slender hollow circular cylinder with wall conduction and buoyancy effects. The heat transfer to non-Newtonian flows over a cylinder in cross flow is experimentally studied by Rao [10] for different non-Newtonian fluids and a comparison with the water (viscous fluid) flow is also experimentally examined. Amoura et al. [11] provided a finite element solution of the Carreau model mixed convection of non-Newtonian fluids between two coaxial rotating cylinders. Moreover, the effects of transpiration on the boundary layer flow and heat transfer over a vertical slender cylinder are addressed by Ishak et al. [12]. Some interesting and important works concerning boundary layer flow of viscous and non-Newtonian fluids are listed in [13–19]. In the present work we have examined the boundary layer flow and heat transfer of a second

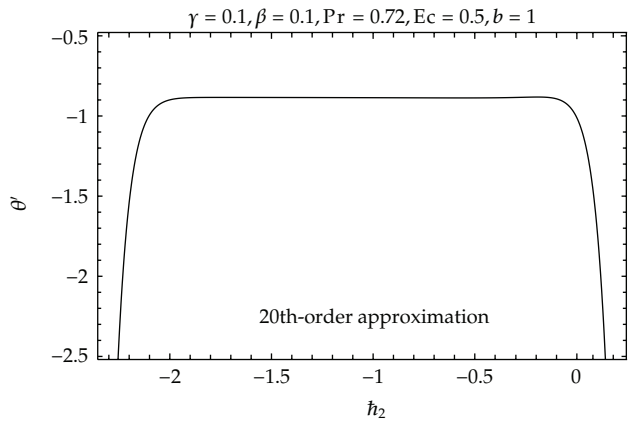

Figure 2: $\hbar$-curves for θ.

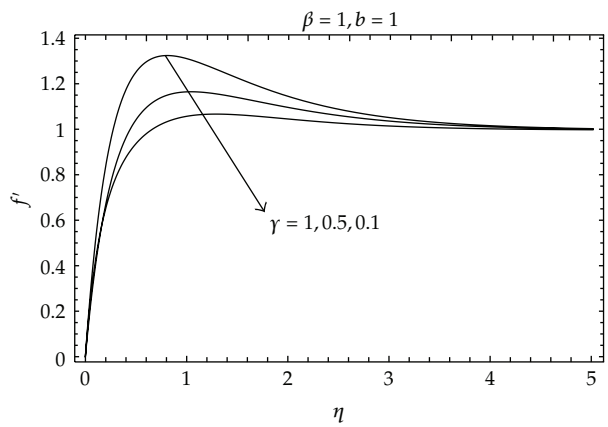

Figure 3: Influence of γ over f'.

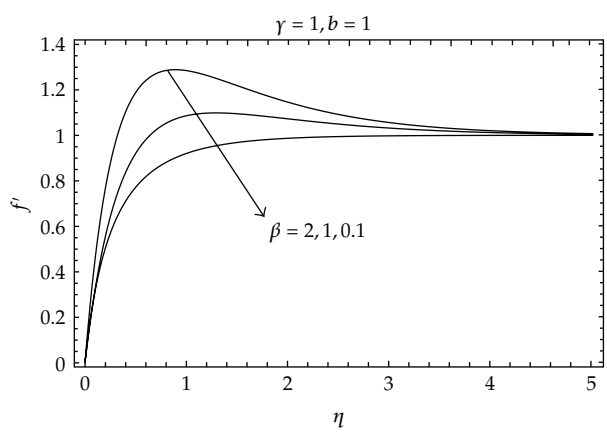

Figure 4: Influence of β over f'.

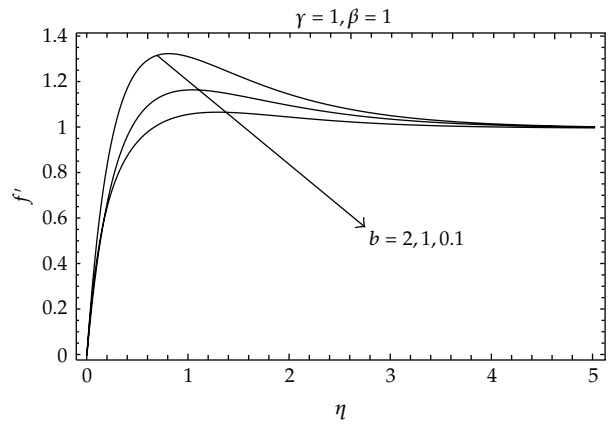

Figure 5: Influence of b over f'.

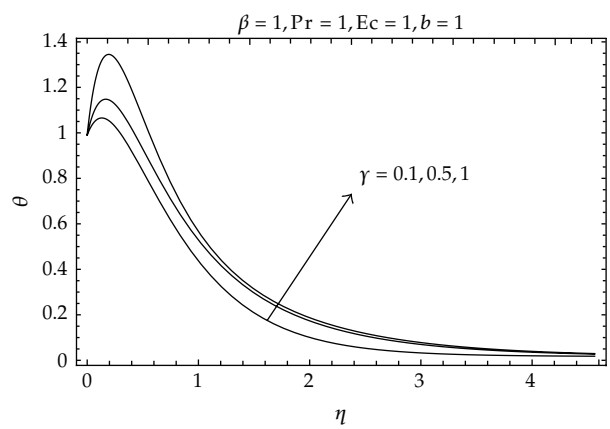

Figure 6: Influence of γ over θ.

grade fluid flow through a horizontal cylinder. The solutions are obtained by implying the analytical technique homotopy analysis method (HAM). A discussion is provided to study the influence of the physical parameters on velocity, the skin friction coefficient, and the local Nusselt number.

2. Formulation

Let us consider the problem of mixed convection boundary layer flow of second grade fluid along a horizontal circular cylinder having radius a. The temperature at the surface of the cylinder is assumed to be a constant T_w and the uniform ambient temperature is taken to be T_∞ such that the quantity $T_w - T_\infty > 0$ for the case of assisting flow, while $T_w - T_\infty < 0$, in case of the opposing flow, respectively. The viscous dissipation effects are also taken into account.

Under these assumptions the boundary layer equations of motion and heat transfer are

$$
\begin{aligned}
&(rw)_r + ru_x = 0,\\
&uu_x + wu_r = U\frac{dU}{dx} + \nu\left(u_{rr} + \frac{1}{r}u_r\right)\\
&\qquad + \frac{\alpha_1}{\rho_\infty}\left[wu_{rrr} + uu_{rrx} + u_x u_{rr} - u_r w_{rr} + \frac{1}{r}(wu_{rr} + uu_{rx} + u_r u_x - u_r w_r)\right],\\
&wT_r + uT_x = \alpha\left(T_{rr} + \frac{1}{r}T_r\right) + \frac{\nu}{c_p}u_r^2 + \frac{\alpha_1}{c_p\rho_\infty}(wu_r u_{rr} + uu_r u_{rx}),
\end{aligned}
\tag{2.1}
$$

where the velocity components along the (x, r) axes are (w, u), ρ is density, ν is the kinematic viscosity, p is pressure, and U is the free stream velocity and is defined as $U = U_\infty(x/l)$.

The corresponding boundary conditions for the problem are

$$
\begin{aligned}
&u(x,a) = 0, \quad u(x,a) \longrightarrow U(x) \quad \text{as } r \longrightarrow \infty,\\
&T(x,a) = T_w(x), \quad T(x,a) \longrightarrow T_\infty \quad \text{as } r \longrightarrow \infty.
\end{aligned}
\tag{2.2}
$$

Introduce the following similarity transformations:

$$
\begin{aligned}
&u = \frac{xU_\infty}{l}f'(\eta), \qquad w = -\frac{a}{r}\left(\frac{\nu U_\infty}{l}\right)^{1/2} f(\eta),\\
&\theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \eta = \frac{r^2 - a^2}{2a}\left(\frac{U_\infty}{\nu l}\right)^{1/2},
\end{aligned}
\tag{2.3}
$$

where the characteristic temperature ΔT is calculated from the relations $T_w - T_\infty = (x/l)^2\Delta T$. With the help of transformations (2.3), (2.1) take the form

$$
\begin{aligned}
&(1 + 2\gamma\eta)f''' + 2\gamma f'' + 1 + ff'' - f'^2 + 4\gamma\beta(f'f'' - ff''')\\
&\quad + \beta(1 + 2\gamma\eta)\left(f''^2 + 2f'f''' - ff^{iv}\right) = 0,\\
&(1 + 2\gamma\eta)\theta'' + 2\gamma\theta' + \Pr(f\theta' - f'\theta) - \Pr \mathrm{Ec}\,\beta\gamma\, ff''^2\\
&\quad + \Pr \mathrm{Ec}(1 + 2\gamma\eta)\left[f''^2 - \beta ff''f''' + \beta f'f''^2\right] = 0,
\end{aligned}
\tag{2.4}
$$

in which $\gamma = (\nu l/U_\infty a^2)^{1/2}$ is the curvature parameter, $\beta = \alpha_1 U_\infty/\rho_\infty \nu l$ is the dimensionless viscoelastic parameter, $\Pr = \mu/\alpha$ is the Prandtl number, $\mathrm{Ec} = U_\infty^2/c_p\Delta T$ is the Eckert number.

The boundary conditions in nondimensional form are defined as

$$
\begin{aligned}
&f(0) = b, \quad f'(0) = 0, \quad f' \longrightarrow 1, \quad \text{as } \eta \longrightarrow \infty,\\
&\theta(0) = 1, \quad \theta \longrightarrow 0, \quad \text{as } \eta \longrightarrow \infty,
\end{aligned}
\tag{2.5}
$$

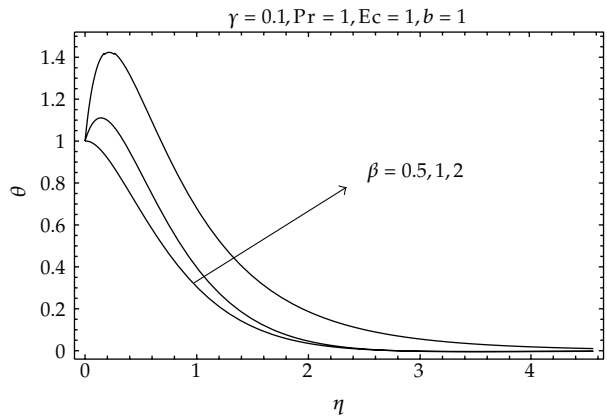

Figure 7: Influence of β over θ.

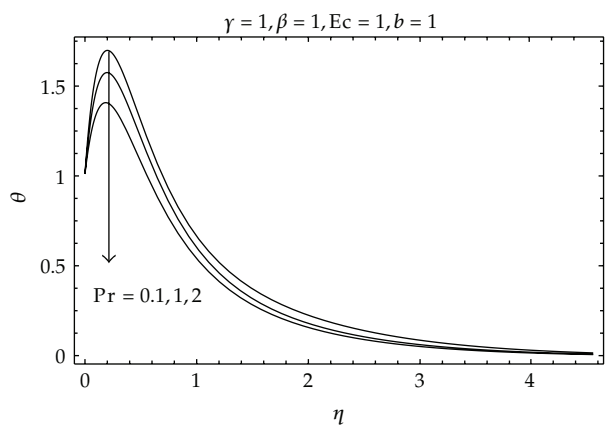

Figure 8: Influence of Pr over θ.

where b is any constant. The dimensionless coefficient of skin friction and the Nusselt number are defined as

$$\frac{1}{2}c_f \operatorname{Re}^{1/2} = f''(0), \qquad \frac{\operatorname{Nu}}{\operatorname{Re}^{1/2}} = -\theta'(0), \tag{2.6}$$

where $\operatorname{Re} = U_\infty x/\nu$ is the local Reynolds number.

3. Solution of the Problem

The solution of the present problem is obtained by using the powerful analytical technique homotopy analysis method (HAM). In the present case we seek the initial guesses to be [20–33]

$$f_0(\eta) = b - 1 + \eta + e^{-\eta}, \qquad \theta_0(\eta) = e^{-\eta}. \tag{3.1}$$

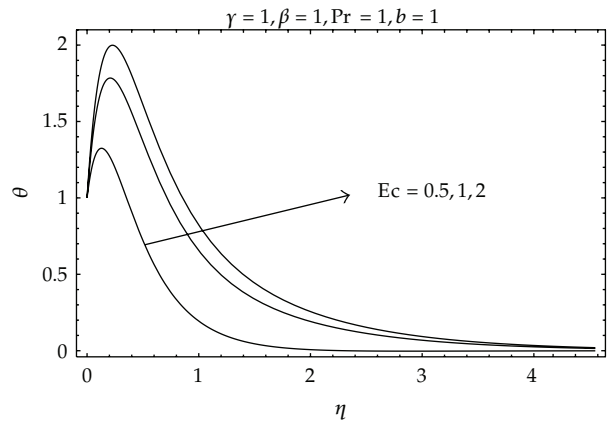

Figure 9: Influence of Ec over θ.

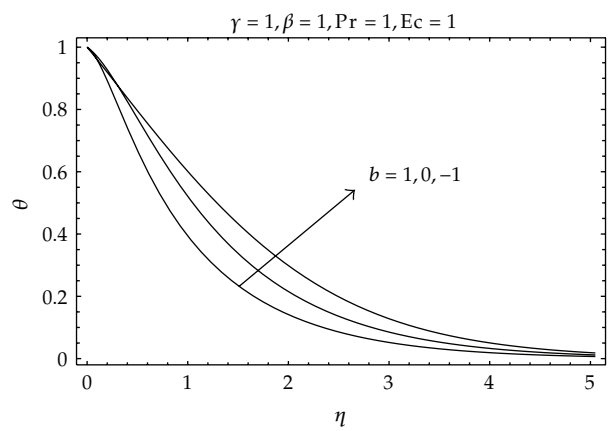

Figure 10: Influence of b over θ.

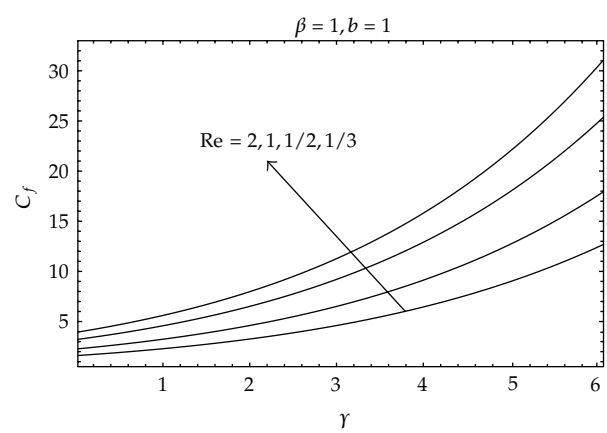

Figure 11: Influence of Re over C_f against γ.

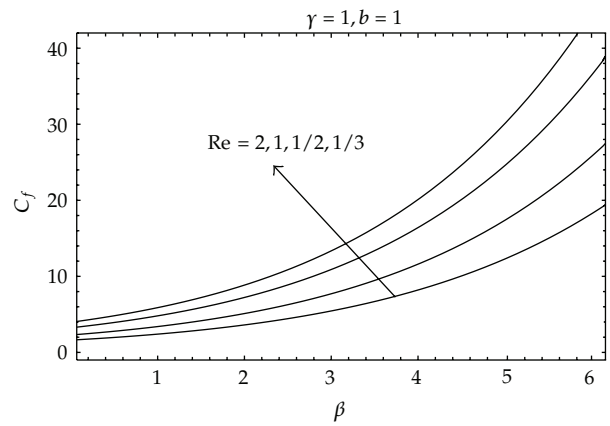

Figure 12: Influence of Re over C_f against β.

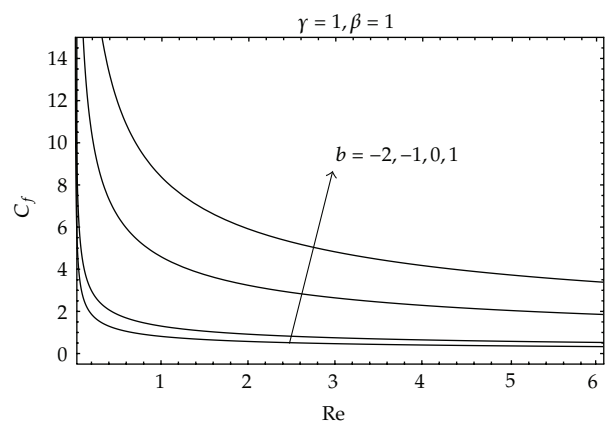

Figure 13: Influence of b over C_f against Re.

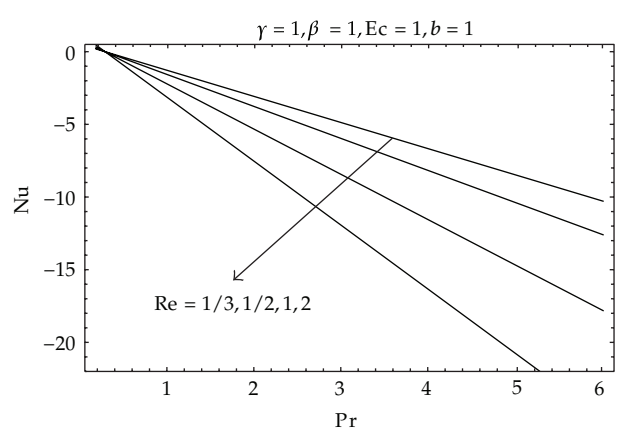

Figure 14: Influence of Re over Nu against Pr.

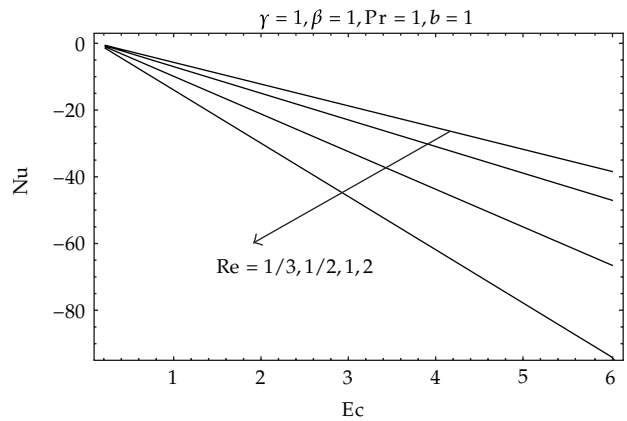

Figure 15: Influence of Re over Nu against Ec.

Table 1: Behavior of shear stress at the surface for different values of the involved parameters.

	$b\backslash\gamma$	$f''(0)$					
		0.5	1.0	1.5	2.0	2.5	3.0
$\beta = 0$	−1	0.9918	1.1942	1.3729	1.5325	1.6763	1.8067
	0	1.4886	1.7244	1.7954	1.9378	2.0656	2.1810
	1	2.0397	2.1751	2.2982	2.4099	2.5116	2.6044
	2	2.7332	2.8029	2.8746	2.9448	3.0119	3.0752
$\beta = 0.2$	−1	0.8626	0.9205	0.95554	0.9794	0.9958	1.0081
	0	1.5946	1.9137	2.0439	2.2662	2.4626	2.6302
	1	4.2201	5.1146	6.8718	9.1792	11.8742	14.465
	2	8.4914	11.7713	15.6358	24.547	29.4885	38.1896
$\beta = 0.4$	−1	0.7798	0.7935	0.7970	0.80716	0.80754	0.8077
	0	1.5822	1.8917	2.0213	2.2349	2.4119	2.6165
	1	4.5183	6.7654	8.7508	12.368	16.0122	24.2009
	2	9.0899	14.1113	19.8594	28.5561	35.0772	46.7566

The corresponding auxiliary linear operators are

$$L_f = \frac{d^3}{d\eta^3} + \frac{d^2}{d\eta^2}, \qquad L_\theta = \frac{d^2}{d\eta^2} + \frac{d}{d\eta}, \tag{3.2}$$

satisfying

$$L_f\left[c_1 + c_2\eta + c_3 e^{-\eta}\right] = 0, \qquad L_\theta\left[c_4 + c_5 e^{-\eta}\right] = 0, \tag{3.3}$$

where c_i $(i = 1, \ldots, 5)$ are arbitrary constants. The zeroth-order deformation equations are

$$\begin{aligned}(1-q)L_f\left[\widehat{f}(\eta;q) - f_0(\eta)\right] &= qH_f\hbar_1 N_f\left[\widehat{f}(\eta;q)\right],\\ (1-q)L_\theta\left[\widehat{\theta}(\eta;q) - \theta_0(\eta)\right] &= qH_\theta\hbar_2 N_\theta\left[\widehat{\theta}(\eta;q)\right],\end{aligned} \tag{3.4}$$

where the auxiliary convergence parameters H_f and H_θ both are taken to be $e^{-\eta}$ and

$$\begin{aligned} N_f\left[\hat{f}(\eta;q)\right] &= (1+2\gamma\eta)\hat{f}''' + 2\gamma\hat{f}'' + 1 + \hat{f}\hat{f}'' - \hat{f}'^2 \\ &\quad + \beta(1+2\gamma\eta)\left(\hat{f}''^2 + 2\hat{f}'\hat{f}''' - \hat{f}\hat{f}^{iv}\right), \\ N_\theta\left[\hat{\theta}(\eta;q)\right] &= (1+2\gamma\eta)\hat{\theta}'' + 2\gamma\hat{\theta}' + \Pr\left(\hat{f}\hat{\theta}' - \hat{f}'\hat{\theta}\right) - \Pr \mathrm{Ec}\,\beta\gamma\,\hat{f}\hat{f}''^2 \\ &\quad + \Pr \mathrm{Ec}(1+2\gamma\eta)\left[\hat{f}''^2 - \beta\hat{f}\hat{f}''\hat{f}''' + \beta\hat{f}'\hat{f}''^2\right]. \end{aligned} \tag{3.5}$$

The appropriate boundary conditions for the zeroth order system are

$$\begin{gathered} \hat{f}(0;q) = b_0, \quad \hat{f}'(0;q) = 0, \quad \hat{f}'(\eta;q) \longrightarrow 1, \quad \text{as } \eta \longrightarrow \infty, \\ \hat{\theta}(0;q) = 1, \quad \hat{\theta}(\eta;q) \longrightarrow 0, \quad \text{as } \eta \longrightarrow \infty. \end{gathered} \tag{3.6}$$

The mth order deformation is

$$\begin{gathered} L_f\left[f_m(\eta) - \chi_m f_{m-1}(\eta)\right] = \hbar_1 R_{mf}(\eta), \\ L_\theta\left[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)\right] = \hbar_2 R_{m\theta}(\eta), \\ f_m(0) = 0, \quad f_m'(0) = 0, \quad f_m(\eta) \longrightarrow 1, \quad \text{as } \eta \longrightarrow \infty, \\ \theta_m(0) = 0, \quad \theta_m(\eta) \longrightarrow 1, \quad \text{as } \eta \longrightarrow \infty, \end{gathered} \tag{3.7}$$

where

$$\chi_m = \begin{cases} 0 & m \le 1 \\ 1 & m > 1. \end{cases} \tag{3.8}$$

With the help of MATHEMATICA, the solutions of (2.4) can be expressed as

$$\begin{aligned} f(\eta) &= \lim_{Q\to\infty} \Sigma_{m=1}^{Q}\left[\sum_{n=1}^{2k-2}\sum_{k=1}^{m} a_{nk}^m e^{-2n\eta}\eta^k\right], \\ \theta(\eta) &= \lim_{Q\to\infty} \Sigma_{m=1}^{Q}\left[\sum_{n=1}^{2k-2}\sum_{k=1}^{m} c_{nk}^m e^{-2n\eta}\eta^k\right] \end{aligned} \tag{3.9}$$

in which the constants a_{nk}^m and c_{nk}^m can be computed through any mathematics software, and here we have shown and discussed the complete results through graphs.

4. Results and Discussion

In this section, we have discussed the analytical solutions of the highly nonlinear equations (2.4) subject to boundary conditions (2.5). The analytical solutions has been calculated with the help of homotopy analysis method. The solutions are finally presented in the form of general series. The convergence of these series solutions have been discussed through plotting the graphs of $\hbar$-curves (see Figures 1 and 2). It is seen that the admissible values of h in which our solutions are convergent are $-1.4 \leq \hbar_1 \leq -0.4$ and $-1.8 \leq \hbar_2 \leq -0.6$. The variation of velocity profile for pertinent parameters is sketched in Figures 3 to 5. Figure 3 is plotted to see the variation of curvature parameter γ on the nondimensional velocity. It is observed that with the increase in γ the velocity profile increases; however, the boundary layer thickness reduces. The maximum velocity is achieved very near to the sheet. The variation of viscoelastic parameter (second grade) β is given in Figure 4. It is depicted here that, with the increase in β, velocity increases and the boundary layer reduces. Thus almost similar effects appear for both γ and β. Almost a similar behavior occurs for the variation of b (see Figure 5). The nondimensional temperature θ for various values of γ, β, Pr, Ec and b is displayed in Figures 6 to 10. It is seen that temperature profile increases with the increases of both γ and β, also the thermal boundary layer reduces with the increase in γ and β (see Figures 6 and 7). It is also observed that when temperature is given near the sheet, the temperature is maximum. The effects of Prandtle number Pr and Eckert number Ec are displayed in Figures 8 and 9. It is observed that temperature profile decreases with the increase in Pr and increases with the increase in Ec. The thermal boundary layer reduces for both the case; however, the behavior of temperature for each case is opposite. The temperature profile decreases as well as the thermal boundary layer thickness reduces with the increase in b (see Figure 10).

The values of skin friction coefficient c_f against γ for different values of Re are plotted in Figure 11. It is seen that with the increase in Re, c_f decreases for all values of γ. In Figure 12 we have sketched c_f against β for various values of Re. It is observed that almost similar effects occur as in case of γ. However, the c_f increases with increase in b (see Figure 13). The local Nusselt number against Pr and Ec for different values of Re is plotted in Figures 14 and 15. In both the figures Nusselt number gives almost similar behavior.

Table 1 is included to check the behavior of boundary derivative (shear rate at the surface), for different values of curvature parameter γ and the second grade parameter β. From Table 1 it is observed that increase in γ increases shear rate at the surface.

Acknowledgments

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